# To Go or Not to Go Green: An Economic Analysis

Yue Zhang, Wenyi Zhang, and Qiang Ling
University of Science and Technology of China
Hefei 230027, China
Email: zhyuesu@mail.ustc.edu.cn, wenyizha@ustc.edu.cn, qling@ustc.edu.cn

***Abstract*—Green, i.e., energy-efficient, communications technologies have been a trend for next-generation cellular communications systems. An imperative question for network operators to address is whether and to what extent one should move to embrace such newly proposed green communications technologies. On one hand, the introduction of green communications technologies saves the operating cost, and on the other hand, it may also lead to some extent of degradation of the quality of service, which would drive users away towards other operators. In this paper, a preliminary economic analysis is developed to address such a tension. An operator chooses to upgrade a proportion of its infrastructure from the legacy technology to some green communications technology, and the goal of analysis is to figure out how large the proportion should be, depending upon system parameters including quality of service, price, and initial user distribution. The analysis reveals that a variety of possibilities exist.**

## I. Introduction

Green, i.e., energy-efficient, communications technologies have been burgeoning in the past a few years. For the common good of the society, these technologies aim at saving the energy consumption and reducing the carbon emission. For the interests of network operators, these technologies help cut the operating cost. A number of schemes and algorithms have been proposed to improve the energy efficiency of cellular networks; see, e.g., [1]-[8].

A common theme among the proposed schemes and algorithms is the adaptation of service availability based on traffic demand fluctuation over time and space. For example, many proposals suggest that a large fraction of those under-utilized base stations (e.g., those in the business district during the mid-night) can enter a sleep mode or even be completely shut down temporarily. Though effectively reducing the energy consumption and thus the operating cost, it is clear that with such adaptation, certain users may indeed experience somewhat compromised quality of service (QoS): for example, if some emergency (say, fire) suddenly arises in a district where many of the base stations have been in sleep mode, the response of the cellular network would be slower than legacy networks in which all the base stations are always operating. Handling bursty traffic demand is a key issue for network operators, and in fact the potentially degraded responsiveness due to the introduction of green communications technologies has been one of the major concerns of leading network operators at the end of the day.

In this paper, we develop a simplified model and analyze it in order to gain some preliminary understanding about whether and to what extent a network operator should upgrade its infrastructure to embrace the green communications technology. We consider a market of two operators. One operator may choose to upgrade a proportion of its infrastructure to green communications technology, while the other operator will take no upgrading action. Users are heterogeneous and make rational decisions when choosing their serving operators, based on their utilities. Depending upon the initial market condition, the user distribution evolves according to several different patterns. Analyzing the profit of the operator who upgrades its infrastructure, we determine the optimal choice of the upgrading proportion. It is revealed that, the optimal proportion critically depends upon system parameters, notably the reduction of the operating cost, the prices of service, the QoS gap between the legacy and the green communications technologies, and the initial user distribution in the market. The operator may choose not to upgrade at all, to upgrade a proportion of, or to upgrade all of its infrastructure.

In a broader context, our work is related to the competition between emerging and incumbent technologies. Adoption of new technologies considering externality has been treated in a number of prior works in economics [9]-[12], and in economic analysis of communication networks [13]-[16]. Specifically, our work is closely related to [15] which characterized the dynamic diffusion process of user distribution and studied users' adoption of a new network technology in the presence of an incumbent technology, and to [16] which considered operators' profits in a 4G network upgrading game.

The remaining part of this paper is organized as follows. Section II describes the system model. Section III-A characterizes the dynamic evolution of user distribution, based on which Section IV analyzes the impact of upgrading proportion on the operator's profit. Section

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V presents numerical results to corroborate our analysis. Finally Section VI concludes this paper.

## II. System Model

### A. Operator Model

Consider two network technology operators, labeled 1 and 2. Operator 1 introduces a certain green communications technology to upgrade a fraction of $\varphi \in [0,1]$ of its infrastructure $I$; Operator 2 is a traditional operator who still uses the legacy technology. With the introduction of the green communications technology, the QoS will be compromised, while the operating cost will also decrease. Hence, although some of the existing users have an incentive to switch to Operator 2 for a better QoS, due to the reduced operating cost, it is still possible for Operator 1 to increase its profit. So the key question to be addressed is how Operator 1 should choose its upgrading proportion $\varphi$ to optimize its profit, under the competitive environment.

We model the one-time upgrading cost of Operator 1 to be proportional to $\varphi$, as $K\varphi$, and its operating cost as

$$\int_0^\infty [(1-\varphi)c_l + \varphi c_g]\, I e^{-St} \mathrm{d}t = \frac{I}{S}[(1-\varphi)c_l + \varphi c_g], \quad (1)$$

where $c_l$ denotes the rate of operating cost for the legacy infrastructure, and $c_g$ denotes the rate of operating cost for the upgraded infrastructure due to introducing the green communications technology. We have $c_l \geq c_g$ to reflect the fact that the introduced green communications technology reduces the operating cost. We consider a large time span, and use $S > 0$ to denote the discount rate over time.

The QoS of Operator $i$ is denoted by $q_i > 0$, and the charged price per user is $p_i > 0$, for $i = 1,2$. The QoS of Operator 1, $q_1$, is determined by $\varphi$ as

$$q_1 = (1-\varphi)q_l + \varphi q_g, \quad (2)$$

where $q_l$ denotes the QoS of the legacy infrastructure and $q_g$ denotes the QoS of the upgraded infrastructure, $q_g \leq q_l$; the QoS of Operator 2 is simply $q_2 = q_l$. In this work, we do not allow the price to be dependent upon the upgrading, which should be a transparent process to users, and thus each $p_i$ keeps a constant.

### B. Network Value and Externality

We adopt the $N \log N$ law [17], which estimates the network value with $N$ users as $\kappa N \log N$. This model has been extensively used in network economic analysis; see, e.g., [16].

We denote the number of users of Operator 1 (resp. Operator 2) at time $t \geq 0$ by $N_1(t)$ (resp. $N_2(t)$), and we assume a fixed population of $N$ users, with $N_1(t) + N_2(t) = N$, i.e., no user exits the market. In our model a user has a positive network externality which derives from other adopters of the same technology [9]. We model the network externality as $N_i(t)/N$, for $i = 1,2$.

### C. User Model

Users can freely make choice between the two operators, depending on cost and QoS, as well as network externality.

*1) Users' Utility:* Due to heterogeneity, users have different preference to the QoS of a technology, and thus we characterize a user's utility with respect to Operator $i$, following [15], as

$$U_i(\theta, \underline{x}(t)) = \theta q_i + x_i(t) - p_i, \quad i = 1,2, \quad (3)$$

where $x_i(t) = N_i(t)/N$, $\underline{x}(t) = (x_1(t), x_2(t))$. The first term represents the standalone benefit from the technology, in which $\theta$, being a uniform random variable over $[0,1]$, captures the preference of the user. The uniform distribution assumption is for tractability and is common in the literature [15]. The second term $x_i(t)$ represents the network externality, which has a positive contribution to the utility. The last term is the price, which has a negative impact on the utility.

*2) Users' Decision:* Each user's choice is affected by its utility function. As we focus on the scenario where no user exits the market, we require at least one of $U_1$ and $U_2$ be nonnegative for each user. Assuming each user to be rational, it chooses

$$\begin{cases} \text{Operator 1} & \text{if } U_1 \geq 0, U_2 \leq U_1 \\ \text{Operator 2} & \text{if } U_2 \geq 0, U_1 \leq U_2. \end{cases} \quad (4)$$

Given $\underline{x}(t)$, we can calculate the equilibrium of the user distribution $(H_1(\underline{x}(t)), H_2(\underline{x}(t)))$, which represents the fraction of users for whom Operator $i$ provides the higher and nonnegative utility. Hence the difference $H_i(\underline{x}(t)) - x_i(t)$ corresponds to the fraction of users that intend to choose Operator $i$ at time $t$. But users' decisions may not be made immediately, and thus we adopt the following diffusion model to describe the variation of user distribution,

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = \gamma(H_i(\underline{x}(t)) - x_i(t)), \quad i = 1,2, \quad (5)$$

where $\gamma < 1$ is the diffusion rate.

## III. Evolution of User Distribution

Given different initial conditions $\underline{x}(0)$, there are different equilibria and thus different evolution curves of user distribution $\underline{x}(t)$ [15]. In this section we characterize the evolution patterns of user distribution.

### A. Classification of Evolution Patterns

Let us denote by $\Theta_i^0(\underline{x}(t))$ the solution $\theta$ of $U_i(\theta, \underline{x}(t)) = 0$, for $i = 1,2$, and by $\Theta_2^1(\underline{x}(t))$ the solution $\theta$ of $U_1(\theta, \underline{x}(t)) = U_2(\theta, \underline{x}(t))$. In explicit form we have

$$\Theta_1^0(\underline{x}(t)) = \frac{p_1 - x_1(t)}{(1-\varphi)q_l + \varphi q_g}, \quad (6)$$

$$\Theta_2^0(\underline{x}(t)) = \frac{p_2 - x_2(t)}{q_l}, \quad (7)$$

$$\Theta_2^1(\underline{x}(t)) = \frac{(p_2 - p_1) + (x_1(t) - x_2(t))}{\varphi(q_l - q_g)}. \quad (8)$$

Since at least one of $U_1$ and $U_2$ is nonnegative for all $\theta \in [0, 1]$ (c.f. the assumption for (4)), we need at least one of $\Theta_1^0(\underline{x}(t))$ and $\Theta_2^0(\underline{x}(t))$ be non-positive. Depending on the values of $\Theta_1^0(\underline{x}(t))$, $\Theta_2^0(\underline{x}(t))$ and $\Theta_2^1(\underline{x}(t))$, we can identify three qualitatively different cases of the evolution patterns of $\underline{x}(t)$. For notational convenience we sometimes omit the time dependency and write $\underline{x}(t)$ simply as $\underline{x}$.

*1) Case 1(a):* $\Theta_1^0(\underline{x}) \geq \Theta_2^0(\underline{x})$, $\Theta_2^0(\underline{x}) \leq 0$

This is illustrated in Figure 1. Note that the region of interest is $\theta \in [0, 1]$, outlined in red.

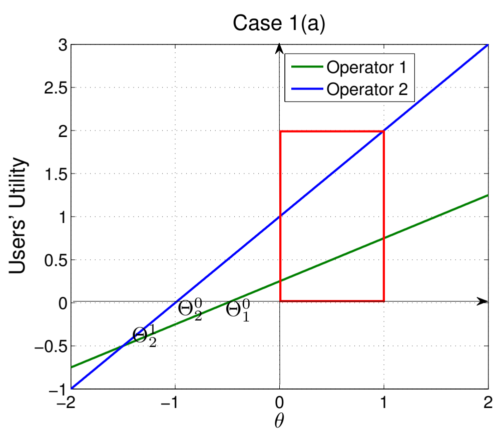


Fig. 1. Case 1(a)

*2) Case 1(b):* $\Theta_1^0(\underline{x}) \leq \Theta_2^0(\underline{x})$, $\Theta_2^1(\underline{x}) \leq 0$ (the condition $\Theta_1^0(\underline{x}) \leq 0$ is suppressed since it is implied by the first two). This is illustrated in Figure 2.

For Case 1, we have $H_1(\underline{x}) = 0$ and $H_2(\underline{x}) = 1$. Based on (4) and (5), the user distribution evolves as $x_1(t) = x_1(0)e^{-\gamma t}$ and $x_2(t) = 1 - x_1(t)$.

*3) Case 2:* $0 \leq \Theta_2^1(\underline{x}) \leq 1$, $\Theta_1^0(\underline{x}) \leq 0$ ($\Theta_1^0(\underline{x}) \leq \Theta_2^0(\underline{x})$ is implied). This is illustrated in Figure 3.

For this case, we have $H_1(\underline{x}) = \Theta_2^1(\underline{x}) = \frac{(p_2-p_1)+(x_1(t)-x_2(t))}{\varphi(q_l-q_g)}$ and $H_2(\underline{x}) = 1 - H_1(\underline{x})$. Based on (4), (5), the user distribution evolves as $x_1(t) = x_1{}^* + (-x_1{}^* + x_1(0))e^{-(1-\frac{2}{\varphi(q_l-q_g)})\gamma t}$ and $x_2(t) = 1 - x_1(t)$, where $x_1{}^* = \frac{1+p_1-p_2}{2-\varphi(q_l-q_g)}$.

*4) Case 3:* $1 \leq \Theta_2^1(\underline{x})$, $\Theta_1^0(\underline{x}) \leq 0$ ($\Theta_1^0(\underline{x}) \leq \Theta_2^0(\underline{x})$ is implied). This is illustrated in Figure 4.

For this case, we have $H_1(\underline{x}) = 1$ and $H_2(\underline{x}) = 0$. Based on (4) and (5), the user distribution evolves as $x_1(t) = (x_1(0) - 1)e^{-\gamma t} + 1$, $x_2(t) = 1 - x_1(t)$.

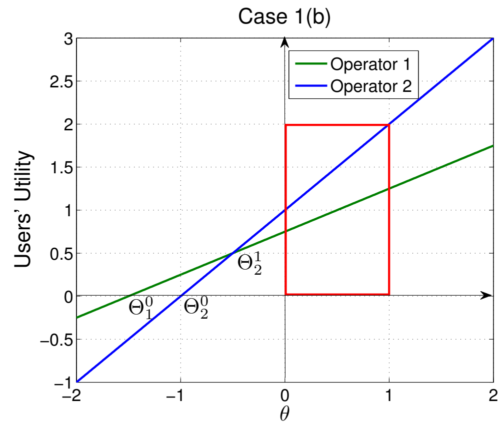


Fig. 2. Case 1(b)

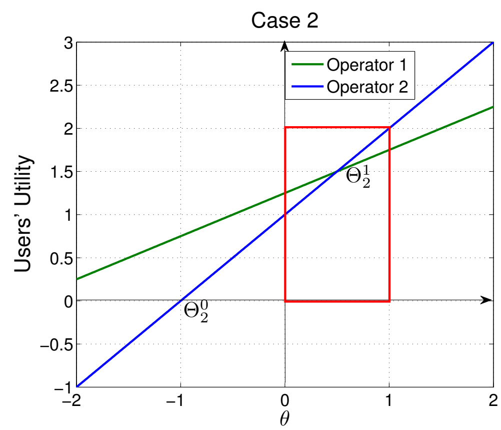


Fig. 3. Case 2

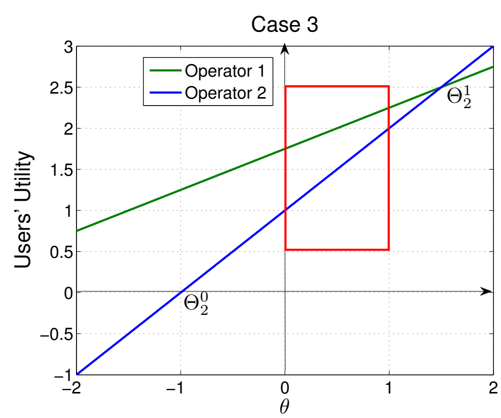


Fig. 4. Case 3

### *B. Switching of Evolution Patterns*

Since $\Theta_1^0(\underline{x})$, $\Theta_2^0(\underline{x})$ and $\Theta_2^1(\underline{x})$ are linear with $\underline{x}(t)$, the evolution of $\underline{x}(t)$ may switch between different cases with time. If the initial condition and its equilibrium are in the same case, then the evolution will remain within this case. Let us call such the "interior condition", which can be described by solving the constraints regarding

$\Theta_1^0(\underline{x}(t))$, $\Theta_2^0(\underline{x}(t))$ and $\Theta_2^1(\underline{x}(t))$ for each case, as follows.

*1) Case 1:* In this case, Operator 2 always takes the whole market in the long run. The interior condition for Case 1(a) is

$$\begin{cases} x_2(0) \geq p_2 \\ \varphi(q_l - q_g)(p_2 - x_2(0)) \geq q_l(p_2 - p_1 + 2x_1(0) - 1), \end{cases} \tag{9}$$

and the interior condition for Case 1(b) is

$$\begin{cases} p_2 - p_1 \leq x_2(0) - x_1(0) \\ \varphi(q_l - q_g)(p_2 - 1) \leq q_l(p_2 - p_1 - 1). \end{cases} \tag{10}$$

*2) Case 2:* In this case, Operators 1 and 2 coexist. The interior condition is

$$\begin{cases} p_1 \leq x_1(0) \\ p_2 - p_1 \geq 1 \\ \varphi(q_l - q_g) \geq p_2 - p_1 + 1 \\ \varphi(q_l - q_g) \leq \frac{p_2 + p_1 - 1}{p_1}. \end{cases} \tag{11}$$

*3) Case 3:* In this case, Operator 1 always takes the whole market in the long run. The interior condition is

$$\begin{cases} p_1 \leq x_1(0) \\ \varphi(q_l - q_g) \leq p_2 - p_1 + (x_1(0) - x_2(0)). \end{cases} \tag{12}$$

From these descriptions of the interior condition, we can trace the switching of the user distribution evolution, and the results are summarized in Table I.

TABLE I
SWITCHING DYNAMICS OF USER DISTRIBUTION EVOLUTION

| Initial condition | Equilibrium | | |
|---|---|---|---|
| Case 1 | Case 1 | | |
| Case 2 | $\varphi(q_l - q_g) \leq 2$ | $x_1(0) < x_1^*$ | Case 1 |
| | | $x_1(0) = x_1^*$ | Case 2 |
| | | $x_1(0) > x_1^*$ | Case 3 |
| | $\frac{p_2+p_1-1}{p_1} \geq \varphi(q_l - q_g) \geq 2$ | Satisfy (11) | Case 2 |
| | | Otherwise | Case 3 |
| Case 3 | Case 3 | | |

## IV. Profit Analysis and Optimal Upgrading

Since in Case 1, the Operator 1 is eventually repelled from the market, which means over a large time span, the profit of Operator 1 vanishes, we mainly discuss the profit in Cases 2 and 3, as follows.

Considering the time discount, the revenue of Operator 1 is

$$R = \int_0^\infty \kappa N_1(t) \log N e^{-St} \mathrm{d}t = \kappa \log N \int_0^\infty N_1(t) e^{-St} \mathrm{d}t, \tag{13}$$

which is the discounted integration of the network value of Operator 1 [16].

The total cost is

$$C = \frac{I}{S}[(1 - \varphi)c_l + \varphi c_g] + K\varphi, \tag{14}$$

including both the operating cost and the upgrading cost.

The profit thus is $P = R - C$. In the sequel, we normalize the profit by the total network value $\kappa N \log N$, and thus $I$ and $K$ are also normalized. With a slight abuse of notation, we still use $I$ and $K$ to denote the infrastructure and the one-time upgrading cost, respectively, after the normalization.

If the evolution of user distribution resides within Case 2 throughout, occurring when (11), i.e., the following set of constraints

$$\begin{cases} p_1 \leq x_1(0) \\ p_2 - p_1 \geq 1 \\ \frac{p_2 - p_1 + 1}{q_l - q_g} \leq \varphi \leq \frac{p_2 + p_1 - 1}{(q_l - q_g)p_1} \\ 0 \leq \varphi \leq 1, \end{cases} \tag{15}$$

holds, then the profit can be evaluated as

$$P_2 = \frac{(c_l - c_g)I - KS}{S}\varphi - \frac{c_l I}{S} + \frac{x_1(0)(q_l - q_g)\varphi}{(S+\gamma)(q_l - q_g)\varphi - 2\gamma} + \frac{\gamma}{S}\frac{p_2 - p_1 - 1}{(S+\gamma)(q_l - q_g)\varphi - 2\gamma}. \tag{16}$$

If the evolution of user distribution resides within Case 3 throughout, occurring when (12), i.e., the following set of constraints

$$\begin{cases} p_1 \leq x_1(0) \\ \varphi \leq \frac{p_2 - p_1 + x_1(0) - x_2(0)}{q_l - q_g} \\ 0 \leq \varphi \leq 1, \end{cases} \tag{17}$$

holds, then the profit can be evaluated as

$$P_3 = \frac{(c_l - c_g)I - KS}{S}\varphi - \frac{c_l I}{S} + \frac{1}{S} + \frac{x_1(0) - 1}{S + \gamma}. \tag{18}$$

For (16), we take its second derivative $P_2''(\varphi)$,

$$P_2''(\varphi) = \frac{4\gamma x_1(0) S(S+\gamma)(q_l - q_g)}{S[(S+\gamma)(q_l - q_g)\varphi - 2\gamma]^3} + \frac{2\gamma(S+\gamma)^2(q_l - q_g)^2(p_2 - p_1 - 1)}{S[(S+\gamma)(q_l - q_g)\varphi - 2\gamma]^3}. \tag{19}$$

With (15), we find that $(S+\gamma)(q_l - q_g)\varphi - 2\gamma = S(q_l - q_g)\varphi + \gamma[(q_l - q_g)\varphi - 2] > 0$ and $p_2 - p_1 - 1 > 0$, so $P_2''(\varphi)$ is positive, which implies that $P_2$ a convex function of $\varphi$. Consequently, the maximum profit in Case 2 is attained at the boundary of (15), namely, either $\varphi = \max\{0, \frac{p_2 - p_1 + 1}{q_l - q_g}\}$ or $\varphi = \min\{1, \frac{p_2 + p_1 - 1}{(q_l - q_g)p_1}\}$.

We take the difference between (16) and (18),

$$P_2 - P_3 = (x_1^* - 1)\left(\frac{1}{S} - \frac{1}{(S + \gamma - \frac{2\gamma}{\varphi(q_l - q_g)})}\right) + (x_1(0) - 1)\left(\frac{1}{(S + \gamma - \frac{2\gamma}{\varphi(q_l - q_g)})} - \frac{1}{S + \gamma}\right), \tag{20}$$

which can be shown to be non-positive for every $\varphi$ satisfying (15).

For notational convenience, we denote $\Phi_2^1 = \frac{p_2 - p_1 + 1}{q_l - q_g}$, $\Phi_2^2 = \frac{p_2 + p_1 - 1}{(q_l - q_g)p_1}$, and $\Phi_3 = \frac{p_2 - p_1 + x_1(0) - x_2(0)}{q_l - q_g}$, respectively.

We remark that there is a gap between the regions of $\varphi$ satisfying (15) and (17), when $\varphi$ is between $\Phi_2^1$ and $\Phi_3$. It can be shown that when $\varphi$ lies in this gap, the evolution pattern is first in Case 2, and then at some time point switches to Case 3. The profit in the gap is somewhat tedious to analyze, given by

$$\begin{aligned} P_{\text{gap}} = & \frac{x_1(0) - x_1^*}{S - \gamma(\frac{2}{\varphi(q_l - q_g)} - 1)} (1 - e^{(\gamma(\frac{2}{\varphi(q_l-q_g)}-1)-S)t^{\#}}) \\ & + \frac{x_1^*}{S}(1 - e^{-St^{\#}}) + \frac{\varphi(q_l - q_g) - 1 - p_2 + p_1}{2(S+\gamma)} e^{-(S+\gamma)t^{\#}} \\ & + \frac{e^{-St^{\#}}}{S} + \frac{(c_l - c_g)I - KS}{S}\varphi - \frac{c_l I}{S}, \qquad (21) \end{aligned}$$

where $t^{\#}$ is determined by $e^{(\frac{2}{\varphi(q_l-q_g)}-1)\gamma t^{\#}} = \frac{\varphi(q_l-q_g)+1-p_2+p_1-2x_1^*}{2x_1(0)-2x_1^*}$. When $P_3(\Phi_3)$ and $P_2(\Phi_2^1)$ are close, $P_{\text{gap}}$ may not be monotone in the gap, whereas our numerical study suggests that the optimal upgrading choice $\varphi^*$ never lies within this gap.

Note that for the same $\varphi$, we have $P_2(\varphi) \leq P_3(\varphi)$ according to (20). When $(c_l - c_g)I - KS \leq 0$, the trend is illustrated in Figure 5; when $(c_l - c_g)I - KS > 0$, the trend is illustrated in Figure 6.

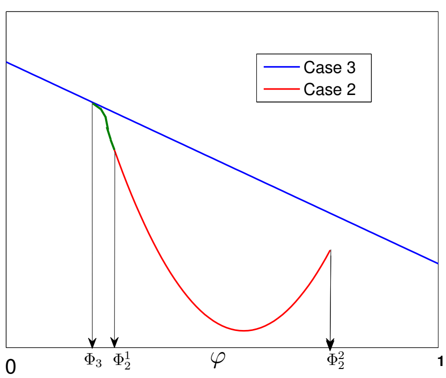


Fig. 5. Illustration of profit about $\varphi$, when $(c_l - c_g)I \leq KS$.

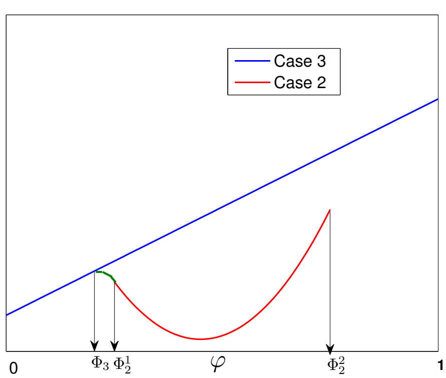


Fig. 6. Illustration of profit about $\varphi$, when $(c_l - c_g)I > KS$.

Comparing the profits attained by the possible optimal choices of $\varphi$, we can obtain our main result in the following, which characterizes the optimal choice $\varphi^*$.

*No upgrading,* $\varphi^* = 0$

When $(c_l - c_g)I - KS < 0$, and $x_1(0) \geq \max\{p_1, \frac{p_1 - p_2 + 1}{2}\}$, we have $\varphi^* = 0$. Intuitively, when green communications technology cannot cut down the operating cost markedly (i.e., $c_l - c_g < KS/I$ is not met), there is no incentive for the operator to consider upgrading its infrastructure.

*Full upgrading,* $\varphi^* = 1$

This only happens when $(c_l - c_g)I - KS > 0$.

- When $\Phi_3 \geq 1$, i.e., $p_2 - p_1 + 2x_1(0) - 1 > q_l - q_g$, and $p_1 \leq x_1(0)$, we have $\varphi^* = 1$. This is when Operator 2's price $p_2$ is exceedingly high so Operator 1 can safely upgrade its infrastructure without concerning about losing users.
- When $0 < \Phi_3 < 1, \Phi_2^1 < 1, \Phi_2^2 > 1$, i.e., $p_2 - p_1 + 1 < q_l - q_g < \frac{p_2 + p_1 - 1}{p_1}$, $p_1 \leq x_1(0)$ and $p_2 - p_1 \geq 1$, we have $\varphi^* = 1$ if $\max\{P_3(\Phi_3), P_2(\Phi_2^1)\} \leq P_2(1)$.

*Partial upgrading,* $0 < \varphi^* < 1$

This also only happens when $(c_l - c_g)I - KS > 0$.

- When $0 < \Phi_3 < 1, \Phi_2^1 < 1, \Phi_2^2 < 1$, i.e., $q_l - q_g > \frac{p_2 + p_1 - 1}{p_1}$, $p_1 \leq x_1(0)$ and $p_2 - p_1 \geq 1$, we have $\varphi^*$ as the value of $\varphi$ attaining $\max\{P_3(\Phi_3), P_2(\Phi_2^1), P_2(\Phi_2^2)\}$.
- When $0 < \Phi_3 < 1, \Phi_2^1 < 1, \Phi_2^2 > 1$, i.e., $p_2 - p_1 + 1 < q_l - q_g < \frac{p_2 + p_1 - 1}{p_1}$, $p_1 \leq x_1(0)$ and $p_2 - p_1 \geq 1$, we have $\varphi^*$ as the value of $\varphi$ attaining $\max\{P_3(\Phi_3), P_2(\Phi_2^1)\}$ if $\max\{P_3(\Phi_3), P_2(\Phi_2^1)\} > P_2(1)$.
- When $0 < \Phi_3 < 1, 1 < \Phi_2^1$, i.e., $p_2 - p_1 + 2x_1(0) - 1 < q_l - q_g < p_2 - p_1 + 1$, $p_1 \leq x_1(0)$ and $p_2 - p_1 \geq 1$, we have $\varphi^* = \Phi_3$.

## V. Numerical Results

### A. Distribution Evolution

The key point to figure out the operator's profit is knowing the evolution pattern of users proportion. From Table I we see that only when the initial condition belongs to Case 2, there can be a pattern switch, so we mainly consider the parameter setting with the initial condition of Case 2.

This dynamic process can be influenced by different initial conditions. Since $\varphi$ affects $q_1$ directly ($q_1 = (1 - \varphi)q_l + \varphi q_g$), we study the impact of $\varphi$ by change $q_1$. Figures 7 and 8 highlight the sensitivity to the quality of Operator 1, i.e., $q_1$. Figure 7 reveals the diffusion dynamics when $\varphi(q_l - q_g) < 2$. We see that for the same values of $p_2 = 0.8, p_1 = 0.3, q_l = 5, q_g = 2, x_1(0) = 0.4$, the differences in $q_1$ can result in drastically different outcomes. From the curves, when

$q_1 < q_2$, Operator 1 may eventually defeat Operator 2. In other words, the higher quality but higher price of the green technology may fail to attract "stingy" users. To avoid being eliminated, Operator 2 can choose a lower price to regain a position in the market. Figure 8 reveals the outcome when $\varphi(q_l - q_g) > 2$. As $\varphi$ increases, the parameter setting satisfies (11), and the final equilibrium changes from Case 3 to Case 2.

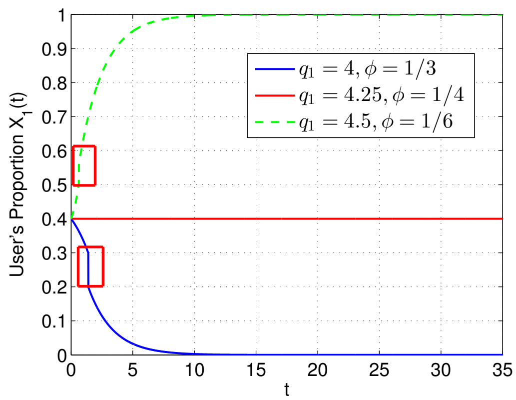


Fig. 7. Effects of $q_1$ on diffusion dynamics I ($\varphi(q_l - q_g) < 2$)

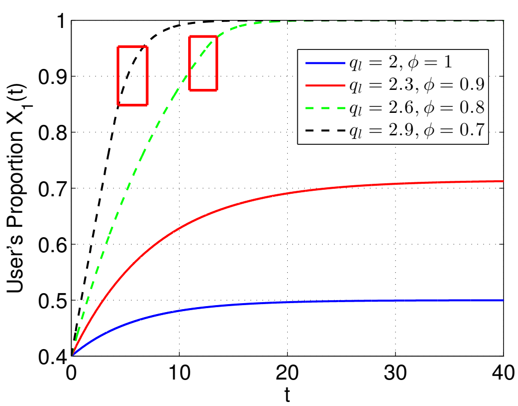


Fig. 8. Effects of $q_1$ on diffusion dynamics II ($\varphi(q_l - q_g) > 2$)

The evolution process also exhibits robustness. In Figure 9, the initial penetration of Operator 1, i.e., $x_1(t_0)$, does not affect the eventual outcome. Therein, we set $p_1 = 0.2, p_2 = 1.3, q_g = 1.7, q_l = 4.2, \varphi = 0.88$. The final equilibrium is $x_1{}^* = \frac{1+p_1-p_2}{2-\varphi(q_l-q_g)} = 0.5$, and the evolution always remains in Case 2.

### B. Value of $\varphi^*$

From the analysis in Section IV, the QoS of the green communication technology $q_g$, the price of Operator 1 $p_1$, and the gap $(c_l - c_g)I - KS$ are key factors influencing the optimal upgrading proportion $\varphi^*$.

When $q_g$ increases, $\Phi_2^1 = \frac{p_2-p_1+1}{q_l-q_g}$, $\Phi_2^2 = \frac{p_2+p_1-1}{(q_l-q_g)p_1}$, and $\Phi_3 = \frac{p_2-p_1+x_1(0)-x_2(0)}{q_l-q_g}$ increase, and the profit

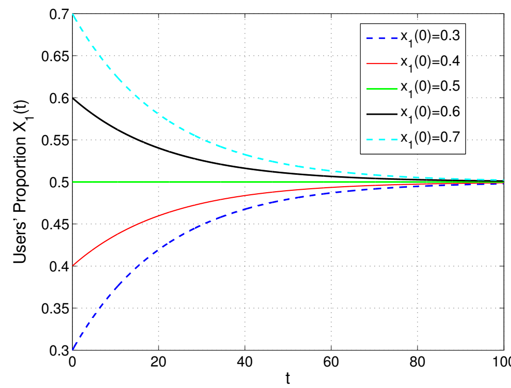


Fig. 9. Robustness of diffusion dynamics

functions $P_2, P_3$ also change accordingly. From Figure 10, the optimal upgrading proportion $\varphi^*$ increases with $q_g$. This reflects the reality that high QoS of green technology attracts the operator to invest on it.

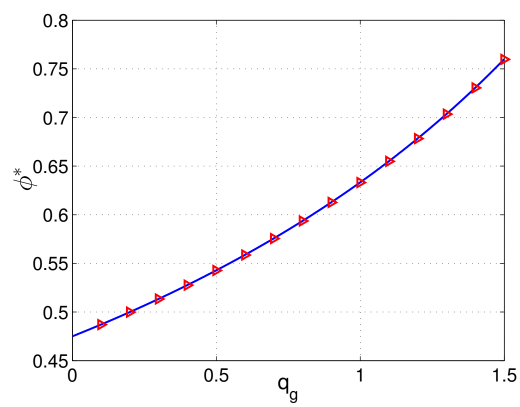


Fig. 10. Relationship between $q_g$ and $\varphi^*$

In Figure 11, we further change the price of Operator 1, to investigate its effect on $\varphi^*$. It is shown that $\varphi^*$ decreases with $p_1$. Note that there is a discontinuous change in $\varphi^*$.

Figure 12 displays the relationship between $(c_l - c_g)I - KS$ and $\varphi^*$. When $(c_l - c_g)I - KS < 0$, $\varphi^* = 0$, and with $(c_l - c_g)I - KS$ increasing, $\varphi^*$ increases accordingly, in a stepwise fashion.

## VI. Conclusion and Future Work

This paper presented a preliminary study of network operators' adoption strategy of green (i.e., energy-efficient) communications technology, focusing on the asymmetric scenario where only one of the operators may upgrade its infrastructure. Heterogenous and rational users make decision upon their choices of operators, based on their utilities which are determined by QoS, price, and network externality. The evolution of user

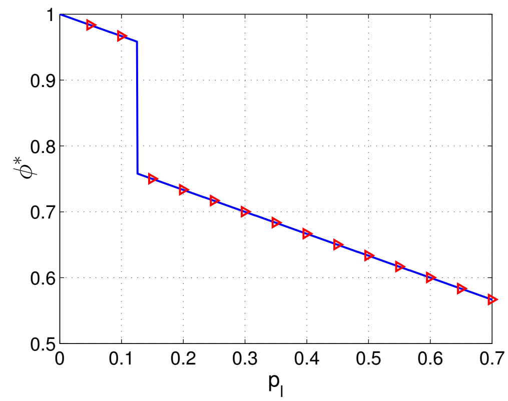


Fig. 11. Relationship between $p_1$ and $\varphi^*$

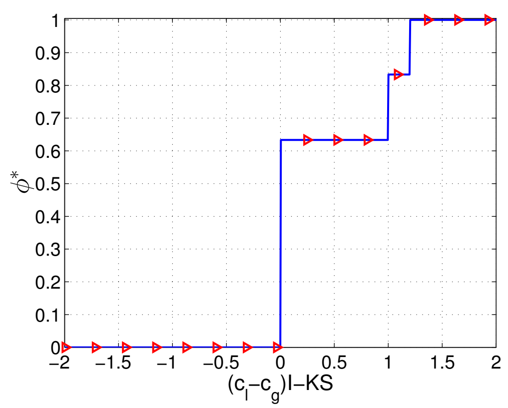


Fig. 12. Relationship between $(c_l - c_g)I - KS$ and $\varphi^*$

distribution affects the resulting profit of the considered operator. Analysis reveals that the choice of the operator depends upon a variety of system parameters as well as the initial condition of market.

A number of generalizations exist to extend this preliminary study, including: considering a symmetric scenario where both operators may upgrade their infrastructures; allowing upgrading to occur at multiple times for each operator, and at the same or different times between operators; allowing prices to be a function of the upgrading proportion; among others.

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