Covering Designs Annotated Bibliography

Abstract

There is a vast literature on covering designs. Here we give some important papers, with a description of their contributions to parameters in the tables.

This is a work in progress, to try to give better references and ensure that original discoverers of particular covering designs receive due credit. Suggestions for additions or corrections are appreciated.

Surveys

These papers gather results for a wide variety of parameters, and are a good starting point, although they are all fairly dated.

References

[GPK95] Daniel M Gordon, Oren Patashnik, and Greg Kuperberg. New constructions for covering designs. *Journal of Combinatorial Designs*, 3(4):269–284, 1995.

This paper was the original motivation to create this website, giving (an immediately dated) table of the best known covering numbers.

[GS06] Daniel M Gordon and Douglas R Stinson. Coverings. In *Handbook of Combinatorial Designs*, pages 391–398. Chapman and Hall/CRC, 2006.

Survey of knowledge in mid-2000's.

[MM92] W. H. Mills and R. C. Mullin. Coverings and packings. In J. H. Dinitz and D. R. Stinson, editors, *Contemporary Design Theory: A Collection of Surveys*, pages 371–399. Wiley, 1992.

Survey of knowledge in the early 1990's.

$C(v, k, 2)$

Exact covering numbers $C(v, k, t)$ for a fixed (k, t) are only fully known for $(3, 2)$ and $(4, 2)$. For $(4, 3)$ and $(5, 2)$ (see [RAB$^+$07]) they are known except for a finite list of possible exceptions.

$C(v, k, 2)$ is also known exactly for most cases with $v/k \leq 13/4$, and all cases where $C(v, k, 2) \leq 13$ (see [GLvR04]).

References

[ABGdH07] R. Julian R. Abel, Iliya Bluskov, Malcolm Greig, and Jan de Heer. Pair covering and other designs with block size 6. *Journal of Combinatorial Designs*, 15(6):511–533, 2007.

Record $C(v, 6, 2)$ for many v.

[BGH00] I. Bluskov, M. Greig, and K. Heinrich. Infinite classes of covering numbers. *Canadian Mathematical Bulletin*, 43(4):385–396, 2000.

Record $C(v, k, 2)$ for $k = 6, 7, 8, 9$, many v.

[GLvR04] M. Greig, P. Li, and G. H. van Rees. Covering designs on 13 blocks revisited. *Util Math*, 70, 06 2004.

Determines which $C(v, k, 2) = 13$.

[RAB$^+$07] R. Julian R. Abel, Ahmed Assaf, Frank E. Bennett, Iliya Bluskov, and Malcolm Greig. Pair covering designs with block size 5. *Discrete Mathematics*, 307(14):1776–1791, 2007.

Settles most open $C(v, 5, 2)$ cases.

$C(v, k, t)$ for $t > 2$

Results are much harder for $t > 2$, but these papers give results for various small k and t.

References

[BBH04] Riccardo Bertolo, Iliya Bluskov, and Heikki Hämäläinen. Upper bounds on the general covering number $C_\lambda(v, k, t, m)$. *Journal of Combinatorial Designs*, 12(5):362–380, 2004.

Record $C(v, 6, 4)$ constructions.

[BH98] Iliya Bluskov and Heikki Hämäläinen. New upper bounds on the minimum size of covering designs. *Journal of Combinatorial Designs*, 6(1):21–41, 1998.

Many record $C(v,5,3)$, $C(v,5,4)$, $C(v,6,4)$, $C(v,6,5)$ and $C(v,7,5)$ bounds.

Lower Bounds

General lower bounds are also difficult, and the proofs tend to be long with many cases. Bounds for $C(v,k,2)$ where v/k is in a given range is discussed in [GLvR04]. There are some similar results for $t = 3$, but the papers are not available online (see the surveys for references). Some exact values for $C(v,k,k-1)$ are given in [ARS03].

Horsley [Hor] uses Fisher's inequality to improve lower bounds for many $C(v,k,2)$, and Horsley and Singh [HS18] extend this to larger t.

Füredi [Für89] improves the lower bound for $C(n^2+n+1, n+1, 2)$ when a finite projective plane does not exist.

The lower bounds link on this page has details about specific lower bound improvements from the late 1990's and early 2000's.

References

[ARS03] David L. Applegate, Eric M. Rains, and N. J. A. Sloane. On asymmetric coverings and covering numbers. *J. Comb. Designs*, 11:218–228, 2003.

Lower bounds for various $C(n,k,k-1)$.

[Für89] Zoltán Füredi. A projective plane is an outstanding 2-cover. *Discrete mathematics*, 74(3):321–324, 1989.

[GLvR04] M. Greig, P. Li, and G. H. van Rees. Covering designs on 13 blocks revisited. *Util Math*, 70, 06 2004.

Determines which $C(v,k,2) = 13$.

[Hor] Daniel Horsley. Generalizing Fisher's inequality to coverings and packings. *Combinatorica*, 37:673–696.

Improvements for lower bounds on many $C(v,k,2)$

[HS18] Daniel Horsley and Rakhi Singh. New lower bounds for t-coverings. *Journal of Combinatorial Designs*, 26(8):369–386, 2018.

Improvements for lower bounds on many $C(v,k,t)$ with t as large as 8.