

Secure Multiparty Computation with Free Branching

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Abstract

We study secure multi-party computation (MPC) protocols for branching circuits that contain multiple sub-circuits (i.e., branches) and the output of the circuit is that of a single “active” branch. Crucially, the identity of the active branch must remain hidden from the protocol participants.

While such circuits can be securely computed by evaluating each branch and then multiplexing the output, such an approach incurs a communication cost linear in the size of the entire circuit. To alleviate this, a series of recent works have investigated the problem of reducing the communication cost of branching executions inside MPC (without relying on fully homomorphic encryption). Most notably, the stacked garbling paradigm [Heath and Kolesnikov, CRYPTO’20] yields garbled circuits for branching circuits whose size only depends on the size of the largest branch. Presently, however, it is not known how to obtain similar communication improvements for secure computation involving *more than two parties*.

In this work, we provide a generic framework for branching multi-party computation that supports *any number of parties*. The communication complexity of our scheme is proportional to the size of the largest branch and the computation is linear in the size of the entire circuit. We provide an implementation and benchmarks to demonstrate practicality of our approach.

1 Introduction

Secure multiparty computation (MPC) [44, 21, 9, 5] is an interactive protocol that allows a group of mutually distrusting parties to jointly compute a function over their private inputs without revealing anything beyond the output of the function. Over the years, significant progress has been made towards improving the efficiency of MPC protocols [11, 43, 25, 22, 3, 24, 34, 26, 12, 23, 16] to make them practically viable.

While a wide variety of techniques for efficiency improvements have been developed in different settings based on the corruption threshold, communication model or security guarantee, a common aspect of most modern efficient protocols in all of these settings is that they rely on a *circuit representation* of the function. A limitation of such protocols, however, is that their total communication complexity is at least linear in the size of the circuit. Known techniques for getting sub-linear communication in the circuit size rely on computationally heavy tools such as fully-homomorphic encryption (FHE) [19] or homomorphic secret sharing (HSS) [7]. While there have been recent advancements in improving the efficiency of these methods, they are still far from being practical in many use cases.

As a result, the efficiency of existing efficient protocols is highly dependent on how succinctly a function can be represented using circuits. This is clearly not ideal, since circuits are often not the

most efficient way of representing many functions. A common example of such functions are ones that include some kind of *conditional* control flow instructions. When evaluating such functions, a circuit-based MPC will incur communication dependent on the size of the *entire* circuit, while in reality we only need to evaluate the “active” path (i.e., the path that is actually executed based on the conditional) in the circuit.

It is therefore useful to design efficient MPC protocols for useful classes of functions, where the total communication between the parties only depends on the “active” parts, rather than the entire circuit.

MPC for Conditional Branches. In this work, we focus on one such class of functions, namely, ones that contain conditional branches. As discussed in [30], a real world example of an application that consists of conditional branches is where a set of servers collectively provide k services and the clients can pay and avail any one of their services (depending on their requirements), without revealing to the servers which service they are availing. Similarly, control flow instructions are also integral to any kind of programming and as observed in [28], many kinds of control flow instructions (including repeated and/or nested loops) can be refactored into conditional branches. Such refactorings often result in a *large* number of conditional branches. For such functions, designing MPC protocols where the total communication only depends on the size of the active branch is very useful.

Recently, in a sequence of works [27, 28], Heath and Kolesnikov made progress in this direction in the two-party setting. They design garbled circuit based *two-party* semi-honest protocols for evaluating functions with conditional branches, where the total communication only depends on the size of the largest branch. In the *multiparty* setting, however, no such protocols are currently known. The recent works of [29, 31] design MPC for conditional branches where they reduce the number of public-key operations required to evaluate conditional branches; however, the total communication in their protocols still depends on the size of all branches. Furthermore, all these protocols only work for Boolean circuits.

Given this state of the art, we consider the following question in this work:

Does there exist an efficient multiparty protocol for securely computing conditional branches, where the total communication only depends on the size of the largest branch?

We remark that all of the above mentioned prior works only focus on the semi-honest setting. The task of designing analogous *maliciously-secure* protocols remains unexplored (both in the two-party and multi-party settings). In this work, we also consider this question.

1.1 Our Contributions

We design the first *multiparty* computation protocols for conditional branches, where the communication complexity only depends on the size of the largest branch. Our protocols can support arbitrary number of parties and corruptions. We present both constant and non-constant round variants.

I. Non-Constant Round Branching MPC. Our first contribution is a *semi-honest* MPC for conditional branches, where the communication complexity only depends on the size of the largest branch. This protocol is capable of computing arithmetic circuits over any field or ring. The round complexity of this protocol depends on the depth of the circuit.

We present this protocol as a generic compiler that can transform a large class of admissible¹ MPC protocols into ones for conditional branches that achieve the aforementioned communication complexity. Several existing concretely efficient protocols including MASCOT [34], SPDZ2k [12], Overdrive [35], TinyOT [18] and [26], [13] can be used with this compiler.

In particular, by instantiating our compiler with a semi-honest admissible (dishonest-majority) MPC protocol with communication complexity $\mathcal{CC}(|C|)$ (where C is the circuit being evaluated), we obtain the following result:

Informal Theorem 1. Let λ be the security parameter. There exists a semi-honest secure MPC for evaluating conditional branches, that can tolerate arbitrary corruptions and that achieves communication complexity of $O(\mathcal{CC}(|C_{\max}|) + n^2k\lambda + n^2|C_{\max}|)$, where k is the number of branches in the conditional.

We also implement this protocol to test its concrete efficiency and compare it to state-of-the-art MPC protocols. More details are provided later in this section.

Extension to Malicious Security. We also present an extension of this protocol to the case of malicious adversaries. Asymptotically, its communication complexity is similar to the semi-honest protocol, except that it incurs a multiplicative overhead dependent on a statistical security parameter.

We view this construction as initial evidence that efficient branching MPC with malicious security is possible. However, we believe that there is significant scope for future improvements towards achieving good concrete efficiency.

II. Constant Round Branching MPC. Our next contribution is a *constant round* MPC for conditional branches, where the communication complexity only depends on the size of the largest branch. This protocol is based on a *multiparty garbling* approach [2] and only supports boolean circuits.

We also present this protocol in the form of a general compiler. Namely, given a MPC protocol with communication complexity $\mathcal{CC}(|C|)$ for evaluating a circuit C , we get the following result:

Informal Theorem 2. Let λ be the security parameter. There exists a constant-round, semi-honest secure MPC for evaluating conditional branches (represented as Boolean circuits), that can tolerate arbitrary corruptions and that achieves communication complexity of $O(|\mathcal{CC}(\lambda|C_{\max}|) + n^2k\lambda + n^2\lambda|C_{\max}|)$, where k is the number of branches in the conditional.

To obtain both of the above results, we adopt a fundamentally different approach as compared to prior works [27, 29, 28, 31] in this area. Specifically, prior works require the parties to locally evaluate *all* the branches. In contrast, in our approach, the parties select the “active” branch and *only execute that branch*. A detailed overview of our approach can be found in the next section.

III. Comparison and Performance Evaluation. To gauge practicality, we implement our non-constant round *semi-honest* compiler and instantiate it using two kinds of protocols:

- *Quadratic Dependence on the Number of Parties:* MP-SPDZ is a common MPC library that contains implementations of the SPDZ protocol [16] and its descendants. All of the protocols in this library have total communication with quadratic dependence on the number of parties.

We instantiate our compiler with an implementation of MASCOT [34] from this library without

¹We require the underlying MPC to be such that it evaluates the circuit in a gate-by-gate manner and maintains an invariant that for every intermediate wire in the circuit, the parties collectively hold a sharing of the value induced on that wire during evaluation.

modification. Our code is agnostic to which protocol the MPC library is configured; this helps demonstrate that our techniques are generic and block-box. We run benchmarks over simulated LAN and WAN settings. We show that our compiled protocol outperforms naïvely evaluating all the branches in parallel using MASCOT for as few as 8 branches.

- *Linear Dependence on the Number of Parties:* We implement an optimized variant of our compiler that incurs a linear additive overhead in the number of parties, instead of a quadratic overhead. We then test the efficiency of our compiler when instantiated with the CDN protocol [13], which only has a linear dependence on the number of parties. For this, we first implement the CDN protocol. To the best of our knowledge, this is the first known implementation of CDN. Similar to the previous case, we show that our compiled protocol (instantiated using CDN) outperforms naïvely evaluating all the branches in parallel using CDN for 8 branches.

2 Technical Overview

Background. All recent works [27, 29, 28, 31] in this area are based on the same principal approach – *the parties evaluate all branches, albeit, only the “active” branch is evaluated on real inputs, while the remaining branches are all evaluated on fake/garbage values.*

For instance, in the two-party setting, [27, 28], which adopt a garbled circuit based approach, one of the parties garbles all the k branches. It then “stacks” these garblings into a compressed form that is proportional to the length of the largest branch in the circuit. Using some additional information sent by the garbler, the evaluator is able to reconstruct k different garbled circuits, only one of which is a valid garbling of the “active” branch, and the remaining are random strings (or some garbage material). Unaware of the active branch, the evaluator evaluates the k garbled circuits w.r.t. different branches to obtain k different output labels. These output labels are then filtered with the help of a “multiplexer” to obtain the correct output. Overall, this approach reduces the communication to only depend on the size of the largest branch (the computation complexity, however, is still large).

In the multiparty setting, both [29, 31], follow the same principal approach. These protocols have separate preprocessing and online phases. They require parties to evaluate all branches (including the inactive ones) in the online phase over 0 or some random values and leverage this fact to get savings in the preprocessing phase. As a result, communication in the preprocessing phase only depends on the size of one branch, but the communication in the online phase still depends on the size of all the branches.

Indeed, it is unclear how to extend the stacked garbling approach used in [27, 28] to get similar savings in communication in the multiparty setting. Recall that the garbler in stacked garbling is required to garble all branches and hence its computation *depends on the size of all branches.* This means that naive approaches that involve distributing the role of the garbler amongst multiple parties are a non-starter as they will incur *communication* proportional to the size of all branches. In order to design a multiparty protocol with similar communication savings as in stacked garbling, we therefore adopt a fundamentally different approach.

Our Approach. In our approach, *the parties select which branch to execute in a “privacy-preserving” manner and only execute that branch.* To facilitate this private selection, both of our constructions (in the non-constant round and constant-round settings) employ a common tool – a variant of oblivious linear evaluation that we refer to as *oblivious inner product* (OIP). In particular, our protocols make use of OIPs with (small) *constant rate.* We show that such OIPs can

be easily constructed using low-rate linearly homomorphic encryption schemes, which are known from a variety of assumptions [17, 8, 15, 42].

In the sequel, we first describe the main ideas underlying our non-constant round constructions. We then proceed to describe our constant-round construction.

2.1 Non-Constant Round Branching MPC

We start with the observation that the problem of computing conditional branches bears some similarities to the problem of private function evaluation (PFE) [32, 39, 40]. Recall that in PFE, one party has the function and the remaining parties provide inputs. This, in some sense is reminiscent of the problem that we have at hand, albeit with some differences. In particular, in our case, while none of the parties actually knows which function/branch is “active”, they all know the set that this branch belongs to. Moreover, the parties collectively hold information about which of these functions to evaluate. This can be viewed as a *distributed variant of PFE*. In light of this observation, we build upon some ideas previously used in the PFE literature.

Private Function Evaluation. In PFE, the function is only known to one of the parties (say party P_1). The security requirements in standard PFE are very similar to that in MPC, with the only additional requirement that the function must remain hidden from all other parties. To achieve this, Mohassel and Sadeghian [39] observe that in order to hide a function that is represented in the form of a circuit, there are two components that need to remain hidden – (1) The wire-configuration of the circuit, i.e., how the gates connect with each other, and (2) the function (i.e., addition or multiplication) implemented by each gate in the circuit. They propose a strategy to conceal the above components of a circuit in order to achieve function privacy (without relying on universal circuits). In particular, they start with MPC protocols that work over some kind of secret shares (additive/threshold/authenticated) and evaluate any given circuit in a gate-by-gate manner. These protocols maintain the invariant that for every intermediate wire in the circuit, all parties hold a sharing of the value induced on that wire during evaluation. Many concretely efficient protocols such as [34, 26, 12, 23, 16], satisfy these requirements. [39] propose the following modifications to such MPC protocols to obtain a PFE protocol:

1. *Hiding Wire Configuration:* Each intermediate wire in the circuit has two end points – (1) one is the *source gate*, for which it acts as the outgoing wire and (2) the other is the *destination gate*, for which it acts as the incoming wire. As discussed earlier, for hiding the wire configuration, we need to hide the gate connections, i.e., we want to hide the mapping between the source and destination of each wire in the circuit. For this, [39] assign two unique labels to each wire w . One is an outgoing label based on its source gate and second is an incoming label based on whether it acts as left or right input wire to its destination gate. Let π denote the mapping between these incoming and outgoing labels, i.e., let $\pi(i) = j$ denote that a wire that has incoming label i has an outgoing label j . In PFE, this mapping π is only known to the function holding party.

In order to hide this mapping, [39] devise a mechanism to mask the outputs value of each gate and unmask them based on π when this value is used for evaluating the destination gate of this wire. This is executed by sampling an input mask and an output mask for every wire in the circuit. Let $\text{in}_1, \dots, \text{in}_W$ and $\text{out}_1, \dots, \text{out}_W$ be the set of these input and output masks, where W is the total number of wires in the circuit. In the preprocessing phase, with the help of the function holding party and the underlying MPC, the parties compute $\Delta_w = \text{in}_w - \text{out}_{\pi(w)}$ for every $w \in [W]$. These Δ_w values are revealed to function holding party in the clear. This processing information helps the parties in using appropriately permuted input and output

masks to mask and unmask wire values during evaluation in the online phase. In more detail, the online phase proceeds as follows:

- Upon evaluating each gate g , the parties use output masks to mask all the outgoing wires of the gate. Let the outgoing wires have labels c and d respectively, and let u_c and u_d denote these masked outputs. These masked outputs are revealed to all parties in the clear.
 - For evaluating a particular gate g , where the two input wires have incoming wire labels a and b , the function holding party computes $A = u_{\pi(a)} + \Delta_a$ and $B = u_{\pi(b)} + \Delta_b$ and sends it to all the parties. The parties subtract their shares of in_a and in_b from these values to get a sharing of the actual values on which to evaluate gate g .
2. *Hiding Gate Functions:* This is relatively easier. Assume that our arithmetic circuit representation of the function only consists of addition and multiplication gates, let $\mathbf{type}_g = 0$ (and $\mathbf{type}_g = 1$ resp.) denote that gate g is an addition gate (and multiplication gate resp.). For each gate g with incoming wires a and b , we can use the underlying MPC to compute both shares of $a + b$ and $a \cdot b$. The function holding party P_1 can secret share \mathbf{type}_g using the underlying MPC and the parties can then choose between shares of $a + b$ and $a \cdot b$ by computing the following using the underlying MPC:

$$(1 - [\mathbf{type}_g])([a + b]) + [\mathbf{type}_g]([a \cdot b]),$$

where we denote $[x]$ as a sharing of a value x using the secret sharing scheme used by the underlying MPC. This allows the parties to evaluate the correct function, without revealing it.

Our Semi-Honest Protocol. In our setting, the parties know the description of all the branches in the conditional and have a secret sharing of the index of the active branch. In order to hide the identity of the active branch, similar to the above approach, we need to hide both the wire configuration and the gate functions of the active branch. We start by listing the barriers in directly adapting the above approach to our setting and then proceed to discuss how we resolve them.

- In the preprocessing phase, computing Δ requires the function holding party to input π to the underlying MPC. In our setting, no party knows the exact value of π .
- In the online phase, A and B values are computed locally by the function holding party in PFE since it already knows the mapping π . This is again a problem in our setting.
- Finally, in order to hide the gate functions in the online phase, the value of each \mathbf{type}_g secret shared by the function holding party. But as above, neither party in our setting knows this value.

In order to overcome the above barriers, we crucially rely on the fact that in our setting, while no single party knows the function (or the mapping π), they all know the set that the function belongs to. In other words, given a set of k branches C_1, \dots, C_k , all the parties can locally compute the mappings π_1, \dots, π_k corresponding to each branch. Moreover, the parties also have a secret sharing of the index of the active branch. Let α be the index of the active branch. Our first idea towards resolving the above barriers is to somehow allow the parties combine their shares of α with π_1, \dots, π_k to get a sharing of π_α . However, since the size of π_1, \dots, π_k depends on the size of all branches, a naive implementation of this computation will incur communication that depends on the size of π_1, \dots, π_k .

We get around this by using a new variant of oblivious linear evaluation, which we refer to as oblivious inner product. We now outline our main ideas:

- **Sharing of α :** We work with a unary representation of the index α . In other words, we assume each party have k secret shares, where the α^{th} share is a sharing of 1, while all others are sharings of 0s. Let these shares be denoted by $[b_1], \dots, [b_k]$
- **Input/Output Masks:** In the preprocessing phase, we use the underlying MPC to sample random input and output masks $\text{in}_1, \dots, \text{in}_W$ and $\text{out}_1, \dots, \text{out}_W$, where W is the number of wires in the largest branch. Each party, now locally permutes its shares of input masks based on the k mappings π_1, \dots, π_k . In more detail, given sharings $[\text{out}_1], \dots, [\text{out}_W]$, for each $m \in [k]$, the parties locally compute sharings $[\text{out}_{\pi_m(1)}], \dots, [\text{out}_{\pi_m(W)}]$. Lets denote each $[\text{out}_{\pi_m(1)}], \dots, [\text{out}_{\pi_m(W)}]$ by $\overrightarrow{[\text{out}_{\pi_m}]}$. If instead of computing shares of π_α , we directly compute re-randomized shares of $\overrightarrow{[\text{out}_{\pi_\alpha}]}$, then the parties can simply compute their shares of Δ_w values as follows

$$\forall w \in [W], [\Delta_w] = [\text{in}_w] - [\text{out}_{\pi_\alpha(w)}]$$

- **Oblivious Inner Product:** For computing re-randomized shares $\overrightarrow{[\text{out}_{\pi_\alpha}]}$, we use a primitive called oblivious inner product (OIP). This is a protocol between two-parties, called the sender and receiver and bears resemblance to oblivious linear evaluation. The sender has inputs m_0, \dots, m_k and the receiver has inputs b_1, \dots, b_k . At the end of the protocol, the receiver learns $m_0 + \sum_{i \in [k]} b_i m_i$ and the sender learns nothing.

We use this primitive and a GMW [21] style approach to obtain shares of $\overrightarrow{[\text{out}_{\pi_\alpha}]}$ as follows: for each pair of parties in the protocol, we run an instance of OIP, where one party acts as the sender and the other acts as the receiver. The inputs of the sender party to this OIP are its shares of $\overrightarrow{[\text{out}_{\pi_1}]}, \dots, \overrightarrow{[\text{out}_{\pi_W}]}$ and a random value X , while the inputs of the receiver are its shares of the unary representation of α . At the end, each party P_i computes its share of $\overrightarrow{[\text{out}_{\pi_\alpha}]}$ by adding the outputs of each OIP instance where it acted as the receiver and subtracting each X sampled in the OIP instance where it acted as the sender. It is easy to see that these resulting shares are indeed shares of $\overrightarrow{[\text{out}_{\pi_\alpha}]}$.

However, note that while the length of the output of each OIP in our case only depends on the size of the largest branch, the length of sender inputs depends on the size of all branches. Therefore, in order to design an MPC protocol where the overall communication is only proportional to the size of the largest branch, we must use an OIP where the communication only depends on the length of receiver inputs and the output, but is independent of the length of sender inputs. We show that such OIPs can be constructed using linearly homomorphic encryption with constant rate.

- **Online Phase:** Now that we have sharing of Δ_w values that was computed using the mapping π corresponding to the active branch, we can compute shares of the A and B values as follows:

$$[A] = \sum_{m \in [k]} [b_1] u_{\pi_1(a)} + [\Delta_a] \quad \text{and} \quad [B] = \sum_{m \in [k]} [b_1] u_{\pi_1(b)} + [\Delta_b]$$

We note that most linearly homomorphic secret sharing schemes allow such computations to be done non-interactively and hence it does not incur any overhead in the communication complexity. Shares of type_g for every gate g can also be computed in a similar manner.

Extension to Malicious Security. While the basic outline of our protocol remains the same even in the malicious setting, we need to do a little more work to make the above protocol secure against a malicious adversary. In particular, we need to ensure that the inputs used by the parties in the OIP instances are consistent with values/shares computed by them using the underlying MPC. For this we propose to add the following consistency checks:

Receiver’s Input Consistency. We start by using an OIP that is secure against a malicious receiver. In order to ensure that receiver uses valid sharings of the active branch, we implement a kind of MAC check using the underlying MPC. In particular, in the OIP execution, the sender samples $k + 1$ random values and appends them to its inputs. Now when the receiver computes the output of the OIP, it also learns an inner product of these random values with its shares of the active branch (we refer to this as the MAC value for this OIP). We now use the underlying MPC to compute the exact same value. In particular, the sender sends the $k + 1$ random values that it sampled in the OIP as input to the underlying MPC, while the receiver sends the MAC value learnt from the output of the OIP. We allow the underlying MPC to now check if the MAC value indeed corresponds to an inner product of the receiver’s shares of the active branch and the random values input by the sender. We note that since the length of the receiver’s input is independent of the size of all branches, computing this MAC value inside the MPC does not incur too much overhead.

Sender’s Input Consistency. Recall that the inputs of the sender to the OIP depend on the size of all branches, and hence we cannot hope to use the kind of check that we used for ensuring receiver consistency. Moreover, since the length of the sender message is much shorter than the length of its inputs, we also cannot hope to use an OIP with malicious sender security that can somehow extract the sender’s inputs. Therefore, instead we continue to work with an OIP that is secure against a semi-honest sender but augment it with a cut-and-choose style approach. In particular, we sample multiple copies of the masks and compute delta values using OIPs for each of those copies. We also ask the sender to commit (using compressive commitments) to the inputs and randomness used for computing each of its sender messages. At the end of all OIP instantiations, we use the underlying MPC to sample a random subset and reveal the shares of masks of all parties for that subset. The senders also send the randomness used by them in the sender messages of this opened subset. Given this information, the parties can verify if the senders behaved honestly and used consistent shares in the opened instances. We use the remaining unopened instances to run multiple copies of the online phase and take a majority to decide the final output. Due to the use of cut-and-choose, the communication complexity of our maliciously secure protocol is proportional to $\delta \times$ the cost of computing the largest branch. Nevertheless, as discussed in the introduction, this is still useful for conditionals with large number of branches.

2.2 Constant Round (Semi-Honest) Protocol

Beaver, Micali, and Rogaway (BMR) [2] proposed a general template for constructing *constant round* MPC from existing generic *non-constant round* MPC. The main observations underlying their technique were – (1) round complexity of more generic non-constant round protocols depends on the depth of the function being computed and (2) garbling [44] a functionality/circuit is a constant depth procedure.

The parties can leverage these observations to first execute a *garbling phase*, where they compute a garbled circuit of the function (that they wished to evaluate) using the non-constant round protocol. This phase will require a constant number of rounds. Given this garbled circuit, they then proceed to the *evaluation phase*, where each party locally evaluates the garbled circuit to learn the output. This phase requires no interaction and hence the overall protocol runs in a constant

number of rounds.

More concretely, in the garbling phase, the parties collectively sample two keys $k_{w,0}, k_{w,1}$ for every wire w in the circuit. The garbled table for each gate g in the circuit with incoming wires a, b and outgoing wire c , consists of the following four rows, corresponding to $\alpha, \beta \in \{0, 1\}$:

$$ct_{\alpha,\beta} = \text{PRF}_{k_{a,\alpha}}(g) + \text{PRF}_{k_{b,\beta}}(g) + k_{c,g(\alpha,\beta)}$$

Branching MPC using BMR Template. The generality of the BMR approach immediately makes it compatible with our non-constant round semi-honest protocol (from Section 2.1). Indeed, in the garbling phase, parties can use that protocol to compute a garbled circuit for the active branch. During the evaluation phase, however, since the parties do not know which branch the garbled circuit corresponds to, they can evaluate it for every branch and obtain the corresponding output wire labels. Note that only the labels obtained by evaluating w.r.t. to the active branch actually correspond to a *valid set of abels*. Finally, via interaction, parties can determine the output corresponding to the “valid” set of output labels. The complexity of this last step is independent of the circuit size and only depends on the number of branches times the output length.

While this yields a simple baseline constant round MPC for conditional branches, it is highly inefficient. Since no party knows the keys $k_{a,\alpha}, k_{b,\beta}$ in their entirety, they must evaluate the PRF (on these keys) inside an MPC protocol. Since, the circuit representations of PRF’s are typically massive, this protocol is unlikely to be concretely efficient. As such, for concrete efficiency, we require a protocol that only makes a *black-box* use of cryptography.

Towards Black-Box use of Cryptography. Damgård and Ishai [14] proposed a variant of the above BMR template that enables parties to evaluate the PRF outside the MPC, thereby only making a black-box use of cryptography.

Specifically, in their approach, each party P_i samples two keys $k_{w,0}^i, k_{w,1}^i$ for every wire w in the circuit. In other words, the cumulative keys associated with every wire is a concatenation of all the parties’ keys. The garbled table for each gate g in the circuit with incoming wires a, b and outgoing wire c , consists of the following $4 \cdot n$ rows, corresponding to $\alpha, \beta \in \{0, 1\}$ and $i \in [n]$:

$$ct_{\alpha,\beta}^i = \bigoplus_{m=1}^n \text{PRF}_{k_{a,\alpha}^m}(g||i) + \bigoplus_{m=1}^n \text{PRF}_{k_{b,\beta}^m}(g||i) + k_{c,g(\alpha,\beta)}^i$$

It is easy to see that unlike the BMR approach, here the parties are only required to evaluate the PRF on their own keys, which can be done locally and the resulting PRF evaluation can be fed as input to the underlying MPC implementing the garbling functionality.

In our setting, however, this approach posits a fundamental barrier. Recall that for evaluating conditional branches, we want to garble the active branch without revealing the index of the active branch. For this, while garbling any gate (say the j^{th} gate), it is imperative that the parties remain oblivious to both the functionality associated with it and its incoming and outgoing wires. As a result, the parties are unaware of which keys $k_{a,\alpha}^i, k_{b,\beta}^i$ to use for computing the corresponding ciphertexts, and hence cannot evaluate the PRF on those keys *locally*. A natural approach to overcome this problem is to perform this evaluation within an MPC; however, we are then back to the realm of non-black-box use of cryptography. As such it is unclear how to directly adapt this approach to our setting, while making a black-box use of cryptography.

Garbling using Key-Homomorphic PRFs. To overcome the above barrier, we explore the work of Ben-Efraim et al. [4] who presented an alternative template for multiparty garbling, using

key-homomorphic PRFs. These are PRFs with the following property: $\text{PRF}_{k_1}(m) \tilde{+} \text{PRF}_{k_2}(m) = \text{PRF}_{k_1 \tilde{+} k_2}(m)$, where $\tilde{+}$ and $\tilde{\cdot}$ are some operations. As before, each party samples two keys for every wire in the circuit and given such a PRF, the parties compute each ciphertext as follows:

$$ct_{\alpha,\beta} = \sum_{m \in [n]} \left(\text{PRF}_{k_{a,\alpha}}^m(g) \tilde{+} \text{PRF}_{k_{b,\beta}}^m(g) \right) \tilde{+} \left(\prod_{m \in [n]} k_{c,g(\alpha,\beta)}^m \right)$$

It is easy to see that similar to the previous approach, each party here is only required to evaluate the PRF on its own key, which can be done locally. At first, it might seem that in our setting, the same problem (as before) still persists. Indeed, for local PRF evaluation, the parties are required to know which key to use, which as discussed earlier is not possible when the parties are required to obviously garble one of the conditional branches. However, we observe that homomorphism of the PRF can be leveraged here to resolve this problem.

Let's assume that the parties start by ordering the gates and wires in every branch in some canonical order. Now, when garbling the j^{th} gate of the active branch, they must choose the appropriate keys from all the keys associated with the j^{th} gate in every branch. We also assume that the parties have a sharing of the unary representation of the index associated with the active branch. The parties can now use multiple instances of OIP (as in our non-constant round protocols) to obtain shares of the keys associated with the two incoming wires of the j^{th} gate in the active branch.

Consider a key homomorphic PRF where both $\tilde{+}$ and $\tilde{\cdot}$ are the same operation associated with the reconstruction algorithm of the secret sharing scheme used in the underlying MPC, i.e., $[\text{PRF}_k(m)] = \text{PRF}_{[k]}(m)$. This PRF can now be used along with the above observation to compute a garbling of the active branch as follows: for simplicity let's assume that each branch is of the same size and has W wires. The parties start by collectively sampling $2W$ keys. For garbling the j^{th} gate, for each $\alpha, \beta \in \{0, 1\}$, they use OIPs to compute shares $[k_{a,\alpha}]$, $[k_{b,\beta}]$ and $[k_{c,g(\alpha,\beta)}]$, where a, b are the incoming and c is the outgoing wire of the j^{th} gate in the active branch and g is the function computed by this gate. Parties can now locally evaluate the PRF on these shares and use the underlying MPC to compute shares of the ciphertexts as follows:

$$[ct_{\alpha,\beta}] = \text{PRF}_{[k_{a,\alpha}]}(j) + \text{PRF}_{[k_{b,\beta}]}(j) + [k_{c,g(\alpha,\beta)}]$$

Upon computing this garbled circuit for the active branch, similar to the baseline solution, parties evaluate it w.r.t. all the branches and then run a “mini-MPC” to filter out the valid labels and determine the final output.

Instantiating Key Homomorphic PRF. Most existing dishonest majority MPC protocols [34, 26, 12, 23, 16] use additive secret sharing. To use the above ideas with such protocols, we need an additively key-homomorphic PRF, i.e., where $\text{PRF}_{k_1}(m) + \text{PRF}_{k_2}(m) = \text{PRF}_{k_1+k_2}(m)$. Unfortunately, key homomorphic PRFs are currently only known from the DDH assumption [41, 6] and those PRFs do not achieve a similar additive homomorphism.

Ben-Efraim et al. [4] observed that instead of a PRF, it suffices to use a (decisional) ring LWE based random function here. This function is of the form: $F = f_k : \mathcal{R}_p \rightarrow \mathcal{R}_p | f_k(a) = a \cdot k + e$, where $p = 2N + 1$ is a prime, N is a power of two, $\mathcal{R}_p = \mathbb{Z}_p[X]/(X^N + 1)$ and a, k , and e are polynomials in the ring and the coefficients of e come from a gaussian distribution. Since a is public, it is easy to see that given additive shares of the key k and error e , it is possible for the parties to locally compute shares of the above function. As is standard when using LWE/RLWE, encrypting using such a random function typically requires multiplying the message (before adding it to the output of this function) with the size of the range from which the message comes from. In the case of garbling, since both the message and keys come from the same distribution, as shown

in [4], this requires choosing the parameters carefully and additionally requires sampling the keys from a gaussian distribution. However, since the parties only need to compute additive shares of these keys and errors, this can be done easily by requiring the parties to sample their shares from appropriate distributions. We defer more details to the technical sections.

3 Preliminaries

3.1 Secure Multiparty Computation

A secure multi-party computation protocol (MPC) is a protocol executed by n parties $\mathcal{P} = \{P_1, \dots, P_n\}$ for a functionality \mathcal{F} . We allow for parties to exchange messages simultaneously. In every round, every party is allowed to exchange messages with other parties using different communication channels, depending on the model. A protocol is said to have k rounds if it proceeds in k distinct and interactive rounds.

3.1.1 Adversarial Behavior

One of the primary goals in MPC is to protect the honest parties against dishonest behavior of the corrupted parties. This is usually modeled using a central adversarial entity, that controls the set of corrupted parties and instructs them on how to operate. That is, the adversary obtains the views of the corrupted parties, consisting of their inputs, random tapes and incoming messages, and provides them with the messages that they are to send in the execution of the protocol.

In this work we consider two types of adversaries. A *semi-honest adversary* is "honest but curious" where it always follows the instructions of the protocol but might try to learn extra information by analyzing the transcript of the protocol later. On the other hand, a *malicious adversary* can deviate from the protocol and instruct the corrupted parties to follow any arbitrary strategy.

We provide the basic definitions for secure multiparty computation according to the real/ideal paradigm [20]. Informally, a protocol is considered secure if whatever an adversary can do in the real execution of protocol, can be done also in an ideal computation, in which an uncorrupted trusted party assists the computation.

3.1.2 Security Definitions

Real World. The real world execution of a protocol $\Pi = (P_1, \dots, P_n)$ begins by an adversary \mathcal{A} selecting any arbitrary subset of parties \mathcal{I} to corrupt. The parties then engage in an execution of a real n -party protocol π . Throughout the execution of π , the adversary \mathcal{A} sends all messages on behalf of the corrupted parties, and may follow an arbitrary polynomial-time strategy. In contrast, the honest parties follow the instructions of π . At the conclusion of the protocol, each honest party outputs all the outputs it obtained in the computations. Malicious parties may output an arbitrary PPT function of the view of \mathcal{A} . This joint execution of Π under $(\mathcal{A}, \mathcal{I})$ in the real model, on input vector $\vec{x} = (x_1, \dots, x_n)$, auxiliary input z and security parameter λ , denoted by $\text{REAL}_{\Pi, \mathcal{I}, \mathcal{A}(z)}(1^\lambda, \vec{x})$, is defined as the output vector of P_1, \dots, P_n and $\mathcal{A}(z)$ resulting from this protocol interaction.

Ideal World. We now present standard definitions of ideal-model computations.

An ideal computation of an n -party functionality \mathcal{F} on input $\vec{x} = (x_1, \dots, x_n)$ for parties (P_1, \dots, P_n) in the presence of an ideal-model adversary \mathcal{A} controlling the parties indexed by $\mathcal{I} \subset [n]$, proceeds via the following steps.

Sending inputs to trusted party: For each $i \notin \mathcal{I}$, P_i sends its input x_i to the trusted party. If $i \in \mathcal{I}$, the adversary may send to the trusted party any arbitrary input for the corrupted party P_i . Let x'_i be the value actually sent as the i^{th} party's input.

Early abort: The adversary \mathcal{A} can abort the computation by sending an abort message to the trusted party. In case of such an abort, the trusted party sends \perp to all parties and halts.

Trusted party answers adversary: The trusted party computes $(y_1, \dots, y_n) = \mathcal{F}(x'_1, \dots, x'_n)$ and sends y_i to party P_i for every $i \in \mathcal{I}$.

Late abort: The adversary \mathcal{A} can abort the computation (after seeing the outputs of corrupted parties) by sending an abort message to the trusted party. In case of such abort, the trusted party sends \perp to all honest parties and halts. Otherwise, the adversary sends a continue message to the trusted party.

Trusted party answers remaining parties: The trusted party sends y_i to P_i for every $i \notin \mathcal{I}$.

Outputs: Honest parties always output the message received from the trusted party and the corrupted parties output nothing. The adversary \mathcal{A} outputs an arbitrary function of the initial inputs x_i s.t. $i \in \mathcal{I}$, the messages received by the corrupted parties from the trusted party and its auxiliary input.

Security Having defined the real and ideal models, we can now define security of protocols according to the real/ideal paradigm.

Definition 3.1. Let $\mathcal{F} : (\{0, 1\}^*)^n \rightarrow (\{0, 1\}^*)^n$ be an n -party functionality and let Π be a probabilistic polynomial-time protocol computing \mathcal{F} . The protocol Π t -securely computes \mathcal{F} , if for every probabilistic polynomial-time real-model adversary \mathcal{A} , there exists a probabilistic polynomial-time simulator \mathcal{S} for the ideal model, such that for every $\mathcal{I} \subset [n]$ of size at most t , it holds that

$$\left\{ \text{REAL}_{\Pi, \mathcal{I}, \mathcal{A}(z)}(1^\lambda, \vec{x}) \right\}_{(\vec{x}, z) \in (\{0, 1\}^*)^{n+1}, \lambda \in \mathbb{N}} \approx_c \left\{ \text{IDEAL}_{\mathcal{F}, \mathcal{I}, \mathcal{S}(z)}(1^\lambda, \vec{x}) \right\}_{(\vec{x}, z) \in (\{0, 1\}^*)^{n+1}, \lambda \in \mathbb{N}}$$

4 Oblivious Inner Product

In this section, we define a variant of oblivious linear evaluation (OLE), which we refer to as *oblivious inner product* (OIP). OIP is a protocol between two parties, called the *sender* and *receiver* respectively. The sender has inputs $(\vec{m}_0, \dots, \vec{m}_k)$ in some domain (say \mathcal{D}^m), and receiver has inputs (b_1, \dots, b_k) in the same domain \mathcal{D} . At the end of the protocol, the receiver should learn $\vec{m}_0 + \sum_{i \in [k]} b_i \vec{m}_i$ and nothing more, while the sender should learn nothing about the receiver inputs b_1, \dots, b_k .

For our constructions, we consider two variants of OIP, a *semi-honest* version and one that is secure against a *malicious receiver*. We now define the syntax and the security guarantees of a two-message OIP protocol in the plain model. The definitions can be naturally extended to the CRS model.

Definition 4.1 (Two-Message Oblivious Inner Product). A two-message oblivious inner product between a receiver R and a sender S is defined by a tuple of 3 PPT algorithms $(\text{OIP}_R, \text{OIP}_S, \text{OIP}_{\text{out}})$. Let λ be the security parameter. The receiver computes msg_R, ρ as the evaluation of $\text{OIP}_R(1^\lambda, (b_1, \dots, b_k))$, where $(b_1, \dots, b_k) \in \mathcal{D}^k$ is the receiver's input. The receiver sends msg_R

to the sender. The sender then computes msg_S as the evaluation of $\text{OIP}_S(1^\lambda, \text{msg}_R, (\vec{m}_0, \dots, \vec{m}_k))$, where $(\vec{m}_0, \dots, \vec{m}_k) \in \mathcal{D}^{m \times (k+1)}$ are sender's inputs. The sender sends msg_S to the receiver. Finally, the receiver computes the output by evaluating $\text{OIP}_{\text{out}}(\rho, \text{msg}_R, \text{msg}_S)$.

A semi-honest OIP satisfies *correctness*, *security against semi-honest receiver and semi-honest sender*, while the malicious variant satisfies *correctness*, *security against semi-honest sender and malicious receiver*, which are defined as follows:

- **Correctness:** For each $(\vec{m}_0, \dots, \vec{m}_k) \in \mathcal{D}^{m \times (k+1)}$ and $(b_1, \dots, b_k) \in \mathcal{D}^k$, the following holds

$$\Pr \left[\begin{array}{l} (\rho, \text{msg}_R) \leftarrow \text{OIP}_R(1^\lambda, (b_1, \dots, b_k)) \\ \text{msg}_S \leftarrow \text{OIP}_S(1^\lambda, \text{msg}_R, (\vec{m}_0, \dots, \vec{m}_k)) \end{array} : \text{OIP}_{\text{out}}(\rho, \text{msg}_R, \text{msg}_S) = \vec{m}_0 + \sum_{i \in [k]} b_i \vec{m}_i \right] = 1$$

- **Security Against Semi-Honest Sender:** The following holds for any $(b_1, \dots, b_k) \in \mathcal{D}^k$ and $(b'_1, \dots, b'_k) \in \mathcal{D}^k$, where $\exists i \in [k]$ s.t. $b_i \neq b'_i$

$$\{(\text{msg}_R, \rho) \leftarrow \text{OIP}_R(1^\lambda, (b_1, \dots, b_k)) : \text{msg}_R\} \approx_c \{(\text{msg}'_R, \rho') \leftarrow \text{OIP}_R(1^\lambda, (b'_1, \dots, b'_k)) : \text{msg}'_R\}.$$

- **Security Against Semi-Honest Receiver:** For every PPT adversary \mathcal{A} corrupting the receiver, there exists a PPT simulator \mathcal{S}_R such that for any choice of $(b_1, \dots, b_k) \in \mathcal{D}^k$ and $(\vec{m}_0, \dots, \vec{m}_k) \in \mathcal{D}^{m \times (k+1)}$, the following holds:

$$\text{OIP}_S(1^\lambda, \text{msg}_R, (\vec{m}_0, \dots, \vec{m}_k)) \approx_c \mathcal{S}_R(1^\lambda, \rho, \text{msg}_R, \vec{m}_0 + \sum_{i \in [k]} b_i \vec{m}_i),$$

where $(\text{msg}_R, \rho) \leftarrow \text{OIP}_R(1^\lambda, (b_1, \dots, b_k))$.

- **Security against a Malicious Receiver:** For every PPT adversary \mathcal{A} corrupting the receiver, there exists a PPT simulator $\mathcal{S}_R = (\mathcal{S}_R^1, \mathcal{S}_R^2)$, such that for any choice of $(\vec{m}_0, \dots, \vec{m}_k) \in \mathcal{D}^{m \times (k+1)}$, the following holds:

$$\left| \Pr [\text{IDEAL}_{\mathcal{S}_R, \mathcal{F}_{\text{OIP}}}(1^\lambda, \vec{m}_0, \dots, \vec{m}_k) = 1] - \Pr [\text{REAL}_{\mathcal{A}, \text{OIP}}(1^\lambda, \vec{m}_0, \dots, \vec{m}_k) = 1] \right| \leq \frac{1}{2} + \text{negl}(\lambda).$$

Where experiments $\text{IDEAL}_{\mathcal{S}_R, \mathcal{F}_{\text{OIP}}}$ and $\text{REAL}_{\mathcal{A}, \text{OIP}}$ are defined as follows:

Exp $\text{IDEAL}_{\mathcal{S}_R, \mathcal{F}_{\text{OIP}}}(1^\lambda, \vec{m}_0, \dots, \vec{m}_k) :$	Exp $\text{REAL}_{\mathcal{S}_R, \mathcal{F}_{\text{OIP}}}(1^\lambda, \vec{m}_0, \dots, \vec{m}_k) :$
<ul style="list-style-type: none"> • $\text{msg}_R \leftarrow \mathcal{A}(1^\lambda)$ • $(b_1, \dots, b_k) \leftarrow \mathcal{S}_R^1(1^\lambda, \text{msg}_R)$ • $\text{out} \leftarrow \mathcal{F}_{\text{OIP}}(\vec{m}_0, \dots, \vec{m}_k, b_1, \dots, b_k)$ • $\text{msg}_S \leftarrow \mathcal{S}_R^2(1^\lambda, \text{out}, \text{msg}_R)$ • Output $\mathcal{A}(\text{msg}_S)$ 	<ul style="list-style-type: none"> • $\text{msg}_R \leftarrow \mathcal{A}(1^\lambda)$ • $\text{msg}_S \leftarrow \text{OIP}_S(1^\lambda, \text{msg}_R, (\vec{m}_0, \dots, \vec{m}_k))$ • Output $\mathcal{A}(\text{msg}_S)$

We present a construction of such OIPs from constant rate linearly homomorphic encryption in Section 9.

5 MPC Interface

As discussed in the introduction, all of our compilers make use of an underlying secure computation protocol with certain properties. In this section, we describe the properties that we want from these underlying protocols.

We model these requirements as a reactive functionality (denoted as \mathcal{F}_{mpc}). At a high level, we require secret sharing based MPC that evaluate a given circuit in a gate-by-gate manner and maintain an invariant that the parties hold a secret sharing of the values induced on each intermediate wire in the circuit. A formal description of this reactive functionality appears in Figure 1.

For ease of notation, in our protocol descriptions, we shall let $[varid]$ denote the value stores by the functionality under $(varid, a)$; and we will write $[z] = [x] + [y]$ as a shorthand for calling **Add** and $[z] = [x] \cdot [y]$ as a shorthand for calling **Multiply**. And by abuse of notation, we will let $varid$ denote the value, x , of the data item held in location $(varid, x)$. We use $[x]_i$ to denote the share of x given to party P_i in the underlying MPC.

To the best of our knowledge, most secret sharing based protocols [34, 26, 12, 16, 13] securely implement this reactive functionality in the presence of a malicious adversary who can corrupt arbitrary number of parties. Moreover, most of these protocols are capable of evaluating circuits over any field/ring.

It is easy to see that any such secret sharing based MPC that evaluates the circuit in a gate-by-gate manner and maintains the invariant that parties hold shares of all intermediate wires in the circuit will trivially have support for the **Initialize Input**, **Initialize constant**, **Add**, **Add by const**, **Multiply**, **Multiply by const**, **Function** and **Output Private Shares** calls. Moreover, since the multiplication in these protocols typically requires parties to actually generate and compute shares of random values, the **Random** call is also implemented by these protocols. We now discuss how the remaining calls can be implemented in both the semi-honest and malicious settings.

Semi-Honest Setting. The only other calls used in our semi-honest protocols are **Random Bit** and **Output**. As observed in some of these protocols, **Random Bit** is also very easy to implement (especially in the semi-honest setting). This is done by requiring each party P_i to randomly sample $b_i \in \{1, -1\}$ and secret share it amongst all the parties. The parties then add all the shares obtained from all parties (let the resulting shares be $[s]$) and then compute $\frac{[s]+1}{2}$. The resulting shares will be of a random bit. **Share Zero** can be realized with semi-honest security by having every party secret share 0 and then requiring each party to locally sum up its shares. Finally, it is easy to see that the **Output** call can also be easily implemented, since the parties actually hold shares of all intermediate values. To reconstruct the output, they can simply broadcast their respective shares to all parties and then run the reconstruction algorithm.

Malicious Setting. While protocols such as SPDZ [16] and its descendants [34, 26, 12] (that use MACs w.r.t. a global key) delegate the check that ensures that these shares are indeed consistent with the “correct” values to the end of the protocol, we show that these protocols still securely implement all remaining calls in the \mathcal{F}_{mpc} functionality.

Intuitively, since these protocols delegate the malicious security/consistency checks to the end of the protocol, the only place where we need to ensure that the shares held by the parties for any particular wire are indeed consistent and correct is when those values are reconstructed or are used outside of this MPC protocol, i.e., in the OIP and when the outcome of OIP is returned to the MPC. The subcalls inside \mathcal{F}_{mpc} that are really affected by this are **Initialize Input**, **Random**, **Share Zero**, **Check Zero** and **Output Shares** and **Output**. As discussed above, **Initialize Input** and **Random** are already implemented by these protocols.

- **Check Zero:** For this sub-call, we observe that given authenticated additive shares $([x_1], [m_1]), ([x_2], [m_2])$, with $m_1 = k * x_1, m_2 = k * x_2$ where k is the global MAC key, parties can compute $[m] = [m_1] - [m_2]$ locally, followed by having each player P_i first commit and then broadcast its share $[m]_i$ to reconstruct $[m]$ and check if $m = \sum_i m_i = 0$.
- **Share Zero:** For this we can augment the semi-honest **Share Zero** protocol described above with an asymptotically efficient batch-wise check to ensure malicious security. Specifically, to verify the outputs of the ℓ semi-honest **Share Zero** calls $[x_1], \dots, [x_\ell]$, parties can publicly sample ℓ random values $\{r_i\}_{i=1}^\ell$ and compute a random linear combination $[r] = \sum_{i=1}^\ell r_i [x_i]$ followed by running the **Check Zero** call on $[r]$ and a trivial sharing of 0 (each party P_i 's share is 0).
- **Output and Output Share:** As discussed above authenticated shares in the above protocols are of the form $([x], [m])$, where $m = k * x$ and k is the global MAC key. For both of these sub-calls, the parties first broadcast their shares $[x]$ and reconstruct. Then the parties can compute $x \cdot [k]$ and run **Check Zero** to check if the resulting shares reconstruct to the same value as the shares $[m]$. This is very similar to “MAC check” subprotocol already implemented in [34].

We note that the above proposed protocols only reveal shares $[x]$ and not $[m]$. Indeed, revealing all shares of both x and m will trivially give away the global MAC key and make the protocol insecure. To make this compatible with our maliciously secure protocol, we assume that when the parties use the shares generated via \mathcal{F}_{mpc} outside of \mathcal{F}_{mpc} (i.e., to compute the OIP messages), they can do so on the “unauthenticated shares”, i.e., on only the $[x]$ part and not on the $[m]$ part. Now, before, using the shares obtained as output of this OIP in \mathcal{F}_{mpc} , we can make them “authenticated” by computing the corresponding $[m]$ shares for this output. This can be done trivially, since the parties hold a secret sharing of the global MAC key. This is a standard approach used in many of the above protocols including MASCOT [34].

Moreover, we remark that the above proposed modification does not cause our compiler or the compiled protocols to be insecure in any way. This is because, the authentication mechanism used on the shares is only specific to \mathcal{F}_{mpc} and not to the primitives used outside of it. As a result, outside of \mathcal{F}_{mpc} , an adversary can easily modify the authenticated shares in whatever way they want. Hence, in principle the following strategies are equivalent – (1) where the computations done outside of \mathcal{F}_{mpc} are performed on authenticated shares. (2) where the computations done outside of \mathcal{F}_{mpc} are performed on unauthenticated shares, but we authenticate the output of those computations before they are used in \mathcal{F}_{mpc} again.

6 Non-Constant Round Semi-Honest Branching MPC

In this section, we present our semi-honest compiler for distributed computation of a circuit with conditional branches.

Let the circuit/function be such that it consists of an initial sub-function f_1 , followed by the k branches and then a sub-function f_2 . We assume that the parties have access to \mathcal{F}_{mpc} (see Figure 1). When evaluated using \mathcal{F}_{mpc} , the output of f_1 is a secret sharing of the inputs to the branching part and a secret sharing of the unary representation of the index associated with the branch that needs to be executed (henceforth referred to as the active branch). The output of the branching part is a secret sharing of the inputs to the function f_2 .

Functionality \mathcal{F}_{mpc}

Initialize Input: On input $(\text{initinp}, \text{varid}, P_i)$ from P_i (for each $i \in [n]$) with a fresh identifier varid the functionality stores $(\text{varid}, [x])$.

Initialize constant: On input $(\text{initconst}, \text{constid}, c)$ from each P_i ($i \in [n]$) with a fresh identifier varid the functionality stores (const, c) .

Random: On command $(\text{rand}, \text{varid})$ from all parties, with a fresh identifier varid , the functionality selects a random value r , stores $(\text{varid}, [r])$ and sends the respective share $[r]_i$ to party P_i (for each $i \in [n]$)

Random Bit: On command $(\text{bitrand}, \text{varid})$ from all parties, with a fresh identifier varid , the functionality selects a random bit $b \in \{0, 1\}$, stores $(\text{varid}, [b])$ and sends the respective share $[b]_i$ to party P_i (for each $i \in [n]$)

ShareZero: On command $(\text{sharezero}, \text{varid})$ from all parties, with a fresh identifier varid , the functionality computes, stores $(\text{varid}, [0])$ and sends the respective share $[0]_i$ to party P_i (for each $i \in [n]$).

Add: On command $(\text{add}, \text{varid}_1, \text{varid}_2, \text{varid}_3)$ from all parties (if $\text{varid}_1, \text{varid}_2$ are present in memory and varid_3 is not), the functionality retrieves $(\text{varid}_1, [x]), (\text{varid}_2, [y])$ and stores $(\text{varid}_3, [x + y])$.

Add by const: On command $(\text{add}, \text{constid}_1, \text{varid}_2, \text{varid}_3)$ from all parties (if $\text{constid}_1, \text{varid}_2$ are present in memory and varid_3 is not), the functionality retrieves $(\text{constid}_1, c), (\text{varid}_2, [x])$ and stores $(\text{varid}_3, [c + x])$.

Multiply: On input $(\text{mult}, \text{varid}_1, \text{varid}_2, \text{varid}_3)$ from all parties (if $\text{varid}_1, \text{varid}_2$ are present in memory and varid_3 is not), the functionality retrieves $(\text{varid}_1, [x]), (\text{varid}_2, [y])$ and stores $(\text{varid}_3, [x \cdot y])$.

Multiply by const: On command $(\text{mult}, \text{constid}_1, \text{varid}_2, \text{varid}_3)$ from all parties (if $\text{constid}_1, \text{varid}_2$ are present in memory and varid_3 is not), the functionality retrieves $(\text{constid}_1, c), (\text{varid}_2, [x])$ and stores $(\text{varid}_3, [c \cdot x])$.

Function: On input $(\text{func}, f, \text{varid}_1, \dots, \text{varid}_n, \text{varid}_{\text{out}})$ from all parties, the functionality retrieves $(\text{varid}_1, [x_1]), \dots, (\text{varid}_n, [x_n])$ and stores $(\text{varid}_{\text{out}}, [f(x_1, \dots, x_n)])$.

Output Shares: On input $(\text{outshare}, \text{varid})$ from all parties, the functionality retrieves $(\text{varid}, [x])$ and outputs all shares $[x]$ to all parties.

Output Private Shares: On input $(\text{outprivshare}, \text{varid})$ from all parties, the functionality retrieves $(\text{varid}, [x])$ and outputs the respective share $[x]_i$ to party P_i (for each $i \in [n]$).

Check Zero: On input $(\text{fcheckzero}, \text{varid}_1, \text{varid}_2)$ from all parties, the functionality retrieves $(\text{varid}_1, [x_1]), (\text{varid}_2, [x_2])$ and outputs 1 w.h.p if $x_1 = x_2$ and otherwise it outputs 0 and aborts.

Output: On input $(\text{out}, \text{varid})$ from all honest parties (if varid is present in memory), the functionality retrieves $(\text{varid}, [x])$ and outputs x to all players.

Figure 1: A Required Ideal Functionality for MPC

Given a circuit C , we assume that the parties decide on some canonical ordering of the gates in the circuit, such that gate i only takes as inputs the values output by the gates $j < i$. We assume w.l.o.g. that the i^{th} gate in C has fan-in 2 and the outgoing wire of any gate can act as the incoming wire for any number of gates.²

For simplicity, we assume that all branches are of the same size and have G gates. Our protocol can be easily extended to the scenario where the branches are of varying sizes by suitably padding the smaller branches with fake gates. Let ℓ be the length of inputs to the branching part of the

²Our compiler can work with circuits that have gates with arbitrary fan-out. In our construction, it suffices to view such gates as having a single outgoing wire that acts as the incoming wire for multiple gates. Hence, we only assign a single label to the outgoing wire of each gate.

function. For evaluating this part, we assume that there are ℓ input gates that are common to all branches. We set both the incoming and outgoing labels for the wires coming out of these gates as $1, \dots, \ell$ respectively. For each branch $m \in [k]$, and each gate i in this branch, we assign outgoing label $i + \ell$ to the wire coming out of this gate and incoming labels $\ell + 2i - 1$ and $\ell + 2i$ respectively to its two incoming wires. Therefore, we assume that the number of unique outgoing labels assigned in a branch are $G + \ell$, while the total number of unique incoming labels assigned in a branch are $W = 2G + \ell$. We present a slightly optimized version of the protocol described in the introduction, namely that only requires parties to sample 1 mask per wire, instead of 2 masks.

Let π be the mapping corresponding to a circuit C that maps incoming labels to the outgoing labels of each wire in C . For instance, $\pi(i)$ corresponds to the outgoing label of the wire with incoming label i . Let C_1, \dots, C_k be the circuit representations of the k branches and let $\{\pi_1, \dots, \pi_k\}$ be the corresponding mappings associated with these branches. Finally, we assume that the circuits and inputs are defined over some field \mathbb{F} .

Protocol. The parties start by invoking $(func, f_1, x_1, \dots, x_n, \text{inp}_1, \dots, \text{inp}_\ell, b_1, \dots, b_k)$ in \mathcal{F}_{mpc} on their original inputs x_1, \dots, x_n , to obtain shares of inputs to the branching part $[\text{inp}_1], \dots, [\text{inp}_\ell]$, where $|\ell|$ is the total input length and shares $[b_1], \dots, [b_k]$, where $b_1 \dots b_k$ is the unary representation of the index associated with the active branch. Given these shares, parties run the protocol presented in Figure 2. The output of this protocol is a secret sharing of the inputs to f_2 (i.e., the last part of the circuit). Let m be the length of these inputs. The parties finally invoke $(func, f_2, y_1, \dots, y_m, \text{out})$ and (out, out) in \mathcal{F}_{mpc} to learn the final output out .

Optimization. A naive implementation of the online phase in the above protocol will result in a round complexity that depends on the maximum number of gates in any particular branch. This can be improved to be proportional to the maximum multiplicative depth of any branch by using a simple optimization. For simplicity, let's assume that all branches have the same depth and each layer of each branch contains the same number of gates. We know that the gates on level ℓ only depend on the outgoing wires of gates on layers $< \ell$. We can therefore evaluate all the gates in a particular level in parallel. This simple idea can also be extended to the case where the branches have different depths and widths. In that case, let x_ℓ (and y_ℓ resp.) be the minimum (and maximum resp.) number of gates on level ℓ in any branch. We can evaluate the first x_ℓ gates in parallel. Then in the next round we can evaluate the $y_\ell - x_\ell + x_{\ell+1}$ gates in parallel. This ensures that the overall round complexity of the online phase will only depend on the depth of the branches.

Complexity Analysis. We now analyze the communication complexity of the above semi-honest protocol. If we use a rate-1 OIP, the communication complexity in the pre-processing phase is $O(n^2|C_{\text{max}}| + n^2k\lambda)$, where $|C_{\text{max}}|$ is the size of the largest branch. In the online phase for each gate we perform both addition and multiplication and then choose between the two. As a result we perform 2 multiplications per gate. The communication complexity of the online phase is $O(2 \times \text{CC}(|C_{\text{max}}|))$, where $\text{CC}(|C_{\text{max}}|)$ is the communication complexity incurred upon evaluating C_{max} using the underlying MPC.

Overall, given the above protocol and optimizations, we obtain the following result.

Theorem 1. Let λ be the security parameter and \mathcal{F} be a function class consisting of functions of the form $f(\vec{x}) = f_2(f_{\text{br}}(f_1(\vec{x})))$, where $f_{\text{br}} := \{g_1, \dots, g_k\}$ is a function consisting of k conditional branches, defined as $f_{\text{br}}(i, \vec{x}) = g_i(\vec{x})$. Assuming the existence of a rate-1 two-message semi-honest secure OIP (see Definition 4.1), there exists an MPC protocol in the \mathcal{F}_{mpc} -hybrid model (see Section 5) for computing any $f \in \mathcal{F}$ that achieves semi-honest security against an arbitrary number of corruptions and incurs a communication overhead of $O(n^2(k\lambda + |C_{\text{max}}|))$.

Semi-Honest Protocol

The protocol is described in the \mathcal{F}_{mpc} -hybrid model. Parties have shares of inputs to the branches, i.e., $[\text{inp}_1], \dots, [\text{inp}_\ell]$ and shares of a unary representation of the active branch, i.e., $[b_1], \dots, [b_k]$.

- **Pre-processing Phase:**

1. **Sample masks:** For each input and gate $g \in [\ell + G]$, parties invoke $(\text{rand}, \text{mask}_g)$ in \mathcal{F}_{mpc} to obtain shares $[\text{mask}_g]$. For each branch $m \in [k]$, let $[\overrightarrow{\text{mask}_{\pi_m}}] = [\text{mask}_{\pi_m(1)}] \parallel \dots \parallel [\text{mask}_{\pi_m(W)}]$.
2. **Shares of zeros:** For each $w \in [W]$ and $i \in [n]$, parties invoke $(\text{sharezero}, X_{w,i})$ in \mathcal{F}_{mpc} to get shares $[X_{w,i}]$, where $X_{w,i} = 0$. For each $i \in [n]$, let $[\overrightarrow{X}_i] = [X_{1,i}] \parallel \dots \parallel [X_{W,i}]$.
3. **Pairwise OIP:** Each pair of parties P_R and P_S ($\forall R, S \in [n]$) engage in a two-message semi-honest OIP as follows, where P_R acts as the receiver and P_S acts as the sender:
 - **Receiver:** P_R computes $(\rho, \text{msg}_R) \leftarrow \text{OIP}_R(1^\lambda, [b_1]_R, \dots, [b_k]_R)$ and sends msg_R to P_S .
 - **Sender:** P_S computes $\text{msg}_S \leftarrow \text{OIP}_S(1^\lambda, \text{msg}_R, [\overrightarrow{X}_R]_S, [\overrightarrow{\text{mask}_{\pi_1}}]_S, \dots, [\overrightarrow{\text{mask}_{\pi_k}}]_S)$ and sends msg_S to P_R .
 - **Output:** P_R computes $\overrightarrow{\text{share}_{R,S}} \leftarrow \text{OIP}_{\text{out}}(\rho, \text{msg}_R, \text{msg}_S)$.
4. **Δ values:** Each party P_i (for $i \in [n]$) computes $[\overrightarrow{\Delta}]_i = \sum_{j \in [n]} \overrightarrow{\text{share}_{j,i}}$, where $[\overrightarrow{\Delta}] = [\Delta_1] \parallel \dots \parallel [\Delta_W]$.

- **Online Phase :**

1. **Inputs:** For each input wire $i \in [\ell]$, parties compute $[u_i] = [\text{inp}_i] + [\text{mask}_i]$. and invoke (out, u_i) in \mathcal{F}_{mpc} to obtain u_i in the clear.
2. **Circuit Evaluation:** For each gate $g \in [G]$, let $\text{left} = \ell + 2g - 1$ and $\text{right} = \ell + 2g$ be the incoming wire labels of its input wires. Let $\text{type}_{m,g}$ be the gate type for gate g in \mathcal{C}_m ($\forall m \in [k]$), where $\text{type}_{m,g} = 0$ denotes an addition gate and $\text{type}_{m,g} = 1$ denotes a multiplication gate. Parties compute the following using \mathcal{F}_{mpc} :
 - (a) For $w \in \{\text{left}, \text{right}\}$, compute $[y_w] = \sum_{m=1}^k (u_{\pi_m(w)} \cdot [b_m]) - [\Delta_w]$.
 - (b) Compute $[\text{type}_g] = \sum_{m=1}^k (\text{type}_{m,g} \cdot [b_m])$
 - (c) Compute $[z_g] = [y_{\text{left}}] + [y_{\text{right}}] + [\text{type}_g] \cdot ([y_{\text{left}}] \cdot [y_{\text{right}}]) - [y_{\text{left}}] - [y_{\text{right}}]$.
 - (d) Compute $[u_{\ell+g}] = [z_g] + [\text{mask}_{\ell+g}]$ and invoke (out, u_s) in \mathcal{F}_{mpc} to obtain u_s in the clear.
3. **Output:** For each output gate g , compute $[z_g] = \sum_{m=1}^k (u_{\pi_m(w)} \cdot [b_m]) - [\Delta_w]$.

Figure 2: Semi-Honest Compiler

In Section 9, we show that a rate-1 two-message semi-honest secure OIP can be constructed from rate-1 linearly homomorphic encryption. Such encryptions are known [17, 8, 15, 42] from a variety of assumptions including LWE, Ring LWE and DDH assumption.

6.1 Security

We now prove security of our semi-honest protocol. We start by describing the simulator and then proceed to argue indistinguishability between the real and ideal world executions.

Simulator. Let \mathcal{A} be the adversary who corrupts a subset $\mathcal{I} \subset [n]$ of the parties and $\mathcal{H} = [n] \setminus \mathcal{I}$

be the set of honest parties. Let \mathcal{S}_R be the simulator associated with security of OIP against semi-honest receiver (see Definition 4.1). Given the output z and inputs $\{x_i\}_{i \in \mathcal{C}}$ of the corrupt parties the simulator proceeds as follows:

- **Computing f_1 .** For each $i \in \mathcal{I}$, $j \in [\ell]$, sample random shares $[\text{inp}_j]_i$ and for each $m \in [k]$, sample random shares $[b_m]_i$ and send all of these shares to the adversary.
- **Pre-processing Phase.**
 - For each $w \in [W]$ and $i \in \mathcal{I}$, sample random $[\text{mask}_w]_i$ and send these values to the adversary.
 - For each $w \in [W]$ and $i \in [n]$ and $j \in \mathcal{I}$, sample random shares $[X_{w,i}]_j$ and sends to the adversary.
 - For each pair of parties P_R and P_S ($\forall R \in \mathcal{I}, S \in \mathcal{H}$), upon receiving a message msg_R from the adversary, sample a random vector of shares $\vec{V}_{R,S}$, compute $\text{msg}_S \leftarrow \mathcal{S}_R(1^\lambda, \text{msg}_R, \vec{V}_{R,S})$ and send msg_S to the adversary on behalf of honest P_S .
 - For each pair of parties P_R and P_S ($\forall R \in \mathcal{H}, S \in \mathcal{I}$), set $[b_1]_R = \dots = [b_k]_R = 0$, compute $\rho, \text{msg}_R \leftarrow \text{OIP}_R(1^\lambda, [b_1]_R, \dots, [b_k]_R)$ and send msg_R to the adversary on behalf of honest P_R . Also, set $\text{share}_{R,S} = [\vec{X}_R]_S + \sum_{i \in [k]} [b_i]_R [\text{mask}_{\pi_i}]_S$.
- **Online Phase.** In the online phase, the simulator mimcs the computation done by \mathcal{F}_{mpc} . Recall from the description of \mathcal{F}_{mpc} (in Figure 1) that this only requires sending messages to the adversary whenever (out, \cdot) is invoked. Since in the online phase, this is invoked on random values, the simulator can easily emulate this by sending a random value to the adversary for each such call.
- **Computing f_2 .** Sends output z to the adversary.

Indistinguishability Argument. We argue indistinguishability via the following sequence of hybrids:

H_0 : This hybrid is identical to the real world execution.

H_1 : This hybrid is very similar to the previous hybrid except that in the preprocessing phase for each pair of parties P_R and P_S ($\forall R \in \mathcal{H}, S \in \mathcal{I}$), we change the way msg_R and $\text{share}_{R,S}$ are computed:

- $\rho, \text{msg}_R \leftarrow \text{OIP}_R(1^\lambda, 0 \dots, 0)$.
- $\text{share}_{R,S} = [\vec{X}_R]_S + \sum_{i \in [k]} [b_i]_R [\text{mask}_{\pi_i}]_S$.

For indistinguishability between hybrids H_0 and H_1 , we consider a sequence of sub-hybrids, where we change the way msg_R and $\text{share}_{R,S}$ are computed for each pair P_R and P_S (where $R \in \mathcal{H}, S \in \mathcal{I}$), one hybrid at a time. In terms of the view of the adversary, the only change in each of these sub-hybrids is in the way msg_R is computed for one pair $R \in \mathcal{I}, S \in \mathcal{H}$. As a result, indistinguishability between each consecutive pair of sub-hybrids follows the security of OIP against a semi-honest sender and by transitivity, it holds that H_0 and H_1 are indistinguishable.

H_2 : This hybrid is very similar to the previous hybrid except that in the preprocessing phase, for each pair of parties P_R and P_S ($\forall R \in \mathcal{I}, S \in \mathcal{H}$), we compute $\text{msg}_S \leftarrow \mathcal{S}_R(1^\lambda, \text{msg}_R, \vec{V}_{R,S})$, using some random share $\vec{V}_{R,S}$.

For indistinguishability between hybrids H_1 and H_2 , we consider a sequence of sub-hybrids, where we change the way msg_S is computed for each pair $R \in \mathcal{I}, S \in \mathcal{H}$, one hybrid at a time.

The only difference between any two consecutive pairs of these sub-hybrids is that in one we compute msg_S using a random vector of shares $\vec{V}_{R,S}$ and the simulator for some receiver R and sender S , while in the other msg_S is computed honestly and the output that the receiver gets is $[\vec{X}_R]_S + \sum_{m \in [k]} [b_m]_R [\overrightarrow{\text{mask}}_{\pi_m}]_S$. Since $[\vec{X}_R]_S$ is a random vector of shares, this output is identically distributed to $\vec{V}_{R,S}$. Given this output indistinguishability between a simulated message msg_S , and an honestly computed message msg_S follows from security of OIP against a semi-honest receiver. As a result, this sub-hybrid is indistinguishable from its previous hybrid and by transitivity, it holds that H_1 and H_2 are indistinguishable.

H_3 : This hybrid is identical to the simulator description.

Indistinguishability between hybrids H_2 and H_3 follows from semi-honest security of the underlying MPC protocol.

7 Non-Constant Round Maliciously Secure Branching MPC

In this section, we present our maliciously secure compiler for distributed computation of a branching circuit. We borrow notations from the previous section. As discussed in the introduction, the basic outline of our maliciously secure protocol remains the same except that we now use a two-message OIP that is secure against malicious receivers. Also, in order to ensure that the sender behaves honestly, we make use of non-interactive commitments.

Protocol. Similar to the semi-honest protocol, the parties start by invoking $(\text{func}, f_1, x_1, \dots, x_n, \text{inp}_1, \dots, \text{inp}_\ell, b_1, \dots, b_k)$ in \mathcal{F}_{mpc} on their original inputs x_1, \dots, x_n , to obtain shares of inputs to the branching part $[\text{inp}_1], \dots, [\text{inp}_\ell]$, where $|\ell|$ is the total input length and shares $[b_1], \dots, [b_k]$, where $b_1 \dots b_k$ is the unary representation of the index associated with the active branch. Given these shares, the parties run the protocol presented in Figures 3 and 4. The output of this protocol is a secret sharing of the inputs to f_2 (i.e., the last part of the circuit). Let m be the length of these inputs. The parties finally invoke $(\text{func}, f_2, y_1, \dots, y_m, \text{out})$ and (out, out) in \mathcal{F}_{mpc} to learn the final output out .

Complexity Analysis. If we use a rate-1 OIP, the communication complexity in the preprocessing phase is $O(\delta \times n^2 |C_{\max}| + n^2 k \lambda)$, where $|C_{\max}|$ is the size of the largest branch and $\delta = \kappa/0.311$. The online phase is repeated for each $q \in [\delta] \setminus Z$, as a result, the communication complexity of the online phase is $O(\delta \times \mathbb{CC}(|C_{\max}|))$, where $\mathbb{CC}(|C_{\max}|)$ is the communication complexity incurred upon evaluating C_{\max} using the underlying MPC.

Overall, the above protocol gives us the following result.

Theorem 2. Let λ be the computational security parameter and κ be the statistical security parameter. Let \mathcal{F} be a function class consisting of functions of the form $f(\vec{x}) = f_2(f_{\text{br}}(f_1(\vec{x})))$, where $f_{\text{br}} := \{g_1, \dots, g_k\}$ is a function consisting of k conditional branches, defined as $f_{\text{br}}(i, \vec{x}) = g_i(\vec{x})$. Assuming the existence of a rate-1 two-message OIP secure against a malicious receiver (see Definition 4.1), there exists an MPC protocol in the \mathcal{F}_{mpc} -hybrid model (see Section 5) for

Preprocessing Phase of the Maliciously Secure Protocol

The protocol is described in the \mathcal{F}_{mpc} -hybrid model. Parties have shares of the inputs to the branches, i.e., $[\text{inp}_1], \dots, [\text{inp}_\ell]$ and shares of a unary representation of the active branch, i.e., $[b_1], \dots, [b_k]$. Let $\delta = \kappa/0.311$, where κ is the statistical security parameter.

Pre-processing Phase:

1. **Sample masks:** For each wire $w \in [W]$, parties invoke $(\text{rand}, \text{mask}_w^1), \dots, (\text{rand}, \text{mask}_w^\delta)$ in \mathcal{F}_{mpc} to obtain shares $[\text{mask}_w^1], \dots, [\text{mask}_w^\delta]$ respectively. For each $q \in [\delta]$, $m \in [k]$, let $[\overrightarrow{\text{mask}}_{\pi_m}^q] = [\text{mask}_{\pi_m(1)}^q] \parallel \dots \parallel [\text{mask}_{\pi_m(W)}^q]$.
2. **Shares of zeros:** For each $q \in [\delta]$, $w \in [W]$ and $i \in [n]$, the parties invoke $(\text{sharezero}, X_{w,i})$ in \mathcal{F}_{mpc} to get shares $[X_{w,i}^q]$, where $X_{w,i}^q = 0$. For each $i \in [n]$, let $[\overrightarrow{X}_i^q] = [X_{1,i}^q] \parallel \dots \parallel [X_{k,i}^q]$.
3. **Pairwise OIP:** Each pair of parties P_R and P_S ($\forall R, S \in [n]$) engage in two-message (malicious receiver) OIPs as follows, where P_R and P_S act as the receiver and sender resp.:
 - **Receiver:** P_R computes $(\rho, \text{msg}_{R \rightarrow S}) \leftarrow \text{OIP}_R(1^\lambda, [b_1]_R, \dots, [b_k]_R)$ and sends msg_R to P_S .
 - **Sender:** For each $q \in [\delta]$, P_S samples random values $r_0^q, \dots, r_k^q \in \mathbb{F}^{k+1}$ and does the following:
 - Computes $\text{msg}_{S \rightarrow R}^q \leftarrow \text{OIP}_S(1^\lambda, \text{msg}_{R \rightarrow S}, [\overrightarrow{X}_R^q]_S \parallel r_0^q, [\overrightarrow{\text{mask}}_{\pi_1}^q]_S \parallel r_1^q, \dots, [\overrightarrow{\text{mask}}_{\pi_k}^q]_S \parallel r_k^q; \rho_{R,S}^{S,q})$.
 - Computes $\text{com}_{R,S}^q \leftarrow \text{Commit}([\overrightarrow{X}_R^q]_S \parallel r_0^q, [\overrightarrow{\text{mask}}_{\pi_1}^q]_S \parallel r_1^q, \dots, [\overrightarrow{\text{mask}}_{\pi_k}^q]_S \parallel r_k^q; \rho_{R,S}^{S,q}; \rho_{R,S}^{\text{com},q})$.
 - It sends $\text{msg}_{S \rightarrow R}^q, \text{com}_{R,S}^q$ to P_R .
 - **Output:** For each $q \in [\delta]$, P_R computes $\overrightarrow{\text{share}}_{R,S}^q \leftarrow \text{OIP}_{\text{out}}(\rho, \text{msg}_{R \rightarrow S}, \text{msg}_{S \rightarrow R})$.
 - **Receiver Consistency check:** For each $q \in [\delta]$, P_S invokes $(\text{initinp}, r_0^q, P_S), \dots, (\text{initinp}, r_k^q, P_S)$ and P_R invokes $(\text{initinp}, \text{mac}_{R,S}^q, P_R)$ in \mathcal{F}_{mpc} . The parties then collectively invoke $(\text{func}, \mathcal{F}_{\text{mac}}^q, r_0^q, r_1^q, \dots, r_k^q, [b_1]_R, \dots, [b_k]_R, \text{out}_{\text{mac}}^q)$, where $\mathcal{F}_{\text{mac}}^q(r_0^q, r_1^q, \dots, r_k^q, [b_1]_R, \dots, [b_k]_R) = r_0^q + \sum_{m \in [k]} [b_m]_R r_m^q$. Finally, the parties invoke $(\text{checkzero}, \text{out}_{\text{mac}}^q, \text{mac}_{R,S}^q)$ to check if $\text{out}_{\text{mac}}^q \stackrel{?}{=} \text{mac}_{R,S}^q$.
4. **Sender Consistency Check:** The parties use \mathcal{F}_{mpc} to sample a random subset $Z \subset [\delta]$ of size $\delta/2$ and then proceed as follows:
 - For each $q \in Z$, they invoke $(\text{outshare}, \text{mask}_{\pi_1}^q), \dots, (\text{outshare}, \text{mask}_{\pi_k}^q)$ and for each $i \in [n], w \in [W]$, they invoke $(\text{outshare}, X_{w,i}^q)$ in \mathcal{F}_{mpc} to obtain all the shares of $\text{mask}_{\pi_1}^q, \dots, \text{mask}_{\pi_k}^q$ and $X_{w,i}^q$.
 - For each $q \in Z$, each pair of parties P_R and P_S ($\forall R, S \in [n]$) do the following:
 - (a) P_S sends $r_0^q, r_1^q, \dots, r_k^q, \rho_{R,S}^{S,q}, \rho_{R,S}^{\text{com},q}$ to P_R .
 - (b) P_R checks if $\text{msg}_{S \rightarrow R}^q \stackrel{?}{=} \text{OIP}_S(1^\lambda, \text{msg}_{R \rightarrow S}, [\overrightarrow{X}_R^q]_S \parallel r_0^q, [\overrightarrow{\text{mask}}_{\pi_1}^q]_S \parallel r_1^q, \dots, [\overrightarrow{\text{mask}}_{\pi_k}^q]_S \parallel r_k^q; \rho_{R,S}^{S,q})$ and $\text{com}_{R,S}^q \stackrel{?}{=} \text{Commit}([\overrightarrow{X}_R^q]_S \parallel r_0^q, [\overrightarrow{\text{mask}}_{\pi_1}^q]_S \parallel r_1^q, \dots, [\overrightarrow{\text{mask}}_{\pi_k}^q]_S \parallel r_k^q; \rho_{R,S}^{S,q}; \rho_{R,S}^{\text{com},q})$.
5. **Δ values:** If all the above checks succeed, then for each $q \in [\delta] \setminus Z$, each party P_i (for $i \in [n]$) computes $[\overrightarrow{\Delta}^j]_i = \sum_{j \in [n]} \overrightarrow{\text{share}}_{j,i}^q$, where $[\overrightarrow{\Delta}^q] = [\Delta_1^q] \parallel \dots \parallel [\Delta_W^q]$.

Figure 3: Pre-processing Phase of the Maliciously Secure Compiler

computing any $f \in \mathcal{F}$ that achieves security with abort against an arbitrary number of malicious corruptions and incurs a communication overhead of $O(n^2(k\lambda + \kappa|C_{\text{max}}|))$.

Rate-1 two-message OIP secure against a malicious receiver can be built from rate-1 linearly

Online Phase of the Maliciously Secure Protocol

Online Phase : For each $q \in [\delta] \setminus Z$, parties compute the following:

1. **Inputs:** For each input wire $i \in [\ell]$, parties compute $[u_i^q] = [\text{inp}_i] + [\text{mask}_i]$ and invoke (out, u_i^q) in \mathcal{F}_{mpc} to obtain u_i^q in the clear.
2. **Circuit Evaluation:** For each gate $g \in [G]$, let $\text{left} = \ell + 2g - 1$ and $\text{right} = \ell + 2g$ be the incoming wire labels of its input wires. Let $\text{type}_{m,g}$ be the gate type for gate g in \mathbb{C}_m ($\forall m \in [k]$), where $\text{type}_{m,g} = 0$ denotes an addition gate and $\text{type}_{m,g} = 1$ denotes a multiplication gate.
 - (a) For $w \in \{\text{left}, \text{right}\}$, compute $[y_w^q] = \sum_{m=1}^k \left(u_{\pi_m(w)}^q \cdot [b_m] \right) + [\Delta_w^q]$.
 - (b) Compute $[\text{type}_g] = \sum_{m=1}^k (\text{type}_{m,g} \cdot [b_m])$
 - (c) Compute $[z_g^q] = (1 - [\text{type}_g])([y_{\text{left}}^q] + [y_{\text{right}}^q]) + [\text{type}_g]([y_{\text{left}}^q] \cdot [y_{\text{right}}^q])$.
 - (d) Compute $[u_{\ell+g}^q] = [z_g^q] + [\text{mask}_{\ell+g}]$ and invoke $(\text{out}, u_{\ell+g}^q)$ in \mathcal{F}_{mpc} to obtain $u_{\ell+g}^q$ in the clear.
3. For each output gate g , compute $[z_g^q] = \sum_{m=1}^k \left(u_{\pi_m(w)}^q \cdot [b_m] \right) + [\Delta_w^q]$.

Output For each output gate g , the parties invoke $(\text{func}, \text{maj}, z_g^1, \dots, z_g^\delta, z_g)$ to get shares $[z_g]$, where the maj is the majority function.

Figure 4: Online Phase of the Maliciously Secure Compiler

homomorphic encryption and non-interactive zero-knowledge.

7.1 Security

We now prove security of our malicious protocol. We start by describing the simulator and then proceed to argue indistinguishability between the view of the adversary in the real protocol execution and the view generated by the simulator.

Simulator. Let $\mathcal{I} \subset [n]$ be the set of corrupt parties and $\mathcal{H} = [n] \setminus \mathcal{I}$ be the set of honest parties. Let $\mathcal{S}_{\mathbb{R}} = (\mathcal{S}_{\mathbb{R}}^1, \mathcal{S}_{\mathbb{R}}^2)$ be the simulators corresponding to the malicious receiver security of OIP. Given the output z and inputs $\{x_i\}_{i \in \mathcal{C}}$ of the corrupt parties the simulator proceeds as follows:

- **Computing f_1 .** When the adversary sends its inputs to \mathcal{F}_{mpc} while invoking func , the simulator receives them and queries the ideal functionality on these inputs to get the final output. For each $i \in \mathcal{I}$, $j \in [\ell]$, the simulator samples random share $[\text{inp}_j]_i$ and for each $m \in [k]$, it samples random share $[b_m]_i$ and sends these shares to the adversary.
- **Pre-processing Phase.**
 - For each $q \in [\delta]$, $w \in [W]$ and $i \in \mathcal{I}$, the simulator samples random shares $[\text{mask}_w^q]_i$ and sends these values to the adversary.
 - For each $q \in [\delta]$, $w \in [W]$ and $i \in [n]$ and $j \in \mathcal{I}$, the simulator samples random shares $[X_{w,i,j}^q]$ and sends to the adversary.
 - It samples a random subset $Z \subset [\delta]$ of size $\delta/2$.

– For each pair of parties P_R and P_S ($\forall R \in \mathcal{H}, S \in \mathcal{I}$), the simulator sets $[b_1]_R = \dots = [b_k]_R = 0$, computes $\rho, \text{msg}_{R \rightarrow S} \leftarrow \text{OIP}_R(1^\lambda, [b_1]_R, \dots, [b_k]_R)$ and sends msg_R to the adversary on behalf of honest P_R .

– For each pair of parties P_R and P_S ($\forall R \in \mathcal{I}, S \in \mathcal{H}$), the simulator proceeds as follows:

* For each $q \in [\delta]$, it samples random values $r_0^q, \dots, r_k^q \in \mathbb{F}^{k+1}$.

* For each $q \in Z$, sample random shares $[\overrightarrow{X}_R^q]_S, [\overrightarrow{\text{mask}}_{\pi_1}^q], \dots, [\overrightarrow{\text{mask}}_{\pi_k}^q]$ and compute

$$\text{msg}_{S \rightarrow R}^q \leftarrow \text{OIP}_S(1^\lambda, \text{msg}_{R \rightarrow S}, [\overrightarrow{X}_R^q]_S \| r_0^q, [\overrightarrow{\text{mask}}_{\pi_1}^q]_S \| r_1^q, \dots, [\overrightarrow{\text{mask}}_{\pi_k}^q]_S \| r_k^q; \rho_{R,S}^{S,q})$$

$$\text{com}_{R,S}^q \leftarrow \text{Commit}([\overrightarrow{X}_R^q]_S \| r_0^q, [\overrightarrow{\text{mask}}_{\pi_1}^q]_S \| r_1^q, \dots, [\overrightarrow{\text{mask}}_{\pi_k}^q]_S \| r_k^q, \rho_{R,S}^{S,q}; \rho_{R,S}^{\text{com},q})$$

* For each $q \in [\delta] \setminus Z$, the simulator samples a random vector of shares $\overrightarrow{V}_{R,S}^q$ and computes $\text{msg}_{S \rightarrow R}^q \leftarrow \mathcal{S}_R^2(1^\lambda, \text{msg}_{R \rightarrow S}, \overrightarrow{V}_{R,S}^q)$ and $\text{com}_{R,S}^q \leftarrow \text{Commit}(0; \rho_{R,S}^{\text{com},q})$.

* For each $q \in [\delta]$, it sends $\text{msg}_{S \rightarrow R}^q$ and $\text{com}_{R,S}^q$ to the adversary on behalf of an honest P_S .

* **Receiver Consistency Check:** For each $q \in Z$, upon receiving $\text{mac}_{R,S}^\delta$, the simulator runs \mathcal{S}_R^1 on $\text{msg}_{R \rightarrow S}$ to extract $[b_1]'_R, \dots, [b_k]'_R$. Check if for each $m \in [k]$, $[b_m]'_R = [b_m]_R$ and if $\text{mac}_{R,S}^\delta = r_0^q + \sum_{m \in [k]} [b_m]_R r_m^q$. If both these checks succeed, output 1 when the parties invoke the *checkzero* function, else output 0 and the simulator signals the ideal functionality to send *abort* to the honest parties and aborts the protocol.

– **Sender Consistency Check:** The simulator sends Z to the adversary. For each $q \in Z$ send all shares of $\text{mask}_{\pi_1}^q, \dots, \text{mask}_{\pi_k}^q$ and $X_{w,i}^q$ used in the OIP instances to the adversary.

* Send all shares of $\text{mask}_{\pi_1}^q, \dots, \text{mask}_{\pi_k}^q$ and $X_{w,i}^q$ used in the OIP instances to the adversary.

* For each pair of parties P_R and P_S (where $R \in \mathcal{H}, S \in \mathcal{I}$), the simulator checks if all the messages and commitments were honestly computed. If not, it signals the ideal functionality to send *abort* to the honest parties and aborts the protocol.

* For each pair of parties P_R and P_S (where $R \in \mathcal{I}, S \in \mathcal{H}$), the simulator proceeds exactly as in the real protocol.

- **Online Phase.** In the online phase, the simulator mimics the computation done by \mathcal{F}_{mpc} , except that it does not compute the majority function. Instead it sends random shares for each output gate to the adversary.

For all other steps in the online phase, based on the description of \mathcal{F}_{mpc} (in Figure 1), the simulator only needs to send messages to the adversary whenever (out, \cdot) is invoked. Since in the online phase, this is only invoked on random values, the simulator can easily emulate this by sending a random value to the adversary for each such call.

- **Computing f_2 .** Outputs the output received from the ideal functionality in the first step to the adversary and send *continue* to the ideal functionality to signal that the honest parties can learn the output.

Indistinguishability Argument. We argue indistinguishability between the real and ideal executions, via the following sequence of hybrids:

H_0 : This hybrid is identical to the real world execution.

H_1 : This hybrid is very similar to the previous one, except that the subset Z is sampled before the parties engage in the pairwise OIP protocols, but is revealed to the parties only during the sender consistency checks.

Hybrids H_0 and H_1 are trivially indistinguishable.

H_2 : This hybrid is very similar to the previous one, except that for each $q \in [\delta] \setminus Z$ and each pair of parties P_R and P_S (where $R \in \mathcal{I}, S \in \mathcal{H}$), we compute $\text{com}_{R,S}^q \leftarrow \text{Commit}(0; \rho_{R,S}^{\text{com},q})$.

Indistinguishability between hybrids H_1 and H_2 follows from a sequence of sub-hybrids, where we change the way $\text{com}_{R,S}^q$ is computed for $q \in [\delta] \setminus Z$ and each pair P_R and P_S (where $R \in \mathcal{I}, S \in \mathcal{H}$), one hybrid at a time. Indistinguishability between each consecutive pair of sub-hybrids follows from the hiding property of the commitment scheme and by transitivity it follows that H_1 and H_2 are indistinguishable.

H_3 : This hybrid is very similar to the previous hybrid except that in the preprocessing phase, for each $q \in [\delta] \setminus Z$ and for each pair of parties P_R and P_S ($\forall R \in \mathcal{I}, S \in \mathcal{H}$), we compute $\text{msg}_{S \rightarrow R}^q \leftarrow \mathcal{S}_R^2(1^\lambda, \text{msg}_R, \vec{V}_{R,S}^q)$, for some random vector of shares $\vec{V}_{R,S}^q$.

For indistinguishability between hybrids H_2 and H_3 , we consider a sequence of sub-hybrids, where we change the way $\text{msg}_{S \rightarrow R}$ is computed for each $q \in [\delta] \setminus Z$ and each pair $R \in \mathcal{I}, S \in \mathcal{H}$, one hybrid at a time.

The only difference between any two consecutive pairs of these sub-hybrids is that in one we compute $\text{msg}_{S \rightarrow R}$ using a random vector of shares $\vec{V}_{R,S}$ and the simulator for some $q \in [\delta] \setminus Z$ and some receiver R and sender S , while in the other, $\text{msg}_{S \rightarrow R}$ is computed honestly and the output that the receiver gets is $[\vec{X}_R]_S + \sum_{m \in [k]} [b_m]_R [\overrightarrow{\text{mask}}_{\pi_m}]_S$. Since $[\vec{X}_R]_S$ is a random vector of shares, this output is identically distributed to $\vec{V}_{R,S}$. Given this output, indistinguishability between a simulated message $\text{msg}_{S \rightarrow R}$, and an honestly computed message $\text{msg}_{S \rightarrow R}$ follows from security of OIP against a malicious receivers. As a result, this sub-hybrid is indistinguishable from its previous hybrid and by transitivity, it holds that H_2 and H_3

H_4 : This hybrid is very similar to the previous hybrid except that in the preprocessing phase for each pair of parties P_R and P_S ($\forall R \in \mathcal{H}, S \in \mathcal{I}$), we change the way msg_R and $\text{share}_{R,S}$:

- $\rho, \text{msg}_{R \rightarrow S} \leftarrow \text{OIP}_R(1^\lambda, 0 \dots, 0)$.
- For each $q \in [\delta] \setminus Z$, $\overrightarrow{\text{share}}_{R,S}^q = [\vec{X}_R^q]_S + \sum_{i \in [k]} [b_i]_R [\overrightarrow{\text{mask}}_{\pi_i}^q]_S$.

For indistinguishability between hybrids H_3 and H_4 , we consider a sequence of sub-hybrids, where we change the way $\text{msg}_{R \rightarrow S}$ and $\text{share}_{R,S}$ are computed for each $q \in [\delta] \setminus Z$ and each pair P_R and P_S (where $R \in \mathcal{H}, S \in \mathcal{I}$), one hybrid at a time. In terms of the view of the adversary, the only change in each of these sub-hybrids is in the way $\text{msg}_{R \rightarrow S}$ is computed for one pair $R \in \mathcal{I}, S \in \mathcal{H}$ and some $q \in [\delta] \setminus Z$. As a result, indistinguishability between each consecutive pair of sub-hybrids follows the security of OIP against a semi-honest sender and by transitivity, it holds that H_3 and H_4 are indistinguishable.

H_5 : This hybrid is identical to the simulator description.

The main difference between H_4 and H_5 is that in H_5 , simulator emulates the \mathcal{F}_{mpc} functionality and hence, the way the output of the honest parties is computed differs in the two protocols. In particular, in H_4 , we take a majority of all the runs of the online phase and only if there exists an output that appears $> \delta/4$ times, do we open the output. In H_5 , the

simulator simply checks if all the opened commitments are consistent to consider the output. Let `noAbort` denote an event where in our cut-and-choose step, all instances in Z are valid. Also let `badMaj` denote the event where $> \delta/4$ instances in the remaining set $[\delta] \setminus Z$ are invalid. To argue indistinguishability between the output of honest parties in the hybrids H_4 and H_5 , we start by recalling Claim 4.3 from [36]. At a high level, this claim essentially states that the probability that $> \delta/4$ instances are invalid and neither gets caught in the opening phase of the cut-and-choose protocol, is $1/2^{0.311\delta}$. More formally

Claim 1. For every $\delta \in \mathbb{N}$, it holds that

$$\Pr[\text{noAbort} \wedge \text{badMaj}] = \frac{\binom{\frac{3\delta}{4}+1}{\frac{\delta}{2}+1}}{\binom{\delta}{\delta/2}} < \frac{1}{2^{\frac{\delta}{4}-1}}$$

and for large enough s (depending on Stirling's approximation), it holds that $\Pr[\text{noAbort} \wedge \text{badMaj}] = \frac{1}{2^{0.311\delta}}$

From the binding property of the commitment scheme, it follows that if any instance in Z is invalid, it will get caught in both H_4 and H_5 . And if all the checks performed on these instances in Z succeed, then indeed, they are all valid/consistent and it is a `noAbort` event. In this case, in hybrid H_5 , the simulator signals the ideal functionality to send the correct output to the honest parties. Therefore, the output of the honest parties in this case will differ from their output in hybrid H_4 only when `noAbort` \wedge `badMaj` happens. For $\delta = \kappa/0.311$, this only happens with exponentially small probability and hence the output of the honest parties is indistinguishable in the two hybrids H_4 and H_5 . Indistinguishability between the view of the adversary in the two hybrids follows from malicious security of the underlying MPC protocol.

8 Constant Round Semi-Honest Branching MPC

In this section we present our constant round semi-honest protocol for distributed computation of a branching circuit.

As discussed in the technical overview, we encrypt keys for the output wires of each gate during garbling using the help of a random function instantiated using the decisional Ring LWE (RLWE) assumption. Let $p = 2N + 1$ be a prime, where N , called the dimension or security parameter, is a power of 2. Let $\mathcal{R}_p = \mathbb{Z}_p[X]/(X^N + 1)$ be the polynomial ring over \mathbb{Z}_p modulo $X^N + 1$. We start by recalling the decisional RLWE assumption stated by Ben-Efraim et al. [4].

Definition 8.1 (Decisional Ring LWE Problem). Any non-uniform PPT adversary cannot distinguish between $\{(a_i, b_i)\}_{i \in I}$ and $\{(a_i, a_i \cdot k + \delta_i)\}_{i \in I}$ with non-negligible probability where $\{a_i\}_{i \in I}$, $\{b_i\}_{i \in I}$ and k are chosen uniformly at random from \mathcal{R}_p and the coefficients of $\{\delta_i\}_{i \in I}$ are sampled from χ , a spherical Gaussian distribution.

By transforming to the Hermite normal form, the decisional RLWE assumption also holds if the key k is chosen from a spherical Gaussian distribution. Similar to the construction of Ben-Efraim et al. [4], sampling both the key and error from χ is key to eliminating the error during decryption in the evaluation phase of our protocol. Specifically, if the mean of the Gaussian distribution χ is $\frac{\sqrt{p}}{2}$ and the standard deviation is sufficiently small, a sample is *not* in the range $[0, \sqrt{p}]$ with negligible probability. Thus, dividing by \sqrt{p} during decryption removes the error and recovers the message if the message was multiplied by \sqrt{p} during encryption.

While we use new public random elements $A_g^{u,v}$ from the ring for every RLWE expansion in our protocol, [4] shows that $8 \cdot f_{\text{out}}$ uniformly random and public elements from the ring suffice, where f_{out} is the maximal fan-out of the circuit, as long as ciphertexts for gates that share inputs wires are computed using distinct sets of elements. Similar to [4], $8 \cdot f_{\text{out}}$ must be less than the bound on the number of RLWE samples $|I|$, for security to hold. We refer the reader to [37, 38] for more details about the decisional RLWE assumption.

Our protocol follows the BMR approach which involves sampling a pair of keys k_w^0, k_w^1 for each wire w in the circuit. A garbled table is then constructed for each gate such that the key corresponding to the value on the output wire is encrypted using the keys corresponding to the input values. Since the position of each ciphertext in the garbled table leaks information about its plaintext, a private random mask bit $\gamma_w \in \{0, 1\}$ is sampled for each wire w and the masks for the input wires are used to permute the rows of the garbled table for each gate. Let the external value β_w on a wire be the plaintext value ρ_w on the wire masked with the mask γ_w i.e., $\beta_w = \rho_w \oplus \gamma_w$. Then, the masks on the input wires are used to permute the rows of the garbled table such that the external values on the input wires can be used to index into the required row of the garbled table. Thus, to ensure that parties decrypt the correct row when evaluating the circuit, the mask for the output wire has to also be included in the ciphertext for each row. We use the approach of Ben-Efraim et al. [4], where the last coordinate of the keys k_w^0, k_w^1 for each wire are set to 0, which slightly reduces security, and the external value is embedded into this coordinate during encryption. We use $k||e$ to denote that the bit e was embedded in the last coordinate of the key k .

The garbling phase is presented in Figure 5 and the evaluation phase is presented in Figure 6. We adopt the same notation as the semi-honest protocol presented in Figure 2. If ℓ be the number of input wires to the branching part of the function, we set the incoming and outgoing labels for these wires to be $1, \dots, \ell$. For each gate g we set the outgoing wire label to be $\ell + g$, the left incoming wire label to be $\ell + 2g - 1$ and the right incoming wire label to be $\ell + 2g$. We also let π_m for each $m \in [k]$ to be the mapping that maps incoming labels to the outgoing labels of each wire for the m -th branch.

Finally, we remark that we require \mathcal{F}_{mpc} to run in constant number of rounds for constant depth circuits to ensure that our protocol has constant number of rounds. This is true for most secret sharing based protocols that evaluate the circuit in a gate-by-gate manner.

Complexity Analysis. We now analyze the communication complexity of the above constant round semi-honest protocol. We assume that the size of the ring \mathcal{R}_p is in $O(\lambda)$. If we use a constant rate semi-honest secure OIP, the communication complexity in the garbling phase is $O(n^2|C_{\text{max}}| + n^2k\lambda\text{CC}(\lambda|C_{\text{max}}|))$, where $|C_{\text{max}}|$ is the size of the largest branch and $\text{CC}(\lambda|C_{\text{max}}|)$ is the communication complexity incurred upon evaluating C_{max} using the underlying MPC. In the evaluation phase, the communication cost incurred is for reconstructing $O(\lambda|C_{\text{max}}|)$ shares corresponding to the garbling material.

Overall, given the above protocol and optimizations, we obtain the following result.

Theorem 3. Let λ be the security parameter and \mathcal{F} be a function class consisting of functions of the form $f(\vec{x}) = f_2(f_{\text{br}}(f_1(\vec{x})))$, where $f_{\text{br}} := \{g_1, \dots, g_k\}$ is a function consisting of k conditional branches, defined as $f_{\text{br}}(i, \vec{x}) = g_i(\vec{x})$. Assuming that a rate-1 two-message semi-honest secure OIP exists (see Definition 4.1) and that the decisional RLWE problem holds (see Definition 8.1), there exists a constant-round MPC protocol in the \mathcal{F}_{mpc} -hybrid model (see Section 5) for computing any $f \in \mathcal{F}$ that achieves semi-honest security against an arbitrary number of corruptions and incurs a communication overhead of $O(n^2\lambda(k + |C_{\text{max}}|))$.

Note that if we instantiate the rate-1 two-message semi-honest secure OIP using RLWE-based

Garbling Phase of the Constant Round Semi-Honest Protocol

The protocol is described in the \mathcal{F}_{mpc} -hybrid model which computes over \mathcal{R}_p . The parties have shares of a unary representation of the active branch, i.e., $[b_1], \dots, [b_k]$. For each gate $g \in [G]$, let $\text{left}_g = \ell + 2g - 1$ and $\text{right}_g = \ell + 2g$ be the incoming wire labels of its input wires and let $\text{out}_g = \ell + g$ be the outgoing wire.

1. **Sample masks:** For each input and gate $g \in [\ell + G]$, the parties invoke $(\text{randbit}, \gamma_g)$ in \mathcal{F}_{mpc} to obtain shares $[\gamma_g]$. For each branch $m \in [k]$, let $[\overrightarrow{\gamma_{\pi_m}}] = [\gamma_{\pi_m(1)}] \parallel \dots \parallel [\gamma_{\pi_m(W)}]$.
2. **Sample keys:** For each $g \in [\ell + G]$, and $j \in \{0, 1\}$ each party P_i (for $i \in [n]$) locally samples its share $[k_g^j]_i \leftarrow \chi^N$ and sets the last coordinate of its share to 0.
3. **Compute LWE expansions:** For each $u, v \in \{0, 1\}$, $g \in [G]$ each party P_i (for $i \in [n]$) locally samples $\delta_{m,g}^{u,v,i} \leftarrow \chi^N$ and computes $[\psi_{m,g}^{u,v}]_i = A_g^{u,v} \cdot ([k_{\pi_m(\text{left}_g)}]_i + [k_{\pi_m(\text{right}_g)}]_i) + \delta_{m,g}^{u,v,i}$. Let $[\overrightarrow{\psi_m}] = [\psi_{m,1}^{0,0}] \parallel [\psi_{m,1}^{0,1}] \parallel [\psi_{m,1}^{1,0}] \parallel [\psi_{m,1}^{1,1}] \parallel \dots \parallel [\psi_{m,G}^{0,0}] \parallel [\psi_{m,G}^{0,1}] \parallel [\psi_{m,G}^{1,0}] \parallel [\psi_{m,G}^{1,1}]$.
4. **Shares of zero:** For each $i \in [n]$ and $j \in [W + 4G]$, the parties invoke $(\text{sharezero}, X_{j,i})$ in \mathcal{F}_{mpc} to get shares $[X_{j,i}]$, where $X_{j,i} = 0$. For each $i \in [n]$, let $[\overrightarrow{X}_i] = [X_{1,i}] \parallel \dots \parallel [X_{W+4G,i}]$.
5. **Pairwise OIP:** Each pair of parties P_R and P_S ($\forall R, S \in [n]$) engage in a two-message semi-honest OIP as follows, where P_R acts as the receiver and P_S acts as the sender:
 - **Receiver:** P_R computes $(\rho, \text{msg}_R) \leftarrow \text{OIP}_R(1^\lambda, [b_1]_R, \dots, [b_k]_R)$ and sends msg_R to P_S .
 - **Sender:** For each $m \in [1, k]$ let $[\overrightarrow{x}_m] = [\overrightarrow{\gamma_{\pi_m}}] \parallel [\overrightarrow{\psi_m}]$. P_S computes $\text{msg}_S \leftarrow \text{OIP}_S(1^\lambda, \text{msg}_R, [\overrightarrow{X}_R]_S, [\overrightarrow{x}_1]_S, \dots, [\overrightarrow{x}_k]_S)$ and sends msg_S to P_R .
 - **Output:** P_R computes $\overrightarrow{\text{share}_{R,S}} \leftarrow \text{OIP}_{\text{out}}(\rho, \text{msg}_R, \text{msg}_S)$.

For each $i \in [n]$, P_i computes $[\overrightarrow{\Gamma}] \parallel [\overrightarrow{\Psi}] = \sum_{j \in [n]} \overrightarrow{\text{share}_{j,i}}$ where $\overrightarrow{\Gamma} = \Gamma_1 \parallel \dots \parallel \Gamma_W$ and $\overrightarrow{\Psi} = \Psi_1^{0,0} \parallel \Psi_1^{0,1} \parallel \Psi_1^{1,0} \parallel \Psi_1^{1,1} \parallel \dots \parallel \Psi_G^{0,0} \parallel \Psi_G^{0,1} \parallel \Psi_G^{1,0} \parallel \Psi_G^{1,1}$.

6. **Garble active branch:** Let $\text{type}_{m,g}$ be the gate type for gate g in C_m ($\forall m \in [k]$), where $\text{type}_{m,g} = 0$ denotes an XOR gate and $\text{type}_{m,g} = 1$ denotes an AND gate. Parties do the following for each $g \in [G]$
 - (a) Compute $[\text{type}_g] = \sum_{m=1}^k (\text{type}_{m,g} \cdot [b_m])$.
 - (b) For each $u, v \in \{0, 1\}$ let $e_{u,v,g}^{\text{xor}} = u \oplus \Gamma_{\text{left}_g} \oplus v \oplus \Gamma_{\text{right}_g} \oplus \gamma_{\text{out}_g}$, $e_{u,v,g}^{\text{and}} = ((u \oplus \Gamma_{\text{left}_g}) \wedge (v \oplus \Gamma_{\text{right}_g})) \oplus \gamma_{\text{out}_g}$, $e_g^{u,v} = \text{type}_g (e_{u,v,g}^{\text{and}} - e_{u,v,g}^{\text{xor}}) + e_{u,v,g}^{\text{xor}}$ and $K_g^{u,v} = e_g^{u,v} (k_{\text{out}_g}^1 - k_{\text{out}_g}^0) + k_{\text{out}_g}^0$. For each $u, v \in \{0, 1\}$, compute $[K_g^{u,v} \parallel e_g^{u,v}]$ using \mathcal{F}_{mpc} .
 - (c) For each $u, v \in \{0, 1\}$ compute $[C_g^{u,v}] = [\Psi_g^{u,v}] + \lceil \sqrt{p} \rceil [K_g^{u,v} \parallel e_g^{u,v}]$.

Figure 5: Garbling phase of the constant round (semi-honest) protocol

linearly homomorphic encryption, then the above theorem yields a protocol that only relies on the hardness of the decisional RLWE.

8.1 Security

We now prove security of our constant round semi-honest protocol. We start by describing the simulator and then proceed to argue indistinguishability between the real and ideal world executions.

Evaluation Phase of the Constant Round Semi-Honest Protocol

The protocol is described in the \mathcal{F}_{mpc} -hybrid model. The parties have shares of the inputs to the branches, i.e., $[\text{inp}_1], \dots, [\text{inp}_\ell]$ and shares of a unary representation of the active branch, i.e., $[b_1], \dots, [b_k]$.

1. For each input wire $w \in [\ell]$ parties compute $[\beta_w] = [\text{inp}_w] \oplus [\gamma_w]$ and invoke $(\text{out}, [\beta_w])$ in \mathcal{F}_{mpc} to obtain β_w . For each $w \in [\ell]$, let $\beta_{1,w} = \dots = \beta_{k,w} = \beta_w$.
2. For each input wire $w \in [\ell]$ parties invoke $(\text{out}, [k_w^{\beta_w}])$ in \mathcal{F}_{mpc} to obtain $k_w^{\beta_w}$. For each $w \in [\ell]$, let $K_{1,w}^{\beta_w} = \dots = K_{k,w}^{\beta_w} = k_w^{\beta_w}$.
3. For each $u, v \in \{0, 1\}$ and $g \in [G]$ parties invoke $(\text{out}, [C_g^{u,v}])$ in \mathcal{F}_{mpc} to obtain $C_g^{u,v}$.
4. For each $m \in [k]$ and $g \in [G]$, parties compute $C_g^{u,v} - A_g^{u,v} \cdot (K_{m,\pi_m(\text{left}_g)}^u + K_{m,\pi_m(\text{right}_g)}^v)$, where $u = \beta_{m,\pi_m(\text{left}_g)}$ and $v = \beta_{m,\pi_m(\text{right}_g)}$, and divide it by $[\sqrt{p}]$ to remove the error and recover $K_{m,\text{out}_g}^{\beta_{m,\text{out}_g}} \parallel \beta_{m,\text{out}_g}$.
5. For each output gate g , parties compute $[z_g] = \sum_{m=1}^k \beta_{m,\text{out}_g} [b_m] \oplus [\gamma_g]$ using \mathcal{F}_{mpc} .

Figure 6: Evaluation phase of the constant round (semi-honest) protocol

Simulator. Let \mathcal{A} be the adversary who corrupts a subset $\mathcal{I} \subset [n]$ of the parties and $\mathcal{H} = [n] \setminus \mathcal{I}$ be the set of honest parties. Let \mathcal{S}_{R} be the simulator associated with the semi-honest security against receiver of the OIP. Given the output z and inputs $\{x_i\}_{i \in \mathcal{I}}$ of the corrupt parties the simulator proceeds as follows:

- **Computing f_1 .** For each $i \in \mathcal{I}$, $j \in [\ell]$, the simulator samples random share $[\text{inp}_j]_i$ and for each $m \in [k]$, it samples random share $[b_m]_i$ and sends all these shares to the adversary.
- **Garbling phase.**
 - For each input and gate $g \in [\ell + G]$, the simulator samples random $[\gamma_g]_i \leftarrow \mathcal{R}_p$ for each $i \in \mathcal{I}$ and sends it to the adversary.
 - For each $j \in [n]$, $w \in [W + 4G]$, and $i \in \mathcal{I}$, the simulator samples random $[X_{j,w}]_i \leftarrow \mathcal{R}_p$ and sends it to the adversary.
 - For each pair of parties P_{R} and P_{S} ($\forall \text{R} \in \mathcal{H}, \text{S} \in \mathcal{I}$), the simulator sets $[b_1]_{\text{R}} = \dots = [b_k]_{\text{R}} = 0$, computes $\rho, \text{msg}_{\text{R}} \leftarrow \text{OIP}_{\text{R}}(1^\lambda, [b_1]_{\text{R}}, \dots, [b_k]_{\text{R}})$ and sends msg_{R} to the adversary on behalf of honest P_{R} .
 - For each pair of parties P_{R} and P_{S} ($\forall \text{R} \in \mathcal{I}, \text{S} \in \mathcal{H}$), upon receiving a message msg_{R} from the adversary, the simulator samples a random vector of shares $\vec{V}_{\text{R},\text{S}}$ and computes $\text{msg}_{\text{S}} \leftarrow \mathcal{S}_{\text{R}}(1^\lambda, \text{msg}_{\text{R}}, \vec{V}_{\text{R},\text{S}})$. It sends msg_{S} to the adversary on behalf of honest P_{S} .
 - When garbling the active branch, the simulator mimics the computation done by \mathcal{F}_{mpc} .
- **Evaluation phase.**
 - For each input wire $w \in [\ell]$, the simulator samples random $\beta_w \leftarrow \{0, 1\}$, and sends it to the adversary.

- For each input wire $w \in [\ell]$, the simulator samples random $k_w^{\beta_w} \leftarrow \mathcal{R}_p$, and also sends it to the adversary.
- For each $g \in [G]$ and $u, v \in \{0, 1\}$, the simulator samples random $C_g^{u,v} \leftarrow \mathcal{R}_p$, randomly sets the last coordinate to either 0 or 1 and sends it to the adversary.

Indistinguishability Argument. We argue indistinguishability via the following sequence of hybrids:

H_0 : This hybrid is identical to the real world execution.

H_1 : This hybrid is very similar to the previous hybrid except that in the garbling phase for each pair of parties P_R and P_S ($\forall R \in \mathcal{H}, S \in \mathcal{I}$), we change the way msg_R and $\overrightarrow{\text{share}}_{R,S}$ are computed:

- $\rho, \text{msg}_R \leftarrow \text{OIP}_R(1^\lambda, 0 \dots, 0)$.
- $\overrightarrow{\text{share}}_{R,S} = [\overrightarrow{X}_R]_S + \sum_{i \in [k]} [b_i]_R [\overrightarrow{x}_i]_S$.

For indistinguishability between hybrids H_0 and H_1 , we consider a sequence of sub-hybrids, where we change the way msg_R and $\overrightarrow{\text{share}}_{R,S}$ are computed for each pair P_R and P_S (where $R \in \mathcal{H}, S \in \mathcal{I}$), one hybrid at a time. In terms of the view of the adversary, the only change in each of these sub-hybrids is in the way msg_R is computed for one pair $R \in \mathcal{I}, S \in \mathcal{H}$. As a result, indistinguishability between each consecutive pair of sub-hybrids follows the security of OIP against a semi-honest sender and by transitivity, it holds that H_0 and H_1 are indistinguishable.

H_2 : This hybrid is very similar to the previous hybrid except that in the garbling phase, for each pair of parties P_R and P_S ($\forall R \in \mathcal{I}, S \in \mathcal{H}$), we compute $\text{msg}_S \leftarrow \mathcal{S}_R(1^\lambda, \text{msg}_R, \overrightarrow{V}_{R,S})$ for some random vector of shares $\overrightarrow{V}_{R,S}$.

The only difference between any two consecutive pairs of these sub-hybrids is that in one we compute msg_S using a random vector of shares $\overrightarrow{V}_{R,S}$ and the simulator for some receiver R and sender S , while in the other msg_S is computed honestly and the output that the receiver gets is $[\overrightarrow{X}_R]_S + \sum_{i \in [k]} [b_i]_R [\overrightarrow{x}_i]_S$. Since $[\overrightarrow{X}_R]_S$ is a random vector of shares, this output is identically distributed to $\overrightarrow{V}_{R,S}$. Given this output indistinguishability between a simulated message msg_S , and an honestly computed message msg_S follows from security of OIP against a semi-honest receiver. As a result, this sub-hybrid is indistinguishable from its previous hybrid and by transitivity, it holds that H_1 and H_2 are indistinguishable.

H_3 : This hybrid is similar to the previous hybrid except that $C_g^{u,v}$ is sampled uniformly at random from \mathcal{R}_p for all $g \in [G]$ and $u, v \in \{0, 1\}$.

Indistinguishability between hybrids H_2 and H_3 follows from a sequence of sub-hybrids, each relying on the decisional RLWE hardness assumption, where we change $C_g^{u,v}$ one at a time.

H_4 : This hybrid is identical to the simulator description.

Indistinguishability between hybrids H_3 and H_4 follows from the semi-honest security of the underlying MPC protocol.

9 OIP from Linearly Homomorphic Encryption

In this section, we show how to construct OIPs from linearly homomorphic encryption.

9.1 Linearly Homomorphic Encryption

We start by recalling the definition of linearly homomorphic encryption.

Definition 9.1 (Linearly Homomorphic Encryption). A linearly homomorphic encryption scheme over a message space \mathcal{M} is defined by a tuple of 4 PPT algorithms (KGen, Enc, Dec, Eval) as follows:

- $(\mathbf{pk}, \mathbf{sk}) \leftarrow \text{KGen}(1^\lambda)$: On input the security parameter λ , the key generation algorithm outputs a public key \mathbf{pk} and a secret key \mathbf{sk} .
- $c \leftarrow \text{Enc}(\mathbf{pk}, x)$: On input the public key \mathbf{pk} and a message $x \in \mathcal{M}$, the encryption algorithm outputs a ciphertext c .
- $x \leftarrow \text{Dec}(\mathbf{sk}, c)$: On input the secret key \mathbf{sk} and the ciphertext c , the decryption algorithm outputs a message $m \in \mathcal{M}$.
- $c_L \leftarrow \text{Eval}(\mathbf{pk}, L, c_1, \dots, c_k)$: On input the public key \mathbf{pk} , a set of ciphertexts c_1, \dots, c_k and a linear function $L : \mathcal{M}^k \rightarrow \mathcal{M}$, the evaluation algorithm outputs another ciphertext c_L .

We proceed to define three main properties of a linearly homomorphic encryption scheme: *correctness*, *homomorphism* and *privacy*.

- **Correctness:** Let $(\mathbf{pk}, \mathbf{sk}) \leftarrow \text{KGen}(1^\lambda)$. Then for any $x \in \mathcal{M}$, it holds that:

$$\Pr [\text{Dec}(\mathbf{sk}, \text{Enc}(\mathbf{pk}, x)) = x] = 1$$

- **Homomorphism:** Let $(\mathbf{pk}, \mathbf{sk}) \leftarrow \text{KGen}(1^\lambda)$. Then for any $x_1, \dots, x_k \in \mathcal{M}^k$ and any linear function $L : \mathcal{M}^k \rightarrow \mathcal{M}$, it holds that:

$$\Pr [\text{Dec}(\mathbf{sk}, \text{Eval}(\mathbf{pk}, L, \{\text{Enc}(\mathbf{pk}, x_i)\}_{i \in [k]})) = L(x_1, \dots, x_k)] = 1$$

- **Circuit Privacy:** There exists a PPT simulator S , such that for every PPT adversary \mathcal{A} with inputs $x_1, \dots, x_k \in \mathcal{M}^k$ and any linear function $L : \mathcal{M}^k \rightarrow \mathcal{M}$ the following distributions are computationally indistinguishable:

$$\text{Eval}(\mathbf{pk}, L, \{c_i\}_{i \in [k]}) \approx_c S(1^\lambda, \mathbf{pk}, L(\{x_i\}_{i \in [k]})),$$

where $(\mathbf{pk}, \mathbf{sk}) \leftarrow \text{KGen}(1^\lambda)$ and $c_i \leftarrow \text{Enc}(\mathbf{pk}, x_i)$ for each $i \in [k]$ were generated honestly by \mathcal{A} .

- **Privacy:** For all n.u. PPT adversaries \mathcal{A} , there exists a negligible function $\mu(\cdot)$ such that:

$$\Pr \left[\begin{array}{l} (\mathbf{pk}, \mathbf{sk}) \leftarrow \text{KGen}(1^\lambda), \\ (x_0, x_1) \leftarrow \mathcal{A}(1^\lambda, \mathbf{pk}), \quad : \mathcal{A}(\text{Enc}(\mathbf{sk}, x_b)) = b \\ b \stackrel{\$}{\leftarrow} \{0, 1\} \end{array} \right] \leq \frac{1}{2} + \mu(\lambda)$$

where $|x_0| = |x_1|$.

Such linearly homomorphic encryption can be obtained from a variety of assumptions [17, 8, 15, 42]. In our implementation we use a variant of the BFV scheme [17].

Two-message Semi-honest OIP from Linearly Homomorphic encryption.

Let $(\text{KGen}, \text{Enc}, \text{Dec}, \text{Eval})$ be a linearly homomorphic encryption. The parties proceed as follows:

Receiver: The receiver with inputs $b_1, \dots, b_k \in \mathcal{M}$, proceeds as follows:

- It computes $(\text{pk}, \text{sk}) \leftarrow \text{KGen}(1^\lambda)$.
- For each $i \in [k]$, it computes $c_i \leftarrow \text{Enc}(\text{pk}, b_i)$.
- It sends $\text{msg}_R = (\text{pk}, c_1, \dots, c_k)$ to the sender.

Sender: The sender with inputs $\vec{m}_0, \dots, \vec{m}_k \in \mathcal{M}^{m \times (k+1)}$, parses $\text{msg}_R = (\text{pk}, c_1, \dots, c_k)$ and computes the following for each $j \in [m]$:

- It computes $c_0 \leftarrow \text{Enc}(\text{pk}, m_0[j])$.
- It sets L_j to be the linear function $L_j(x_1, \dots, x_k) = \sum_{i \in [k]} x_i \cdot \vec{m}_i[j]$, where $\vec{m}_i[j]$ is the j^{th} element in the vector \vec{m}_i .
- It computes $c_{L_j} \leftarrow \text{Eval}(\text{pk}, L_j, c_1, \dots, c_k) + c_0$.

It sends $\text{msg}_S = (c_{L_1}, \dots, c_{L_m})$ to the receiver.

Output: The receiver parses $\text{msg}_S = (c_{L_1}, \dots, c_{L_m})$ and for each $j \in [m]$, it computes $\vec{x}[j] \leftarrow \text{Dec}(\text{sk}, c_{L_j})$ and outputs \vec{x} .

Figure 7

9.2 Constructing OIP

We now describe a simple construction of a semi-honest OIP using linearly homomorphic encryption.

Lemma 1. Let $(\text{KGen}, \text{Enc}, \text{Dec}, \text{Eval})$ be a linearly homomorphic encryption, then the construction in Figure 7 is a two-message semi-honest oblivious inner product protocol.

Proof. Correctness of this construction follows trivially from the correctness and homomorphic property of the underlying linearly homomorphic encryption scheme. Security against semi-honest sender follows from privacy of the encryption scheme and security against semi-honest receiver follows from circuit privacy. \square

Remark. Note that if the linearly homomorphic encryption scheme has a constant rate, then the length of the sender message in the above construction of OIP only depends on the length of the output and not on the inputs of the sender. Also, the above construction is only semi-honest secure. For our maliciously secure MPC, we also require an OIP that is secure against a malicious receiver. Such an OIP can be easily constructed by attaching non-interactive zero-knowledge proofs of knowledge to the receiver messages.

10 Implementation

We implement and benchmark our semi-honest non-constant round protocol from Section 6. The code is publicly available at <https://github.com/rot256/research-branching-mpc>. In addition to the code and instructions used for benchmarking, the repository also contains the raw data used in this paper and scripts used to create the plots.

10.1 How We Benchmark

Underlying MPC. We implement our semi-honest compiler on top of two different multi-party computation protocols.

1. *Quadratic Dependence on the Number of Parties.* A semi-honest variant of MASCOT [34] (MASCOT without sacrificing and message authentication codes) over the prime field $\mathbb{F}_{2^{16}+1} = \mathbb{Z}/(0x10001 \mathbb{Z})$ provided by MP-SPDZ [33] (called “`semi-party.x`”). We simply invoke the MP-SPDZ implementation as a black-box: wrapping each instance of “`semi-party.x`” in a program which provides provides inputs/outputs to the party. Since MP-SPDZ provides a universal interface our implementation is agnostic with regards to the underlying MPC implementation: any reactive MPC in MP-SPDZ which allows computation over $\mathbb{F}_{2^{16}+1}$ could be swapped in with ease.
2. *Linear Dependence on the Number of Parties.* A batched semi-honest version of CDN [13] where we instantiate the linearly homomorphic encryption using the same ring LWE parameters described above. We implement this ourselves again using the Lattigo (more information below) library for the RLWE components.

CDN Implementation. We implement a semi-honest batched version of CDN, instantiating the linearly homomorphic encryption using the same parameters described above (the same as the OIP). To reduce the overhead (computational/communication) induced by the homomorphic encryption we execute multiplications in batches of 2^{12} (the dimension of the ring used for RLWE), by packing 2^{12} independent shares (over $0x10001$) into a single ciphertext and execute the CDN multiplication protocol on these in parallel. The decryption threshold is the full set of parties. The CDN implement is included in the same repository. To the best of our knowledge, this is the first known implementation of CDN.

Instantiating OIP and Ring LWE Parameters. In our implementation, we use an optimized version of OIP. We observe that the $O(n^2)$ overhead incurred from the use of pairwise-OIPs can be driven down to $O(n)$, if instead of a regular linearly homomorphic encryption, we use a threshold linearly homomorphic encryption (TLHE). TLHE are linearly homomorphic encryptions that comprise of a single public-key and where each party holds a “share” of the secret key. This share of the secret key can be used by the parties to decrypt to a share of the plaintext. As shown in [10], the keys for RLWE based threshold linearly homomorphic encryption can be setup very efficiently by the parties in a couple of rounds. At a high level, this observation allows us to reuse the sender and receiver messages of each party across multiple OIP instantiations and as a result, overall, each party only needs to send one receiver message and one sender message.

Recall that in our semi-honest protocol, the receiver and sender messages in all OIP instances are computed using the same shares of the index associated with the active branch and the masks. Each party can compute its receiver message by encrypting its shares of b_1, \dots, b_k . Similarly, for the sender message, each party can compute an inner-product of these encryptions received from all parties and its shares of the permuted masks. Finally, all parties can add all the sender messages (which are also ciphertexts) received from all parties. This gives them an encryption of the permuted masks for the active branch. Now each party can run threshold decryption using its share of the secret-key to obtain a sharing of the resulting inner-product.

We use BFV [17] over a cyclotomic ring of index 2^{13} and dimension 2^{12} , i.e. $R[X]/(X^{2^{12}} + 1)$ where: $Q_1 := 0x7ffffec001, Q_2 := 0x8000016001, P := 0x40002001, N := Q_1 Q_2 P, R := \mathbb{Z}/(N\mathbb{Z})$. This gives us a linearly homomorphic encryption scheme for vectors $\vec{v} \in (\mathbb{F}_{2^{16}+1})^{2^{12}}$,

which additionally allows (full) threshold decryption. We use the `Lattigo` [1] library to implement all the RLWE components.

Benchmarking Platform. All benchmarks were run on a laptop with an Intel i7-11800H CPU (@ 2.3 GHz) and 64 GB of RAM. All networking is over the loopback interface and network latency was simulated using traffic control (`tc`) on Linux. We also do not restrict the bandwidth when comparing running times – note that this constitutes a relative “worst-case scenario” for our results: as our technique reduces communication, the relative performance gain for many branches would only increase by restricting bandwidth.

How The Branches Were Generated. During our benchmark each branch contained 2^{16} uniformly random gates: each gate is a multiplication/addition gate with probability $1/2$. We benchmark using “layered circuits”, meaning each level contains 2^{12} gates which can be evaluated in parallel (to reduce the number of rounds). Subject to the layering constraint, the wiring is otherwise random: the inputs to each gate are sampled uniformly at random from all previous outputs (not just those in the last layer). We believe this distribution over circuits form a realistic benchmark for the expected performance across many real-world applications.

Averaging. We run all benchmarks 10 times and take the average.

10.2 Comparison of Communication Complexity

In Fig. 8 and Fig. 9, we compare the communication complexity of our technique to the naïve baseline solution of evaluating each branch in parallel using the underlying MPC. For the baseline solution we do not consider the additional overhead of multiplexing the output, i.e., selecting the output of the active branch.

We observe that our technique improves communication over the baseline for both CDN and MASCOT with 3 parties when the number of branches is ≥ 8 . For less than 8 branches the communication overhead of the RLWE-based OIP and the need to evaluate universal gates (requiring the base-MPC to compute 3 multiplications) outweighs the communication saving of only executing the active branch. Upon reflection 8 branches is about the lowest number of branches we could hope to see savings for: recall that each branch contains $\approx 2^{15}$ multiplications³, therefore the parallel execution of 6 branches requires the same number of multiplications as that of the 2^{16} universal gates used in our technique. As expected we also observe that the communication of our technique remains (nearly⁴) constant for any number of branches.

Lastly we fix the number of branches to 16 and plot (in Fig. 10) the communication complexity of our technique for a varying number of parties, as expected the communication of our compiler applied to MASCOT increases quadratically, while our technique preserves the linearly increasing communication of CDN; constant per-party communication (and computation).

10.3 Comparison of Running Time

From Fig. 8 and Fig. 9, we observe that for sufficiently many branches our technique also reduces running time over the baseline for both CDN and semi-honest MASCOT. This is also expected: after the relatively high constant overhead of our technique, the marginal cost of adding another branch (of length ℓ) is that of: (1) $O(\ell)$ linear operations in the underlying MPC. (2) $O(\ell)$

³Since the type of each gate in each branch is sampled uniformly at random.

⁴It grows slightly, since the unary representation of the selection wire must be shared/computed. However the computation of the branch completely dominates the communication.

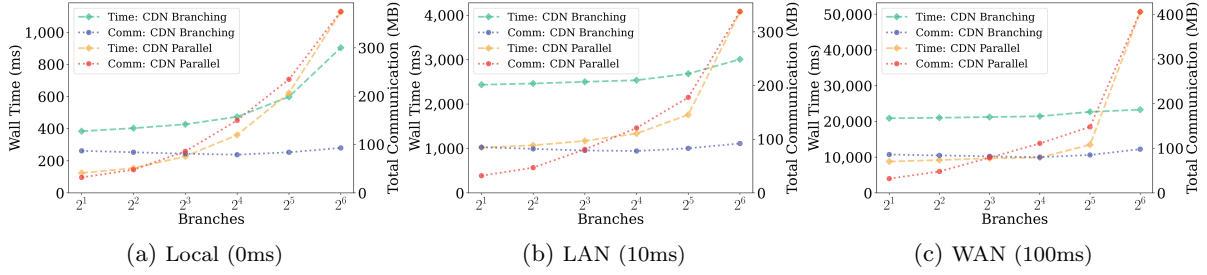


Figure 8: Running time of Branching MPC with CDN.

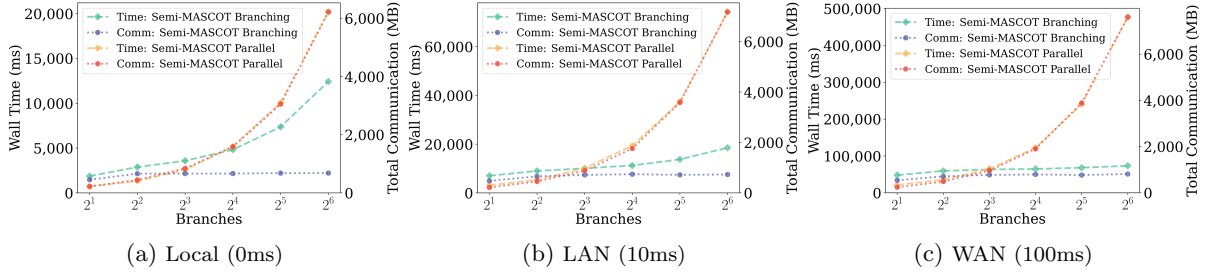


Figure 9: Running time of Branching MPC with Semi-Honest MASCOT.

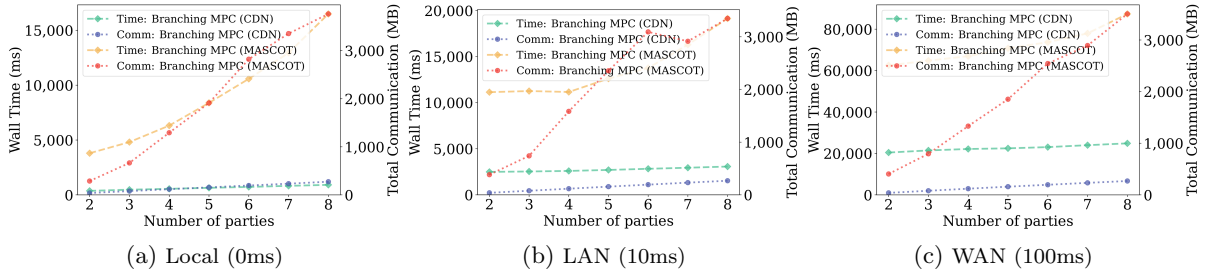


Figure 10: Running time of Branching MPC for Different Number of Parties.

$\langle ciphertext \rangle \times \langle plaintext \rangle$ operations in the RLWE based homomorphic encryption scheme. (3) $O(\ell)$ $\langle ciphertext \rangle + \langle ciphertext \rangle$ operations in the RLWE based homomorphic encryption scheme.

The first one introduces a very small cost (essentially that of reading the branch), the second is dominated by the cost of doing a number theoretic transform (NTT) on the plaintext (the players local share), which again is essentially that of computing a small constant number of fixed-size FFTs. We note that the NTTs are computed on random shares and could be relegated to a pre-computation phase. The final ciphertext/ciphertext addition is just a constant number of entry-wise additions of vectors in a small prime field – the cost of which is miniscule. Looking at Fig. 8 and Fig. 9 we observe that this marginal computational cost (of doing NTTs) has a higher influence when the network latency is low and quickly becomes insignificant as the latency increases.

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References

- [1] Lattigo v2.2.0. Online: <http://github.com/ldsec/lattigo>, July 2021. EPFL-LDS.
- [2] Donald Beaver, Silvio Micali, and Phillip Rogaway. The round complexity of secure protocols (extended abstract). In *22nd ACM STOC*, pages 503–513. ACM Press, May 1990.
- [3] Gabrielle Beck, Aarushi Goel, Abhishek Jain, and Gabriel Kaptchuk. Order-C secure multi-party computation for highly repetitive circuits. In Anne Canteaut and François-Xavier Standaert, editors, *EUROCRYPT 2021, Part II*, volume 12697 of *LNCS*, pages 663–693. Springer, Heidelberg, October 2021.
- [4] Aner Ben-Efraim, Yehuda Lindell, and Eran Omri. Efficient scalable constant-round MPC via garbled circuits. In Tsuyoshi Takagi and Thomas Peyrin, editors, *ASIACRYPT 2017, Part II*, volume 10625 of *LNCS*, pages 471–498. Springer, Heidelberg, December 2017.
- [5] Michael Ben-Or, Shafi Goldwasser, and Avi Wigderson. Completeness theorems for non-cryptographic fault-tolerant distributed computation (extended abstract). In *20th ACM STOC*, pages 1–10. ACM Press, May 1988.
- [6] Dan Boneh, Kevin Lewi, Hart William Montgomery, and Ananth Raghunathan. Key homomorphic PRFs and their applications. In Ran Canetti and Juan A. Garay, editors, *CRYPTO 2013, Part I*, volume 8042 of *LNCS*, pages 410–428. Springer, Heidelberg, August 2013.
- [7] Elette Boyle, Niv Gilboa, and Yuval Ishai. Breaking the circuit size barrier for secure computation under DDH. In Matthew Robshaw and Jonathan Katz, editors, *CRYPTO 2016, Part I*, volume 9814 of *LNCS*, pages 509–539. Springer, Heidelberg, August 2016.
- [8] Guilhem Castagnos and Fabien Laguillaumie. Linearly homomorphic encryption from DDH. In Kaisa Nyberg, editor, *CT-RSA 2015*, volume 9048 of *LNCS*, pages 487–505. Springer, Heidelberg, April 2015.
- [9] David Chaum, Claude Crépeau, and Ivan Damgård. Multiparty unconditionally secure protocols (abstract) (informal contribution). In Carl Pomerance, editor, *CRYPTO’87*, volume 293 of *LNCS*, page 462. Springer, Heidelberg, August 1988.
- [10] Megan Chen, Carmit Hazay, Yuval Ishai, Yuriy Kashnikov, Daniele Micciancio, Tarik Riviere, abhi shelat, Muthu Venkitasubramaniam, and Ruihan Wang. Diogenes: Lightweight scalable RSA modulus generation with a dishonest majority. Cryptology ePrint Archive, Report 2020/374, 2020. <https://eprint.iacr.org/2020/374>.
- [11] Koji Chida, Daniel Genkin, Koki Hamada, Dai Ikarashi, Ryo Kikuchi, Yehuda Lindell, and Ariel Nof. Fast large-scale honest-majority MPC for malicious adversaries. In Hovav Shacham and Alexandra Boldyreva, editors, *CRYPTO 2018, Part III*, volume 10993 of *LNCS*, pages 34–64. Springer, Heidelberg, August 2018.

- [12] Ronald Cramer, Ivan Damgård, Daniel Escudero, Peter Scholl, and Chaoping Xing. SPDZ2k: Efficient MPC mod 2^k for dishonest majority. Cryptology ePrint Archive, Report 2018/482, 2018. <https://eprint.iacr.org/2018/482>.
- [13] Ronald Cramer, Ivan Damgård, and Jesper Buus Nielsen. Multiparty computation from threshold homomorphic encryption. In Birgit Pfitzmann, editor, *EUROCRYPT 2001*, volume 2045 of *LNCS*, pages 280–299. Springer, Heidelberg, May 2001.
- [14] Ivan Damgård and Yuval Ishai. Constant-round multiparty computation using a black-box pseudorandom generator. In Victor Shoup, editor, *CRYPTO 2005*, volume 3621 of *LNCS*, pages 378–394. Springer, Heidelberg, August 2005.
- [15] Ivan Damgård and Mats Jurik. A generalisation, a simplification and some applications of Paillier’s probabilistic public-key system. In Kwangjo Kim, editor, *PKC 2001*, volume 1992 of *LNCS*, pages 119–136. Springer, Heidelberg, February 2001.
- [16] Ivan Damgård, Valerio Pastro, Nigel P. Smart, and Sarah Zakarias. Multiparty computation from somewhat homomorphic encryption. In Reihaneh Safavi-Naini and Ran Canetti, editors, *CRYPTO 2012*, volume 7417 of *LNCS*, pages 643–662. Springer, Heidelberg, August 2012.
- [17] Junfeng Fan and Frederik Vercauteren. Somewhat practical fully homomorphic encryption. Cryptology ePrint Archive, Report 2012/144, 2012. <https://eprint.iacr.org/2012/144>.
- [18] Tore Kasper Frederiksen, Marcel Keller, Emmanuela Orsini, and Peter Scholl. A unified approach to mpc with preprocessing using ot. In Tetsu Iwata and Jung Hee Cheon, editors, *Advances in Cryptology – ASIACRYPT 2015*, pages 711–735, Berlin, Heidelberg, 2015. Springer Berlin Heidelberg.
- [19] Craig Gentry. Fully homomorphic encryption using ideal lattices. In Michael Mitzenmacher, editor, *41st ACM STOC*, pages 169–178. ACM Press, May / June 2009.
- [20] Oded Goldreich. *Foundations of Cryptography: Basic Applications*, volume 2. Cambridge University Press, Cambridge, UK, 2004.
- [21] Oded Goldreich, Silvio Micali, and Avi Wigderson. How to play any mental game or A completeness theorem for protocols with honest majority. In Alfred Aho, editor, *19th ACM STOC*, pages 218–229. ACM Press, May 1987.
- [22] S. Dov Gordon, Daniel Starin, and Arkady Yerukhimovich. The more the merrier: Reducing the cost of large scale MPC. In Anne Canteaut and François-Xavier Standaert, editors, *EUROCRYPT 2021, Part II*, volume 12697 of *LNCS*, pages 694–723. Springer, Heidelberg, October 2021.
- [23] Vipul Goyal, Hanjun Li, Rafail Ostrovsky, Antigoni Polychroniadou, and Yifan Song. ATLAS: Efficient and scalable MPC in the honest majority setting. In Tal Malkin and Chris Peikert, editors, *CRYPTO 2021, Part II*, volume 12826 of *LNCS*, pages 244–274, Virtual Event, August 2021. Springer, Heidelberg.
- [24] Vipul Goyal, Antigoni Polychroniadou, and Yifan Song. Unconditional communication-efficient MPC via hall’s marriage theorem. In Tal Malkin and Chris Peikert, editors, *CRYPTO 2021, Part II*, volume 12826 of *LNCS*, pages 275–304, Virtual Event, August 2021. Springer, Heidelberg.

- [25] Vipul Goyal and Yifan Song. Malicious security comes free in honest-majority MPC. Cryptology ePrint Archive, Report 2020/134, 2020. <https://eprint.iacr.org/2020/134>.
- [26] Carmit Hazay, Emmanuela Orsini, Peter Scholl, and Eduardo Soria-Vazquez. Concretely efficient large-scale MPC with active security (or, TinyKeys for TinyOT). In Thomas Peyrin and Steven Galbraith, editors, *ASIACRYPT 2018, Part III*, volume 11274 of *LNCS*, pages 86–117. Springer, Heidelberg, December 2018.
- [27] David Heath and Vladimir Kolesnikov. Stacked garbling - garbled circuit proportional to longest execution path. In Daniele Micciancio and Thomas Ristenpart, editors, *CRYPTO 2020, Part II*, volume 12171 of *LNCS*, pages 763–792. Springer, Heidelberg, August 2020.
- [28] David Heath and Vladimir Kolesnikov. LogStack: Stacked garbling with $O(b \log b)$ computation. In Anne Canteaut and François-Xavier Standaert, editors, *EUROCRYPT 2021, Part III*, volume 12698 of *LNCS*, pages 3–32. Springer, Heidelberg, October 2021.
- [29] David Heath, Vladimir Kolesnikov, and Stanislav Peceny. MOTIF: (almost) free branching in GMW - via vector-scalar multiplication. In Shiho Moriai and Huaxiong Wang, editors, *ASIACRYPT 2020, Part III*, volume 12493 of *LNCS*, pages 3–30. Springer, Heidelberg, December 2020.
- [30] David Heath, Vladimir Kolesnikov, and Stanislav Peceny. Garbling, stacked and staggered - faster k-out-of-n garbled function evaluation. In Mehdi Tibouchi and Huaxiong Wang, editors, *Advances in Cryptology - ASIACRYPT 2021 - 27th International Conference on the Theory and Application of Cryptology and Information Security, Singapore, December 6-10, 2021, Proceedings, Part II*, volume 13091 of *Lecture Notes in Computer Science*, pages 245–274. Springer, 2021.
- [31] David Heath, Vladimir Kolesnikov, and Stanislav Peceny. Masked triples - amortizing multiplication triples across conditionals. In Juan Garay, editor, *PKC 2021, Part II*, volume 12711 of *LNCS*, pages 319–348. Springer, Heidelberg, May 2021.
- [32] Jonathan Katz and Lior Malka. Constant-round private function evaluation with linear complexity. In Dong Hoon Lee and Xiaoyun Wang, editors, *ASIACRYPT 2011*, volume 7073 of *LNCS*, pages 556–571. Springer, Heidelberg, December 2011.
- [33] Marcel Keller. MP-SPDZ: A versatile framework for multi-party computation. In *Proceedings of the 2020 ACM SIGSAC Conference on Computer and Communications Security*, 2020.
- [34] Marcel Keller, Emmanuela Orsini, and Peter Scholl. MASCOT: Faster malicious arithmetic secure computation with oblivious transfer. In Edgar R. Weippl, Stefan Katzenbeisser, Christopher Kruegel, Andrew C. Myers, and Shai Halevi, editors, *ACM CCS 2016*, pages 830–842. ACM Press, October 2016.
- [35] Marcel Keller, Valerio Pastro, and Dragos Rotaru. Overdrive: Making SPDZ great again. In Jesper Buus Nielsen and Vincent Rijmen, editors, *EUROCRYPT 2018, Part III*, volume 10822 of *LNCS*, pages 158–189. Springer, Heidelberg, April / May 2018.
- [36] Yehuda Lindell and Benny Pinkas. Secure two-party computation via cut-and-choose oblivious transfer. In Yuval Ishai, editor, *TCC 2011*, volume 6597 of *LNCS*, pages 329–346. Springer, Heidelberg, March 2011.

- [37] Vadim Lyubashevsky, Chris Peikert, and Oded Regev. On ideal lattices and learning with errors over rings. In Henri Gilbert, editor, *EUROCRYPT 2010*, volume 6110 of *LNCS*, pages 1–23. Springer, Heidelberg, May / June 2010.
- [38] Vadim Lyubashevsky, Chris Peikert, and Oded Regev. A toolkit for ring-LWE cryptography. In Thomas Johansson and Phong Q. Nguyen, editors, *EUROCRYPT 2013*, volume 7881 of *LNCS*, pages 35–54. Springer, Heidelberg, May 2013.
- [39] Payman Mohassel and Seyed Saeed Sadeghian. How to hide circuits in MPC an efficient framework for private function evaluation. In Thomas Johansson and Phong Q. Nguyen, editors, *EUROCRYPT 2013*, volume 7881 of *LNCS*, pages 557–574. Springer, Heidelberg, May 2013.
- [40] Payman Mohassel, Seyed Saeed Sadeghian, and Nigel P. Smart. Actively secure private function evaluation. In Palash Sarkar and Tetsu Iwata, editors, *ASIACRYPT 2014, Part II*, volume 8874 of *LNCS*, pages 486–505. Springer, Heidelberg, December 2014.
- [41] Moni Naor, Benny Pinkas, and Omer Reingold. Distributed pseudo-random functions and KDCs. In Jacques Stern, editor, *EUROCRYPT'99*, volume 1592 of *LNCS*, pages 327–346. Springer, Heidelberg, May 1999.
- [42] Chris Peikert, Vinod Vaikuntanathan, and Brent Waters. A framework for efficient and composable oblivious transfer. In David Wagner, editor, *CRYPTO 2008*, volume 5157 of *LNCS*, pages 554–571. Springer, Heidelberg, August 2008.
- [43] Ryan Wails, Aaron Johnson, Daniel Starin, Arkady Yerukhimovich, and S. Dov Gordon. Stormy: Statistics in tor by measuring securely. In Lorenzo Cavallaro, Johannes Kinder, XiaoFeng Wang, and Jonathan Katz, editors, *ACM CCS 2019*, pages 615–632. ACM Press, November 2019.
- [44] Andrew Chi-Chih Yao. How to generate and exchange secrets (extended abstract). In *27th FOCS*, pages 162–167. IEEE Computer Society Press, October 1986.