A New PPML Paradigm for Quantized Models

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***Abstract*—Model quantization has become a common practice in machine learning (ML) to improve efficiency and reduce computational/communicational overhead. However, adopting quantization in privacy-preserving machine learning (PPML) remains challenging due to the complex internal structure of quantized operators, which leads to inefficient protocols under the existing PPML frameworks.**

In this work, we propose a new PPML paradigm that is tailor-made for and can benefit from quantized models. Our main observation is that lookup tables can ignore the complex internal constructs of any functions which can be used to simplify the quantized operator evaluation. We view the model inference process as a sequence of quantized operators, and each operator is implemented by a lookup table. We then develop an efficient private lookup table evaluation protocol, and its online communication cost is only $\log n$, where n is the size of the lookup table. On a single CPU core, our protocol can evaluate 2^{15} tables with 8-bit input and 8-bit output per second.

The resulting PPML framework for quantized models offers extremely fast online performance. The experimental results demonstrate that our quantization strategy achieves substantial speedups over SOTA PPML solutions, improving the online performance by $40 \sim 60\times$ w.r.t. convolutional neural network (CNN) models, such as AlexNet, VGG16, and ResNet18, and by $10 \sim 25\times$ w.r.t. large language models (LLMs), such as GPT-2, GPT-Neo, and Llama2.

I. Introduction

Machine Learning (ML) technology has re-shaped the way we analyze data, leading to the breakthroughs in various sectors such as healthcare, finance, transportation, and science. At the same time, due to the nature of ML, extensive datasets with sensitive information is collected and processed, raising significant privacy concerns. This has led to an urgent call for the development of Privacy-Preserving Machine Learning (PPML) techniques. Secure Multi-Party Computation (MPC) has emerged as a pivotal cryptographic primitive within the realm of PPML. In a nutshell, MPC allows multiple parties to jointly evaluate a function while keeping their inputs private.

In practice, the communication cost is often the performance bottleneck of an MPC-based PPML. For instance, ResNet-50 [26], with 50 convolution layers and 98MB parameters, requires over 3.8 billion fix-point (or floating-point) MPC operations to complete one model inference task, which produces nearly 2GB of communication, even adopting the most efficient MPC protocols. Therefore, exploring the possibility of reducing the communication cost is the key to speed up a PPML platform.

Quantization. The quantization technique [55], [16], [15], [29] maps high-precision floating-point values to a smaller set of discrete finite values, and it has been widely adopted to speed-up model inference in practice. For large models, quantization is an essential compression technique that could potentially reduce the model size by two to four times without compromising its accuracy. This reduction in data size means that a quantized model requires less memory bandwidth to fetch and store the data, which can be a critical performance bottleneck. Less memory usage also means that more of the model can fit into faster caches, reducing the need to access slower main memory. Therefore, the quantization technique is particularly effective for accelerating model inference using GPUs/NPUs with limited I/O bandwidth.

Difficulty of adopting quantization to PPML. Several attempts have been made to adopt the quantization technique to the context of PPML. However, none of the existing solutions are quite successful, and naive adoption cannot save the communication cost in general. The main reason is as follows. Although the (intermediate) data between different model operators is quantized in a more succinct representation format, within each operator, the quantized data should be first recovered to its original high-precision format before the operation, and re-quantize back afterwards. Such a precision recovery step typically requires secure multiplication with the private scaler (with high-precision) as well as a module switch operation from a smaller module, e.g., 2^8, to a bigger module, e.g., 2^{64}. Note that the workload of the module switch is usually equivalent to the expensive most significant bit extraction in the MPC setting. For instance, Dalskov *et. al* [17] propose an MPC-based platform that supports quantized model inference, but the resulting PPML scheme needs even more communication than the unquantized version.

As a toy example, suppose we want to perform the convolution operation $\boldsymbol{z} \leftarrow \mathsf{Conv}(\boldsymbol{x}, \boldsymbol{w})$, where $\boldsymbol{x}, \boldsymbol{w}$ and $\boldsymbol{z}$ consist of ℓ-bit fix-point variables. For ℓ'-bit quantization, we choose proper ℓ-bit fix-point scale factor s_0, s_1, s_2 and ℓ'-bit offsets b_0, b_1, b_2 such that $\boldsymbol{x} = s_0(\boldsymbol{x'} - b_0)$, $\boldsymbol{w} = s_1(\boldsymbol{w'} - b_1)$, and $\boldsymbol{z} = s_2(\boldsymbol{z'} - b_2)$. The quantized convolution operation Conv^* takes inputs as $\boldsymbol{x'}$ and $\boldsymbol{y'}$, and it shall output $\boldsymbol{z'} := \frac{1}{s_2} \cdot \mathsf{Conv}(s_0(\boldsymbol{x'} - b_0), s_1(\boldsymbol{w'} - b_1)) + b_2$. It is easy to see that the operation Conv^* requires extra steps on top of the original Conv. If s_i are kept in private, Conv^* usually costs more than

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the unquantized convolution.

As another line of work, to speed up quantized model inference, Riazi *et. al* [47], Agrawal *et. al* [4] and Keller *et. al* [32] propose to treat the quantization scalers as public variables, and the value of those scalers is limited to the perfect power of 2 to avoid secure multiplication. However, we emphasize that this type of approach might cause severe privacy leakage, also the restriction of the choices of quantization scalers has a negative impact on overall model accuracy.

This prompts our main research question:

> *Does there exist an efficient PPML framework that is tailor-made for and can benefit from quantized models?*

In this work, we answer this question affirmatively by proposing a new PPML paradigm.

A new paradigm. As mentioned above, model quantization is particularly effective for operators with limited I/O bandwidth; that is, the input/output of the operators is encoded in some compressed format. We observe that operators with such characteristics can be efficiently evaluated by lookup tables. For operators with n-bit input and m-bit output, the table size is bounded by $2^n \cdot m$ bits. For common quantized models, $n = 4$-bit or 8-bit. Our work focuses on the two-party (2PC) privacy-preserving model inference setting, where one party, called the server, holds the model in plaintext, and another party, called the client, holds the data. Our paradigm lets the model server prepare the lookup tables for each operator in the offline phase, and then the client privately evaluates those lookup tables in order to obtain the model inference result.

In the literature, several works [30], [25] use lookup tables for the evaluation of non-linear functions, e.g. the activation functions; whereas, in this work, we show that even linear functions can be accelerated by lookup tables in the quantization setting. Take the multiplication operation $y = x \cdot w$ as an example. Let $x = s_0(x' - b_0)$, $w = s_1(w' - b_1)$, and $y = s_2(y' - b_2)$. In our setting, the model holder knows w', $\{s_i\}_{i\in\{0,1,2\}}$, and $\{b_i\}_{i\in\{0,1,2\}}$; therefore, we can re-write the operation as $y' = f(x') := \frac{1}{s_2} \cdot s_1(w' - b_1) \cdot s_0(x' - b_0) - b_2$, where all variables besides x' are hard-coded into the function f. Since x' is only 8 bits for 8-bit quantization, the lookup table consists of 256 elements; as we will show later, the online communication cost of this private lookup table evaluation is only 8 bits, which is much less than the cost of the conventional secure (quantized) multiplication.

While applying our technique to PPML, our framework supports lookup-based operator fusion, i.e., the multiple lookup tables can be fused into a single lookup table, and thus the overall cost only equals to single lookup table evaluation.

Private lookup table evaluation. There are several works [28], [19], [20], [8] on lookup table evaluation and their usage in the context of PPML. For instance, FLUTE [8] utilizes a boolean circuit to represent the lookup table, and their online communication only depends on the output length of the function; more specifically, if the function output is ℓ bits, the online communication of FLUTE is 2ℓ bits, regardless the input length (or table size). However, those works assume the lookup table is public to everyone, which is not suitable for our case. As mentioned before the model holder will embed the model weights/parameters into the lookup table; therefore, in our work, we study the *private* lookup table evaluation problem, where the lookup table is considered as a private input. Our technique is based on the secret shifting protocol proposed by Lu *et.al* [38], where their original protocol is only designed for binary vectors. We extend the protocol to support vectors over large ring (or field), and apply this shift technique to realize our private lookup evaluation scheme. Its online communication is as low as $\log n$ bits , where n is the table size. On a single CPU core, our protocol can evaluate 2^{15} numbers of lookup tables with 8-bit input and 8-bit output per second.

Performance. We apply our framework to various machine-learning models. For the convolutional neural network (CNN) models, such as AlexNet, VGG16, and ResNet18, our benchmark shows that our 8-bit quantized PPML framework (single-core CPU only) is $40 \sim 60\times$ faster than the typical 2PC SOTA – Cryptoflow2 [46], and $5 \sim 15\times$ faster than the typical 3PC SOTA – Falcon [52] and Bicoptor [54], even though they use GPU acceleration. For the large language models, such as GPT-2, GPT-Neo, and Llama2, our 8-bit quantized framework (single-core CPU only) achieves $10 \sim 25\times$ performance improvement compared to the SOTA works – CrypTen [34] and Sigma [25].

Paper organization. Section II introduces the preliminary including notations and the primitives to construct our framework. In Section III, we propose our PPML paradigm for quantized models. In Section IV, we construct the private lookup table evaluation scheme. In Section V, we give the concrete PPML framework and realize multiple specific operators in both the outsourcing setting and C/S setting. Performance evaluation and comparison of SOTA can be found in Section VI.

II. Preliminaries

Notation. The frequently used notations are shown in Table I. We denote n-dimension vector as $\boldsymbol{a} := (a_0, \ldots, a_{n-1})$, and a_i be the i^{th} element of $\boldsymbol{a}$. For notation simplicity, we override the multiplication between a vector and a scalar as $\boldsymbol{a} \cdot b := (a_0 \cdot b, \ldots, a_{n-1} \cdot b)$; similarly, we override the addition between a vector and a scalar as $\boldsymbol{a} + b := (a_0 + b, \ldots, a_{n-1} + b)$. We denote $[n]$ as the index set $\{0, \ldots, n-1\}$. We denote the matrix as the uppercase letter $\mathbf{M} := (m_{i,j})_{i\in[n_1], j\in[n_2]}$ with $n_1 \times n_2$ dimension, and denote the element in the i^{th} row and j^{th} column of $\mathbf{M}$ as $m_{(i,j)}$. It can also be represented as $\mathbf{M} := (\boldsymbol{m}_j)_{j\in[n_2]}$ where $\boldsymbol{m}_j$ is j^{th} column vector, and $\mathbf{M} := (\boldsymbol{m}_i)^T_{i\in[n_2]}$ where $\boldsymbol{m}_i$ is i^{th} row vector. We define $\mathsf{shift}(\boldsymbol{m}, i)$ as the operation of right circular shifting the vector $\boldsymbol{m}$ by i positions. We use $\mathcal{T}^{\ell_x,\ell_y} := (t_0, \ldots, t_{2^{\ell_x}-1})$ to denote a lookup table with ℓ_x bits input and ℓ_y bits output. When the semantics are clear, we omit the superscript of $\mathcal{T}$. For the evaluation of $\mathcal{T}$ at position x, we represent it as $\mathcal{T}(x)$. We view the lookup table as a vector and denote its r^{th} item as

$t_r \in \mathbb{Z}_{2^{\ell_y}}$. For an operator op, we denote its quantized operator as op^*. Let (k,n)-OT denote the k-out-of-n OT. We consider 2-out-of-2 secret shares and define the secret share $[\![\cdot]\!]^\ell$ over ring $\mathbb{Z}_{2^\ell}$ as $[\![x]\!]^\ell := ([\![x]\!]_1^\ell \in \mathbb{Z}_{2^\ell}, [\![x]\!]_2^\ell \in \mathbb{Z}_{2^\ell})$ where $x = [\![x]\!]_1^\ell + [\![x]\!]_2^\ell \pmod{2^\ell}$. For simplicity, we use $[\![x]\!]$ when the semantics are clear. We denote the shared vector as $[\![\boldsymbol{x}]\!] := ([\![x_0]\!], \ldots, [\![x_{n-1}]\!])$.

TABLE I: Notations

Notations	Descriptions
$\boldsymbol{a}$	The vector $\boldsymbol{a} := (a_0, \ldots, a_{n-1})$.
$\mathbf{M} := (m_{i,j})_{i\in[n_1], j\in[n_2]}$	The $n_1 \times n_2$ dimension matrix M.
op^*	The quantized operator for op.
$[n]$	The index set $\{0, \ldots, n-1\}$.
$[\![x]\!]^\ell := ([\![x]\!]_0^\ell, [\![x[\!]_1^\ell)$	The 2PC secret shares of x over $\mathbb{Z}_{2^\ell}$ where $x = [\![x]\!]_0^\ell + [\![x]\!]_1^\ell \pmod{2^\ell}$.
$\mathsf{shift}(\boldsymbol{m}, i)$	Right circular shift the vector $\boldsymbol{m}$ by i positions.
$\mathcal{T}^{\ell_x,\ell_y}$	The lookup table with ℓ_x-bit input and ℓ_y-bit output.
$\mathcal{T}(x)$	Evaluate lookup table $\mathcal{T}$ at position x.
(k,n)-OT	k-out-of-n OT.

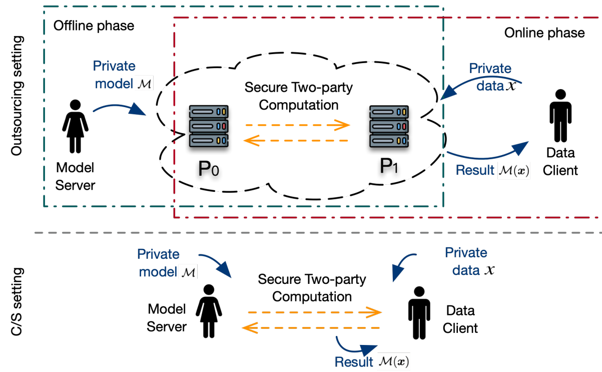


Fig. 1: Our system architecture.

x		y			
x_0	x_1	y_0	y_1	y_2	y_3
0	0	0	0	0	0
0	1	0	0	0	1
1	0	0	1	0	0
1	1	1	0	0	1

Fig. 2: An example of lookup table for $y = \mathsf{op}(x) := x^2$

System Architecture and Threat Model. As shown in Fig. 1. Our PPML framework can be deployed in outsourcing and client/server (C/S) settings. We assume all the participants are semi-honest where the adversary may attempt to extract private information from her views but she must follow the protocol. In particular, our framework contains four participants, denoted by $\mathcal{P} := \{\mathcal{C}, \mathcal{S}, \mathcal{P}_0, \mathcal{P}_1\}$. Among them, $\mathcal{C}$ is the data client, $\mathcal{S}$ is the model server, $\mathcal{P}_0$ and $\mathcal{P}_1$ are the computing nodes in the outsourcing setting.

In our settings, we assume that the machine learning model is prepared in-prior. In other words, the model inference server can quantize the model parameters in advance, and uses the quantized models as an input in the preprocessing phase. In contrast, the user input will only be determined in the online phase.

Without loss of generality, we define the machine learning model as a sequence of operators: $\mathcal{M} := \{\mathsf{op}^{(0)}, \ldots, \mathsf{op}^{(N-1)}\}$ where op corresponds to the operator of each layer, and the input vector (for the tensor which is the high-dimension matrix, we convert it to vector) as $\boldsymbol{x}$. In our setting, the operator op is embedded with all the model weights. For instance, the convolution can be written as $\mathsf{op}^{(i)}(\boldsymbol{x}) := \mathsf{Conv}(\boldsymbol{w}, \boldsymbol{x})$, where $\boldsymbol{w}$ are the corresponding model weights. We denote the evaluation of $\boldsymbol{x}$ in $\mathcal{M}$ as $\boldsymbol{y} = \mathcal{M}(\boldsymbol{x})$.

- In the outsourcing setting, the machine learning model server $\mathcal{S}$ inputs the private model $\mathcal{M}$, and the data client $\mathcal{C}$ inputs the private data $\boldsymbol{x}$. They employ two computing nodes $\{\mathcal{P}_0, \mathcal{P}_1\}$ to perform secure model inference $\mathcal{M}(\boldsymbol{x})$. The computing node P_j for $j \in \{0,1\}$ holds the secret share $[\![\boldsymbol{x}]\!]_j$ and will not collude with other parties.
- In the C/S setting, instead of employing the computing nodes, the data client $\mathcal{C}$ and the model server $\mathcal{S}$ directly perform secure model evaluation $\mathcal{M}(\boldsymbol{x})$, where $\mathcal{C}$ hold the secret share $[\![\boldsymbol{x}]\!]_0$ and $\mathcal{S}$ hold the secret share $[\![\boldsymbol{x}]\!]_1$.

Oblivious transfer. The oblivious transfer is a fundamental cryptographic primitive in which one party (the sender) inputs a list of private messages and another party (the receiver) inputs private indexes. The receiver then obtains the messages corresponding to the indexes without any additional information. We denote (k,n)-OT^ℓ as the OT with k-dimension input indexes and n-dimension message list where each message is ℓ length. We utilize $\Pi_{(1,2)\text{-}\mathsf{OT}^\ell}$ to denote the protocol of $(1,2)$-OT^ℓ [24], [22]. In $\Pi_{(1,2)\text{-}\mathsf{OT}^\ell}$, the sender inputs message $m_0 \in \mathbb{Z}_{2^\ell}$ and $m_1 \in \mathbb{Z}_{2^\ell}$ into $\Pi_{(1,2)\text{-}\mathsf{OT}^\ell}$; the receiver inputs index $i \in \{0,1\}$ and receive m_i from $\Pi_{(1,2)\text{-}\mathsf{OT}^\ell}$. Random OT (ROT) [7] is a special case of OT where the selective index is randomly generated by protocol. In an $(n-1,n)$-ROT^ℓ, the sends holds a list of messages, and the receiver holds an index j and all the messages except for the j^{th} message. We utilize $\Pi_{(n-1,n)\text{-}\mathsf{ROT}^\ell}$ [11] to denote the protocol of $(n-1,n)$-ROT^ℓ. It sends $(m_0, \ldots, m_n)$ to the sender and sends an index $j \in [n]$ with $n-1$ messages m_i for $i \in [n]\backslash\{j\}$ to the receiver.

lookup table. The look up table $\mathcal{T}$ for operation $\mathsf{op}^* : \{0,1\}^{\ell_x} \to \{0,1\}^{\ell_y}$ traverse all possible inputs of op^*. It accepts ℓ_x bits input and output ℓ_y bits. The r^{th} item of lookup table $\mathcal{T}$ stores the evaluation result of op^* with input r. For instance, Fig. 2 illustrates the example of operation $\mathsf{op}(x) := x^2$. The corresponding r^{th} item of lookup table t_x store r^2.

Secure two-party computation. Our PPML framework fo-

Protocol $\Pi^{\ell,n}_{\mathsf{vole}}(\boldsymbol{x}, y)$

Input : $\boldsymbol{x} := (x_0, \ldots, x_{n-1}) \in (\mathbb{Z}_{2^\ell})^n$ input by $\mathcal{C}$ and $y \in \mathbb{Z}_{2^\ell}$ input by $\mathcal{S}$.
Output : $\mathcal{C}$ receives $[\![\boldsymbol{z}]\!]_0 := ([\![z_0]\!]_0, \ldots, [\![z_{n-1}]\!]_0) \in (\mathbb{Z}_{2^\ell})^n$, $\mathcal{S}$ receives $[\![\boldsymbol{z}]\!]_1 := ([\![z_0]\!]_1, \ldots, [\![z_{n-1}]\!]_1) \in (\mathbb{Z}_{2^\ell})^n$, where $x_i \cdot y = z_i$ for $i \in [n]$.

Protocol:

- $\mathcal{S}$ bit-extract y as $(y_j)_{j\in[\ell]}$ where $y = \sum_{j=0}^{\ell-1} 2^j \cdot y_j$.
- For $j \in [\ell]$, $\mathcal{C}$ and $\mathcal{S}$ invoke $\Pi_{(1,2)\text{-}\mathsf{OT}^{\ell\cdot n}}$:
  - $\mathcal{C}$ picks $\boldsymbol{r}_j := (r_{i,j})_{i\in[n]} \in (\mathbb{Z}_{2^\ell})^n$, inputs $m_0 = -r_{0,j}|| \ldots || - r_{n-1,j}$ and $m_1 = x_0 \cdot 2^j - r_{0,j}|| \ldots ||x_{n-1} \cdot 2^j - r_{n-1,j}$.
  - $\mathcal{C}$ inputs the chooes bit y_j and receives output r'_j.
- $\mathcal{C}$ parses r'_j as $r'_j = r'_{0,j}|| \ldots ||r'_{n-1,j}$ and computes $[\![z_i]\!]_0 = \sum_{j=1}^{\ell} r'_{i,j}$, $\mathcal{P}_1$ sets $[\![z_i]\!]_1 = \sum_{j=0}^{\ell} r_{i,j}$.

Fig. 3: The VOLE protocol based on OT

cuses on secure two-party computation (2PC). We define the addition on the secret share as $[\![z]\!] = [\![x]\!] + [\![y]\!]$ and it holds that $z = x + y$ with secret shared form. It can be obtained by locally adding $[\![x]\!]_i$ and $[\![y]\!]_i$ for P_i. We use $[\![z]\!] = c \cdot [\![x]\!]$ to denote the scale of a public value, where $z = c \cdot x$. it can be obtained by locally executing $[\![z]\!]_i = c \cdot [\![x]\!]_i$ by P_i. Each computation party can add a private input into the secret share. Specifically, we use $[\![z]\!] = [\![x]\!] + c$ to denote one of parties add private value c to the secret share $[\![x]\!]$, namely, P_i locally sets $[\![z]\!]_i = [\![x]\!]_i + c$ where P_{1-i} sets $[\![z]\!]_i = [\![x]\!]_i$. We define the secret share protocol and the reconstruct protocol as follows.

- $[\![x]\!]^\ell \leftarrow \Pi^\ell_{\mathcal{C}/\mathcal{S}}(\mathbb{P}, x)$: We define the secret share for C/S setting as $\Pi^\ell_{\mathcal{C}/\mathcal{S}}(\mathbb{P}, x)$, where $\mathbb{P} \in \{\mathcal{C}, \mathcal{S}\}$ holds x and secret shares x to $\mathcal{C}$ and $\mathcal{S}$. Before execution, $\mathcal{C}$ and $\mathcal{S}$ generate a correlated seed η. If $\mathbb{P} = \mathcal{C}$, $\mathcal{C}$ and $\mathcal{S}$ pick $[\![x]\!]^\ell_1 \leftarrow \mathbb{Z}_{2^\ell}$ with same seed η. $\mathcal{C}$ locally sets $[\![x]\!]^\ell_0 = x - [\![x]\!]^\ell_1$. Similarly, If $\mathbb{P} = \mathcal{S}$, $\mathcal{C}$ and $\mathcal{S}$ pick $[\![x]\!]^\ell_0 \leftarrow \mathbb{Z}_{2^\ell}$ together. $\mathcal{S}$ locally sets $[\![x]\!]^\ell_1 = x - [\![x]\!]^\ell_0$.
- $[\![x]\!]^\ell \leftarrow \Pi^\ell_{\mathsf{out}}(\mathbb{P}, x)$: We define the secret share for outsourcing setting as $\Pi^\ell_{\mathsf{out}}(\mathbb{P}, x)$, where $\mathbb{P} \in \{\mathcal{C}, \mathcal{S}\}$ holds x and secret shares x to $\mathcal{P}_0$ and $\mathcal{P}_1$. Before execution, $\mathbb{P}$ and $\mathcal{P}_0$ generate a correlated seed η. $\mathbb{P}$ and $\mathcal{P}_0$ pick $[\![x]\!]^\ell_0 \leftarrow \mathbb{Z}_{2^\ell}$ with same seed η. $\mathbb{P}$ calculates $[\![x]\!]^\ell_1 = x - [\![x]\!]^\ell_0$ and sends it to $\mathcal{P}_1$.
- $x \leftarrow \Pi^\ell_{\mathsf{rec}\to\mathbf{P}}([\![x]\!]^\ell)$. We define the reconstruction of $[\![x]\!]^\ell$ as $\Pi^\ell_{\mathsf{rec}\to\mathbf{P}}([\![x]\!]^\ell)$. $\mathbf{P}$ is the set of parties to receive x. The holders of $[\![x]\!]_0$ and $[\![x]\!]_1$ send them to the parties in $\mathbf{P}$. The parties in $\mathbf{P}$ reconstruct $x = [\![x]\!]_0 + [\![x]\!]_1$.

Vector Oblivious Linear Evaluation. Oblivious Linear Evaluation (OLE)[6], [33] is a foundational component in various secure 2PC [49], [45]. In the OLE protocol[6], one party inputs value x and the other inputs value y. They jointly evaluate $z = x \cdot y$, and each party obtains the share $[\![z]\!]$ of the result.

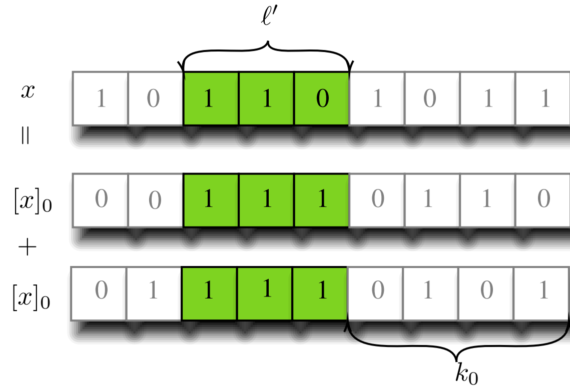


Fig. 4: The case of cut protocol.

Considering the vector case, the primitive so-called Vector Oblivious Linear Evaluation (VOLE) [49], accepts a vector $\boldsymbol{x} := (x_0, \ldots, x_n)$ input by one party and a value y by another party, and output a shared vector of $\boldsymbol{z} := (x_0 \cdot y, \ldots, x_n \cdot y)$. The OLE can be viewed as a special case of VOLE. We follow the OT-based multiplication protocol [31] to realize VOLE. Fig. 3 depicts the procedure of VOLE protocol Π_{vole}.

Fixed-point encoding. For the floating-point data used in PPML, we use its fixed-point structure. Specifically, for a fixed point value x with k bit decimal and the effective number of bits ℓ, if $x \geq 0$, we encode it as $\lfloor x \cdot 2^k \rfloor$; if $x < 0$, we encode it as $2^\ell + \lfloor x \cdot 2^k \rfloor$. For a fixed-point value x with decimal, we utilize encode to denote the procedure of fixed-point encoding, and decode to denote the procedure of decoding to the fixed-point value.

Significant bits extraction. For a shared value $[\![x]\!]^\ell$, it can be locally extracted of partial significant bits to a lower-precision value [54] with a probabilistic 1-bit carry error. For instance, a fixed-point value with $\ell - \ell'$ decimal bits can drop the decimal bits to obtain an integer. More specifically, we use the function $\mathsf{cut}(x, k, \ell')$ to denote the procedure which drops the first k bits and the last $\ell - \ell' - k$ bits of x. Formally, let $x := \sum_{i=0}^{\ell-1} x_i \cdot 2^i$, where x_i is the i^{th} bit of x from small endian, we have $\mathsf{cut}(x, k, \ell') := \sum_{i=k}^{k+\ell'} x_i \cdot 2^{i-k}$. Based on cut function, we can construct significant bits chop protocol $[\![x]\!]^{\ell'} \leftarrow \Pi^{\ell'}_{\mathsf{cut}}([\![x]\!]^\ell, k)$ as follows (See Fig 5). $\mathcal{P}_0$ sets $[\![x]\!]^{\ell'}_0 = \mathsf{cut}([\![x]\!]^\ell_0, k, \ell')$ and $\mathcal{P}_1$ sets $[\![x]\!]^{\ell'}_1 = \mathsf{cut}([\![x]\!]^\ell_1, k, \ell')$. Fig. 4 illustrates the case of Π_{cut}, the drop of the lower k bits will introduce at most one bit of error (due to two number addition at most case one bit carry), while the drop of higher $\ell - k - \ell'$ bits will not cause any error.

III. Our new framework

In this work, we design a new PPML framework. Our framework achieves performance improvement through two key points. On the one hand, we design our framework tailored for model quantization scenarios. On the other hand, we enable the model server to perform more operations based on the specific model to enhance the performance of privacy-preserving model evaluation. We address the potential

Protocol $\Pi_{\mathsf{cut}}^{\ell'}([\![x]\!]^{\ell}, k)$

Input : $[\![x]\!]_0 \in \mathbb{Z}_{2^\ell}$ input by $\mathcal{C}$ and $[\![x]\!]_1 \in \mathbb{Z}_{2^\ell}$ input by $\mathcal{S}$.
Output : $\mathcal{C}$ receives $[\![x']\!]_0^{\ell} \in \mathbb{Z}_{2^{\ell'}}$, $\mathcal{S}$ receives $[\![x']\!]_1^{\ell} \in \mathbb{Z}_{2^{\ell'}}$, where $x' = \mathsf{cut}(x, k, \ell')$.

Protocol:

- $\mathcal{C}$ locally calculates $[\![x']\!]_0^{\ell'} = \mathsf{cut}([\![x]\!]_0^{\ell}, k, \ell')$.
- $\mathcal{S}$ locally calculates $[\![x']\!]_1^{\ell'} = \mathsf{cut}([\![x]\!]_1^{\ell}, k, \ell')$.

Fig. 5: The low precision cut protocol

problems of quantized model evaluation in adopting the MPC technique. Consequently, we introduce our PPML paradigm which can evaluate the quantized PPML efficiently.

A. *Quantization scheme.*

Quantization is the process of mapping continuous infinite values to a smaller set of discrete finite values. In the context of simulation and embedded computing, it is about approximating real-world values with a digital representation that introduces limits on the precision and range of a value.

We formalize the quantization scheme. A quantization scheme is a tuple $\mathcal{Q} := (\mathcal{G}, \mathcal{E}, \mathcal{F}, \mathcal{D})$. Considering the function f to be quantized, the calibration dataset d, and n-dimension input vector $\boldsymbol{x}$ with ℓ-bit precision, the quantization scheme $\mathcal{Q}^{d,\ell,\ell'}$ for ℓ'-bit quantization contains four steps:

- $(\mathcal{F}, \mathcal{E}, \mathcal{D}) \leftarrow \mathcal{G}(f, d)$: the quantization generation $\mathcal{G}$ accept the function f, a calibration dataset d and generate a quantized function f', an encode function $\mathcal{E}$, and a decode function $\mathcal{D}$.
- $\boldsymbol{x}' \leftarrow \mathcal{E}(\boldsymbol{x})$: The encode function $\mathcal{E}$ encode the input $\boldsymbol{x} \in \mathbb{Z}_{2^\ell}^n$ to quantized vector $\boldsymbol{x}' \in \mathbb{Z}_{2^{\ell'}}^n$.
- $\boldsymbol{y}' \leftarrow f'(\boldsymbol{x}')$: The quantized function f' is performed on the quantized vector $\boldsymbol{x}'$ and return a quantized output $\boldsymbol{y}'$.
- $\boldsymbol{y} \leftarrow \mathcal{D}(\boldsymbol{y}')$: The decode function $\mathcal{D}$ recover the quantized vector $\boldsymbol{y}'$ to the vector $\boldsymbol{y}$ which is lay on the original precision.

The quantization scheme is designed for IO communication and storage reduction. To measure the magnitude of the reduction, we define the following properties:

Definition 1 (ρ-succinctness)**.** We say the quantization scheme $\mathcal{Q}^{d,\ell,\ell'}$ is ρ-succinct, if the precision bits ℓ of the original data x and the precision bits ℓ' of its quantized data x' hold that:

$$\frac{\ell'}{\ell} = \rho.$$

Definition 2 (ε-accuracy-loss)**.** We say the quantization scheme $\mathcal{Q}^{d,\ell,\ell'}$ for the calibration dataset $d := \{\boldsymbol{x}_0, \ldots, \boldsymbol{x}_{N-1}\}$ is ε-accuracy-loss, if the encode function $\mathcal{E}$, the quantified function $\mathcal{F}$ and the decode function $\mathcal{D}$ hold that:

$$\frac{|\{\mathcal{D}(f'(\mathcal{E}(\boldsymbol{x}_i))) \neq f(\boldsymbol{x}_i); i \in [N]\}|}{N} < \varepsilon;$$

where $|\cdot|$ denotes the number of elements in a collection.

Quantization in machine learning. Previous work [55], [16], [15], [29] has demonstrated the effectiveness of quantization techniques in machine-learning scenarios. We give a toy example to illustrate how the quantization scheme works on the machine learning model. As shown in Fig. 6, we utilize a simple convolutional neural network model $f := (\mathsf{FC}, \mathsf{ReLU}, \mathsf{Conv})$, with input tensor $\boldsymbol{x}$, the weight $\boldsymbol{w}_1$ for the convolution Conv and $\boldsymbol{w}_2$ for the full connection FC, where $f(\boldsymbol{x}) = \mathsf{FC}(\boldsymbol{w}_2, \mathsf{ReLU}(\mathsf{Conv}(\boldsymbol{w}_1, \boldsymbol{x})))$. We define its quantized model as $f' := (\mathsf{FC}^*, \mathsf{ReLU}^*, \mathsf{Conv}^*)$. We show more details about the quantization scheme $\mathcal{Q}_{\mathsf{ML}}^{d,\ell,\ell'} := (\mathcal{G}, \mathcal{E}, \mathcal{F}, \mathcal{D})$ for the neural network model f.

Encode function $\mathcal{E}$: Before input f', all the data will be encoded into the quantized data. In the typical ML quantization, the encode function $\mathcal{E}$ is defined as $\boldsymbol{x}' = \mathcal{E}(\boldsymbol{x}) = \lfloor \frac{1}{s} \cdot \boldsymbol{x} \rfloor - b$. Considering the element x_i in the vector $\boldsymbol{x}$ which is a high-precision fixed-point value, it will be scaled by $\frac{1}{s}$ to a low-precision integer. b is an offset which is the so-called zero-point to shift the central value to zero. Note that each input vector $\boldsymbol{x}$ utilizes the single scale factor s and offset b for all elements it contains.

Quantized function f': To convert the original model f to a quantized model f', we convert each operator of f to a quantized operator. For the convolution operator $\boldsymbol{y} = \mathsf{Conv}(\boldsymbol{w}_1, \boldsymbol{x})$, the corresponding quantized operator Conv^* inputs with the quantized vector $\boldsymbol{x}'$ and $\boldsymbol{w}_1'$ and output $\boldsymbol{y}'$. Assume that s_0 and b_0 are the scale factor of $\boldsymbol{x}$, s_1 and b_1 are the scale factor of $\boldsymbol{w}_1$, s_2 and b_2 are the scale factor of $\boldsymbol{y}$. We have $s_2(\boldsymbol{y}' + b_2) = \mathsf{Conv}(s_1(\boldsymbol{w}_1' + b_1), s_0(\boldsymbol{x}' + b_0))$. From this, the quantized operator for Conv can be deduced, namely, $\boldsymbol{y}' = \mathsf{Conv}^*(\boldsymbol{w}_1', \boldsymbol{x}') = \frac{s_1 s_0}{s_2} \cdot \mathsf{Conv}(\boldsymbol{w}_1' + b_1, \boldsymbol{x}' + b_0) - b_2$. For the next layer, to evaluate $\boldsymbol{y} = \mathsf{ReLU}(\boldsymbol{x})$, the quantized vector holds that $s_2(\boldsymbol{y}' + b_2) = \mathsf{ReLU}(s_1(\boldsymbol{x}' + b_1))$. The quantized operator ReLU^* can be calculated by $\mathsf{ReLU}^*(x') = \frac{1}{s_2}\mathsf{ReLU}(s_1(\boldsymbol{x}'+b_1)) - b_2$. Similarly, we can infer the quantized operator for full connection FC^*. Consequently, instead of evaluating $f := (\mathsf{Conv}, \mathsf{ReLU}, \mathsf{FC})$ with vector $\boldsymbol{x}$, we evaluate $f' := (\mathsf{Conv}^*, \mathsf{ReLU}^*, \mathsf{FC}^*)$ with the quantized vector $\boldsymbol{x}'$ layer by layer.

Decode function $\mathcal{D}$: All the quantized data will be de-quantized (decode to the original precision) before output. It is the inverse operator of the encode function, which is $\boldsymbol{y} = \mathcal{D}(\boldsymbol{y}') = s(\boldsymbol{y}' + b)$. Each integer element y_i' in the vector $\boldsymbol{y}'$ will scale a high-precision fixed-point value s to obtain a high-precision fixed-point value.

Quantization generation $\mathcal{G}$: To measure the quantization arguments $\{(s_0, b_0), \ldots, (s_m, b_m)\}$, the model server needs to evaluate the model in the calibration data-set d. This evaluation is used to generate a priori data ranges for the intermediate results of each operator in the model. These data ranges are then used to determine the scale factor s and the offset b. We formalize $\mathcal{G}$ as follows.

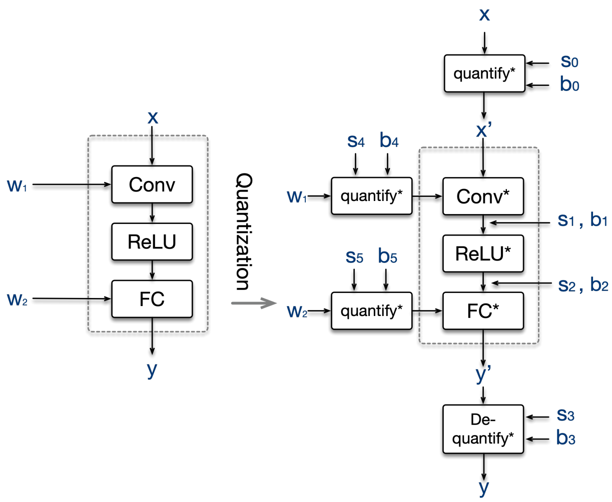


Fig. 6: Case of quantization.

- Evaluate f with the calibration data-set $d := (\boldsymbol{x}_0, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_{n-1})$. For each operator op, record the maximum and minimum element of output, denoted by t_{max} and t_{min}.
- For op with t_{max} and t_{min}, calculate $s = \frac{t_{\mathsf{max}} - t_{\mathsf{min}}}{2^{\ell'}}$ and $b = \frac{t_{\mathsf{max}} + t_{\mathsf{min}}}{2 \cdot s}$. Calculate s and b for each output wire of the operators.

The challenge of adopting quantization in PPML. Although the quantized model f' can significantly reduce the size of input data and the temporary variables, it still can not speed up privacy-preserving machine learning. The main reason is that each quantized operator, e.g., Conv^*, contains a multiplication of $\frac{s_1 s_0}{s_2}$ which lies on the original precision (In practice, equals dequantization and re-quantization). The overall cost of evaluation quantization over typical MPC is even higher than that of the evaluation of the original model. A line of work [47], [4], [32], makes the scale factor public to avoid this problem, which causes massive data leakage on the model.

B. Our PPML paradigm

Revisit the structure of the quantized operator, the input $\boldsymbol{x}'$ and output $\boldsymbol{y}'$ keep a small range (the low precision integer), while s keeps a big range. The quantization scheme is designed to reduce the data communication size of the IO, i.e., between the GPU and memory. With this in mind, we ask whether there exists an MPC operation whose cost depends solely on the input and output sizes, without considering the intermediate computational steps. Interestingly, we observe that if we view the quantized operator as a lookup table, the scale factor s will be hidden within it. For simplicity, we assume the quantized operator op^* acts independently on each element. Next, we will discuss the operations based on single elements. For vectorized operators, we will address them separately within the context of specific operators. In particular, let the quantized operator op^* accept input $x' \in \mathbb{Z}_{2^{\ell'}}$ and its hidden parameters s is known to the model server, the model server can locally generate the lookup table as a vector $\mathcal{T} := (\mathsf{op}^*(0), \ldots, \mathsf{op}^*(2^{\ell'} - 1))$ by evaluating $2^{\ell'}$ times of op^*.

Quantization scheme in PPML. We propose the PPML-based quantization scheme $\hat{\mathcal{Q}}_{\mathsf{ML}}^{d,\ell,\ell'}$. The input vector is secret sharing as $[\![x]\!]^{\ell}$, and the quantized function encodes $[\![x]\!]^{\ell}$ to a much succinct secret sharing $[\![x]\!]^{\ell'}$. Considering that PPML involves fixed-point encoding and multiplication will introduce double scaling ($\hat{x} = x \cdot 2^k$, $\hat{y} = y \cdot 2^k$, $\hat{z} = \hat{x} \cdot \hat{y} = x \cdot y \cdot 2^{2k}$ is double scaled), more bits are needed to represent the fixed-point secret sharing in PPML compared to plaintext. For example, 32-bit fixed-point numbers typically use 64-bit encoding to accommodate a single multiplication. Intuitively, for a typical 64-bit PPML, our quantization scheme $\hat{\mathcal{Q}}_{\mathsf{ML}}^{d,\ell,\ell'}$ achieves 0.125-succinctness under 8-bit quantization. Additionally, unlike conventional PPML fixed-point computations that potentially introduce fractional calculation errors, our PPML-based quantization scheme achieves evaluation results identical to plaintext, because the computations are performed on integers, with internal fractional calculations encoded into the lookup table. We test the accuracy loss of the 8-bit quantized convolutional neural networks using the deep learning inference SDK – TensorRT. As shown in Table II, our quantization scheme $\hat{\mathcal{Q}}_{\mathsf{ML}}^{d,\ell,\ell'}$ holds 0.2%-accuracy-loss.

TABLE II: The accuracy of the 8-bit quantized model compared to the original model.

	Squeeze Net	ResNet	AlexNet CIFAR	AlexNet Tiny	VGG CIFAR	VGG Tiny
Original Accuracy	58.19	80.24	91.52	58.63	92.72	68.08
8-bit Accuracy	58.10	80.39	91.53	58.46	92.73	68.18

Quantized model (lookup table) generation. The model server $\mathcal{S}$ will first quantize the machine learning model with the calibration data set, obtaining scale factors (s_i, b_i) for each layer. Then the $\mathcal{S}$ generates the lookup table for each quantized operator. We observe that besides the scale factor s, $\mathcal{S}$ can also embed the model weight into the lookup table, which makes the dual-input operator, e.g., convolution, matrix multiplication, to single-input. Taking $\mathsf{Mult}^*(w', x') := \frac{1}{s_2}\mathsf{Mult}(s_1(w' + b_1), s_0(x' + b_0)) - b_2$ as an example, the model server knows about s_1, w', b_1, s_0, b_0, s_2 and b_2, such that w' can be part of operator. The operator is converted to a single-input function $\mathsf{Mult}^*(x')$. To generate the lookup table, the model server traverses all possible values of input and evaluates the operator with such values. For $x' \in \{0, \ldots, 2^{\ell'} - 1\}$, the corresponding lookup table is $\mathcal{T}^{\ell',\ell'} := (\mathsf{Mult}^*(w', 0), \ldots, \mathsf{Mult}^*(w', 2^{\ell'} - 1))$. So far, the model server obtains the lookup table for each model operator of f. We denote the overall lookup table set for model f which contains m operators as the Q-model $\mathcal{M} := (\mathcal{T}^{(0)}, \ldots, \mathcal{T}^{(m-1)})$, and its i^{th} lookup table represent the i^{th} quantized operator.

Q-model evaluation. Instead of evaluating the operator with the white box function, the model evaluation turns to the black box with the lookup table. In each layer of the ML model, the model server inputs the private lookup table $\mathcal{T}^{\ell',\ell'} := (\mathsf{op}^*(0), \ldots, \mathsf{op}^*(2^{\ell'}))$, and all parties input the shared value $[\![x']\!]^{\ell'}$ to query $\mathcal{T}$, resulting in a new shared index $[\![y']\!]^{\ell'} = [\![\mathsf{op}^*(x')]\!]^{\ell'} = [\![\mathcal{T}(x')]\!]^{\ell'}$. We directly adopt the lookup evaluation technique on the single-input-single-output (SISO) operator, e.g., activate operator, batch normalization operator, etc. These operators are performed on each element, leading to a single input of the lookup table. For the multiple-input-single-output (MISO) operator, e.g., convolution, and matrix multiplication, we provide the construction in the next section. By querying the lookup table of $\mathcal{M}$ layer by layer, all parties finally obtain the shared output.

lookup tables fusion. We observe that multiple lookup tables can be fused into a single lookup table, resulting in a single fused table rather than multiple table evaluation. The lookup fusion technique is suitable for any SISO operator, or an SISO operator connecting to an MISO operator. Formally, considering a sequence of lookup table $x_2 = \mathcal{T}^{(1)}(x_1), x_3 = \mathcal{T}^{(2)}(x_2), \ldots, x_{n+1} = \mathcal{T}^{(n)}(x_n)$, it holds that $x_{n+1} = \mathcal{T}^{(n)}(\ldots, \mathcal{T}^{(2)}(\mathcal{T}^{(1)}(x_1)), \ldots)$. All the lookup tables can be combined by a single lookup table $x_{n+1} = \mathcal{T}^*(x) := \mathcal{T}^{(n)}(\ldots, \mathcal{T}^{(2)}(\mathcal{T}^{(1)}(x)), \ldots)$. If $\mathcal{T}^{(n)}$ is an MISO lookup table, the fusion concludes an overall MISO lookup table. That is, $\mathcal{T}^*(x, y) := \mathcal{T}^{(n)}(\mathcal{T}^{(1)}(x), \mathcal{T}^{(2)}(y))$. Notice that the fusion of SISO and MISO does not introduce larger lookup tables, while the fusion of MISO and MISO will exponentially increase the size of the lookup table.

Dequantization fusion. The other optimization is that we can further reduce the de-quantify phase by fusing the dequantization phase with the previous layer's lookup table. If we view the dequantization scheme as a lookup table, its input range is $\mathbb{Z}_{2^{\ell'}}$ and output range is $\mathbb{Z}_{2^{\ell}}$, such that the lookup table size is $2^{\ell'} \cdot \ell$ (Considering ℓ' is small, it is acceptable). Similarly, we use the lookup fusion technique to combine the de-quantify function with the last operator, resulting in a new lookup table with $\mathbb{Z}_{2^{\ell'}}$ input and $\mathbb{Z}_{2^{\ell}}$ output. Note that this technique is not suitable for the quantization operator, as the input of the quantizing function is ℓ-bit, which would lead to an unacceptably large lookup table of size $2^{\ell} \cdot \ell'$.

IV. Private lookup Table Evaluation.

A. The Existing lookup Table Overview

Our PPML framework is based on secure lookup evaluation. Looking forward to a suitable component, we review the existing lookup table evaluation scheme. In general, we classify the lookup table evaluation into two types: i. the scheme where online communication cost only depends on the output size of lookup; ii. the scheme where online communication costs are only dependent on the input size of the lookup.

lookup Table with Output-length-dependent Cost. Brüggemann [8] *et. al* propose a lookup table construction named FLUTE where the online phase communication costs are only dependent on the lookup table output size. The FLUTE approach utilizes a boolean circuit to represent the lookup table and adopts the ABY2.0 [43] protocol to evaluate the boolean circuit securely. Since the online phase of ABY2.0 only depends on the output size ℓ', the lookup table evaluation only requires concrete $2\ell'$ bits communication. FLUTE provides a fast online phase when the output of the lookup table is small. However, FLUTE is not suitable for our framework, as its lookup table structure needs to be public. For our PPML framework, the lookup table encoding the scale factor s, the offset b and the weight w needs to be processed in secret.

lookup Table with Input-length-dependent Cost. Another type of lookup evaluation schemes [28], [19], [20] holds the property that the online phase communication costs are only dependent on the lookup table input size, by introducing an offset r on the lookup table. In particular, for a lookup table $\mathcal{T}^{\ell_x,\ell_y}$ whose input size is 2^{ℓ_x}, the preprocessing phase involving two parties jointly generating the shifted shared lookup table $[\![\mathcal{T}']\!]$ whose item is $[\![t_i]\!]^{\ell_y} = [\![\mathcal{T}(i+r)]\!]^{\ell_y}$ for $i \in \{0, \ldots, 2^{\ell_x} - 1\}$ (Circular shifting $\mathcal{T}$ by r position to obtain $\mathcal{T}'$) and the secret shared offset $[\![r]\!]^{\ell_x}$. In the online phase, given the secret shared input $[\![x]\!]$, two parties reconstruct $\delta = x - r$ and set result $[\![y]\!]^{\ell_y} = [\![t_\delta]\!]^{\ell_y}$ (it is easy to see $t_\delta = \mathcal{T}(\delta + r) = \mathcal{T}(x)$). The communication cost of the online phase only contains $2\ell_x$ bits communication of reconstruction which corresponds to the input size. Coincidentally, similar to the output depending on the scheme, these works [28], [19], [20] are also incompatible with our PPML framework, where their settings assume the lookup table is public in the preprocessing phase to generate $[\![t_i]\!]^{\ell_y}$.

B. Our Private lookup Table Evaluation Scheme.

We first study the private lookup table evaluation scheme. In our scheme, instead of the public lookup table, we view the lookup table as private input. In our setting, one of the parties inputs the private lookup table $\mathcal{T}^{\ell_x,\ell_y}$, and all parties input a secret shared value $[\![x]\!]^{\ell_x}$. As the lookup table evaluation, all parties receive the shared result $[\![\mathcal{T}(x)]\!]^{\ell_y}$.

Our scheme is based on the aforementioned approach where online communication cost only depends on the input size of the lookup. We observe that this approach is partially compatible with the private lookup table since the shifted lookup table $[\![\mathcal{T}']\!]$ in the online phase is secretly shared and leaks no information about the original lookup table. The remaining challenge is how to generate $[\![\mathcal{T}']\!]^{\ell_y}$ and $[\![r]\!]^{\ell_x}$ with a private lookup table $\mathcal{T}$. Formally, we define the private lookup table evaluation as two phases: (i) private shifted lookup table pair generation, in the preprocessing phase, all parties jointly generate the shifted shared lookup table pair where one of the parties inputs the private lookup table. (ii) lookup table evaluation, in this phase, we follow the works [28], [19], [20] where all parties are only required to perform $2 \cdot \ell_x$ bits communication to reconstruction. Formally, we formally define the circular shifted lookup pair. We give the definition as follows.

Definition 3. Let $\mathcal{T}^{\ell_x,\ell_y}$ be a lookup table with ℓ_x-bit input and ℓ_y-bit output. We say a 2PC circular shift lookup pair for

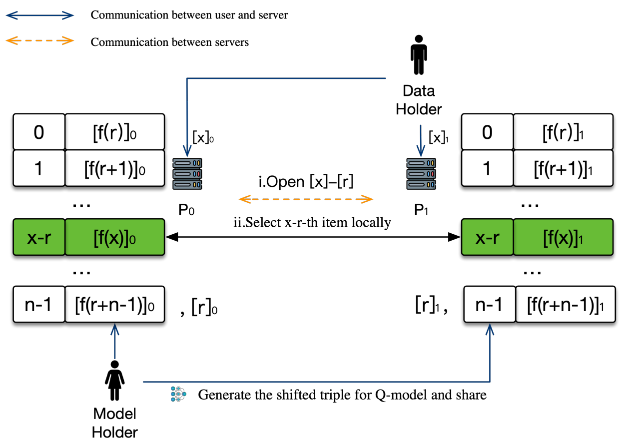


Fig. 7: Secure ML Operator Evaluation with lookup Table.

Protocol $\Pi^{\ell_x,\ell_y}_{\mathsf{lookup}}(\mathcal{T}^{\ell_x,\ell_y}, [\![x]\!]^{\ell_x})$

Input : $\mathcal{S}$ inputs a lookup table $\mathcal{T}^{\ell_x,\ell_y} \in (\mathbb{Z}_{2^{\ell_y}})^{2^{\ell_x}}$ in the preprocessing phase. $\mathcal{S}$ and $\mathcal{C}$ inputs a secret shared index $[\![x]\!]^{\ell_x}$ in the online phase.
Output : The secret shared lookup table evaluation result $[\![\mathcal{T}(x)]\!]^{\ell_y}$.

Preprocessing:

1) $\mathcal{C}$ and $\mathcal{S}$ invoke $\Pi_{(2^{\ell_x}-1,2^{\ell_x})\text{-ROT}}$:
   - $\mathcal{S}$ receives $(\boldsymbol{m}_i | i \in [2^{\ell_x}], \boldsymbol{m}_i \in (\mathbb{Z}_{2^{\ell_y}})^{2^{\ell_x}})$.
   - $\mathcal{C}$ receives $[\![r]\!]_0$ and $(\boldsymbol{m}_i | i \in [2^{\ell_x}] \backslash \{[\![r]\!]_0\})$.
2) $\mathcal{C}$ picks random $\boldsymbol{m}_{[\![r]\!]_0} \leftarrow (\mathbb{Z}_{2^{\ell_y}})^{2^{\ell_x}}$.
3) $\mathcal{C}$ and $\mathcal{S}$ generate the matrix $\mathbf{M} \in (\mathbb{Z}_{2^{\ell_y}})^{2^{\ell_x} \times 2^{\ell_x}}$ by setting $\boldsymbol{m}_i$ as the i^{th} column vectors.
4) $\mathcal{C}$ and $\mathcal{S}$ right circular shift the i^{th} row of $\mathbf{M}$ by i positions locally for $i \in [2^{\ell_x}]$.
5) $\mathcal{S}$ and $\mathcal{C}$ computes $v_i = \sum_{j=0}^{2^{\ell_x}-1} m_{(i,j)}$ and $u_i = \sum_{j=0}^{2^{\ell_x}-1} (-m_{(j,i)})$ for $i \in [2^{\ell_x}]$, and denotes $\boldsymbol{v} := (v_0, \ldots, v_{2^{\ell_x}-1})$ and $\boldsymbol{u} := (u_0, \ldots, u_{2^{\ell_x}-1})$.
6) $\mathcal{C}$ computes $w_i = v_i + u_{[\![r]\!]_0+i}$, and denotes $\boldsymbol{w} := (w_0, \ldots, w_{2^{\ell_x}-1})$.
7) $\mathcal{S}$ picks random $[\![r]\!]_1 \in \mathbb{Z}_{2^{\ell_x}}$ and right circular shift $\mathcal{T}$ by $[\![r]\!]_1$ positions to obtain $\hat{\mathcal{T}}$.
8) $\mathcal{S}$ sends $\boldsymbol{s} = \hat{\mathcal{T}} + \boldsymbol{u}$ to $\mathcal{C}$ and sets $[\![\mathcal{T}']\!]_1 := \boldsymbol{v}$.
9) $\mathcal{C}$ computes $[\![\mathcal{T}']\!]_0 := \mathsf{shift}(\boldsymbol{s}, [\![r]\!]_0) - \boldsymbol{w}$.

Online:

1) $\mathcal{S}$ and $\mathcal{C}$ calculates $[\![\delta]\!]^{\ell_x} = [\![x]\!]^{\ell_x} - [\![r]\!]^{\ell_x}$.
2) $\mathcal{C}$ and $\mathcal{S}$ invoke $\delta \leftarrow \Pi^{\ell_x}_{\mathsf{rec}\to\{\mathcal{C},\mathcal{S}\}}([\![\delta]\!]^{\ell_x})$.
3) $\mathcal{C}$ and $\mathcal{S}$ set output $[\![\mathcal{T}(x)]\!]^{\ell_y} = [\![\mathcal{T}'(\delta)]\!]^{\ell_y}$.

Fig. 8: The lookup evaluation protocol Π_{lookup}.

lookup table $\mathcal{T}^{\ell_x,\ell_y}$ is $([\![\mathcal{T}']\!]^{\ell_y}, [\![r]\!]^{\ell_x})$, if it holds that $\mathcal{T}'(x) = \mathcal{T}(x+r)$.

Circular Shift Lookup Pair in Outsourcing Setting. The circular shift lookup table pair can be generated easily in the outsourcing setting. Since the employed computation parties $\mathcal{P}_0$ and $\mathcal{P}_1$ will not collude with the model holder $\mathcal{S}$, $\mathcal{S}$ can directly generate $r \in \mathbb{Z}_{2^{\ell_x}-1}$ and locally circular shift $\mathcal{T}$ to r position and obtain $\mathcal{T}'$. Consequently, $\mathcal{S}$ secret share $\mathcal{T}'$ and r to $\mathcal{P}_0$ and $\mathcal{P}_1$.

Circular Shift Lookup Pair in C/S Setting. The circular shift lookup pair generation has more challenges in the C/S setting. Recently, Lu [38] *et. al* propose a vector oblivious shift evaluation (VOSE) scheme that can secure shift a n-dimension binary vector $\mathcal{T} \in (\mathbb{Z}_2)^n$ to a random position r in the two-party setting. In VOSE, $\mathcal{P}_0$ inputs a binary vector $\mathcal{T} := (t_0, \ldots, t_{n-1}) \in (\mathbb{Z}_2)^n$ and receives $[\![\mathcal{T}]\!]_0^1$, $\mathcal{P}_1$ receives $[\![\mathcal{T}]\!]_1^1$ and offset r where $[\![\mathcal{T}]\!]_0^1 \oplus [\![\mathcal{T}]\!]_1^1 = \mathsf{shift}(\mathcal{T}, r)$. We realize that applying their definition to the ring $\mathbb{Z}_{2^{\ell_y}}$ satisfies our requirements. We adopt the technique of Lu *et. al* to our circular shift lookup pair generation. In our setting, we define the VOSE in the arbitrary ring $\mathbb{Z}_{2^\ell}$, namely, $\mathcal{S}$ inputs a vector $\mathcal{T} \in (\mathbb{Z}_{2^\ell})^n$ and receives $[\![\mathcal{T}]\!]_0$, $\mathcal{C}$ receives $[\![\mathcal{T}]\!]_1$ and offset r where $[\![\mathcal{T}]\!]_0 + [\![\mathcal{T}]\!]_1 = \mathsf{shift}(\mathcal{T}, r)$. Using VOSE, we realize the circular shift lookup pair generation. We let $\mathcal{S}$ locally circular shift $\mathcal{T}$ to r_1 position and input the shifted lookup table to VOSE. $\mathcal{C}$ input another position r_0 to VOSE. Since $\mathcal{S}$ locally shift $\mathcal{T}$ to r_1 position and VOSE shift r_0 position, the overall shifted position is $r_0 + r_1$. After that $\mathcal{C}$ receive $[\![\mathcal{T}']\!]_0$ and $\mathcal{S}$ receive $[\![\mathcal{T}']\!]_1$ where $\mathcal{T}' = \mathsf{shift}(\mathcal{T}, r_0 + r_1)$. Setting $[\![r]\!]_0 = r_0$ and $[\![r]\!]_1 = r_1$, we obtain the circular shift lookup pair generation.

Our VOSE protocol is also inspired by Lu *et. al* [38], At a high level, the VOSE protocol contains two parts. In the first part, all parties generate a random VOSE, where $\mathcal{T}$ is a random vector, rather than determined by $\mathcal{P}_0$. In the second part, all parties construct VOSE based on random VOSE.

Random Vector Oblivious Shift Evaluation over Ring. In our random VOSE scheme, $\mathcal{C}$ receives two n-dimension random vectors $\boldsymbol{u} \in (\mathbb{Z}_{2^\ell})^n$ and $\boldsymbol{v} \in (\mathbb{Z}_{2^\ell})^n$; $\mathcal{S}$ receives a offset r and a vector $\boldsymbol{w} \in (\mathbb{Z}_{2^\ell})^n$, and it holds $\boldsymbol{w} = \mathsf{shift}(\boldsymbol{u}, r) + \boldsymbol{v}$. We realize random VOSE from $\ell \cdot n$ length $n-1$ out of n random OT. Specifically, we describe the process as follows.

- $\mathcal{S}$ and $\mathcal{C}$ invoke an instance of $\Pi_{(n-1,n)\text{-ROT}}$. After the protocol, $\mathcal{S}$ receives n messages $(\boldsymbol{m}_0, \ldots, \boldsymbol{m}_{N-1})$ and $\boldsymbol{m}_i \in (\mathbb{Z}_{2^\ell})^n$. $\mathcal{C}$ receives r and all messages except for $\boldsymbol{m}_r$. We allow $\mathcal{C}$ pick random $\hat{\boldsymbol{m}}_r \leftarrow (\mathbb{Z}_{2^\ell})^n$. Viewing $(\boldsymbol{m}_0, \ldots, \boldsymbol{m}_{n-1})$ as a $n \times n$-dimension matrix $\mathbf{M}$, $\mathcal{S}$ obtains the complete $\mathbf{M}$, while $\mathcal{C}$ can obtain the $\hat{\mathbf{M}}$ with dummy r^{th} column $\hat{\boldsymbol{m}}_r$.
- $\mathcal{S}$ and $\mathcal{C}$ right circular shift the i^{th} row of $\mathbf{M}$ ($\hat{\mathbf{M}}$ for $\mathcal{C}$) by i positions for $i \in [n]$, and denote the new matrix as $\mathbf{M}' := (m'_{(i,j)})_{i\in[n],j\in[n]}$ (or $\hat{\mathbf{M}}' := (\hat{m}'_{(i,j)})_{i\in[n],j\in[n]}$).
- $\mathcal{S}$ sets $v_i = \sum_{j=0}^{n-1} m'_{(i,j)}$ and $u_i = \sum_{j=0}^{n-1} (-m'_{(j,i)})$ for $i \in [n]$ to generate $\boldsymbol{v} := (v_0, \ldots, v_{n-1})$ and $\boldsymbol{u} := (u_0, \ldots, u_{n-1})$. Note that v_i is the sum of i^{th} row of $\mathbf{M}'$ and u_i is the sum of i^{th} column of $\mathbf{M}'$.
- Similarly, $\mathcal{C}$ calculates $\hat{v}_i = \sum_{j=0}^{n-1} \hat{m}'_{(i,j)}$ and $\hat{u}_i = \sum_{j=0}^{n-1} (-\hat{m}'_{(j,i)})$ for $i \in [n]$. Let $w_i = \hat{v}_i + \hat{u}_{r+i} \mod 2^\ell$, it holds that $w_i = \hat{v}_i - \hat{m}'_{(i,i+r)} + m'_{(i,i+r)} + \hat{u}_{r+i} + m_{(i,i+r)} - m'_{(i,i+r)} = v_i + u_{r+i}$. Considering $m'_{(i,i+r)}$ is

Quantization scheme $\Pi^{\mathsf{out}}_{\mathsf{quantize}}(\boldsymbol{x}, \mathcal{E}, f', \mathcal{D})$

Input : The fractional precision k is common input; the fixed-point encode of input $\hat{\boldsymbol{x}} = \mathsf{encode}(\boldsymbol{x}) \in \mathbb{Z}_{2^\ell}$ is input by $\mathcal{C}$; the scale factor $\hat{s} = \mathsf{encode}(\frac{1}{s}) \in \mathbb{Z}_{2^\ell}, b \in \mathbb{Z}_{2^{\ell'}}$ for encoding function $\mathcal{E}$ is input by $\mathcal{S}$; the quantized lookup table set for model $f' := (\mathsf{op}_0, \ldots, \mathsf{op}_{N-1}, \hat{\mathsf{op}})$ is input by $\mathcal{S}$, where $\hat{\mathsf{op}}$ is the last lookup table which is combined with the dequantization $\mathcal{D}$.
Output : model evaluation result $\boldsymbol{z} := \mathcal{D}(f'(\mathcal{E}(\boldsymbol{x})))$.

Input Quantization:

- (Preprocessing) $\mathcal{S}$ picks random vector $\boldsymbol{r} \leftarrow (\mathbb{Z}_{2^\ell})^n$ and invokes $\Pi^\ell_{\mathsf{out}}(\mathcal{S}, \boldsymbol{r})$,$\Pi^\ell_{\mathsf{out}}(\mathcal{S}, \hat{s})$ and $\Pi^\ell_{\mathsf{out}}(\mathcal{S}, \boldsymbol{r} \cdot \hat{s})$, where the computing node P_i for $i \in \{0,1\}$ holds $(\llbracket \boldsymbol{r} \rrbracket^\ell_i, \llbracket s \rrbracket^\ell_i, \llbracket \boldsymbol{r} \cdot \hat{s} \rrbracket^\ell_i)$;
- $\mathcal{C}$ invokes $\Pi^\ell_{\mathsf{out}}(\mathcal{C}, \hat{\boldsymbol{x}})$ and the computing node P_i for $i \in \{0,1\}$ holds $\llbracket \hat{\boldsymbol{x}} \rrbracket^\ell_i$;
- P_i for $i \in \{0,1\}$ does:
  - calculate $\llbracket \boldsymbol{\delta} \rrbracket^\ell_i = \llbracket \hat{\boldsymbol{x}} \rrbracket^\ell_i - \llbracket \boldsymbol{r} \rrbracket^\ell_i$ and invoke $\Pi^\ell_{\mathsf{rec} \to \{\mathcal{P}_0, \mathcal{P}_1\}}(\llbracket \boldsymbol{\delta} \rrbracket^\ell)$ to reconstruct $\boldsymbol{\delta}$.
  - calculate $\llbracket \boldsymbol{x}' \rrbracket^\ell_i = \boldsymbol{\delta} \cdot \llbracket \hat{s} \rrbracket^\ell_i + \llbracket \boldsymbol{r} \cdot \hat{s} \rrbracket^\ell_i$.
  - invoke cut function $\llbracket x'_i \rrbracket^{\ell'} \leftarrow \Pi^{\ell'}_{\mathsf{cut}}(\llbracket x'_i \rrbracket^\ell, 2k)$ for $i \in [n]$ locally.

Model evaluation:

For the operators $(\mathsf{op}_0, \ldots, \mathsf{op}_{N-1})$ with each element $\llbracket x' \rrbracket^{\ell'}$ of input vector $\llbracket \boldsymbol{x}' \rrbracket^{\ell'}$ and output $\llbracket \mathsf{op}^*_j(x') \rrbracket^{\ell'}$,

- (Preprocessing) $\mathcal{S}$ invokes $\Pi^{\ell'}_{\mathsf{out}}(\mathcal{S}, \mathsf{op}^*(j + r))$ for $j \in \{0, \ldots, 2^{\ell'} - 1\}$ and $\Pi^\ell_{\mathsf{out}}(\mathcal{S}, r)$, where P_i for $i \in \{0,1\}$ holds the share of lookup table $(\llbracket \mathcal{T}'(0) \rrbracket_i, \ldots, \llbracket \mathcal{T}'(2^{\ell'} - 1) \rrbracket_i)$ and the offset $\llbracket r \rrbracket_i$.
- P_i for $i \in \{0,1\}$does:
  - calculate $\llbracket \delta \rrbracket^{\ell'}_i = \llbracket x' \rrbracket_i - \llbracket r \rrbracket_i$ and invoke $\Pi^{\ell'}_{\mathsf{rec} \to \{\mathcal{P}_0, \mathcal{P}_1\}}(\llbracket \delta \rrbracket)$ to reconstruct δ.
  - set $\llbracket y' \rrbracket^{\ell'}_i = \llbracket \mathcal{T}'(\delta) \rrbracket_i$.

Output Dequantization:

For the operator $\hat{\mathsf{op}}^*$ which combines the dequantization operator with the last operator and each element x' of input vector $\boldsymbol{x}'$,

- (Preprocessing) $\mathcal{S}$ invokes $\llbracket \mathcal{T}^{\ell',\ell}(j) \rrbracket \leftarrow \Pi^\ell_{\mathsf{out}}(\mathcal{S}, \mathsf{op}^*(j + r))$ for $j \in \{0, \ldots, 2^{\ell'} - 1\}$ and $\llbracket r \rrbracket^{\ell'} \leftarrow \Pi^{\ell'}_{\mathsf{out}}(\mathcal{S}, r)$.
- P_i for $i \in \{0,1\}$ does:
  - calculate $\llbracket \delta \rrbracket^{\ell'}_i = \llbracket x' \rrbracket^{\ell'}_i - \llbracket r \rrbracket^{\ell'}_i$ and invoke $\Pi^{\ell'}_{\mathsf{rec} \to \{\mathcal{P}_0, \mathcal{P}_1\}}(\llbracket \delta \rrbracket)$ to reconstruct δ.
  - set $\llbracket \hat{z} \rrbracket^\ell_i = \llbracket \mathcal{T}(\delta) \rrbracket^{\ell'}_i$ and send $\llbracket \hat{z} \rrbracket^\ell_i$ to $\mathcal{C}$.
  - $\mathcal{C}$ calculate $z = \mathsf{decode}(\llbracket \hat{z} \rrbracket^\ell_0 + \llbracket \hat{z} \rrbracket^\ell_1)$ and combine all element to vector $\boldsymbol{z}$.

Fig. 9: The quantization PPML scheme in the outsourcing setting

the only item shift by m'_r which correspond to $\hat{m}'_r$ of $\mathcal{C}$, the calculation of w_i eliminates the same item of m'_r and $\hat{m}'_r$ so that $\mathcal{S}$ can correctly calculate w_i. While any other v_i or u_i is random to $\mathcal{S}$. $\mathcal{S}$ set $\boldsymbol{w} := (w_0, \ldots, w_{n-1})$.

Obviously, $\boldsymbol{w}, \boldsymbol{u}$ and $\boldsymbol{v}$ satisfy $\boldsymbol{w} = \mathsf{shift}(\boldsymbol{u}, r) + \boldsymbol{v}$.

Private lookup Evaluation from VOSE. Based on the random VOSE, we construct our private lookup evaluation protocol for $\mathcal{T}^{\ell_x, \ell_y}$. Fig. 8 illustrates the specific procedure of lookup evaluation protocol Π_{lookup}. Random VOSE output vectors $\boldsymbol{u} \in (\mathbb{Z}_{2^{\ell_y}})^{2^{\ell_x}}$ and $\boldsymbol{v} \in (\mathbb{Z}_{2^{\ell_y}})^{2^{\ell_x}}$ to $\mathcal{S}$, $r_0 \in \mathbb{Z}_{2^{\ell_x}}$ and a vector $\boldsymbol{w} \in (\mathbb{Z}_{2^{\ell_y}})^{2^{\ell_x}}$ to $\mathcal{C}$, such that $\boldsymbol{w} = \mathsf{shift}(\boldsymbol{u}, r_0) + \boldsymbol{v}$. We let $\mathcal{S}$ picks random offset $r_1 \in \mathbb{Z}_{2^{\ell_x}}$ and locally shift $\mathcal{T}$ by r_1 position to obtain $\hat{\mathcal{T}}$. Consequently, $\mathcal{S}$ sends $\boldsymbol{s} = \hat{\mathcal{T}} + \boldsymbol{u}$ to $\mathcal{C}$. Then $\mathcal{C}$ sets $\llbracket \mathcal{T}' \rrbracket_1 = \mathsf{shift}(\boldsymbol{s}, r_0) - \boldsymbol{w}$ and $\mathcal{S}$ sets $\llbracket \mathcal{T}' \rrbracket_0 = \boldsymbol{v}$. Clearly, $\llbracket \mathcal{T}' \rrbracket_1 + \llbracket \mathcal{T}' \rrbracket_0 = \mathsf{shift}(\boldsymbol{s}, r_0) - \mathsf{shift}(\boldsymbol{u}, r_0) = \mathsf{shift}(\hat{\mathcal{T}}, r_0) = \mathsf{shift}(\mathcal{T}, r_1 + r_0)$. For the security proof of our protocol Π_{lookup}, we refer the reader to Appendix. A.

V. PPML FOR QUANTIZED MODELS.

In this section, we give a concrete construction for our PPML framework. We first propose the general PPML framework of our outsourcing setting and C/S setting, in which all the operator is viewed as SISO operators. Next, we discuss the special case of the MISO operators.

A. The Outsourcing Setting.

We first talk about how to apply our paradigm to the outsourcing setting. Compared to the C/S structure, the offline of our outsourcing structure is much cheaper. We define the execution procedure of quantization, lookup table evaluation, and de-quantization as follows. Fig. 9 formally illustrates our quantization framework.

- **Input Quantization.** In the outsourcing setting, the data client holds the input vector $\boldsymbol{x} \in (\mathbb{Z}_{2^\ell})^n$ and inputs it in the online phase, while the model server holds the scale factor $s \in \mathbb{Z}_{2^\ell}$, offset $b \in \mathbb{Z}_{2^{\ell'}}$ and input in the preprocessing phase. They would like to employ two computation parties $\mathcal{P}_0$ and $\mathcal{P}_1$ to evaluate $\mathcal{E}(\boldsymbol{x}) :=$

$\frac{1}{s} \cdot \boldsymbol{x} - b \in \mathbb{Z}_{2^{\ell'}}$. At the preprocessing phase, we let the model holder $\mathcal{S}$ first encode $\frac{1}{s}$ as the fixed-point encoding, namely $\hat{s} = \mathsf{enc}(\frac{1}{s})$ (Note that enc scale $\frac{1}{s}$ up to k bits), and secret share the VOLE triple $([\![\boldsymbol{r}]\!]^{\ell}, [\![\hat{s}]\!]^{\ell}, [\![\boldsymbol{r}\hat{s}]\!]^{\ell})$ to $\mathcal{P}_0$ and $\mathcal{P}_1$. In the online phase, the data client encode $\boldsymbol{x}$ as $\hat{\boldsymbol{x}} = \mathsf{enc}(\boldsymbol{x})$ (Similarly, enc scale $\boldsymbol{x}$ up to k bits) and secret share $\hat{\boldsymbol{x}}$ to $\mathcal{P}_0$ and $\mathcal{P}_1$. $\mathcal{P}_0$ and $\mathcal{P}_1$ reveal $\boldsymbol{\delta} = \hat{\boldsymbol{x}} - \boldsymbol{r}$ and calculate $[\![\boldsymbol{x}']\!]^{\ell} = \boldsymbol{\delta} \cdot [\![\hat{s}]\!]^{\ell} + [\![\boldsymbol{x}\hat{s}]\!]^{\ell}$ which is equals to $[\![\hat{\boldsymbol{x}} \cdot \hat{s}]\!]^{\ell} := (\hat{x_0} \cdot \hat{s}, \ldots, \hat{x_{n-1}} \cdot \hat{s})$. Consequently, we apply $[\![x_i']\!]^{\ell'} \leftarrow \Pi^{\ell'}_{\mathsf{cut}}([\![\hat{x}_i \cdot \hat{s}]\!]^{\ell}, 2k)$ for $i \in [n]$ to chop ℓ' significant bits (Note that the last $2k$ bits is the fractional part introduced by the multiplication of $\hat{x}$ and $\hat{s}$). For the additive part of $[\![\boldsymbol{x}']\!]^{\ell'} + [\![b]\!]^{\ell'}$, it can be evaluated locally.

- **Q-model Evaluation.** For the lookup table evaluation of each element x' in the vector $\boldsymbol{x}'$, i.e., the lookup table $\mathcal{T}^{\ell',\ell'}$ with input $[\![x']\!]^{\ell'}$ and output $[\![y]\!]^{\ell'} = [\![\mathcal{T}(x')]\!]^{\ell'}$, we adopt the aforementioned shift pair (The procedure is shown in Fig. 7), we let the model server $\mathcal{S}$ shift $\mathcal{T}$ with a random offset r, namely, $\mathcal{T}'(i) = \mathcal{T}(i + r)$ for $i \in \mathbb{Z}_{2^{\ell'}}$. In the preprocessing phase, $\mathcal{S}$ secret share $\mathcal{T}'$ and r to $\mathcal{P}_0$ and $\mathcal{P}_1$. In the online phase, P_j for $i \in \{0, 1\}$ calculates $[\![\delta]\!]_j = [\![x']\!]_j - [\![r]\!]_j$ and reconstruct Δ. P_j then locally sets output as $[\![y]\!]^{\ell'}_j = [\![\mathcal{T}'(\delta)]\!]_j$. Obviously, $\mathcal{T}'(\delta) = \mathcal{T}(\delta + r) = \mathcal{T}(x')$.
- **Output De-quantization.** As mentioned before, the de-quantization function $\hat{\boldsymbol{y}} = \mathcal{E}(\boldsymbol{y}') = s(\boldsymbol{y}' - b)$, where $\boldsymbol{y}' \in (\mathbb{Z}_{2^{\ell'}})^n$ and $\hat{\boldsymbol{y}} \in (\mathbb{Z}_{2^{\ell}})^n$, can be combined with the previous operator $\hat{\boldsymbol{y}} = s(\mathsf{op}^*(\boldsymbol{y}')-b)$. The model server $\mathcal{S}$ generates the lookup table $\mathcal{T} \in \mathbb{Z}_{2^{\ell}}^{2^{\ell'}}$ and evaluate it as like the model evaluation. Upon calculating the result $[\![\hat{\boldsymbol{y}}]\!]^{\ell}$, $\mathcal{P}_0$ and $\mathcal{P}_1$ reconstruct $\hat{\boldsymbol{y}}$ to the data client $\mathcal{C}$. $\mathcal{C}$ invoke decode function $\boldsymbol{y} = \mathsf{decode}(\hat{\boldsymbol{y}})$ to obtain the fixed-point result $\boldsymbol{y}$.

B. Client/Server structure

Compared to outsourcing setting, the C/S setting has more challenges. In the outsourcing setting, we assume that the server and the computing nodes will not collude, such that the revealed data $\boldsymbol{\delta} = \boldsymbol{x} - \boldsymbol{r}$ will not leak information of $\boldsymbol{x}$. In contrast, in the C/S setting, no matter the multiplication triple $(\boldsymbol{r}, \hat{s}, \boldsymbol{r} \cdot \hat{s})$ or the shift pair $(\mathcal{T}', r)$ can not be directly generated by server $\mathcal{S}$, because the knowledge of $\boldsymbol{r}$ will lead $\mathcal{S}$ learn $\boldsymbol{x}$ from $\boldsymbol{\delta}$. For this concern, we utilize the VOLE protocol Π_{vole} and our 2PC private lookup evaluation protocol Π_{lookup} to realize PPML inference.

- **Input Quantization.** In the C/S setting, the data client holds the input vector $\boldsymbol{x} \in (\mathbb{Z}_{2^{\ell}})^n$ and the model sever holds the scale factor $s \in \mathbb{Z}_{2^{\ell}}$ and $b \in \mathbb{Z}_{2^{\ell'}}$. They directly invoke a 2PC protocol to evaluate $\mathcal{E}(\boldsymbol{x}) := \frac{1}{s} \cdot \boldsymbol{x} - b \in \mathbb{Z}_{2^{\ell'}}$. Similarly, the model holder encode $\hat{s} = \frac{1}{s}$ at first. At the preprocessing phase, $\mathcal{C}$ generate random shared vector $\boldsymbol{r} := (r_0, \ldots, r_{n-1})$. In particular, $\mathcal{C}$ picks random value $r_i \in \mathbb{Z}_{2^{\ell}}$ for $i \in [n]$. They adopt $([\![\hat{s} \cdot r_i]\!]^{\ell})_{i \in [n]} \leftarrow \Pi_{\mathsf{vole}}(\hat{s}, (r_0, \ldots, r_{n-1}))$, where $\mathcal{C}$ input $(r_i)_{i \in [n]}$ and $\mathcal{S}$ input $\hat{s}$. $\mathcal{S}$ secret share $\hat{s}$ at the same round. In the online phase, $\mathcal{C}$ sends $\boldsymbol{\delta} = \boldsymbol{x} - \boldsymbol{r}$ to $\mathcal{S}$. Similar to the outsourcing setting, all parties calculate $[\![\boldsymbol{x}']\!]^{\ell} = \boldsymbol{\delta} \cdot [\![\hat{s}]\!]^{\ell} + [\![\boldsymbol{x}\hat{s}]\!]^{\ell}$ and perform $\Pi^{\ell'}_{\mathsf{cut}}$ to obtain $[\![\boldsymbol{x}']\!]^{\ell'}$.
- **Q-model Evaluation/Output De-quantization.** We adopt the private lookup table evaluation scheme for the model evaluation in the C/S setting. For each operator op^* with input $[\![\boldsymbol{x}]\!]^{\ell'}$. $\mathcal{C}$ and $\mathcal{S}$ invoke $[\![y_i]\!]^{\ell'} \leftarrow \Pi^{\ell',\ell'}_{\mathsf{lookup}}((\mathsf{op}^*(0), \ldots, \mathsf{op}^*(2^{\ell'} - 1)), [\![x_i]\!]^{\ell'})$ for each $i \in [n]$. All parties set the output vector $\boldsymbol{y} := (y_0, \ldots, y_{n-1})$. For the operator $\hat{\mathsf{op}}^* : \mathbb{Z}_{2^{\ell'}} \to \mathbb{Z}_{2^{\ell}}$ witch contains both model operator and dequantization, $\mathcal{C}$ and $\mathcal{S}$ invoke $[\![\hat{z}_i]\!]^{\ell'} \leftarrow \Pi^{\ell',\ell}_{\mathsf{lookup}}((\hat{\mathsf{op}}^*(0), \ldots, \hat{\mathsf{op}}^*(2^{\ell'} - 1)), [\![x_i]\!]^{\ell'})$. Consequently, $\mathcal{S}$ sends $[\![\hat{z}_i]\!]_1$ to $\mathcal{C}$. $\mathcal{C}$ reconstruct $\hat{\boldsymbol{z}} := (\hat{z}_0, \ldots, \hat{z}_{n-1})$ and invoke decode to recover the fixed-point value.

Quantization scheme $\Pi^{\mathcal{C}/\mathcal{S}}_{\mathsf{quantize}}(\boldsymbol{x}, \mathcal{E}, f', \mathcal{D})$

Input : The fractional precision k is common input; $\hat{\boldsymbol{x}} = \mathsf{encode}(\boldsymbol{x}) \in \mathbb{Z}_{2^{\ell}}$ input by $\mathcal{C}$; $\hat{s} = \mathsf{encode}(\frac{1}{s}) \in \mathbb{Z}_{2^{\ell}}, b \in \mathbb{Z}_{2^{\ell'}}$ for encoding function $\mathcal{E}$ input by $\mathcal{S}$; $f' := (\mathsf{op}_0, \ldots, \mathsf{op}_{N-1}, \hat{\mathsf{op}})$ input by $\mathcal{S}$, $\hat{\mathsf{op}}$ is combined with the dequantization $\mathcal{D}$.
Output : The model evaluation result $\boldsymbol{z} := \mathcal{D}(f'(\mathcal{E}(\boldsymbol{x})))$.

Quantize:

- (Preprocessing) $\mathcal{C}$ picks random $\boldsymbol{r} := (r_0, \ldots, r_{n-1}) \leftarrow (\mathbb{Z}_{2^{\ell}})^n$;
- (Preprocessing) $\mathcal{S}$ and invokes $[\![\hat{s}]\!]^{\ell} \leftarrow \Pi_{\mathcal{C}/\mathcal{S}}(\hat{s})$;
- (Preprocessing) $\mathcal{S}$ and $\mathcal{C}$ invokes $[\![\boldsymbol{r}']\!] := ([\![r_0']\!], \ldots, [\![r_{n-1}']\!])_{i \in n} \leftarrow \Pi^{\ell}_{\mathsf{vole}}(\boldsymbol{r}, \hat{s})$;
- $\mathcal{C}$ calculates $\boldsymbol{\delta} = \boldsymbol{x} - \boldsymbol{r} := (\delta_0, \delta_1, \ldots, \delta_{N-1})$ and sends to $\mathcal{S}$.
- $\mathcal{C}$ and $\mathcal{S}$ calculate $[\![x_i']\!]^{\ell} = \delta_i \cdot [\![\hat{s}]\!]^{\ell} + [\![r_i']\!]^{\ell}$ for $i \in [n]$;
- $\mathcal{C}$ and $\mathcal{S}$ invoke cut function $[\![x_i']\!]^{\ell'} \leftarrow \Pi^{\ell'}_{\mathsf{cut}}([\![x_i']\!]^{\ell}, 2k)$ for $i \in [n]$ locally and set $[\![\boldsymbol{x}']\!]^{\ell'} = ([\![x_i']\!]^{\ell'})_{i \in [n]}$.

Model evaluation/De-quantization:

- For the quantized operator op^* with input $[\![\boldsymbol{x}']\!]^{\ell'} := ([\![x_i']\!]^{\ell'})_{i \in [n]}$ and output $[\![\mathsf{op}^*(x')]\!]^{\ell'}$, $\mathcal{C}$ and $\mathcal{S}$ invoke $[\![y_i]\!]^{\ell'} \leftarrow \Pi^{\ell',\ell'}_{\mathsf{lookup}}((\mathsf{op}^*(0), \ldots, \mathsf{op}^*(2^{\ell'} - 1)), [\![x_i]\!]^{\ell'})$ for $i \in [n]$.
- For the last operator $\hat{\mathsf{op}}^*$ which is combined with the de-quantization, $\mathcal{C}$ and $\mathcal{S}$ invoke $[\![\hat{z}_i]\!]^{\ell} \leftarrow \Pi^{\ell',\ell}_{\mathsf{lookup}}((\mathsf{op}^*(0), \ldots, \mathsf{op}^*(2^{\ell'} - 1)), [\![x_i]\!]^{\ell'})$ for $i \in [n]$.
- $\mathcal{S}$ sends $[\![\hat{\boldsymbol{z}}]\!]^{\ell}_1 := ([\![\hat{z}_0]\!]^{\ell}_1, \ldots, [\![\hat{z}_{n-1}]\!]^{\ell}_1)$ to $\mathcal{C}$.
- $\mathcal{C}$ reconstruct $\hat{\boldsymbol{z}} = [\![\hat{\boldsymbol{z}}]\!]^{\ell}_0 + [\![\hat{\boldsymbol{z}}]\!]^{\ell}_1$ and recover the fixed-point value $\boldsymbol{z} = \mathsf{decode}(\hat{\boldsymbol{z}})$.

Fig. 10: The quantization PPML scheme in C/S setting

C. lookup for MISO.

Above we describe how to evaluate the quantized operator with the lookup table and convert SISO operators of PPML,

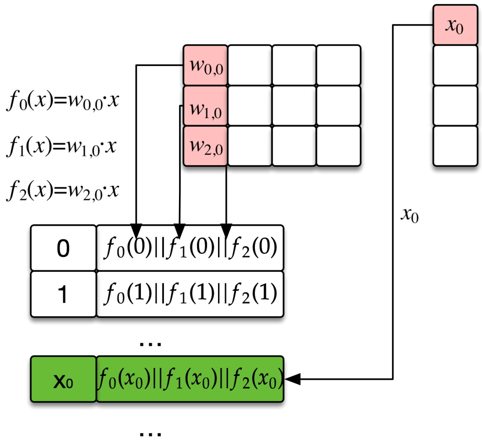


Fig. 11: A illustration example of lookup table evaluation for matrix multiplication.

e.g., activate function, to the lookup table. There remain some operators which are the MISO structure. Since the lookup table size grows exponentially as the input size increases, the lookup scheme is not applicable for the multiple input function. We analyze those MISO operators as follows.

Multiple lookup table with a single index. For the convolution layer or the matrix multiplication layer, each private input will multiplicate multiple weights, which can be viewed as a query multiple lookup table with a single index. In particular, as shown in Fig. 11, let $W := (w_{i,j})_{i \in \mathbb{Z}_3, j \in \mathbb{Z}_4}$ be a 3×4 dimension matrix, and $\boldsymbol{x} := (x_0, \ldots, x_3)$ be a 4 dimension vector. In the calculation of $W \times \boldsymbol{x}$, x_0 will multiplicate both $w_{0,0}$, $w_{0,1}$ and $w_{0,2}$. Naively, it can be realized by performing Π_{lookup} three times. Given $f_0(x_0) = w_{0,0} \cdot x_0$, $f_1(x_0) = w_{0,1} \cdot x_0$ and $f_2(x_0) = w_{0,2} \cdot x_0$, we observe that we can combine three functions to an overall function $F(x_0) := f_0(x_0)||f_1(x_0)||f_2(x_0)$. Its corresponding lookup table has ℓ' bits input and $3\ell'$ bits output. Considering the online phase of Π_{lookup} only depends on the input size, the online phase cost for N times query lookup table with a single index equals the cost of a single query.

Addition. For the addition $y = x_0 + x_1$, where $x_0 = s_0(x_0' + b_0)$ and $x_1 = s_1(x_1' + b_1)$ are the temporary variable from two different wires which are unknown to both $\mathcal{C}$ and $\mathcal{S}$, $y = s_2(y' + b_2)$ is the output of addition, its quantized function is $y' = \frac{s_0}{s_2}(x_0' + b_0) + \frac{s_1}{s_2}(x_1' + b_1) - b_2$. Addition can be realized using two times invoking lookup table with $f_0(x_0') := \frac{s_0}{s_2}(x_0' + b_0)$ and $f_1(x_1') := \frac{s_1}{s_2}(x_1' + b_1) - b_2$.

Convert the dual-input operator to the SISO structure. For the multiplication $y = x_0 \cdot x_1$, its quantified function is $y' = \frac{s_0 \cdot s_1}{s_2}(x_0' + b_0)(x_1' + b_1) - b_2$. A naive idea is to evaluate $x' = (x_0' + b_0)(x_1' + b_1)$ using 2PC multiplication in $\mathbb{Z}_{2^{\ell'}}$, after that all parties evaluate $y' = \frac{s_0 \cdot s_1}{s_2} \cdot x' - b_2$ with lookup table. Nevertheless, this approach will incur a significant error caused by overflow of $(x_0' + b_0)(x_1' + b_1)$. Considering $(x_0' + b_0)(x_1' + b_1) > 2^{\ell'}$, in lookup table, $\frac{s_0 \cdot s_1}{s_2}$ will scale it back to $[0, 2^{\ell'} - 1]$ while separate calculation will lose the overflow part, leading to a big error. The potential solution is to deal $(x_0' + b_0)(x_1' + b_1)$ in $\mathbb{Z}_{2^{2\ell'}}$, however, it still enlarged the range of lookup table input. Our solution is to convert the dual-input operator to the SISO structure, and it can be used to deal with arbitrary dual-input operators op^*. We set the previous layer of op^* to be $\ell'/2$ output, by encoding the corresponding $\ell'/2$-bit into the previous lookup table. Before inputting two $\ell'/2$-bit values to op^*, we combine them to ℓ'-bit value.

Convolutional Neural Network. We implemented typical convolutional neural network (CNN) models in our framework, including LeNet, VGG16, and ResNet18. All SISO-type operators, such as batch normalization, ReLU, sigmoid, and so on, can be directly evaluated using our paradigm. For the MISO-type operators, we discuss their implementation as follows.

- Convolution(Conv)/Fully Connection(FC)/General Matrix Multiplication(GeMM): The convolution operator can be transferred to the general matrix multiplication. For instance, the naive method explicitly lowers the convolution to GeMM, commonly known as im2col. Furthermore, the full connection can also be represented as the GeMM form. As mentioned before, viewing the weight as part of the lookup table, our framework can deal with GeMM as a "multiple lookup table query with a single index". For the sum part of GeMM, due to each item keeping the same scale factor, it can be calculated by the sum of each quantized value. In particular, we take two-dimensional multiplication as an example. Considering $z = z_1 + z_2 = x_1 \cdot y_1 + x_2 \cdot y_2$, where x_1, x_2 are in the same vector so that they keep the same scale factor s_1, b_1, the same to y_1, y_2 with scale factor s_2, b_2. It holds that $z' = \frac{s_1 \cdot s_2}{s_3}((x_1' - b_1)(y_1' - b_2) + (x_2' - b_1)(y_2' - b_2)) + b_3$. Considering $z_1' = \frac{s_1 \cdot s_2}{s_3}(x_1' - b_1)(y_1' - b_2) + b_3$ and $z_2' = \frac{s_1 \cdot s_2}{s_3}(x_2' - b_1)(y_2' - b_2) + b_3$, we have $z' = z_1' + z_2' - b_3$. Without loss of generality, for n-dimension inner product, the quantized output holds $z' = \sum_{i=0}^{N-1} z_i' - (n-1) \cdot b_3$. For the secret form $[z']^{\ell'} = \sum_{i=0}^{N-1} [z_i']^{\ell'} - (n-1) \cdot b_3$, the part minus $(n-1) \cdot b_3$ can be evaluated locally by $\mathcal{S}$.
- Max Pooling or Average Pooling: The max pooling is an expensive operator in our framework since evaluating n-dimensional max pooling is equivalent to performing $n-1$ comparisons. We need to convert the dual-input operator to an SISO structure to evaluate the comparison. Given an example, for 8-dimension max pooling with 8-bit output, we let the previous layer output 4-bit vector, such as $[x_0]^4, \ldots, [x_7]^4$. We evaluate the comparison between each two shares and obtain 4-dimension 4-bit shared vector. We perform comparison layer by layer until the dimension is reduced to 1. In the last layer, we utilize the 8-bit output lookup table to recover 8-bit quantization. The average pooling is much cheaper, the sum part of it can be evaluated by locally adding the quantized value since all the elements are in the same vector, which keeps the same scale factor. To avoid wrap-around of the secret, we need to perform division before summation. For the division, we use the SISO lookup table.

TABLE III: Online performance and offline performance in C/S setting for CNN model comparison with CryptoFlow2 [46].(We take $\ell = 64$ and $\ell' = 8$. The bandwidth is 377 MBps and 40 MBps in the LAN and the WAN setting respectively and the latency is 0.3ms and 80ms respectively.)

Model	Protocol	LAN	WAN	Comm.
ImageNet_ SqueezeNet	CryptoFlow2	44.3s	293.6s	26.07GB
	Our Online	1.66s	5.88s	0.077GB
	Our Offline	80.54s	440.20s	108.0GB
ImageNet_ ResNet50	CryptoFlow2	619.4s	3611.6s	370.8GB
	Our Online	7.32s	13.46s	0.45GB
	Our Offline	352.40s	1586.59s	481.8GB

Large Language Models. For the Large Language Models (LLM), we implement GPT-2 models. In LLM, the GeLU operator is SISO-type, allowing us to apply our paradigm directly. For the matrix multiplication which all the input is unknown to the model server, we convert the dual-input operator to the SISO structure. For the softmax and layer normalization, we discuss the implementation as follows.

- Softmax: For the softmax operator $\mathsf{Softmax}(\boldsymbol{x}) := (\frac{e^{x_i}}{\sum_{i=0}^{n-1} e^{x_i}})_{i\in[n]}$ (The scale factor of $\boldsymbol{x}$ is $\hat{s}$ and $\hat{b}$), we follow the typical PPML implementation [21] and modify it for quantization scheme. We first perform $\mathsf{max}(\boldsymbol{x})$ to find out the max element of $\boldsymbol{x}$, denoted by $\hat{x}$. After that we define temporary variable $y_i = e^{x_i - \hat{x}} = e^{s(x_i' - b - \hat{x} + b)}$. Considering $e^{x_i' - \hat{x}} \leq 1$, we can take the scale factor $\hat{s}$ and $\hat{b}$ for $\boldsymbol{y} := (y_0, \ldots, y_{n-1})$ as $\hat{s} \cdot 2^{\ell'}/n = 1$ and $\hat{b} = 0$, such that $\sum_{i=0}^{n-1} y_i'$ will not wrap around. We have $\hat{s} \cdot y_i' = e^{s(x_i' - \hat{x})}$ and $\frac{e^{x_i}}{\sum_{i=0}^{n-1} e^{x_i}} = \frac{y_i'}{\sum_{i=0}^{n-1} y_i'}$. Based on these logics, we evaluate the $\mathsf{Softmax}(\boldsymbol{x})$ as follows. i. evaluate $y_i' = \frac{1}{\hat{s}} \cdot e^{s(x_i' - \hat{x})}$ with lookup table. ii. calculate $\hat{y} = \sum_{i=0}^{n-1} y_i'$ in quantized value. iii. perform $z_i' = \frac{1}{s_2} \cdot \frac{y_i'}{\hat{y}} + b_2$ with lookup table to obtain the quantized output, where s_2, b_2 is the scale factor for $\boldsymbol{z} := (z_0, \ldots, z_{n-1})$.
- Word2Vec/Gather: Word2Vec is used for LLM tasks to produce word embeddings, which are dense vector representations of words. The Word2Vec is realized by the operator Gather, which picks a row vector from a matrix. For GPT-2, the word matrix has 50257 row vectors for each potential word. Each row vector involves 768 quantized elements in GPT-2-base or 1600 quantized elements in GPT-2-XL. We utilize our lookup table to evaluate Gather, where the lookup table input is in $\mathbb{Z}_{50257}$ and the output is in $\mathbb{Z}_{2^{\ell'}}$.

VI. Implementation and Benchmark.

In this section, we benchmark our lookup-based quantization PPML framework. We realize two types of machine learning models – the convolutional neural network for image classification models and the large language models.

TABLE IV: Online performance (for both C/S setting and outsourcing setting) comparison with Piranha-Falcon [53], [52] and Bicoptor [54] for CNN model. (We take $\ell = 64$ and $\ell' = 8$. The bandwidth is 5Gbps/100Mbps in the LAN and the WAN setting respectively and the latency is 0.2ms/40ms respectively.)

Model	Batch Size	Protocol	LAN	WAN	Plaintext Time
CIFAR10_ AlexNet	1650	P-Falcon	16.72s	297.45s	0.32s
		Bicoptor	5.00s	99.83s	
		Ours	0.98s	22.86s	
Tiny_ AlexNet	510	P-Falcon	30.47s	513.48s	0.10s
		Bicoptor	7.12s	179.53s	
		Ours	1.02s	34.18s	
CIFAR10_ VGG16	240	P-Falcon	54.28s	968.43s	0.55s
		Bicoptor	15.17s	336.64s	
		Ours	2.30s	22.32	
Tiny_ VGG16	60	P-Falcon	55.02s	967.74	0.15s
		Bicoptor	15.35s	336.09s	
		Ours	3.12s	20.01s	

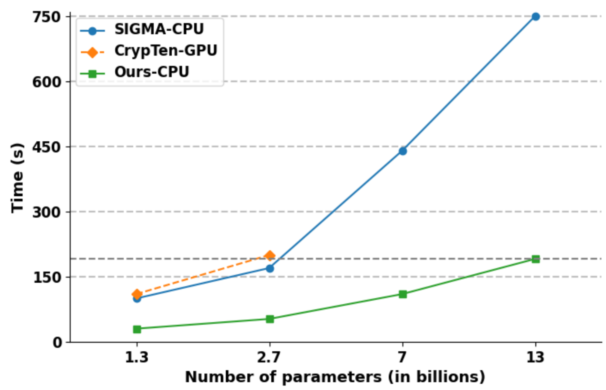
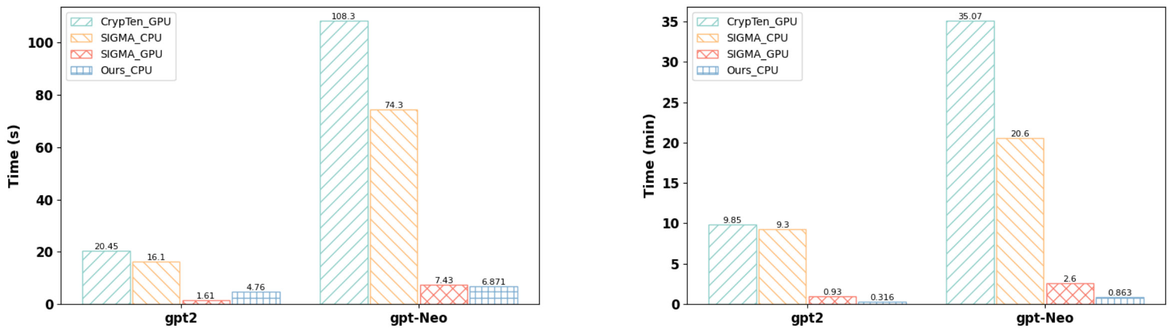


Fig. 12: Online performance comparison with Sigma [25] and CrypTen [34] for LLM (1.3B/2.7B/7B/13B corresponds to GPT-Neo1.3B, GPT-Neo2.7B, Llama2-7B, Llama2-13B respectively) in the LAN setting with 9.4 Gbps bandwidth and 0.05 ms latency. We take $\ell = 64$ and $\ell' = 8$. The sequence length of the input is 128 and the token output is 1.

A. Benchmark setting

We implement our protocols in C++. For the Π_{OT}, we utilize the OT library – libOTe [1]. Our experiments are performed in the server that runs Ubuntu 18.04.2 LTS with Intel(R) Xeon(R) Silver 4214 CPU @ 2.20GHz, 48 CPUs, 128 GB Memory. In our benchmark, we set the security parameter $\lambda = 128$. Since most of the code is not available, as a baseline, we use the data in CryptoFlow2 [46], Bicoptor [54], Piranha [53], CrypTen [34], Sigma [25], and use the software to simulate the operating environment in these papers. The benchmarks of our private lookup table evaluation in both outsourcing and C/S settings can be found in Appendix. B.



(a) LAN setting

(b) WAN setting

Fig. 13: Online performance comparison for LLM with Sigma [25] and CrypTen [34].(The bandwidth is 9.4 Gbps/293 Mbps in the LAN and the WAN setting respectively and the latency is 0.05 ms / 60 ms respectively. The sequence length of input is 128 and output one token.)

B. Convolutional neural network evaluation.

For the CNN model, we realize the typical CNN model, like SqueezeNet, ResNet50, AlexNet, and VGG16. We use their 8-bit quantized version in ONNX Model Zoo [2](For SqueezeNet, we use TensorRT to manually generate). We compare SqueezeNet and ResNet50 with the 2PC framework CryptoFlow2 [46]. We consider the same setting of CryptoFlow2 which performs SqueezeNet and ResNet50 in the ImageNet with the batch size 1 (The dimension of the input image is $224 \times 224 \times 3$). The performance is shown in TABLE. III. Our framework achieves more than $40\times$ performance improvement of the online phase compared to CryptoFlow2 in the LAN setting for both SqueezeNet and ResNet50; achieves over $60\times$ performance improvement in the WAN setting. As a trade-off, our protocol introduces a heavy offline phase. Nevertheless, our framework is $2\times$ faster than CryptoFlow2 even adding the offline cost. We compare the performance of the online phase with the Falcon [52] and Bicoptor [54] for AlexNet and VGG. They are 3PC based on the PPML GPU-platform Piranha [53]. It is worth mentioning that Bicoptor utilizes 8-bit ReLU to realize CNN. We follow their setting where the data set is CIFAR10 (32×32 input) and Tiny ImageNet ($64\times64\times3$ input). Our CPU-based framework realizes more than $5\times$ performance improvement compared to Bicoptor and more than $15\times$ improvement compared to Falcon in both LAN and WAN settings for both AlexNet and VGG16, even though they use GPU acceleration.

C. LLM model evaluation.

For the LLM model, we realize GPT-2-Base, GPT-Neo1.3B, GPT-Neo2.7B, Llama2-7B and Llama2-13B. The 8-bit quantized model for all of these can be found in Hugging Face [3]. We compare the online phase performance of our framework with Sigma [25] and CrypTen [34]. All of their implementation is based on GPU. Fig. 12 illustrates the performance comparison for GPT-Neo1.3B, GPT-Neo2.7B, Llama2-7B, and Llama2-13B models in the LAN setting. Compared to the CPU version of Sigma, our framework achieves more than $5\times$ performance improvement for all benchmarked LLM models. Even though CrypTen uses GPU acceleration, our framework is over $4\times$ faster than CrypTen for GPT-Neo2.7B. The performance of GPT-2 and GPT-Neo1.3B is depicted in Fig. 13. Our framework is slightly slower than the GPU version of Sigma for GPT-2 in the LAN setting. Affected by the benefit of optimization such as operator fusion, our framework outperforms the GPU version of Sigma for GPT-Neo1.3B in the LAN setting. Considering the WAN setting, our framework outperforms all of the other frameworks. Our framework is $10\times$ faster than the CPU version of Sigma for both GPT-2 and GPT-Neo1.3B in the LAN setting. This improvement will be further amplified over the WAN setting.

VII. Related Works

MPC-based PPML. The current works on secure multi-party computation in PPML mainly focus on two-party, three-party, and four-party settings. The representative works for two-party setting are SecureML [41], Delphi [39], Chameleon [48], CryptFlow2 [46], ABY2.0 [43], Cheetah [27], and Li et al. [37]. For three-party setting, there are SecureNN [51], Falcon [52], ABY^3 [40], ASTRA [12], BLAZE [44], CryptFlow [35]. SWIFT [42], FLASH [9], and Trident [13] are considering four-party setting. Recently, Orca [30] applied function secret sharing on the PPML where the three parties share the function secret share key in the offline phase. and the two parties perform secure computation in the online phase. Edabits [23] implements PPML in multiparty settings and support the security against malicious adversaries. For the GPU setting, CrypTen [34], Piranha [34] and Orca [30] construct the GPU platform for PPML. Recently, THE-X [14], MPCformer [36], Privformer [5] and Sigma [25], Grotto [50] focus on the large language models and also use GPU to accelerate performance. Note that Sigma utilizes the lookup table evaluation to perform non-linear function evaluation.

VIII. Conclusion

In this work, we propose a PPML framework for quantized models. We managed to push majority of the workload to the preprocessing phase, resulting in an extremely fast online performance. Our benchmarks demonstrated that our framework offers $40 \sim 60\times$ performance improvement over SOTA for CNN models and $10 \sim 25\times$ performance improvement over SOTA for LLMs.

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APPENDIX

A. Security of Private lookup Table Evaluation

Universal Composability (UC). Our protocols ensure security within the standard semi-honest setting. In this scenario, the adversary may attempt to extract private information from legitimate messages but must adhere strictly to the protocol's procedure. The security proof is based on the Universal Composability (UC) framework [10], which follows the simulation-based security paradigm. In the UC framework, protocols are executed across multiple interconnected machines. The network adversary Adv is allowed to partially control the communication tapes of all uncorrupted machines, observing messages sent to/from uncorrupted parties and influencing message sequences. Then, a protocol Π is considered UC-secure in realizing a functionality $\mathcal{F}$ if, for every probabilistic polynomial-time (PPT) adversary Adv targeting an execution of Π, there exists another PPT adversary known as a simulator Sim attacking the ideal execution of $\mathcal{F}$ such that the executions of Π with Adv and that of $\mathcal{F}$ with Sim are indistinguishable to any PPT environment $\mathcal{Z}$.

UC for private lookup evaluation. Next, we prove the security of our private lookup table protocol Π_{lookup}. We first provide the functionality for the private lookup table evaluation in Fig. 14.

Functionality $\mathcal{F}_{\mathsf{lookup}}$

$\mathcal{F}_{\mathsf{lookup}}$ interacts with $\mathcal{C}$, $\mathcal{S}$ and the simulator Sim. Let f denote the function encoded to the lookup table.

Input:

- Upon receiving $(\mathsf{Input}, \mathsf{sid}, (\mathcal{T}^{\ell_x,\ell_y}, x_1))$ from $\mathcal{S}$, record $(\mathcal{T}^{\ell_x,\ell_y}, x_1)$ and send $(\mathsf{Input}, \mathsf{sid}, \mathcal{S})$ to Sim, where $\mathcal{T}^{\ell_x,\ell_y} \in (\mathbb{Z}_{2^{\ell_y}})^{\ell_x}, x_1 \in \mathbb{Z}_{2^{\ell_x}}$.
- Upon receiving $(\mathsf{Input}, \mathsf{sid}, x_0)$ from $\mathcal{C}$, record x_0 and send $(\mathsf{Input}, \mathsf{sid}, \mathcal{C})$ to Sim, where $x_0 \in \mathbb{Z}_{2^{\ell_x}}$.

Execution:

- If both $\mathcal{T}^{\ell_x,\ell_y}, x_0, x_1$ are recorded, compute $y = \mathcal{T}^{\ell_x,\ell_y}(x_0 + x_1)$.
- Upon receiving $(\mathsf{Output}, \mathsf{sid}, y' \in \mathbb{Z}_{2^{\ell_y}})$ from Sim, if $\mathcal{S}$ is corrupted calculate $y_0 = y - y'$ and $y_1 = y'$, otherwise calculate $y_1 = y - y'$ and $y_0 = y'$.
- Send $(\mathsf{Output}, \mathsf{sid}, y_0)$ to $\mathcal{C}$ and $(\mathsf{Output}, \mathsf{sid}, y_1)$ to $\mathcal{S}$.

Fig. 14: The Ideal Functionality $\mathcal{F}_{\mathsf{lookup}}$ for private lookup table evaluation.

The idea world execution $\mathsf{Ideal}_{\mathcal{F}_{\mathsf{lookup}},\mathsf{Sim},\mathcal{Z}}(1^\lambda)$. In the ideal world, the parties $\mathcal{P} := \{\mathcal{C}, \mathcal{S}\}$ only communicate with the ideal functionality $\mathcal{F}_{\mathsf{lookup}}$ with the executed function f. Both parties send their private data to $\mathcal{F}_{\mathsf{lookup}}$, and $\mathcal{F}_{\mathsf{lookup}}$ calculates and output the result to $\mathcal{C}$ and $\mathcal{S}$.

The real world execution $\mathsf{Real}_{\Pi_{\mathsf{lookup}},\mathsf{Adv},\mathcal{Z}}(1^\lambda)$. In the real world, the parties $\mathcal{P} := \{\mathcal{C}, \mathcal{S}\}$ communicate with each other, it executes the protocol Π_{lookup}. Our protocols work in the pre-processing model, but we analyze the offline and online protocols together as a whole.

Theorem 1. *Protocol* Π_{lookup} *UC-secure realizes functionality* $\mathcal{F}_{\mathsf{lookup}}$ *in the* $\mathcal{F}_{(N-1,N)\text{-}\mathsf{OT}}$*-hybrid model against semi-honest PPT adversaries with statical corruption, namely it holds:*

$$\mathsf{Real}_{\Pi_{\mathsf{lookup}},\mathsf{Adv},\mathcal{Z}}(1^\lambda) \approx \mathsf{Ideal}_{\mathcal{F}_{\mathsf{lookup}},\mathsf{Sim},\mathcal{Z}}(1^\lambda)$$

Proof. Before proving Theorem 1, we replace $\Pi_{(2^{\ell_x}-1,2^{\ell_x})\text{-}\mathsf{ROT}}$ with functionality $\mathcal{F}_{(2^{\ell_x}-1,2^{\ell_x})\text{-}\mathsf{ROT}}$ in Chase *et. al* [11]. To prove Theorem 1, we construct a PPT simulator $\mathcal{S}$, such that no non-uniform PPT environment $\mathcal{Z}$ can distinguish between the ideal world $\mathsf{Ideal}_{\mathcal{F}_{\mathsf{lookup}},\mathcal{S},\mathcal{Z}}(1^\lambda)$ and the real world $\mathsf{Real}_{\Pi_{\mathsf{lookup}},\mathsf{Adv},\mathcal{Z}}(1^\lambda)$. We consider the following cases:

Case 1: $\mathcal{C}$ is corrupted. We construct the simulator Sim which internally runs Adv, forwarding messages to/from $\mathcal{Z}$ and simulates the interface of honest Sim.

- Upon receiving $(\mathsf{Input}, \mathsf{sid})$ from $\mathcal{F}_{\mathsf{lookup}}$, Sim starts simulation.
- Sim emulates $\mathcal{F}_{(N-1,N)\text{-}\mathsf{ROT}}$ and forward the output $\boldsymbol{m}_i \in (\mathbb{Z}_{2^{\ell_y}})^{2^{\ell_x}}$ for $i \in [2^{\ell_x}] \backslash \{[\![r]\!]_0\}$ and $[\![r]\!]_0$ to $\mathcal{C}$.

- Sim generates the matrix $\mathbf{M}$ by using the $\{\boldsymbol{m}_i\}_{i\in[2^{\ell_x}]}$ as the column vectors, and right circular shift the i^{th} row of $\mathbf{M}$ by i positions locally for $i \in [2^{\ell_x}]$.
- Sim computes $v_i = \sum_{j=0}^{2^{\ell_x}-1} m_{(i,j)}$ and $u_i = \sum_{j=0}^{2^{\ell_x}-1} -m_{(j,i)}$ to generate $\boldsymbol{u} := (u_0, \ldots, u_{2^{\ell_x}-1})$ and $\boldsymbol{v} := (v_0, \ldots, v_{2^{\ell_x}-1})$.
- Sim picks random vector $\boldsymbol{s}$ and acts as $\mathcal{S}$ to send $\boldsymbol{s}$ to $\mathcal{C}$.
- Sim picks random $[\![\delta]\!]_1^{\ell_x}$ and acts as $\mathcal{S}$ to send it to $\mathcal{C}$.
- Upon receiving $[\![\delta]\!]_0^{\ell_x}$ from $\mathcal{C}$, Sim extracts $[\![x]\!]_0^{\ell_x} = [\![\delta]\!]_0^{\ell_x} + [\![r]\!]_0^{\ell_x}$.
- Sim calculate $\delta := [\![\delta]\!]_0^{\ell_x} + [\![\delta]\!]_1^{\ell_x}$ and $[\![\mathcal{T}']\!]_0$ with the values $\boldsymbol{s}, [\![r]\!]_0^{\ell_x}, \boldsymbol{w}$.
- Sim inputs $(\mathsf{Input}, \mathsf{sid}, [\![x]\!]_0^{\ell_x})$ to $\mathcal{F}_{\mathsf{lookup}}$.
- Sim inputs $(\mathsf{Output}, \mathsf{sid}, [\![\mathcal{T}'(\delta)]\!]_0)$ to $\mathcal{F}_{\mathsf{lookup}}$.

Informally, we discuss the indistinguishable. Obviously, in the above simulation, considering $\mathcal{S}$ will input $[\![x]\!]_1^{\ell_x}$ to $\mathcal{F}_{\mathsf{lookup}}$ calculates $y = \mathcal{T}([\![x]\!]_0^{\ell_x} + [\![x]\!]_1^{\ell_x})$ and output $y - [\![\mathcal{T}'(\delta)]\!]_0$ to $\mathcal{S}$. When $\mathcal{C}$ sets output as $[\![\mathcal{T}'(\delta)]\!]_0$, we get the same output in the real world. To illustrate the indistinguishable of temporary value, we prove that the ideal world $[\![\delta]\!]_1^{\ell_x}$, $\boldsymbol{s}$ are generated randomly. For $\boldsymbol{s}$, it is easy to see that it is uniform random in the real world since the vector $\boldsymbol{u}$ can be viewed as a random vector. So the values $[\![\delta]\!]_1^{\ell_x}$, $\boldsymbol{s}$ both keep the same distribution between the real world and ideal world and can not be distinguished.

Case 2: $\mathcal{S}$ is corrupted. We construct the simulator Sim which internally runs Adv, forwarding messages to/from $\mathcal{Z}$ and simulates the interface of honest $\mathcal{C}$.

- Upon receiving $(\mathsf{Input}, \mathsf{sid})$ from $\mathcal{F}_{\mathsf{lookup}}$, Sim starts simulation.
- Sim emulates $\mathcal{F}_{(N-1,N)\text{-}\mathsf{ROT}}$ and forward the output $\boldsymbol{m}_i \in (\mathbb{Z}_{2^{\ell_y}})^{2^{\ell_x}}$ for $i \in [2^{\ell_x}]$ to $\mathcal{C}$.
- Sim calculate $\boldsymbol{v}, \boldsymbol{u}, \boldsymbol{w}$ using the output of $\mathcal{F}_{(N-1,N)\text{-}\mathsf{ROT}}$.
- Upon receiving $\boldsymbol{s}$ for $\mathcal{S}$, Sim extracts $\hat{\mathcal{T}} = \boldsymbol{s} - \boldsymbol{u}$.
- Sim picks random $[\![\delta]\!]_0^{\ell_x}$ and acts as $\mathcal{C}$ to send it to $\mathcal{S}$.
- Upon receiving $[\![\delta]\!]_1^{\ell_x}$ from $\mathcal{S}$, Sim calculates $\delta = [\![\delta]\!]_0^{\ell_x} + [\![\delta]\!]_1^{\ell_x}$.
- Sim inputs $(\mathsf{Input}, \mathsf{sid}, (\hat{\mathcal{T}}, [\![\delta]\!]_1^{\ell_x}))$ as like $\mathcal{S}$ to $\mathcal{F}_{\mathsf{lookup}}$.
- Sim inputs $(\mathsf{Output}, \mathsf{sid}, v_\delta)$ to $\mathcal{F}_{\mathsf{lookup}}$.

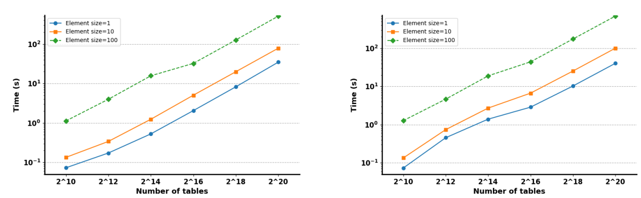
Informally, we discuss the indistinguishable. For the output, in above simulation, $\mathcal{F}_{\mathsf{lookup}}$ will calculate $y = \hat{\mathcal{T}}([\![\delta]\!]_1^{\ell_x} + [\![x]\!]_0^{\ell_x})$. Since $\hat{\mathcal{T}}(x) = \mathcal{T}(x + [\![r]\!]_1^{\ell_x})$ and $[\![\delta]\!]_1^{\ell_x} = [\![x]\!]_1^{\ell_x} - [\![r]\!]_1^{\ell_x}$, it equals to $\mathcal{T}([\![x]\!]_1^{\ell_x} - [\![r]\!]_1^{\ell_x} + [\![x]\!]_0^{\ell_x} + [\![r]\!]_1^{\ell_x})$, which is $y = \mathcal{T}(x)$. $\mathcal{F}_{\mathsf{lookup}}$ sends $y - v_\delta$ to $\mathcal{C}$, while the corrupted $\mathcal{S}$ hold v_δ due to δ received from Sim. Furthermore, $[\![\delta]\!]_0^{\ell_x}$ in the ideal world is randomly generated which is indistinguishable from the real world.

This concludes the proof. □

B. Other benchmarks

In this section, we give more benchmarks. The bandwidth is 5Gbps/40Mbps in the LAN and the WAN setting respectively and the latency is 0.05 ms / 60 ms respectively.

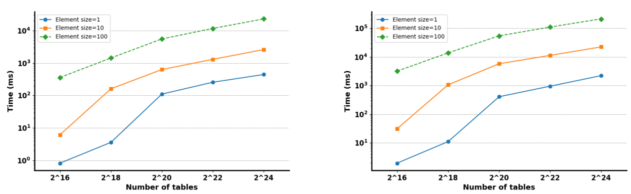
Offline performance for C/S setting. Figure 15 shows the offline performance for the C/S setting with different element sizes in LAN and WAN settings. The element size in the legend represents the number of elements in each line of the lookup table. While the element size is 1, the lookup table can be used for computing functions such as ReLU or Maxpool for individual elements. However, when the element size is 10, 100, or higher, these lookup tables are employed for matrix multiplication with the dimension of 10, 100, or higher. The same input is multiplied with multiple different elements in one row or column during matrix multiplication, which allow these tables to be efficiently combined into a single table line for offline processing. As the number of tables and the element size increase, the corresponding runtime also increases. Despite some fluctuations, the growth in time is largely linearly related to the increase in the number of tables. Additionally, it is evident that the time used for WAN does not exhibit a significant increase compared to LAN time. This means that during this runtime process, network communication does not significantly contribute to the overall time consumption. Given our device and code limitations, there is theoretically room for further improvement in runtime, especially considering that many computations can be parallelized and better memory management.



(a) LAN (b) WAN

Fig. 15: The running time of offline phase for C/S setting compared with different element sizes in LAN and WAN setting.

Offline performance for outsourcing setting. Figure 16 shows the offline performance for outsourcing settings with different element sizes in LAN and WAN settings. Due to the impact of factors such as network and memory, the initial part of the curve exhibits slight fluctuations. However, it still demonstrates a linear relationship. In the context of outsourcing, two computational nodes primarily handles the lookup table from the server, while the server providing the machine learning model performs relatively less complex computations compared with C/S setting. Notably, the impact of network communication restrictions is more pronounced in this scenario, as the WAN time significantly exceeds the LAN time.



(a) LAN (b) WAN

Fig. 16: The running time of offline phase for outsourcing setting compared with different element size in LAN and WAN setting.