Mobile Data Trading: A Behavioral Economics Perspective

Junlin Yu, Man Hon Cheung, Jianwei Huang, and H. Vincent Poor

***Abstract*—Motivated by the recently launched 2CM data trading platform of China Mobile Hong Kong, we study the optimal user mobile data trading problem under the future demand uncertainty. We consider a brokerage-based market, where sellers and buyers propose their selling and buying prices and quantities to the trading platform, respectively. The platform acts as a broker, which facilitates the trade by matching the supply and demand. To understand users' realistic trading behaviors, we use prospect theory (PT) from behavioral economics in the modeling, which leads to a challenging non-convex optimization problem. Nevertheless, we are able to determine the unique optimal solution in closed-form, by utilizing the unimodal structure of the objective function. When comparing with the benchmark expected utility theory (EUT), we show that a PT user with a low reference point is more willing to buy mobile data. Moreover, when the probability of high demand is low, comparing with an EUT user, a PT user is more willing to buy mobile data due to the probability distortion.**

I. Introduction

With the increasing computation and communication capabilities of mobile devices, global mobile data traffic has been growing tremendously in the past few years [1], [2]. In order to alleviate the tension between mobile data demand and network capacity, mobile service providers have been experimenting with several innovative pricing schemes, such as usage-based pricing, shared data plans, and sponsored data pricing [3]–[5]. However, the above schemes do not fully take advantage of the heterogeneous demands across mobile users. Recently, China Mobile Hong Kong (CMHK) launched the first 4G data trading platform in the world, called the 2nd exChange Market (2CM), which allows its users to trade their 4G mobile data directly with each other. In this platform, a seller can list his desirable selling price (within a predefined range), together with the amount of data to be sold (up to his monthly service plan quota). If there is a buyer who is willing to buy (part of) the data at the listed price, the platform will clear the transaction and transfer the corresponding data to the buyer's monthly quota limit.

In the current form of 2CM, only a seller can list his price and selling quantity, but not a buyer. This means that a buyer needs to frequently check the platform to see whether he is willing to buy according to the current (lowest) selling price. This motivates us to propose a new market mechanism that is based on the widely used Walrasian auction in the stock markets [6], [7], in which both sellers and buyers can submit their selling and buying prices and quantities to the platform. The platform clears the market whenever the buying price of a buyer is no smaller than the selling price of a seller.

In this paper, we focus on a single user's trading decision under the future demand *uncertainty*, given the prices and quantities of other sellers and buyers. More specifically, we focus on the following questions: *(i) Should a user choose to be a seller or a buyer? (ii) How much should he sell or buy?*

One way of answering the above questions is to compute the maximum expected utilities for being a seller and a buyer, considering his future demand uncertainty and the satisfaction loss for exceeding the monthly data quota. Then by comparing these utilities, the user can decide whether to be a seller or a buyer. Such an approach relies on the *expected utility theory* (EUT), which has been widely used in studying decision problems *under uncertainty* [8]. However, substantial empirical evidence suggests that predictions based on EUT can significantly deviate from real world observations, due to the complicated psychological aspect of human decision-making. Researchers in behavioral economics showed that *prospect theory* (PT) provides a psychologically more accurate description of the decision making under uncertainty, and can explain quite a few human behaviors that seem to be illogical under EUT [9].

PT shows that a decision maker evaluates an outcome significantly differently from what people have commonly assumed in EUT in several aspects: (1) *Impact of reference point*: A PT decision maker's evaluation is based on the *relative* gains or losses comparing to a reference point, instead of the absolute values of the outcomes. (2) *Asymmetric value function*: A PT decision maker tends to be *risk averse* when considering gains and *risk seeking* when considering losses. Furthermore, the PT decision maker is *loss averse*, in the sense that he strongly prefers avoiding losses to achieving gains. (3) *Probability distortion*: A PT decision maker tends to *overweigh* low probability events and *underweigh* high probability events. As PT has been shown to be more accurate than EUT in predicting human behaviors [9]–[11], it has been successfully applied to have a better understanding of financial markets [12] and labor markets [13].

In this paper, we aim to understand a user's realistic trading behavior in a mobile data market, considering the user's future demand uncertainty. Specifically, we formulate the problem as a two-stage optimization problem, where the user will decide

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whether to be a seller or a buyer in Stage I, and his selling price and quantity (as a seller) or buying price and quantity (as a buyer) in Stage II. We will discuss the practical insights by comparing the analysis under PT and EUT (which is a special case of PT with properly chosen system parameters).

The research of using behavioral economics (and PT in particular) to understand user decisions in networking is at its infancy stage. Li *et al.* in [14], [15] considered a linear value function with the probability distortion, and compared the equilibrium strategies of a two-user random access game under EUT and PT. Xiao *et al.* in [16] and Wang *et al.* in [17] considered a linear value function with the probability distortion, and characterized the unique Nash Equilibrium of an energy exchange game among microgrids under PT. Yu *et al.* in [18] considered the general S-shaped value function in studying a secondary wireless operator's spectrum investment problem. To the best of our knowledge, this paper is the first work that captures all three characteristics of PT when modeling and analyzing a wireless networking problem. As a result, we are able to gain a more thorough understanding of the user's optimal decisions and derive more insights.

Our key contributions are summarized as follows:

- *Behavioral economics modeling of uncertainty*: We use prospect theory to model the user's trading behavior under future demand uncertainty. We consider all three key characteristics of PT, and derive key insights that characterize the optimal selling and buying decisions.
- *Non-convex optimization*: Despite the non-convexity of the user's decision problem, we are able to obtain a closed-form characterization of the unique optimal solution. We further evaluate how different behavioral characteristics (i.e., reference point, probability distortion, and S-shaped valuation) affect this optimal decision.
- *Engineering insights*: Comparing with the results under EUT, we show that a PT user with a low reference point is more willing to buy mobile data and less willing to sell mobile data. Moreover, a PT user is more willing to buy mobile data when the probability of high demand is low, mainly due to the probability distortion.

The rest of this paper is organized as follows. In Section II, we describe the data usage trading platform and formulate the user's utility functions under both EUT and PT. In Section III, we compute the unique optimal user decision. In Section IV, we numerically evaluate the sensitivity of the user's optimal decision with respect to several model parameters. We conclude the paper in Section V.

II. System Model

We consider a mobile data trading market with a large number of users. Each user makes the trading decision in two stages. In Stage I, he decides whether to sell or to buy in the market, or not participate in the market. In Stage II, he decides the price and quantity as a seller or as a buyer, depending on his choice in Stage I. For simplicity, we assume that the user makes the trading decision only once in a billing cycle, although different users may make decisions at different times[1]. Since the number of users in the market is large, a single user's choice will have a negligible impact on the market.

Table I: A snapshot of the data trading platform.

Buyers' Market		Sellers' Market	
Price (per GB)	Available (GB)	Price (per GB)	Available (GB)
\$16	60	\$20	50
\$13	20	\$21	45
\$11	30	\$24	20
...	...	...	...

More specifically, in Stage I, a user makes a decision $a \in \mathcal{A} = \{s, b\}$, where s and b correspond to being a seller and a buyer, respectively. In Stage II, a seller determines his offer $\{q_s, \pi_s\}$, which means that he is willing to sell q_s (GBs) of data at a unit price of π_s (dollars per GB). A buyer determines his bid $\{q_b, \pi_b\}$, which means that he is willing to buy a total of q_b of data at a unit price of π_b.

Table I shows an example of the market, which includes the prices and quantities of users who have made their decisions. In this example, the highest buying price from the buyers' market ($\pi_b^{\max} = \$16$) is lower than the lowest selling price from the sellers' market ($\pi_s^{\min} = \$20$). This is because those selling offers with prices less than \$16 have already been cleared by the market, and so are those buying requests with prices higher than \$20. Under a large network assumption, it is reasonable to assume that the quantity associated with the maximum buyer price and the quantity associated with the minimum seller price are both large enough. This means that for a single user who wants to complete the trade immediately, he only needs to consider the maximum buyer price $\pi_b^{\max}$ and minimum seller price $\pi_s^{\min}$, and ignore all other prices[2]. In particular, if a user chooses to be a seller in Stage I, his selling price in Stage II will be $\pi_s = \pi_b^{\max}$, so that he can sell the data immediately with the maximum price that some existing buyer can accept. Similarly, if a user chooses to be a buyer in Stage I, he will set his buying price in Stage II as $\pi_b = \pi_s^{\min}$, so that he can buy the data immediately with the minimum price that some existing seller can offer. Hence we will ignore the users' pricing decisions in the rest of the paper.

The key issue that the user needs to consider is the future data demand uncertainty. If his total monthly data consumption d exceeds his monthly data quota Q, he will incur a *satisfaction loss*. For simplicity, we consider a linear satisfaction loss function,

$$L(y) = \begin{cases} 0, & y \geq 0, \\ \kappa y, & y < 0, \end{cases} \tag{1}$$

where

$$y = Q - d. \tag{2}$$

When y is negative, it means that the quota is exceeded. The linear coefficient κ represents the usage-based pricing imposed

[1]In the future work, we will consider the multi-period scenario, where each user can make multiple sequential trading decisions in a single billing cycle.

[2]If a user does not need to complete the trade immediately, he may choose to list a selling price higher than $\pi_s^{\min}$ or list a buying price lower than $\pi_b^{\max}$. We will consider this more general case in our future work.

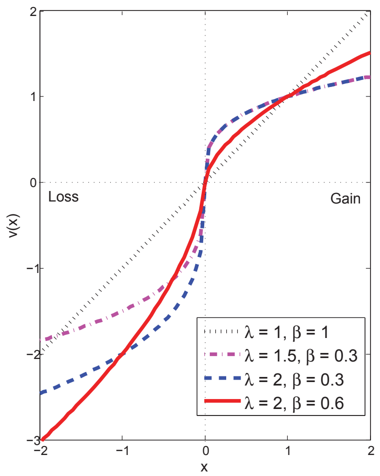

Figure 1: The S-shaped asymmetrical value function $v(x)$ in PT.

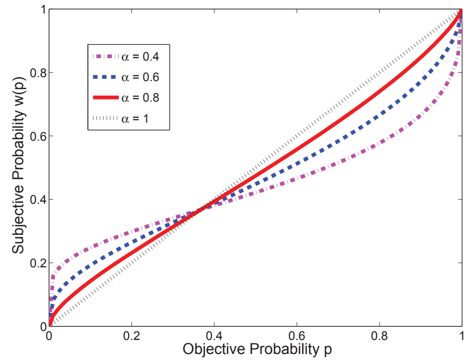

Figure 2: The probability distortion function $w(p)$ in PT.

by the operator[3]. By selling or buying data in the market, a user can change his monthly data quota (for the current month only), and hence will change the expected value of the satisfaction loss.

Next we derive the user's expected utilities of being a seller and a buyer, under both EUT and PT.

A. Utilities under Expected Utility Theory (EUT)

We assume that the user's total monthly data consumption has I possible values $\{d_i: i = 1, ..., I\}$, with the corresponding probabilities $\{p_i: i = 1, ..., I\}$ such that $\sum_{i=1}^{I} p_i = 1$. If a user buys q_b GBs of data, his expected utility is

$$U(b, q_b) = \sum_{i=1}^{I} p_i \left[-\pi_s^{\min} q_b + L(Q + q_b - d_i) \right], \tag{3}$$

where $\pi_s^{\min} q_b$ is the cost for buying the data at the minimum selling price $\pi_s^{\min}$, and $L(Q + q_b - d_i)$ is the satisfaction loss if the total data consumption is d_i.

If a user sells q_s GBs of data in the market, then his expected utility is

$$U(s, q_s) = \sum_{i=1}^{I} p_i \left[\pi_b^{\max} q_s + L(Q - q_s - d_i) \right], \tag{4}$$

where $\pi_b^{\max} q_s$ is the revenue for selling the data at the maximum buying price $\pi_b^{\max}$.

B. Utilities under Prospect Theory (PT)

Here, we consider the three features of PT, namely S-shaped *value function* $v(x)$, *probability distortion function* $w(p)$, and *reference point* R_p [9], [20].

A higher reference point R_p indicates that the user has a high expectation on the utility, and he considers an outcome as a *loss* if it is less than his expectation. A lower reference point R_p indicates that the user has a low expectation, and he considers an outcome as a *gain* if it is above his expectation.

[3]We have assumed a two-part pricing tariff, where the user pays a fixed fee for the data consumption up to a monthly quota, and a linear usage-based cost for any extra data consumption. Such a pricing model is widely used by major operators [19]. For example, for a 4G CMHK user, $\kappa = \$60$ with a monthly data quota of 1 GB.

As we will see next, whether an outcome is considered as a loss and gain will significantly affect the user's subjective valuation of the outcome.

Figure 1 illustrates the value function $v(x)$, which maps an outcome x to the user's subjective valuation $v(x)$. Notice that all the outcomes are measured relatively to the reference point, which is normalized to $x = 0$. Behavioral studies show that the function $v(x)$ is S-shaped, which is concave in the gain region (i.e., $x > 0$, the outcome is larger than the reference point) and convex in the loss region (i.e., $x < 0$, the outcome is smaller than the reference point). Moreover, the impact of loss is larger than the gain, i.e., $|v(-x)| > v(x)$ for any $x > 0$. A commonly used value function in the PT literature is [9]

$$v(x) = \begin{cases} x^{\beta}, & x \ge 0, \\ -\lambda(-x)^{\beta}, & x < 0, \end{cases} \tag{5}$$

where $0 < \beta \le 1$ and $\lambda \ge 1$. Here β is the **risk aversion parameter**, where a *smaller* β means that the value function is more concave in the gain region, hence the user is more *risk averse* in gains. Similarly, a *smaller* β means that the value function is more convex in the loss region, hence the user is more *risk seeking* in losses. The valuation of the loss region is further characterized by the **loss penalty parameter** λ, where a *larger* λ indicates that the user is more *loss averse*.

Figure 2 illustrates the probability distortion function $w(p)$, which captures humans' psychological over-weighting of low probability events and under-weighting of high probability events. A commonly used probability distortion function in the PT literature is [20]

$$w(p) = \exp(-(-\ln p)^{\alpha}), \; 0 < \alpha \le 1, \tag{6}$$

where p is the real probability of an outcome, and $w(p)$ is the corresponding subjective probability. Here α is the **probability distortion parameter**, which reveals how a person's subjective evaluation distorts the objective probability. A *smaller* α means a *larger* distortion.

Considering the above three features in PT, a buyer's expected utility is

$$U(b, q_b) = \sum_{i=1}^{I} w(p_i) v \left(-\pi_s^{\min} q_b + L(Q + q_b - d_i) - R_p \right), \tag{7}$$

Table II: Buyer's optimal buying quantity in Stage II under EUT

Condition	Optimal Buying Quantity q_b^*
$\pi_s^{\min} < \kappa p$	$q_b^* = d_h - Q$
$\pi_s^{\min} \geq \kappa p$	$q_b^* = 0$

Table III: Buyer's optimal buying quantity in Stage II under PT with $R_p = 0$

Condition	Optimal Buying Quantity q_b^*
$\pi_s^{\min} < \kappa \left[\frac{w(p)}{w(p)+w(1-p)}\right]^{\frac{1}{\beta}}$	$q_b^* = d_h - Q$
$\pi_s^{\min} \geq \kappa \left[\frac{w(p)}{w(p)+w(1-p)}\right]^{\frac{1}{\beta}}$	$q_b^* = 0$

and a seller's expected utility is

$$U(s, q_s) = \sum_{i=1}^{I} w(p_i) v\left(\pi_b^{\max} q_s + L(Q - q_s - d_i) - R_p\right). \tag{8}$$

We note that the utility functions under EUT ((3) and (4)) are special cases of those under PT ((7) and (8)), with the parameter choices of $\lambda = \beta = \alpha = 1$ and $R_p = 0$[4].

In the next section, we will study the user's optimal trading decision in a large market.

III. Solving The Two-stage Optimization Problem

In this section, we use backward induction to solve the two-stage sequential optimization problem. First, we derive the user's optimal selling or buying amount in Stage II. Then, we consider whether the user chooses to be a seller or a buyer in Stage I, by comparing his maximum achievable utilities under both cases.

To simplify the presentation and better illustrate the insights, we assume $I = 2$ for the rest of the paper. More specifically, there are two possible realizations of a user's monthly data consumption (or simply called *demand*): d_h and d_l, with $d_h > Q > d_l > 0$[5]. The probability of observing a high demand d_h is p, and the probability of observing low demand d_l is $1-p$. The analysis and insights can be generalized to the case of $I > 2$ with slightly more complicated algebraic manipulations.

We further focus on two choices of reference points. The first choice is $R_p = 0$, which reflects the user's expectation of observing the low demand and hence having no excessive demand. The second choice is $R_p = \kappa(Q - d_h) < 0$, which reflects the user's expectation of observing the high demand and paying for the corresponding excessive demand (without trading). Hence the same outcome is more likely to be considered as a gain under $R_p = \kappa(Q - d_h)$ than under $R_p = 0$.

A. Stage II

1) Buyer's Problem: To solve the Stage II problem, we first consider the buyer's problem, where the buyer decides

[4]In fact, as long as $\lambda = \beta = \alpha = 1$, choosing a non-zero value of R_p will just induce a constant shift of the EUT utilities in (3) and (4), without affecting the optimal decision under EUT.

[5]The analysis for the case where both d_h and d_l are higher (or lower) than the monthly quota Q is relatively trivial, and hence is omitted here due to space limitations.

Table IV: Buyer's optimal buying quantity in Stage II under PT with $R_p = \kappa(Q - d_h)$

Condition	Optimal Buying Quantity q_b^*
$\pi_s^{\min} < \frac{\kappa w(p)}{w(1-p)+w(p)}$	$q_b^* = d_h - Q$
$\pi_s^{\min} \geq \frac{\kappa w(p)}{w(1-p)+w(p)}$ & $\beta = 1$	$q_b^* = 0$
$\pi_s^{\min} \geq \frac{\kappa w(p)}{w(1-p)+w(p)}$ & $0 < \beta < 1$	$q_b^* = \frac{\kappa(Q-d_h)}{\left[\frac{w(p)(\kappa-\pi_s^{\min})^{\beta}}{w(1-p)\pi_s^{\min}}\right]^{\frac{1}{\beta-1}} + \pi_s^{\min}}$

the buying quantity at the lowest seller price $\pi_s^{\min}$. The corresponding optimization problem is

$$u(b) = \max_{q_b \geq 0} U(b, q_b), \tag{9}$$

where $U(b, q_b)$ is the utility function in (7) with $I = 2$ possible demands. As we have mentioned before, EUT is a special case of PT under $\lambda = \beta = \alpha = 1$ and $R_p = 0$.

Theorem 1. *The buyer's optimal buying quantity under EUT is summarized in Table II. The buyer's optimal buying quantities under PT with high reference $R_p = 0$ and low reference $R_p = \kappa(Q - d_h)$ are summarized in Table III and Table IV, respectively.*

The proof of Theorem 1 is given in Appendix A.

Tables II, III, and IV show how the optimal buying quantity depends on the minimum seller price $\pi_s^{\min}$. In each table, we observe a **buyer's threshold price**, below which the optimal buying amount equals $d_h - Q$:

$$\bar{\pi}_b^{EUT} = \kappa p \text{ (Table II)}, \tag{10}$$

$$\bar{\pi}_b^{PT1} = \kappa \left[\frac{w(p)}{w(p) + w(1-p)}\right]^{\frac{1}{\beta}} \text{ (Table III)}, \tag{11}$$

$$\bar{\pi}_b^{PT2} = \frac{\kappa w(p)}{w(1-p) + w(p)} \text{ (Table IV)}. \tag{12}$$

In Tables II and III, we observe that the optimal buying quantity is discontinuous at the buyer's threshold price. This is due to the linearity of utility function in the EUT case and the convexity of utility function in the PT case with $R_p = 0$. Details are given in Appendix A.

From Tables II to IV, we have the following observations.

Observation 1. *When the probability distortion parameter $\alpha = 1$, a PT buyer with a high reference point $R_p = 0$ (Table III) has a smaller threshold price than an EUT buyer (Table II), i.e., $\bar{\pi}_b^{PT1} < \bar{\pi}_b^{EUT}$. This means that comparing with an EUT buyer, a PT buyer with a high reference point is less willing to purchase mobile data.*

Observation 2. *When the probability distortion parameter $\alpha = 1$, a PT buyer with a low reference point $R_p = \kappa(Q - d_h)$ (Table IV) has the same threshold price as an EUT buyer (Table II), i.e., $\bar{\pi}_b^{PT2} = \bar{\pi}_b^{EUT}$. However, the optimal buying quantity q_b^* of the PT buyer (Table IV) is higher than the EUT buyer under the same price $\pi_s^{\min}$. This means that comparing with an EUT buyer, a PT buyer with a low reference point is more willing to purchase mobile data.*

Notice that buying data reduces the risk that the data consumption exceeds the quota. When a buyer has a high

Table V: Seller's Optimal Selling quantity in Stage II under PT with $R_p = 0$

Condition	Optimal Selling Quantity q_s^*
$1 > \frac{\lambda(\kappa-\pi_b^{\max})^\beta w(p)}{\pi_b^{\max\beta} w(1-p)} \left(1 + \frac{\kappa(d_h-Q)}{(\kappa-\pi_b^{\max})(Q-d_l)}\right)^{\beta-1}$	$q_s^* = Q - d_l$
$1 \leq \frac{\lambda(\kappa-\pi_b^{\max})^\beta w(p)}{\pi_b^{\max\beta} w(1-p)} \left(1 + \frac{\kappa(d_h-Q)}{(\kappa-\pi_b^{\max})(Q-d_l)}\right)^{\beta-1}$ & $\beta = 1$	$q_s^* = 0$
$1 \leq \frac{\lambda(\kappa-\pi_b^{\max})^\beta w(p)}{\pi_b^{\max\beta} w(1-p)} \left(1 + \frac{\kappa(d_h-Q)}{(\kappa-\pi_b^{\max})(Q-d_l)}\right)^{\beta-1}$ & $0 < \beta < 1$	$q_s^* = \frac{\frac{\kappa}{\kappa-\pi_b^{\max}}(d_h-Q)}{\left(\frac{w(1-p)\pi_b^{\max\beta}}{w(p)\lambda(\kappa-\pi_b^{\max})^\beta}\right)^{\frac{1}{\beta-1}}-1}$

Table VI: Seller's Optimal Selling quantity in Stage II under PT with $R_p = \kappa(Q - d_h)$

Condition	Optimal Selling Quantity q_s^*
$\lambda w(p)[(\kappa - \pi_b^{\max})(Q - d_l)]^\beta < w(1-p)\{[(\pi_b^{\max} - \kappa)Q + \kappa d_h - \pi_b^{\max} d_l]^\beta - [\kappa(d_h - Q)]^\beta\}$	$q_s^* = Q - d_l$
$\lambda w(p)[(\kappa - \pi_b^{\max})(Q - d_l)]^\beta \geq w(1-p)\{[(\pi_b^{\max} - \kappa)Q + \kappa d_h - \pi_b^{\max} d_l]^\beta - [k(d_h - Q)]^\beta\}$	$q_s^* = 0$

Table VII: Seller's Optimal Selling quantity in Stage II under EUT

Condition	Optimal Selling Quantity q_s^*
$\pi_b^{\max} > \kappa p$	$q_s^* = Q - d_l$
$\pi_b^{\max} \leq \kappa p$	$q_s^* = 0$

expectation (e.g., $R_p = 0$), he is more likely to encounter losses than gains under uncertainty. As we have mentioned in Section II, a smaller β means the buyer is more *risk seeking* in losses and will not buy data. When a buyer has a low expectation (e.g., $R_p = \kappa(Q - d_h) < 0$), the buyer is more likely to encounter gains than losses. As we have mentioned in Section II, a smaller β (which is the case for a PT user comparing with an EUT user) means the buyer is more *risk averse* in gains and will buy an amount equal to $d_h - Q$, which will completely eliminate the risk that the data consumption exceeds the updated quota d_h.

2) Seller's Problem: Next we consider the seller's problem, where he needs to decide the selling quantity q_s at the highest buyer price π_b^{max}:

$$u(s) = \max_{0 \leq q_s \leq Q} U(s, q_s), \tag{13}$$

where $U(s, q_s)$ is the utility function in (8) with $I = 2$ possible demands. As we have mentioned, EUT is a special case of PT under $\lambda = \beta = \alpha = 1$ and $R_p = 0$.

Theorem 2. *The seller's optimal selling quantity q_s^* under EUT is summarized in Table VII. The seller's optimal selling quantity q_s^* under PT with high reference $R_p = 0$ and low reference $R_p = \kappa(Q - d_h)$ are summarized in Table V and Table VI, respectively.*

The proof of Theorem 2 is given in our online technical report [21].

Tables VII, V, and VI show how the optimal selling quantity depends on the maximum buyer price $\pi_b^{\max}$. In each table, we observe a **seller's threshold price**, above which the optimal selling amount equals $Q - d_l$. The seller's threshold prices $\bar{\pi}_s^{EUT}$, $\bar{\pi}_s^{PT1}$, and $\bar{\pi}_s^{PT2}$ are the unique solutions of the following three equations:

$$\bar{\pi}_s^{EUT} = \kappa p \text{ (Table V)}, \tag{14}$$

$$\frac{\lambda(\kappa-\bar{\pi}_s^{PT1})^\beta w(p)}{(\bar{\pi}_s^{PT1})^\beta w(1-p)} \left(1+\frac{\kappa(d_h-Q)}{(\kappa-\bar{\pi}_s^{PT1})(Q-d_l)}\right)^{\beta-1} = 1 \text{ (Table VI)}, \tag{15}$$

$$w(1-p)\{[(\bar{\pi}_s^{PT2}-\kappa)Q + \kappa d_h - \bar{\pi}_s^{PT2} d_l]^\beta - [\kappa(d_h-Q)]^\beta\} = \lambda w(p)[(\kappa - \bar{\pi}_s^{PT2})(Q - d_l)]^\beta \text{ (Table VII)}. \tag{16}$$

In Tables V and VII, we observe that the optimal selling quantity q_s^* is discontinuous at the seller's threshold price. This is due to the linearity of utility function in the EUT case and the unimodality of utility function in the PT case with $R_p = \kappa(Q - d_h)$. Details are given in our technical report [21].

From Tables V-VII, we have the following observations.

Observation 3. *When the probability distortion parameter $\alpha = 1$, a PT seller with a high reference point $R_p = 0$ (Table VI) has a smaller threshold price than an EUT seller (Table V), i.e., $\bar{\pi}_s^{PT1} < \bar{\pi}_s^{EUT}$. This means that comparing with an EUT seller, a PT seller with a high reference point is more willing to sell mobile data.*

Observation 4. *When the probability distortion parameter $\alpha = 1$, a PT seller with a low reference point $R_p = \kappa(Q - d_h)$ (Table VII) has a larger threshold price than an EUT seller (Table V), i.e., $\bar{\pi}_s^{PT2} > \bar{\pi}_s^{EUT}$. This means that comparing with an EUT seller, a PT seller with a low reference point is less willing to sell mobile data.*

Contrary to buying data, selling data increases the risk that the data consumption exceeds the quota. When a seller has a high expectation (e.g., $R_p = 0$), he is more likely to encounter losses than gains under uncertainty. As we have mentioned in Section II, a smaller β means the seller is more *risk seeking* in losses and will sell an amount equal to $Q - d_l$. When a seller has a low expectation (e.g., $R_p = \kappa(Q - d_h) < 0$), the seller is more likely to encounter gains than losses. As we have mentioned in Section II, a smaller β means the seller is more *risk averse* in gains and will not sell data.

B. Stage I

In Stage I, the user decides whether to be a seller or a buyer, by comparing the maximum utilities that he can achieve in both cases (based on the calculation in Stage II). He needs to solve the following optimization problem:

$$\max_{a \in \{s,b\}} u(a), \tag{17}$$

where $u(b)$ and $u(s)$ are defined in (9) and (13), respectively. In the case of EUT, we can compute the closed-form optimal solution of problem (17).

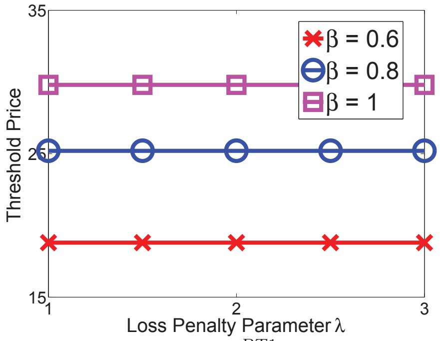

Figure 3: Buyer's threshold price $\bar{\pi}_b^{PT1}$ versus loss penalty parameter λ with different β.

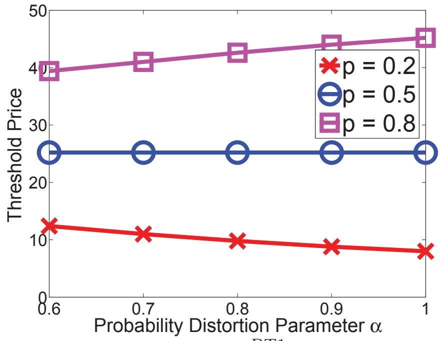

Figure 4: Buyer's threshold price $\bar{\pi}_b^{PT1}$ versus probability distortion parameter α with different p.

Table VIII: user's Optimal Decision in Stage I under EUT

Condition	The user's Optimal Decision (a^*, q_a^*)
$\kappa p > \pi_s^{\min}$	$(b, d_h - Q)$
$\pi_s^{\min} \geq \kappa p \geq \pi_b^{\max}$	$(s, 0)$ or $(b, 0)$
$\pi_b^{\max} > \kappa p$	$(s, Q - d_l)$

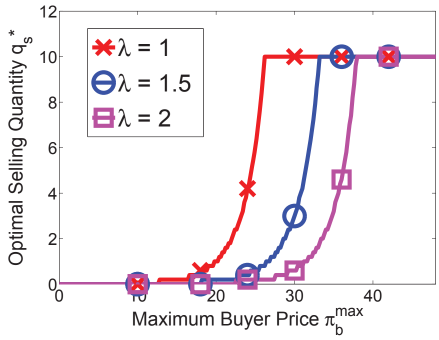

Figure 5: Seller's selling quantity q_s^* versus maximum buyer price $\pi_b^{\max}$ with different λ.

Theorem 3. *The user's optimal decision in Stage I under EUT is summarized in Table VIII.*

The proof of Theorem 3 is given in our technical report [21]. The results in Table VIII depend on the probability p of high demand as well as the market prices $\pi_s^{\min}$ and $\pi_b^{\max}$. When p is high ($\kappa p > \pi_s^{\min}$), the user chooses to be a buyer and buys an amount equal to $d_h - Q$. When p is medium ($\pi_b^{\max} \leq \kappa p \leq \pi_s^{\min}$), the user will not participate in the market, since neither selling nor buying will bring a higher utility. When p is small ($\pi_b^{\max} > \kappa p$), the user chooses to be a seller, and sells an amount equal to $Q - d_l$.

In the case of PT, we compare the corresponding $U(b, q_b^*)$ and $U(s, q_s^*)$ with q_b^* and q_s^* obtained in Tables III, IV, V, and VI, to find the optimal Stage I solution of the problem. Different from the EUT case, the optimal selling or buying quantity may not be equal to the difference between monthly quota and high/low demand. We will further illustrate the results in the next section.

IV. Numerical Results

In this section, we illustrate the impact of the PT model parameters (λ, β, and α), market parameters ($\pi_s^{\min}$ and $\pi_b^{\max}$), and demand uncertainty parameter (p) on the user's optimal decision. Due to space limitations, we will only consider a high reference point $R_p = 0$ for the PT case.

Comparing with the EUT benchmark, numerical results illustrate the following insights for a PT user: (i) Risk seeking under a high reference point: A PT buyer is risk seeking and is less willing to buy mobile data. A PT seller is also risk seeking and is more willing to sell mobile data. (ii) Probability distortion: When the probability of high demand is low, a PT buyer is risk averse and is more willing to buy mobile data comparing with an EUT buyer. On the other hand, when the probability of high demand is high, a PT buyer is risk seeking and is less willing to buy mobile data comparing with an EUT buyer.

Impact of the loss penalty parameter λ and the risk aversion parameter β on a buyer's threshold price $\bar{\pi}_b^{PT1}$ in (11): Here we assume $p = 0.5$ and $\alpha = 1$. Figure 3 illustrates the results stated in Theorem 1, where $\bar{\pi}_b^{PT1}$ is increasing in β for a fixed value of λ, and does not change in λ for a fixed value of β. Note that a higher threshold price means that the buyer is more willing to buy mobile data. This is because under a reference point $R_p = 0$, the buyer will not enounter a gain. In this case, a smaller β means that the user is more risk seeking, and is hence less willing to purchase mobile data to reduce the risk that the demand exceeds the quota. Meanwhile, notice that λ only affects the loss region in (2). As the user will never encounter a gain in this case, the threshold price is independent of λ.

Impact of the probability distortion parameter α on a buyer's threshold price $\bar{\pi}_b^{PT1}$ in (11): Figure 4 considers three different probabilities of high demand: high ($p = 0.8$), medium ($p = 0.5$), and low ($p = 0.2$). Here we assume $\beta = 0.8$ and $\lambda = 2$. We can see that $\bar{\pi}_b^{PT1}$ decreases in α when $p = 0.2$, is independent of α when $p = 0.5$, and increases in α when $p = 0.8$. As a smaller α means that the buyer will overweigh the low probability more, he becomes more risk averse when p is small. Similarly, since a smaller α means that the buyer will underweigh the high probability more, he is more risk seeking when the p is large.

Impact of the loss penalty parameter λ and the risk aversion parameter β on a seller's optimal selling quantity q_s^* in Table VI: Figure 5 illustrates how the seller's selling quantity q_s^* changes with the maximum buyer price $\pi_b^{\max}$ and λ. Here we assume that $\alpha = 1$ and $\beta = 0.8$. Figure 5 shows that as $\pi_b^{\max}$ increases, q_s^* increases accordingly until reaching

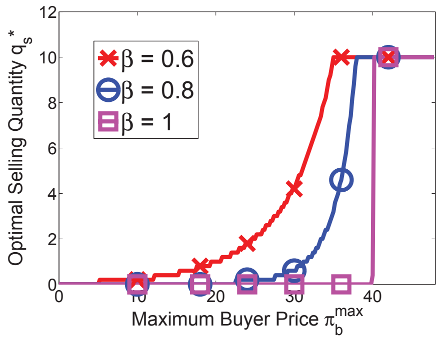

Figure 6: Seller's selling quantity q_s^* versus maximum buyer price $\pi_b^{\max}$ with different β.

a fixed value. This is because as $\pi_b^{\max}$ increases, the seller gains more revenue from the trade, and he wants to sell more. However, he will not sell more than $Q-d_l$, because his revenue from the trade cannot cover his satisfaction loss otherwise. Figure 5 also shows that under the same value of $\pi_b^{\max}$, q_s^* is non-increasing in λ. This is because, as λ increases, the seller becomes more loss averse, hence he will sell less in order to avoid a heavy loss when the demand is high.

Figure 6 illustrates how the seller's selling quantity q_s^* changes with the maximum buyer price $\pi_b^{\max}$ and β. Here we assume that $\alpha = 1$ and $\lambda = 2$. Figure 6 shows that under a fixed $\pi_b^{\max}$, q_s^* is non-increasing in β. This is because, under a high reference point $R_p = 0$, the seller will encounter either a small gain or a large loss. In this case, a smaller β means that the user is more risk seeking, and hence becomes more willing to sell mobile data.

V. Conclusion and Future Work

In this paper, we have considered a mobile data trading market that is motivated by the CMHK's 2CM platform. We have considered a large market regime, and analyzed the optimal trading decision of a single user. We have compared and contrasted the user's optimal decisions under prospect theory (PT) and expected utility theory (EUT), and have highlighted several key insights. Comparing with an EUT user, a PT user with a low reference point is more willing to buy mobile data and less willing to sell mobile data. Moreover, when the probability of high demand is low, a PT user is more willing to buy mobile data comparing with an EUT user. On the other hand, when the probability of high demand is high, a PT user is less willing to buy mobile data.

This study demonstrated that a more realistic behavioral modeling based on PT can shed important insights in understanding some seemingly illogical human behavior. In our future work, we will study how a user makes multiple sequential trading decisions in the same billing period, with updated estimation of future demand. We are also conducting a market survey to evaluate the prediction power of our analysis based on realistic user data.

Appendix

A. Proof of Theorem 1

For all three cases, we divide the feasible interval of buying quantity q_b into two subintervals, $[0, d_h - Q]$ and $[d_h - Q, \infty)$, and analyze the optimal buying quantity q_b^* that maximizes $U(b, q_b)$ within each subinterval. Such a division is based on the fact that the satisfaction loss $L(Q + q_b - d_h) = 0$ when $q_b \in [d_h - Q, \infty)$.

1) Buyer's Problem Under EUT (Table II):

- Case I: $q_b \in [0, d_h - Q]$. In this case, from (1), the satisfaction loss under low demand is $L(Q+q_b-d_l) = 0$, and the satisfaction loss under high demand is $L(Q+q_b-d_h) = \kappa(Q + q_b - d_h)$. The expected utility from (3) is
$$U(b, q_b) = (\kappa p - \pi_s^{\min})q_b + \kappa p(Q - d_h), \quad (18)$$
which is a linear function in q_b. It is increasing in q_b when $\pi_s^{\min} < \kappa p$, and decreasing in q_b when $\pi_s^{\min} > \kappa p$. The optimal buying quantity is then $q_b^* = d_h - Q$ when $\pi_s^{\min} < \kappa p$, and $q_b^* = 0$ when $\pi_s^{\min} > \kappa p$. When $\pi_s^{\min} = \kappa p$, the utility is independent of q_b. Without loss of generality, we assume that $q_b^* = 0$ when $\pi_s^{\min} = \kappa p$.
- Case II: $q_b \in [d_h - Q, \infty)$. In this case, the satisfaction loss under both low demand and high demand equals to 0, and the utility $U(b, q_b) = -\pi_s^{\min} q_b$. Since the utility function $U(b, q_b)$ is linearly decreasing in q_b, we have $q_b^* = d_h - Q$ in this case.

Combing the above analysis, we have the following result for the EUT case in Table II:

- When $\pi_s^{\min} < \kappa p$, $q_b^* = d_h - Q$, $U(b, Q - d_h) = -\pi_s^{\min}(Q - d_h)$.
- When $\pi_s^{\min} \geq \kappa p$, $q_b^* = 0$, $U(b, 0) = -\kappa p(Q - d_h)$.

2) Buyer's Problem Under PT with $R_p = 0$ (Table III):

- Case I: $q_b \in [0, d_h - Q]$. In this case, from (1), the satisfaction loss under low demand is $L(Q+q_b-d_l) = 0$, and the satisfaction loss under high demand is $L(Q+q_b-d_h) = \kappa(Q + q_b - d_h)$. The expected utility from (3) is
$$\begin{aligned} U(b, q_b) = & -\lambda(\pi_s^{\min} q_b - \kappa(Q + q_b - d_h))^\beta w(p) \\ & -\lambda(\pi_s^{\min} q_b)^\beta w(1 - p). \end{aligned} \quad (19)$$
The second order partial derivative of $U(b, q_b)$ with respect to q_b is
$$\begin{aligned} \frac{\partial U^2(b, q_b)}{\partial^2 q_b} = & -\lambda\beta(\beta - 1)\Big[{\pi_s^{\min}}^2(\pi_s^{\min} q_b)^{\beta-2} w(1 - p) \\ & +(\pi_s^{\min} - \kappa)^2[(\pi_s^{\min} - \kappa)q_b - \kappa Q + \kappa d_h]^{\beta-2} w(p)\Big] > 0, \end{aligned} \quad (20)$$
which implies that $U(b, q_b)$ is a convex function in q_b, and the optimal solution must lie at one of the boundary points[6]. Hence $q_b^* = d_h - Q$ if $U(b, 0) < U(b, d_h - Q)$, and $q_b^* = 0$ if $U(b, 0) \geq U(b, d_h - Q)$.

[6] In the case $\beta = 1$ and $U(b, 0) = U(b, d_h - Q)$, we will choose $q_b^* = 0$ without loss of generality.

- Case II: $q_b \in [d_h - Q, \infty)$. In this case, the satisfaction loss under both low demand and high demand equals to 0, and the expected utility is

$$U(b, q_b) = -\lambda[w(p) + w(1-p)](\pi_s^{\min} q_b)^\beta. \quad (21)$$

Since the first order partial derivative $\partial U(b, q_b)/\partial q_b < 0$, $U(b, q_b)$ is a decreasing function of q_b, and $q_b^* = d_h - Q$ in this case.

Combing the above analysis, we have the following result for the PT case with $R_p = 0$ in Table III:

- When $\pi_s^{\min} < \kappa[\frac{w(p)}{w(p)+w(1-p)}]^{\frac{1}{\beta}}$, $q_b^* = d_h - Q$.
- When $\pi_s^{\min} \geq \kappa[\frac{w(p)}{w(p)+w(1-p)}]^{\frac{1}{\beta}}$, $q_b^* = 0$.

3) Buyer's Problem Under PT with $R_p = \kappa(Q - d_h)$ (Table IV):

- Case I: $q_b \in [0, d_h - Q]$. In this case, the satisfaction loss under low demand is $L(Q + q_b - d_l) = 0$, and the satisfaction loss under high demand is $L(Q + q_b - d_h) = \kappa(Q + q_b - d_h)$. The expected utility is

$$U(b, q_b) = -\lambda(\pi_s^{\min} q_b + \kappa(d_h - Q))^\beta w(1-p) - \lambda((\kappa - \pi_s^{\min})q_b)^\beta w(p). \quad (22)$$

The second order partial derivative of $U(b, q_b)$ with respect to q_b is

$$\frac{\partial U^2(b, q_b)}{\partial^2 q_b} = \beta(\beta-1)\{w(p)(\kappa - \pi_s^{\min})^\beta q_b^{\beta-2} + w(1-p)(\pi_s^{\min})^2[-\pi_s^{\min} q_b + \kappa(d_h - Q)]^{\beta-2}\} < 0, \quad (23)$$

so $U(b, q_b)$ is a strictly concave function of q_b. As a result, the optimal solution q_b^* satisfies the first order condition, or lies at one of the boundary points.

We consider the first order partial derivative of $U(b, q_b)$ with respect to q_b:

$$\frac{\partial U(b, q_b)}{\partial q_b} = \beta[w(p)(\kappa - \pi_s^{\min})^\beta q_b^{\beta-1} + w(1-p)(\pi_s^{\min})[-\pi_s^{\min} q_b + \kappa(d_h - Q)]^{\beta-1}]. \quad (24)$$

  - If $\beta = 1$, $\partial U(b, q_b)/\partial q_b$ is independent of q_b. When $\pi_s^{\min} < \frac{\kappa w(p)}{w(p)+w(1-p)}$, $\partial U(b, q_b)/\partial q_b > 0$, so $q_b^* = d_h - Q$. When $\pi_s^{\min} \geq \frac{\kappa w(p)}{w(p)+w(1-p)}$, $\partial U(b, q_b)/\partial q_b \leq 0$, so $q_b^* = 0$.
  - If $0 < \beta < 1$, solving $\partial U(b, q_b)/\partial q_b = 0$, we have $\tilde{q}_b = \frac{\kappa(Q-d_h)}{\left[\frac{w(p)(\kappa-\pi_s^{\min})^\beta}{w(1-p)\pi_s^{\min}}\right]^{\frac{1}{\beta-1}} + \pi_s^{\min}} > 0$. If $\tilde{q}_b < d_h - Q$, then the optimal solution $q_b^* = \tilde{q}_b$. Otherwise, $q_b^* = d_h - Q$.

- Case II: $q_b \in [d_h - Q, \infty)$. In this case, the satisfaction losses under both low demand and high demand equals to 0, and the expected utility is

$$U(b, q_b) = -\lambda[w(p) + w(1-p)](\pi_s^{\min} q_b + \kappa(d_h - Q))^\beta. \quad (25)$$

Since the first order partial derivative $\partial U(b, q_b)/\partial q_b < 0$, the utility function $U(b, q_b)$ is a decreasing function of q_b, so $q_b^* = d_h - Q$ in this case.

Combing the above analysis, we have the following result for the PT case with $R_p = \kappa(Q - d_h)$ in Table IV:

- When $\pi_s^{\min} < \kappa\frac{w(p)}{w(p)+w(1-p)}$, $q_b^* = d_h - Q$.
- When $\pi_s^{\min} \geq \kappa\frac{w(p)}{w(p)+w(1-p)}$ & $\beta = 1$, $q_b^* = 0$.
- When $\pi_s^{\min} \geq \kappa\frac{w(p)}{w(p)+w(1-p)}$ & $0 < \beta < 1$, $q_b^* = \frac{\kappa(Q-d_h)}{\left[\frac{w(p)(\kappa-\pi_s^{\min})^\beta}{w(1-p)\pi_s^{\min}}\right]^{\frac{1}{\beta-1}} + \pi_s^{\min}}$.

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