Report from Dagstuhl Seminar 13031

Computational Counting

Edited by

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Abstract

Dagstuhl Seminar 13031 "Computational Counting" was held from 13th to 18th January 2013, at Schloss Dagstuhl – Leibniz Center for Informatics. A total of 43 researchers from all over the world, with interests and expertise in different aspects of computational counting, actively participated in the meeting.

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1 Executive Summary

Peter Bürgisser
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Introduction

Computational complexity is typically concerned with decision problems, but this is a historical accident, arising from the origins of theoretical computer science within logic. Computing applications, on the other hand, typically involve the computation of numerical quantities. These applications broadly fall into two types: optimisation problems and counting problems. We are interested in the latter, broadly interpreted: computing sums, weighted sums, and integrals including, for example, the expectation of a random variable or the probability of an event. The seminar covered all aspects of computational counting, including applications, algorithmic techniques and complexity. Computational counting offers a coherent set of problems and techniques which is different in flavour from other algorithmic branches of computer science and is less well-studied than its optimisation counterpart.

Specific topics covered by the meeting include

- Techniques for exact counting, including moderately exponential algorithms for intractable problems, fixed parameter tractability, and holographic algorithms and reductions;

- techniques for approximate counting including Markov Chain Monte Carlo (MCMC);
- computational complexity of counting, including complexity in algebraic models; and
- applications, for example to models in statistical physics, and to constraint satisfaction.

The questions addressed include: What algorithmic techniques are effective for exact counting and approximate counting? Do these techniques remain effective in the presence of weights (including negative and complex weights)? What inherent limitations arise from computational complexity? Are there inherent limitations for specific techniques such as MCMC? Our nominated application areas prompted many of those questions and hopefully will benefit from the answers.

Although each of these topics is important in its own right, the real goal of this seminar was to bring them together to allow cross-fertilisation. Here is an example. A key issue for MCMC is the rate at which a Markov chain converges to equilibrium, which determines the length of simulation needed to get a good estimate. An important insight has been that this mixing rate is connected to the phenomenon of phase transitions in statistical physics. But it also seems likely that phase transitions are connected with computational intractability more generally, i.e., resistance to all efficient approximation algorithms, not just those based on MCMC. A further example is provided by the way algebra pervades several of our topics – holographic algorithms, complexity of counting, and constraint satisfaction – and yet the connections between these are only now being explored. For example, algebraic methods permit semi-automatic generation of reductions between counting problems, and open up the speculative possibility of resolving the P versus NP question positively through "accidental algorithms".

We are interested in the complexity of counting in different models of computation. Counting in models of arithmetic circuits is intimately connected with the permanent versus determinant problem. The latter has recently triggered the study of several specific counting problems such as the computation of Littlewood-Richardson coefficients. Another direction of research that is relevant to the meeting is the classification of counting problems in computational algebraic geometry (counting irreducible factors, connected components, etc).

Two key applications areas, statistical physics and constraint satisfaction, have a central role. The problem of computing and approximating weighted sums already arises frequently in statistical physics, where such sums are referred to as partition functions. Constraint Satisfaction is a wide class of problems which arose in the context of AI – many computer science problems can be cast in this framework. Weights are not traditionally considered in CSP, but with this addition, many applications can be viewed in terms of counting CSPs.

Participation and Programme

The seminar brought together 43 researchers from Canada, China, Europe, India, Israel, Japan and the United States with interests and expertise in different aspects of computational counting. Among them there was a good mix of senior participants, postdoctoral researchers and PhD students. Altogether, there were 32 talks over the week.

If the spread of talks at the meeting is a reliable guide, the most active topics in the field at the moment are: algorithms and complexity in algebraic models, the complexity of Counting CSPs (Constraint Satisfaction Problems), and holographic algorithms and the holant framework. Other topics covered included: graph polynomials, MCMC (Markov Chain Monte Carlo) algorithms, parameterised complexity, phase transitions/decay of correlation and its relation to computational complexity, streaming algorithms, and exponential-time exact algorithms. In addition to the technical presentations listed in the online programme, there were tutorial-style talks on topics featured in the Seminar. On Monday afternoon, Tyson Williams introduced the audience to holant problems and holographic transformations, and on Tuesday, Thore Husfeldt provided a similar service for newcomers to ETH and #ETH (the "Exponential Time Hypothesis" and its counting analogue).

One of the main aims of the seminar was to bring together researchers from different, but related fields, covering all aspects of computational counting with the goal of fostering the exchange of knowledge and to stimulate new research. This goal was fully achieved according to our opinion and the participant's feedback. The programme was as usual a compromise between allowing sufficient time for participants to present their work, while also providing unstructured periods for informal discussions. New contacts and maybe even friendships were made.

Snow and an early sunset did not the prevent the traditional Wednesday "hike" from taking place, though they did curtail it somewhat. The scenery was enhanced by the recent snowfall.

The organisers and participants thank the staff and the management of Schloss Dagstuhl for their assistance and support in the arrangement of a very successful and productive meeting.

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3 Overview of Talks

3.1 The complexity of the noncommutative determinant

Markus Bläser (Universität des Saarlandes, DE)

We consider the complexity of computing the determinant over arbitrary finite-dimensional algebras. We first consider the case that A is fixed. We obtain the following dichotomy: If $A/\mathrm{rad}A$ is noncommutative, then computing the determinant over A is hard. "Hard" here means #P-hard over fields of characteristic 0 and ModP_p-hard over fields of characteristic $p > 0$. If $A/\mathrm{rad}A$ is commutative and the underlying field is perfect, then we can compute the determinant over A in polynomial time.

We also consider the case when A is part of the input. Here the hardness is closely related to the nilpotency index of the commutator ideal of A. The commutator ideal of A is the ideal generated by all elements of the form $xy - yx$ with $x, y \in A$. We prove that if the nilpotency index of the commutator ideal is linear in n, where $n \times n$ is the format of the given matrix, then computing the determinant is hard. On the other hand, we show the following upper bound: Assume that there is an algebra $B \subseteq A$ with $B = A/\mathrm{rad}(A)$. (If the underlying field is perfect, then this is always true.) The center $Z(A)$ of A is the set of all elements that commute with all other elements. It is a commutative subalgebra. We call an ideal J a complete ideal of noncommuting elements if $B + Z(A) + J = A$. If there is such a J with nilpotency index $o(n/\log n)$, then we can compute the determinant in subexponential time. Therefore, the determinant cannot be hard in this case, assuming the counting version of the exponential time hypothesis.

Our results answer several open questions posed by Chien et al.

3.2 Fast and Slow Mixing in the Ferromagnetic Potts Model

Magnus Bordewich (University of Durham, GB)

Joint work of Bordewich, Magnus; Greenhill, Catherine; Patel, Viresh

The Potts model is a statistical physics model of magnetism closely related to the Tutte polynomial of a graph in combinatorics. One element of interest is the Glauber dynamics of the model – a Markov chain process on a vertex colourings of the underlying graph. Each step of the Markov chain involves recolouring a single vertex. The state of the chain converges to a stationary distribution which is a weighted distribution on all colourings of the graph. This convergence can either happen in polynomial time in the number of vertices of the graph (rapid mixing), or it can take an exponential number of steps (torpid mixing). In this talk we explore what properties of the graph and the parameters of the model lead to fast or slow mixing.

3.3 On the average number of roots of a real sparse polynomial

Irénée Briquel (Université d'Orléans, FR)

The average number of real zeros of a random real polynomial is a well-studied problem. For instance, when the coefficients are identically tossed following a normal distribution, the number of zeros is asymptotically logarithmic in the degree. We here consider random sparse polynomials: polynomials for which a (small) number of nonzero coefficients are tossed. A criterion from Descartes tells us that the number of positive zeros of a sparse polynomial cannot be greater than the number of nonzero coefficients, no matter what the degree is. But no better result is known in the average. Yet, we suspect that the number could be much smaller in the average. We will discuss the tools we could use to estimate this number of real zeros and what implications it could have in complexity theory.

3.4 Geometric Complexity Theory and Counting

Peter Bürgisser (Universität Paderborn, DE)

Joint work of Bürgisser, Peter; Ikenmeyer, Christian
Main reference P. Bürgisser, C. Ikenmeyer, "Explicit Lower Bounds via Geometric Complexity Theory," in Proc. of STOC 2013, ACM, 2013.
URL http://arXiv.org/abs/1210.8368v2

We prove the lower bound $3/2m^2 - 2$ on the border rank of m x m matrix multiplication by exhibiting explicit representation theoretic (occurence) obstructions in the sense of Mulmuley and Sohoni's geometric complexity theory (GCT) program. While this bound is weaker than the one recently obtained by Landsberg and Ottaviani, these are the first significant lower bounds obtained within the GCT program. Behind the proof is the new combinatorial concept of obstruction designs H, which encode highest weight vectors f_H in $\mathrm{Sym}^d \otimes 3C^n$ and provide new insights into Kronecker coefficients. Deciding whether f_H equals the zero polynomial is not easy: we show that for a simple family of obstruction designs H_n, this is equivalent to the Alon-Tarsi Conjecture on Latin squares. The Alon-Tarsi Conjecture is also relevant for the permanent versus determinant problem. Kumar showed that this conjecture implies restrictions on the possible candidates for occurences obstructions. This excludes an asymptotic approach to the problem.

3.5 The complexity of counting CSP with complex weights

Xi Chen (Columbia University, US)

We prove a complexity dichotomy theorem for all Counting Constraint Satisfaction Problems (#CSP) with complex weights. We give three conditions for tractability. Let F be any finite set of complex functions, then we show that the #CSP defined by F is solvable in polynomial time if all three conditions are satisfied; and is #P-hard otherwise. Our dichotomy generalizes a long series of important results on counting problems.

In this talk, we will focus on some of the most interesting ingredients of our algorithm, including a new polynomial-time operation over relations that share a common Mal'tsev polymorphism. We will then describe the framework of utilizing this operation to solve any #CSP efficiently when all three tractability conditions are satisfied.

3.6 Weighted counting of k-matchings is #W[1]-hard

Radu Curticapean (Universität des Saarlandes, DE)

Joint work of Bläser, Markus; Curticapean, Radu
Main reference M. Bläser, R. Curticapean, "Weighted Counting of k-Matchings Is #W[1]-Hard," in Proc. of the 7th Int'l Symp. on Parameterized and Exact Computation (IPEC'12), LNCS, Vol. 7535, pp. 171–181, Springer, 2012.
URL http://dx.doi.org/10.1007/978-3-642-33293-7_17

In the seminal paper for parameterized counting complexity (Flum, Grohe 2002), the following problem is conjectured to be #W[1]-hard: Given a bipartite graph G and a number k, which is considered as a parameter, count the number of matchings of size k in G.

We prove hardness for a natural weighted generalization of this problem: Let $G = (V, E, w)$ be an edge-weighted graph and define the weight of a matching as the product of weights of all edges in the matching. We show that exact evaluation of the sum over all such weighted matchings of size k is #W[1]-hard for bipartite graphs G.

As an intermediate step in our reduction, we also prove #W[1]-hardness of the problem of counting k-partial cycle covers, which are vertex-disjoint unions of cycles including k edges in total.

3.7 Enumeration complexity of query problems: constant delay and quantifier elimination methods

Arnaud Durand (University Paris-Diderot, FR)

In this talk, we present some results on the enumeration complexity for query problems and CSP. We first survey the main complexity measures defined in the context of enumeration and comment on some connexions between enumeration and counting. We then focus on the notion of constant delay enumeration. This class contains all problems for which solutions can be enumerated with a polynomial (linear in our case) precomputation and a constant delay between two consecutive solutions. This notion was introduced in the context of query answering and, in this setting, "constant" means "depending on the formula size". We show that several natural classes of queries can be enumerated with constant delay. Surprisingly, in all cases, methods based on quantifier elimination are developed to obtain these upper bounds.

3.8 Generating random regular graphs and digraphs

Martin Dyer (University of Leeds, GB)

We consider the problem of sampling a regular graph uniformly at random, in time polynomial in the size of the graph. We review various approaches to this problem, which work in different ranges for the degree of the graph as a function of its size. In particular, we consider Markov chain methods for generating such a graph. Jerrum and Sinclair gave a general solution, but we consider more natural chains, which have applications to the design of peer-to-peer networks. These methods use "switches" and "flips", which are simple modifications of the graph. Finally, we consider the extension of these methods to regular digraphs.

3.9 The Potts/Tutte connection with an external field

Jo Ellis-Monaghan (Saint Michael's College – Colchester, US)

Joint work of Ellis-Monaghan, Jo; Moffatt, Iain
Main reference J. Ellis-Monaghan, I. Moffatt, "The Tutte-Potts connection in the presence of an external magnetic field," Advances in Applied Mathematics, Vol. 47, Issue 4, October 2011, pp. 772–782, 2011.
URL http://dx.doi.org/10.1016/j.aam.2011.02.004

The Potts model in statistical mechanics is a rapidly emerging and increasingly applicable predictive model for complex systems in which very simple interactions at the microscale level determine the macroscale properties of the system. This model plays an important role in the theory of phase transitions and critical phenomena in physics, and has applications as widely varied as tumor migration, foam behaviors, and social demographics.

Here we define the V-polynomial, which lifts the classical relationship between the Tutte polynomial of graph theory and the zero field Potts model to encompass external magnetic fields. The classical relationship between the Tutte polynomial of graph theory and the zero field Potts model has resulted in valuable interactions between the disciplines. Unfortunately, it does not include the external magnetic fields that appear in most Potts model applications. Thus the current work unifies an important segment of Potts model theory and brings previously successful combinatorial machinery, including complexity results, to bear on a wider range of statistical mechanics models.

3.10 On fixed-polynomial size circuit lower bounds for uniform polynomials in the sense of Valiant

Hervé Fournier (University Paris-Diderot, FR)

We consider the problem of fixed-polynomial lower bounds on the size of arithmetic circuits computing uniform families of polynomials. Our first result is that for all k, there exist polynomials with coefficients in MA having no arithmetic circuits of size $O(n^k)$ over the complex field (i.e. allowing any complex constant). We also investigate links between fixed-polynomial size circuit bounds in the Boolean and arithmetic settings. In particular, NP without n^k size circuits or NP=MA imply lower bounds on the circuit size of uniform polynomials in VNP over the complex field.

3.11 On the connection between interval size functions and path counting

Andreas Goebel (University of Liverpool, GB)

Joint work of Evangelos Bampas, Aris Pagourtzis, Aris Tentes

We investigate the complexity of hard counting problems that belong to the class #P but have easy decision version; several well-known problems such as #Perfect Matchings, #DNFSat share this property. We focus on classes of such problems which emerged through two disparate approaches: one taken by Hemaspaandra et al. who defined classes of functions that count the size of intervals of ordered strings, and one followed by Kiayias et al. who defined the class TotP, consisting of functions that count the total number of paths of NP computations. We provide inclusion and separation relations between TotP and interval size counting classes, by means of new classes that we define in this work. Our results imply that many known #P-complete problems with easy decision are contained in the classes defined in Hemaspaandra et al.—but are unlikely to be complete for these classes under certain types of reductions. We also define a new class of interval size functions which strictly contains FP and is strictly contained in TotP under reasonable complexity-theoretic assumptions. We show that this new class contains some hard counting problems.

3.12 Approximating the partition function of planar two-state spin systems

Leslie Ann Goldberg (University of Liverpool, GB)

Joint work of Goldberg, Leslie Ann; Jerrum, Mark; McQuillan, Colin
Main reference L.A. Goldberg, M. Jerrum, C. McQuillan, "Approximating the partition function of planar two-state spin systems," arXiv:1208.4987v2 [cs.CC].
URL http://arxiv.org/abs/1208.4987v2

We consider the problem of approximating the partition function of the hard-core model on planar graphs of degree at most 4. We show that when the activity lambda is sufficiently large, there is no fully polynomial randomised approximation scheme for evaluating the partition function unless NP=RP. The result extends to a nearby region of the parameter space in a more general two-state spin system with three parameters. We also give a polynomial-time randomised approximation scheme for the logarithm of the partition function.

3.13 Factoring bivariate lacunary polynomials without heights

Bruno Grenet (ENS-Lyon, FR)

Joint work of Arkadev Chattophyay, Pascal Koiran, Natacha Portier, Yann Strozecki

The lacunary, or supersparse, representation of a multivariate polynomial P is a list of pairs (c, e) where c is a coefficient of P and e is a vector of exponent. Each pair defines a term of the polynomial, and P equals the sum of all these terms. The factorization of lacunary polynomials has been investigated in a series of papers by Cucker, Koiran and Smale (J. Symb. Comput., 1999), Lenstra (Number Theory in Progress, 1999), and Kaltofen and Koiran (ISSAC 2005 & 2006). In this paper, we are interested in more elementary proofs for some of these results. We focus on Kaltofen and Koiran's results dealing with linear factors of bivariate lacunary polynomials. In particular, we give a polynomial-time algorithm to find linear factors of bivariate polynomials that is not based on heights of algebraic numbers. This simplification allows us to give a similar result for some fields of positive characteristic. Our main technical result is an upper bound on the valuation of polynomials of the form $P(X, 1 + X)$ where P is a bivariate lacunary polynomial, and can be viewed as a generalization of a result of Hajós.

3.14 A Complete Dichotomy Rises from the Capture of Vanishing Signatures

Heng Guo (University of Wisconsin – Madison, US)

Joint work of Cai, Jin-Yi; Williams, Tyson
Main reference J.-Y. Cai, H. Guo, T. Williams, "A Complete Dichotomy Rises from the Capture of Vanishing Signatures," arXiv:1204.6445v1 [cs.CC].
URL http://arxiv.org/abs/1204.6445v1

We prove a complexity dichotomy theorem for Holant problems over an arbitrary set of complex-valued symmetric constraint functions F on Boolean variables. This extends and unifies all previous dichotomies for Holant problems on symmetric constraint functions taking values in a field of characteristic zero. We define and characterize all symmetric vanishing signatures. They turned out to be essential to the complete classification of Holant problems. The dichotomy theorem has an explicit tractability criterion. The Holant problem defined by a set of constraint functions F is solvable in polynomial time if it satisfies this tractability criterion, and is #P-hard otherwise. The tractability criterion can be intuitively stated as follows: the set F is tractable if (1) every function in F has arity at most 2, or (2) F is transformable to an affine type, or (3) F is transformable to a product type, or (4) F is vanishing, combined with the right type of binary functions, or (5) F belongs to a special category of vanishing type Fibonacci gates. The proof of this theorem utilizes many previous dichotomy theorems on Holant problems and Boolean #CSP.

3.15 The Parity of Directed Hamiltonian Cycles

Thore Husfeldt (IT University of Copenhagen, DK)

We present a deterministic algorithm that given any directed graph on n vertices computes the parity of its number of Hamiltonian cycles in $O(1.618^n)$ time and polynomial space.

3.16 Approximate Counting of Matchings in Uniform Hypergraphs

Marek Karpinski (Universität Bonn, DE)

Joint work of Karpinski, Marek; Rucinski, Andrzej; Szymanska, Edyta
Main reference M. Karpinski, A. Rucinski, E. Szymanska, "Approximate Counting of Matchings in Sparse Uniform Hypergraphs," arXiv:1204.5335v1 [cs.DS].
URL http://arxiv.org/abs/1204.5335v1

We give a fully polynomial *randomized approximation scheme (FPRAS)* for counting matchings in k-uniform hypergraphs without *wide* edges, or equivalently hypergraphs whose intersection graphs are claw-free. The method generalizes to the weighted monomer-polymer (trimer, tetramer, pentamer, etc.) problems for the corresponding class of hypergraphs.

The method adopts and generalizes the canonical path method of Jerrum and Sinclair to those restricted hypergraph classes. We prove also that the problem of counting matchings in k-uniform hypergraphs without the above restriction is approximation hard for all $k \geq 6$. It leaves very interesting open problems on approximate counting matchings in arbitrary k-uniform hypergraphs for $k = 3, 4$ and 5, and also some connected issues on approximate counting independent sets in some classes of intersection graphs.

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4 M. Karpinski, A. Rucinski and E. Szymanska. *Approximate Counting of Matchings in Sparse Uniform Hypergraphs*. Proc. 13th SIAM ANALCO (2013), pp. 71–78
5 A. Sly. *Computational Transition at the Uniqueness Threshold*. Proc. 51st IEEE FOCS (2010), pp. 287–296

3.17 Enumerating monomials

Meena Mahajan (The Institute of Mathematical Sciences – Chennai, IN)

Joint work of de Rugy, Nicolas; Saurabh, Nitin; Sreenivasaiah, Karteek; Strozecki, Yann

In this talk, I discuss some ongoing work on the following problems:

Given an arithmetic circuit C computing a polynomial p,

1. deterministically enumerate (without repetition) all monomials of p,
2. and given also a term m, compute the coefficient of m in p.

We look for restrictions that allow us to achieve these tasks with preprocessing time / delay / computation time polynomial in the size of C.

Joint work with Nicolas de Rugy, Nitin Saurabh, Karteek Sreenivasaiah and Yann Strozecki. Funded by an IFCPAR (CEFIPRA) project.

3.18 A computational framework for the study of partition functions and graph polynomials

Johann Makowsky (Technion – Haifa, IL)

Partition functions and graph polynomials have found many applications in combinatorics, physics, biology and even the mathematics of finance. Studying their complexity poses some problems. To capture the complexity of their combinatorial nature, the Turing model of computation and Valiant's notion of counting complexity classes seem most natural. To capture the algebraic and numeric nature of partition functions as real or complex valued functions, the Blum-Shub-Smale (BSS) model of computation seems more natural. As a result many papers use a naive hybrid approach in discussing their complexity or restrict their considerations to sub-fields of $\mathbb{C}$ which can be coded in a way to allow dealing with Turing computability.

In this paper we propose a unified natural framework for the study of computability and complexity of partition functions and graph polynomials and show how classical results can be cast in this framework.

3.19 Lower bounds for restricted arithmetic computations

Guillaume Malod (University Paris-Diderot, FR)

Joint work of Dvir, Zeev; Malod, Guillaume; Perifel, Sylvain; Yehudayoff, Amir
Main reference Z. Dvir, G. Malod, S. Perifel, A. Yehudayoff, "Separating multilinear branching programs and formulas," in Proc. of the 44th Symp. on Theory of Computing Conference (STOC'12), pp. 615–624, ACM, 2012.
URL http://dx.doi.org/10.1145/2213977.2214034

Valiant's theory contains classes VPe, VPws, VP, VNP which we can use to classify the complexity of polynomials such as the permanent or the determinant. As in other areas of computational complexity, no class separation is known. I will present several lower bound results: Nisan's beautiful lower bounds for non-commutative computations, Raz's multilinear techniques, and an application separating formulas from branching programs, from a joint work with Dvir, Perifel and Yehudayoff.

3.20 Approximating Holant problems

Colin McQuillan (University of Liverpool, GB)

Main reference C. McQuillan, "Approximating Holant problems by winding," arXiv:1301.2880v1 [cs.CC].
URL http://arxiv.org/abs/1301.2880v1

I will discuss the complexity of approximately evaluating Holant problems. I will present a hardness result for Holant problems with edge weights, and an FPRAS for Holant problems with even, odd, and not-all-equal constraints [1].

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3.21 Structural Tractability of Counting of Solutions to Conjunctive Queries

Stefan Mengel (Universität Paderborn, DE)

Joint work of Durand, Arnaud; Mengel, Stefan;
Main reference A. Durand, S. Mengel, "Structural Tractability of Counting of Solutions to Conjunctive Queries," accepted for the 16th Int'l Conf. on Database Theory (ICDT2013).
URL http://arxiv.org/abs/1303.2059

Conjunctive queries (CQs) are a fundamental class of logical queries. The corresponding decision problem consist of evaluating an existential conjunctive first-order formula over a finite structure. This is equivalent to answering Select-Project-Join queries in database theory and has several equivalent definitions, in particular, in terms of constraint satisfaction or homomorphism problems. Generally, answering CQs is NP-hard, but there has been huge progress in identifying so-called 'islands of tractability', i.e. subclasses of queries which can be evaluated in polynomial time.

I will survey recent joint results with Arnaud Durand on the complexity of counting solutions to CQs and discuss how the situation differs from #CSP.

3.22 The complexity of approximating conservative counting CSP

David Richerby (University of Liverpool, GB)

Joint work of Chen, Xi; Dyer, Martin; Goldberg, Leslie Ann; Jerrum, Mark; Lu, Pinyan, McQuillan, Colin; Richerby, David
Main reference X. Chen, M. Dyer, L.A. Goldberg, M. Jerrum, P. Lu, C. McQuillan, D. Richerby, "The complexity of approximating conservative counting CSPs," arXiv:1208.1783v2 [cs.CC].
URL http://arxiv.org/abs/1208.1783v2

We discuss the complexity of approximately solving the weighted counting constraint satisfaction problem (#CSP) in the conservative case, where all unary weight functions are assumed to be available. We define the notions of weak log-modularity and weak log-supermodularity. For this presentation, we restrict to constraint languages with at-most binary functions. If such a constraint language F is weakly log-modular, then #CSP(F) can be solved exactly in polynomial time; otherwise, if it is weakly log-supermodular, then #CSP(F) is equivalent to the problem #BIS of counting independent sets in bipartite graphs; otherwise, #CSP(F) is NP-hard to approximate.

3.23 Effective De Rham Cohomology

Peter Scheiblechner (Hochschule Luzern, CH)

Main reference P. Scheiblechner, "Effective de Rham cohomology: the hypersurface case," in Proc. of the 37th Int'l Symp. on Symbolic and Algebraic Computation (ISSAC'12), pp. 305–310, ACM, 2012.
URL http://dx.doi.org/10.1145/2442829.2442873

A long standing open problem in computational algebraic geometry is to find an algorithm which computes the topological Betti numbers of a semialgebraic set in single exponential time. There has been recent progress on the corresponding problem over the complex numbers. A fundamental theorem of Grothendieck states that the Betti numbers of a smooth complex variety can be computed via its algebraic de Rham cohomology, which is defined in terms of algebraic differential forms on the variety. In this talk, we discuss single exponential degree bounds on these differential forms and their importance for the algorithmic computation of Betti numbers.

References

1 P. Scheiblechner. Effective de Rham cohomology – the Hypersurface Case. Proc. ISSAC 2012. arXiv:1112.2489v1.
2 P. Scheiblechner. Effective de Rham cohomology – the General Case. submitted, 2012. arXiv:1203.5706v1.

3.24 Inapproximability of the Partition Function for the Spin Models (Antiferromagnetic Ising, Hard-Core Models, and more)

Daniel Stefankovic (University of Rochester, US)

Recent inapproximability results of Sly (2010), together with an approximation algorithm presented by Weitz (2006) establish a picture for the computational complexity of approximating the partition function of the hard-core model. Let $L_c(T_D)$ denote the critical activity for the hard-model on the infinite D-regular tree. Weitz presented an FPTAS for the partition function when $L < L_c(T_D)$ for graphs with constant maximum degree D. In contrast, Sly showed that for all $D >= 3$, there exists $\epsilon > 0$ such that (unless RP=NP) there is no FPRAS for approximating the partition function on graphs of maximum degree D for activities L satisfying $L_c(T_D) < L < L_c(T_D) + \epsilon$.

We prove the complementary result that for the antiferromagnetic Ising model without external field that, unless RP=NP, for all $D >= 3$, there is no FPRAS for approximating the partition function on graphs of maximum degree D when the inverse temperature lies in the non-uniqueness regime of the infinite tree T_D. Our results extend to a region of the parameter space for general 2-spin models. Our proof works by relating certain second moment calculations for random D-regular bipartite graphs to the tree recursions used to establish the critical points on the infinite tree. We will also report on the progress for multi-spin systems.

3.25 Counting Arbitrary Subgraphs in Data Streams

He Sun (MPI für Informatik – Saarbrücken, DE)

Joint work of Kane, Daniel; Mehlhorn Kurt; Sauerwald Thomas
Main reference D. Kane, K. Mehlhorn, T. Sauerwald, H. Sun, "Counting Arbitrary Subgraphs in Data Streams," in Pro. of the 39th Int'l Colloquium on Automata, Languages and Programming (ICALP'12), LNCS, Vol. 7392, pp. 598–609, Springer, 2012.
URL http://dx.doi.org/10.1007/978-3-642-31585-5_53

We study the subgraph counting problem in data streams. We provide the first non-trivial estimator for approximately counting the number of occurrences of an *arbitrary* subgraph H of constant size in a (large) graph G. Our estimator works in the turnstile model, i.e., it can handle both edge-insertions and edge-deletions and is applicable in a distributed setting. Prior to this work, estimators were known for only a few non-regular graphs in the case of edge-insertions, leaving the problem of counting general subgraphs in the turnstile model wide open. Further we demonstrate the applicability of our estimator by analyzing its concentration for several graphs H and the case where G is a power law graph.

3.26 What are the Chances that P=NP?

Leslie Valiant (Harvard University, US)

Main reference L.G. Valiant, "Some observations on holographic algorithms," in Proc. 9th Latin American Theoretical Informatics Symp. (LATIN'10), LNCS, Vol. 6034, pp. 577–590, Springer, 2010.
URL http://dx.doi.org/10.1007/978-3-642-12200-2_50

We argue that while there has been substantial recent progress in the theory of holographic algorithms, these results have not removed the possibility that holographic methods might be sufficient to yield polynomial time algorithms for all of #P. They do, however, exclude some possibilities. Known negative results dichotomize into those that are representation dependent, such as the Cai-Lu collapse theorem, and those that are not. In pursuit of the latter, we discuss the notion of an "elementary holographic transformation to matchgrids," which is sufficient to express many of the known polynomial time holographic algorithms. We discuss a lower bound argument that shows that #SAT, the Boolean satisfiability counting problem, cannot be solved by such an elementary transformation. The constraints on elementarity can be evaded by using interpolation from many individual elementary transformations, or by using exponential size fields. We give examples of such elementarity evasive algorithms for various parity problems related to graph coloring, connected independent sets and forests.

3.27 Analysis of message-passing iterative decoders via zeta functions

Pascal Vontobel (HP Labs – Palo Alto, US)

Joint work of Pfister, Henry; Vontobel, Pascal
Main reference to be submitted to IEEE Trans. Inf. Theory

Graph-based codes and message-passing iterative decoders have become increasingly popular in the last fifteen years. It is fair to say that these codes and decoding algorithms (and ideas related to them) have thoroughly changed much of modern communications. Before this backdrop, a good understanding of these types of communication techniques is obviously highly desirable, especially the understanding of iterative decoding of finite-length codes.

We will focus on a particular message-passing iterative decoder, the so-called sum-product algorithm (SPA) decoder. As the name SPA suggests, this algorithm is counting certain objects, and, indeed, when the underlying factor graph has no cycles then it is clear what the SPA is counting. However, when the underlying factor graph has cycles then the situation is much fuzzier.

In this talk we use graph zeta functions for analyzing the SPA decoder. This approach allows us to connect and count computation tree pseudo-codewords and graph-cover pseudo-codewords, two central objects in the SPA analysis.

3.28 The Complexity of Planar Boolean #CSP with Complex Weights

Tyson Williams (University of Wisconsin – Madison, US)

Joint work of Williams, Tyson; Guo, Heng
Main reference H. Guo, T. Williams, "The Complexity of Planar Boolean #CSP with Complex Weights," arXiv:1212.2284v1 [cs.CC].
URL http://arxiv.org/abs/1212.2284v1

We prove a complexity dichotomy theorem for symmetric complex-weighted Boolean #CSP when the constraint graph of the input must be planar. The problems that are #P-hard over general graphs but tractable over planar graphs are precisely those with a holographic reduction to matchgates. This generalizes a theorem of Cai, Lu, and Xia for the case of real weights. We also obtain a dichotomy theorem for a symmetric arity 4 signature with complex weights in the planar Holant framework, which we use in the proof of our #CSP dichotomy. In particular, we reduce the problem of evaluating the Tutte polynomial of a planar graph at the point $(3, 3)$ to counting the number of Eulerian orientations over planar 4-regular graphs to show the latter is #P-hard. This strengthens a theorem by Huang and Lu to the planar setting. Our proof techniques combine new ideas with refinements and extensions of existing techniques. These include planar pairings, the recursive unary construction, the anti-gadget technique, and pinning in the Hadamard basis.

3.29 Dichotomy for Holant* Problems with Domain Size 3

Mingji Xia (MPI für Informatik – Saarbrücken, DE)

Joint work of Cai, Jin-Yi; Lu, Pinyan
Main reference J.-Y. Cai, P. Lu, M. Xia, "Dichotomy for Holant* Problems with a Function on Domain Size 3," arXiv:1207.2354v1 [cs.CC].
URL http://arxiv.org/abs/1207.2354v1

This talk is about the complexity of Holant* problems defined by one symmetric ternary function in variables from the domain of size 3. Holographic reductions are used for both algorithms and #P-hardness proofs. Three kinds of tractable problems are introduced one by one following the corresponding cases of Boolean domain Holant* problems. Two segments of #P-hardness proof are shown in details, because the second one is closely related with the discovery of the third kinds of tractable problems. One is realizing a binary function of rank 2, and the other is realizing a binary function of rank 2 such that its eigenvector corresponding to eigenvalue 0 is not isotropic.

3.30 Approximation Classification of Complex-Weighted Counting CSPs

Tomoyuki Yamakami (University of Fukui, JP)

We deal with counting constraint satisfaction problems, which are particularly composed of complex-valued constraints over Boolean variables. We show a complete classification theorem on the computational complexity of approximately solving those problems when auxiliary unary constraints are freely available besides input constraints. To clarify the roles of extra free unary constraints, we then present an alternative proof of the classification theorem in which unary constraints are required only at the very end of its argument. This talk is based on recent papers [1, 2].

References

1 Tomoyuki Yamakami. Approximate counting for complex-weighted Boolean constraint satisfaction problems. Information and Computation, 219 (2012) 17–38.
2 Tomoyuki Yamakami. Constant unary constraints and symmetric real-weighted counting CSPs. In: Proc. of the 23rd International Symposium on Algorithms and Computation (ISAAC 2012), Lecture Notes in Computer Science, Springer-Verlag, vol.7676 pp. 237–246, 2012.

3.31 Approximate Counting via Correlation Decay on Planar Graphs

Chihao Zhang (Shanghai Jiao Tong Univ., CN)

Joint work of Yin, Yitong; Zhang, Chihao
Main reference Y. Yin, C. Zhang, "Approximate Counting via Correlation Decay on Planar Graphs," arXiv:1207.3564v1 [cs.DS].
URL http://arxiv.org/abs/1207.3564v1

We show for a broad class of counting problems, correlation decay (strong spatial mixing) implies FPTAS on planar graphs. The framework for the counting problems considered by us is the Holant problems with arbitrary constant-size domain and symmetric constraint functions. We define a notion of regularity on the constraint functions, which covers a wide range of natural and important counting problems, including all multi-state spin systems, counting graph homomorphisms, counting weighted matchings or perfect matchings, the subgraphs world problem transformed from the ferromagnetic Ising model, and all counting CSPs and Holant problems with symmetric constraint functions of constant arity.

The core of our algorithm is a fixed-parameter tractable algorithm which computes the exact values of the Holant problems with regular constraint functions on graphs of bounded treewidth. By utilizing the locally tree-like property of apex-minor-free families of graphs, the parameterized exact algorithm implies an FPTAS for the Holant problem on these graph families whenever the Gibbs measure defined by the problem exhibits strong spatial mixing. We further extend the recursive coupling technique to Holant problems and establish strong spatial mixing for the ferromagnetic Potts model and the subgraphs world problem. As consequences, we have new deterministic approximation algorithms on planar graphs and all apex- minor-free graphs for several counting problems.

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