Energy Efficient Low Complexity Joint Scheduling and Routing for Wireless Networks

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Abstract—**We consider a multihop wireless network with multiple users. The channels may experience fading. The power is consumed only during transmission and is a general monotonically increasing function of rate. We provide low complexity algorithms for joint routing, scheduling and power control which ensure stability of the queues, certain minimum rates and/or upper bound on the end-to-end, mean delays.**

Keywords—Multihop wireless networks, scheduling, routing, minimizing power, QoS.

I. Introduction

A multihop wireless network (MWN) is a collection of mobile nodes which are connected with each other by wireless media. A MWN has many applications, mainly in military services, where there is no pre-existing infrastructure. With emergering electronics, MWN is not confined to military and disaster management systems but also widely used in vehicle-to-vehicle communication systems, home networking and peer-to-peer networks.

Wide-spread use of a MWN is mainly due to the advantages that it offers, such as, easy deployment, better coverage at lower cost (where it is hard to wire) and higher throughput (due to shorter hops). All these benefits occur at the cost of routing complexity, path management, additional delay due to passage of data in multiple hops and limitations in the transmission range due to wireless channel inherent characteristics such as fading, path loss, shadowing and interference.

If the transmitting node and the receiving node are far apart, then the data may need to traverse to the destination in more than one hop. Here, a real challenge lies in routing and scheduling of the wireless links, which can become complicated due to limited battery at intermediate nodes and half duplex and other interference constraints. Furthermore, the applications carried may need some quality of service (QoS) constraints such as mean end-to-end delay, minimum mean rate guarantee or stability of its queues in the network. Thus, for limited energy MWN systems (such as sensor networks) , minimizing total average power consumed by the network while providing QoS is an important consideration.

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The multihop QoS problem can be solved in either a distributed or a centralized manner. Pioneering work on joint routing, scheduling and power control was provided in [5] which maximizes a utility function under average power constraint. As this problem is intractable, a heuristic sub-optimal algorithm was provided. [11] considered the problem of ensuring a fair utilization of network resources by jointly optimizing routing, scheduling and power control and obtained an efficient sub-optimal solution when the nodes may be powered by energy harvesting sources. [8] extended the solution in [5] to a multihop network where different nodes have multiple antennas and presented efficient, fair algorithms.

Back-pressure based algorithms have been used in [7], [9], [10]. These algorithms are centralized, use channel and queue length information and ensure stability. Upper bounds on mean delays are provided in [10] via Lyapunov approach but these are rather loose and under heavy traffic the mean delays obtained may violate any given mean delay constraints.

A large survey on multihop networks in general, with emphasis on QoS is presented in [6].

For this problem one may consider using Markov Decision Processes (MDP). However due to complex coupled queue dynamics in multihop networks MDP, or approximate MDP techniques have a large state space and the computations become unrealistic even for small networks [6]. Furthermore, MDP techniques may not provide any insights in the structure of the optimal policy and require huge signaling overheads.

A. Problem Statement

We consider the problem of joint link scheduling, routing and power control, so as to ensure certain end-to-end QoS for individual flows. The QoS may be the upper bound on the end-to-end mean delays, minimum rate guarantees or just the stability of the queue. While ensuring the QoS, we would like to minimize the average power consumed in the network.

B. Our Contribution

We present computationally efficient algorithms, for routing, scheduling and power control for a multihop network which minimizes total average power consumed while providing end-to-end QoS to individual users. We assume that the transmit rate is a general monotonically increasing function of the power invested at each node. Although our algorithms are sub-optimal in power, these are power-efficient and guarantee end-to-end QoS.

We also provide conditions for finiteness of stationary queue length moments, rate of convergence to the stationary distribution and good approximation to mean end-to-end delays for our algorithms. We are not aware of any other work that provides such results for multihop wireless networks.

As against the previous work our algorithms are in closed form or have very low computational complexity. Our algorithms do not require queue length information or packet arrival distribution. These algorithms are built on our previous work in [13], [14] and [15]. These algorithms are optimal in the class of algorithms requiring only channel gains. In these works it is shown that these algorithms are overall optimal or close to optimal via computations on explicit examples. We are not aware of any other work on multihop wireless networks that ensures QoS except stability, which again does not minimize power.

The paper is organised as follows. In Section II, we describe the system model. Section III results to the single user multi-hop scenario. Section V extends our results to the multiuser, multihop system. Section VI develops algorithms to ensure minimum average rate guarantee and/or upper bounds on the end-to-end mean delays to all the users for all the above cases. Section VII concludes the paper.

II. SYSTEM MODEL

We consider a wireless network, modelled as a connected directed graph $\mathcal{G}(\mathcal{N}, \mathcal{L})$, where $\mathcal{N} = \{1, \ldots, N\}$ and $\mathcal{L} = \{1, \ldots, L\}$ represent the set of nodes and the set of directed links respectively. A subset of nodes in the network transmits data to another subset of nodes. Each source has a unique destination.

We consider a discrete time slotted system. A stream of packets that are transmitted from a source node to its destination node is called a *flow*. We denote the set of user flows as $\mathcal{F} := \{1, 2, \ldots, M\}$. A flow carries data packets, for which we may either only need to ensure the stability of all the queues in the network or need to guarantee that the flow gets a minimum end-to-end average rate or an upper bound on end-to-end average delay.

Let A_f^t be the number of packets generated by flow f in slot t at its source and placed in an infinite buffer queue. We assume that all packets are of same size. Also, let $\{A_f^t, \ t \geq 0\}$ to independent, identically distributed (*iid*), independent of streams for the other flows. We assume that all the links in the network are half- duplex. Also, a node can either transmit or receive data at a given time instant. This assumption is not critical for our algorithm. Actually we will only assume that the set of links $\mathcal{L}$ is divided into independent sets $S_1, ..., S_d$ such that all links within a set can simultaneously transmit with negligible interference to each other. However, the links in two different sets will not be allowed to transmit simultaneously. A link can belong to multiple sets. This is a commonly made assumption in multihop wireless networks [4].

Let H_{ij}^t be the channel gain in a slot t for link (i, j) from node i to node j. We assume that instantaneous channel gain knowledge is available at node i in the beginning of every time slot and we also assumed that the channel gain is constant in a time slot. We also assume that the channel gain process $\{H_{ij}^t, \ t \geq 0\}$ is *iid* on all links and independent of the link channel gain processes of the other links. The channel gain H_{ij}^t takes values on a finite set of values. This can be a good approximation of continuous distributions, (e.g., Rayleigh, Rician, Nakagami) if we take the finite set to be large enough. Let $P_{ij}^t(f)$ be the power spent by node i to transmit $R_{ij}^t(f)$ packets to node j in time slot t for flow f. Then, often,

$$R_{ij}^t(f) = \frac{1}{2} \log_2(1 + G_{ij} P_{ij}^t(f) H_{ij}^t(f)/\sigma_{ij}^2), \qquad (1)$$

where σ_{ij}^2 is the receiver noise variance and G_{ij} is a constant that depends on the modulation and coding used. Our polices will not require fragmentation of packets. Also, instead of (1), $R_{ij}^t(f)$ could as well be a general, nonnegative, monotonic continuous, non-decreasing function $g_{ij}(P_{ij}^t(f), H_{ij}^t)$ of power $P_{ij}^t(f)$ and H_{ij}^t.

Let q_{ij}^t be the queue length at node i for transmission on link (i, j) in the beginning of slot t. Then

$$q_{ij}^{t+1} = (q_{ij}^t + A_{ij}^t - R_{ij}^t)^+, \forall i, j, \ t \geq 0, \qquad (2)$$

where A_{ij}^t is the amount of data arriving at node i in the beginning of slot t and R_{ij}^t is the data transmitted on link (i, j).

Our objective is to minimize the total average power consumed by the system such that the QoS of different flows is met. Our challenges lie in scheduling the links, routing the flows and allocating the transmit power on each link. We transmit the data from a flow on a single path instead of splitting it on multiple paths. This will avoid unnecessary delays while splitting and merging (re-ordering) at the destination.

We consider a centralized setup, i.e., system has all the information, including channel gain statistics and average external arrival rates. This will be needed only for developing the algorithms for routing, scheduling and power control. The actual implementation will require little information.

III. OPTIMAL POLICIES: STABILITY

In this section, we consider joint routing, scheduling and power allocation policies which minimize,

$$\limsup_{n\to\infty} \frac{1}{n} E \sum_{(i,j)} \sum_{t=0}^{n-1} \sum_{f=1}^{M} P_{ij}^t(f), \tag{3}$$

such that the long term average queue length

$$E[q_{ij}] < \infty, \ \forall (i,j) \in \mathcal{L}. \tag{4}$$

We provide efficient sub-optimal solutions of this problem. For computational tractability we limit ourselves to policies which do not exploit queue length information. We first consider a single flow case and later on generalize to the multiuser system.

A. Single User, Single Hop

We first study the single user, single hop case and provide optimal polices. We then extend these polices to the single user, multihop case. Now we keep the notation of Section II but omit the sub/super scripts $f, (i,j)$. Let $E[A]$ and $E[A^\alpha], \alpha \geq 1$ be the mean and the αth moment of the number of a arrivals from the source node in one slot. In the absence of fading, an average power optimal policy (which does not consider the queue length) which provides the stability of the queue is,

$$R^t = \begin{cases} \lceil E[A] + \delta \rceil, & \text{w.p. } p, \\ \lfloor E[A] + \delta \rfloor, & \text{otherwise,} \end{cases} \tag{5}$$

where w.p. p denotes with probability p, $\delta > 0$, is a small positive quantity and p is chosen such that $E[R] = \lceil E[A] + \delta \rceil p + \lfloor E[A] + \delta \rfloor (1-p) = E[A] + \epsilon$, and ϵ is a small positive quantity. Also, for positive real x, $\lceil x \rceil$ ($\lfloor x \rfloor$) is the smallest (largest) integer $\geq$ ($\leq$) x. Policy (5) ensures that an integer number of packets are transmitted in a slot. The policies obtained by us will be close to optimal (depending on the small ϵ, δ chosen) but we will ignore this point.

Since (2) is the usual Lindley equation [1], with this policy, the queue length process $\{q_k\}$ has a unique stationary distribution. Furthermore, if $E[A_k^{\alpha+1}] < \infty$ for $\alpha \geq 1$, then $E[q_k^\alpha]$ under stationarity is also finite.

For (5), the average power consumption is

$$E[P] = \phi(\lceil E[A] + \delta \rceil) p + \phi(\lfloor E[A] + \delta \rfloor)(1-p). \tag{6}$$

Now we consider the channel with fading. To ensure stability we take $E[R] = E[A] + \epsilon$, where $\epsilon > 0$ is a small constant. Let $h^u, 1 \leq u \leq L$ be the channel gain with probability of occurrence p^l and $h^u < h^{u+1}$ for $u, u+1 \in \{1, ..., L\}$. Let the power consumption when a packets are transmitted and the channel gain is h be $\phi(a,h)$. We assume that $\phi(a,h) < \phi(a+1,h)$ and $\phi(a,h) > \phi(a,h')$ for $h' > h$.

Optimizing (3) is a discrete optimization problem. A simple iterative algorithm to obtain the optimal solution is as follows. Let $\mathbf{R} = [R(1), ..., R(L)]$ be the rate vector at some point in the computation of the algorithm, where $R(u)$ is the number of packets that are transmitted when the channel gain is h^u. If $\sum_{u=1}^{L} R(u) p^u < E[A] + \epsilon$ then we update the rate vector by increasing one of the components of the rate vector by one while the rest of the components remain same. The component chosen to increase is $\arg\min Diff(u) = \phi(R(u)+1, h^u) - \phi(R(u), h^u)$. We repeat this till $E[R] \geq E[A] + \epsilon$. Then we stop. At the end, if $E[R] = E[A] + \epsilon$ we take that policy. If $E[R] > E[A] + \epsilon$ then we increase the last rate only with a certain probability so that the average service rate $E[R] = E[A] + \epsilon$.

Often the optimal solution is even simpler. For example, let $R = \frac{1}{2}\log_2(1 + \frac{ph}{\sigma^2})$ where σ^2 is the noise power. Let the packet size be n bits. If $\frac{h^L}{h^1} \leq 2^{2n}$, then the optimal solution has the following simple structure. Find an integer m such that $\sum_{i=1}^{m} p^{L+1-i} \leq \overline{p} = E[A] - \lfloor E[A] \rfloor < \sum_{i=1}^{m+1} p^{L+1-i}$. Let $p' = \overline{p} - \sum_{i=1}^{m} p_{L+1-i}$. Then the optimal policy is,

$$R^t = \begin{cases} \lceil E[A] \rceil, & \text{if } H^t = h^i, \ i \geq L+1-m, \\ \lceil E[A] \rceil, & \text{if } H^t = h^{L-m-1}, \ \text{w.p. } (1-p'), \\ \lfloor E[A] \rfloor, & \text{if } H^t = h^{L-m-1}, \ \text{w.p. } p', \\ \lfloor E[A] \rfloor, & \text{if } H^t = h^i, \ i \leq L-m-1. \end{cases} \tag{7}$$

The condition $\frac{h^L}{h^1} \leq 2^{2n}$ is quite a reasonable assumption and is satisfied for most practical systems.

IV. SINGLE USER, MULTIHOP

Now we extend the optimal policy of Section III to the case where a single flow traverses multiple hops to reach the destination, where every link in the network may experience fading.

A. Single User, Multihop: No Fading

Initially, we consider the case, when the network experiences no fading. Let us assume that the route for the flow has been fixed. We will comment on it later. We replace the stability constraint of all the queues on the route with $E[R_{ij}] \geq E[A] + \epsilon$.

Let $S_1, ..., S_d$ be the independent sets of links [8]. We need restrict this set to the links on the route selected. We schedule the independent sets such that each S_k is active for $\gamma_k > 0$ fraction of time and $\sum_{z=1}^{d} \gamma_k = 1$. The allocation of slots to the different links will be done as follows. A central authority generates *iid* random variables Y_t in the beginning of each slot t with probability $P[Y_t = k] = \gamma_k, \ k = 1, ..., d$. If $Y_t = k$

then slot t is assigned to the independent set S_k. If in a slot a link has channel gain H_{ij}^t and the link is active in that slot then it will transmit R_{ij}^t packets from its queue in that slot, which is a function of γ_ks and H_{ij}^t.

Define for link (i,j), $\Gamma_{ij} = \sum_{k:(i,j)\epsilon S_k}^{d} \gamma_k$. To get the optimal power, we solve the optimization problem,

$$\min \sum_{(i,j)} \phi_{ij}(\frac{E[A]+\epsilon}{\Gamma_{ij}})$$

subject to,

$$\sum_{k=1}^{d} \gamma_k = 1, \qquad 0 < \gamma_k < 1, \ \forall k,$$

where $\phi_{ij}(a)$ is the power needed to transmit a packets on link (i,j). If ϕ_{ij} are convex, then it is a convex optimization problem. The corresponding optimal rates are

$$R_{ij}^t = \begin{cases} \lceil \frac{E[A]+\delta}{\Gamma_{ij}} \rceil & \text{w.p. } p_{ij}, \\ \lfloor \frac{E[A]+\delta}{\Gamma_{ij}} \rfloor & \text{otherwise} \end{cases}, \tag{8}$$

where p_{ij} is such that $E[R] = E[A]+\epsilon = \lceil \frac{E[A]+\delta}{\Gamma_{ij}} \rceil p_{ij} + \lfloor \frac{E[A]+\delta}{\Gamma_{ij}} \rfloor (1-p_{ij})$. The total average power consumed by the network on the given route, can be made close to

$$E[P] = \sum_{(i,j)} \phi_{ij}(\lceil \frac{E[A]}{\Gamma_{ij}} \rceil) p_{ij} + \phi_{ij}(\lfloor \frac{E[A]}{\Gamma_{ij}} \rfloor)(1-p_{ij}). \tag{9}$$

Thus, we should select the route which minimizes (9). But this involves searching in an exponential number of routes. One sub-optimal solution is to give each link (i,j) the cost $\phi_{ij}(\frac{E[A]+\epsilon}{d})$ and use Dijkstra's (say) algorithm to obtain the optimal route. In particular, this will be a good solution for (7), especially at large $E[A]$. One can further improve over it by using greedy iterative algorithms over the cost function (9). In such algorithms, a link consuming one of the highest powers from the currently considered route is selected (say link (i,j)) and removed from the network. Then the least cost path from the source to destination is computed again via Dijkstra's algorithm. If this provides lower (9) then it is kept otherwise we go back to the previous path. Again the process is repeated.

B. Single User, Multihop: With Fading

Next we consider the case when the channels experience fading. Let $P[H_{ij}^t = h_{ij}^u] = p_{ij}^u$ and β_{ij}^{cu} be the probability that link (i,j) will transmit c packets when it is allowed to transmit in channel state h_{ij}^u. Then, for a fixed route, to minimize power, we optimize over β_{ij}^{cu} and γ_j. Over the links on the route,

$$\min \sum_{(i,j)} \sum_{u} \sum_{c} \phi(c, h_{ij}^u) p_{ij}^u \beta_{ij}^{cu} \Gamma_{ij} \tag{10}$$

subject to,

$$\Gamma_{ij} \sum_{u} \sum_{c} c p_{ij}^u \beta_{ij}^{cu} \geq E[A] + \epsilon, \tag{11}$$

$$0 \leq \beta_{ij}^{cu} \leq 1, \ \sum_{c} \beta_{ij}^{cu} \leq 1, \tag{12}$$

$$\sum_{k=1}^{d} \gamma_k = 1, \ 0 < \gamma_k < 1, \ \forall k. \tag{13}$$

This is a non-convex optimization problem. However, if we fix γ_k's, it becomes a Linear Program (LP) in β_{ij}^{cu}. We iterate greedily over γ_k's and solve LPs to obtain a better solution. On the other hand, if β_{ij}^{cu} are (approximately) known (e.g.(7)), then it is an LP in γ_k's.

Considering the example (7), a good route can be selected by obtaining the least cost algorithm via Dijkstra algorithm [2], by taking the link cost for link (i,j) as $E[\frac{1}{H_{ij}}]$.

Let A_{ij}^t be the number of packets arriving to queue i for link (i,j) in slot t. The sequence $\{R_{ij}^t\}$ provided by the above algorithm is *iid* for each (i,j). However $\{A_{ij}^t, t \geq 0\}$ is not necessarily *iid* if i is not the source node. In the following we study the process $\{q_{ij}^t, t \geq 0\}$ for the above queueing system. Let $q^t = (q^t(l), l = 1, \ldots, L_1)$ be the queue length process along the given route. Also let $\mathcal{P}^t(l)$ be the distribution of q_l^t and $\mathcal{P}^t = (\mathcal{P}^t(l), l = 1, ..., L_1)$.

Theorem 1. *For the above policy $\{q^t\}$ process has a unique stationary distribution π. Starting from any initial conditions, q^t converges in total variation to π. Also, if $E[A^{\alpha+1}] < \infty$ then $E_\pi[q^\alpha(l)] < \infty$ for $l = 1, \ldots, L_1$. Furthermore, $\|\mathcal{P}^t - \pi\| \leq \beta_1 \beta_2^{-\alpha-1}$ where $0 < \beta_2 < 1$, β_1 is a positive constant and $\|.\|$ denotes the total variation norm.*

Proof: If i is the source node and link (i,j) is the first link on the path, then $A_{ij}^t = A^t$ is an *iid* sequence and we have ensured that $E[R_{ij}^t] \geq E[A] + \epsilon$. Thus from the usual results on $GI/GI/1$ queues ([1]), $q_{ij}^t = q^t(1)$ has a unique stationary distribution and $q^t(1)$ converges in total variation to the stationary distribution. Also if $E[A^{\alpha+1}] < \infty$ for an $\alpha \geq 1$, then $E[(q_{ij}^t)^\alpha] < \infty$ under stationarity.

For the second queue $q^t(2)$ on the route, let the input process be $\{A^t(2)\}$ and the transmission rate process be $\{R^t(2)\}$, which is given by the optimal policy. From the first queue result, we also have that $\{A^t(2)\}$, which is output process of the first queue, also has a stationary distribution. Furthermore, although it is not *iid*, it is a regenerative process with regeneration epochs the time slots when the first queue is empty. Let its regeneration length be $\tau(1)$. Under the above assumptions we also have $E[(\tau(1))^\alpha] < \infty$ if $E[A^\alpha] < \infty$ for $\alpha \geq 1$ [1]. Since $\{R^t(2)\}$ is *iid*, $\{(A^t(2), R^t(2))\}$ is also regenerative with the same regeneration epochs. Also, $\tau(1)$ is aperiodic. Then, from [16], if

$$E\left[\sum_{t=1}^{\tau(1)} A^t(2)\right] < E\left[\tau(1)\right] E\left[R^t(2)\right], \quad (14)$$

the process $\{A^t, q^t(1), q^t(2)\}$ is also regenerative with regeneration length $\tau(2)$ with finite mean. Since at regeneration epochs, the first queue is empty and $\tau(1)$ is a stopping time,

$$E\left[\sum_{t=1}^{\tau(1)} A^t(2)\right] = E\left[\sum_{t=1}^{\tau(1)} A^t\right] = E\left[\tau(1)\right] E\left[A\right]$$

and hence (14) is satisfied if $E[A] < E[R^t(2)]$, which has been ensured by our algorithm. Also, from [16], if for $\alpha \geq 1$,

$$E\left[\left(\sum_{t=1}^{\tau(1)} A^t(2)\right)^{\alpha+1}\right] = E\left[\left(\sum_{t=1}^{\tau(1)} A^t\right)^{\alpha+1}\right] < \infty, \quad (15)$$

then $E\left[(q(2))^\alpha\right] < \infty$. But, (15) holds when $E\left[A^{\alpha+1}\right] < \infty$. Under the same conditions, from [16] we also get $E[\tau(2)^{\alpha+1}] < \infty$ (see also [16]).

This argument extends directly to the queues $q^t(l), l \geq 3$. Thus we get that $E_\pi[q^t(l)^\alpha] < \infty$ for $l = 1, ..., L_1$ and the regeneration length τ of the $\{q^t\}$ process satisfies $E[\tau^{\alpha+1}] < \infty$. Then the rate of convergence $\|\mathcal{P}^t - \pi\| \leq \beta_1 \beta_2^{-\alpha-1}$ follows from the results on regenerative processes [16]. ■

It is useful to compute the mean queue length $E_\pi[q(1)]$, under stationarity. Since $\{A^t, t \geq 0\}$ is *iid* and $\{R^t(1)\}$ are *iid*, $E[q(1)]$ can be directly approximated from the approximations for $GI/GI/1$ queues ([12], [3])

$$E[q] \approx \frac{\rho d E[A](C_A^2 + C_R^2)}{2(1-\rho)}. \quad (16)$$

where

$$\rho = \frac{E[A]}{E[R(1)]}, C_A^2 = \frac{Var[A]}{E[A]^2}, C_R^2 = \frac{Var(R(1))}{E[R(1)]^2} \quad (17)$$

$$d = \begin{cases} \exp[-\frac{2(1-\rho)}{3\rho}\frac{(1-C_R^2)^2}{C_R^2 + C_A^2}], \text{ if } C_R^2 < 1, \\ \exp[-(1-\rho))\frac{(C_R^2 - 1)}{C_R^2 + 4C_A^2}], \text{ if } C_R^2 \geq 1. \end{cases} \quad (18)$$

For $E_\pi[q(l)], l > 1, \{A^t(l), t \geq 0\}$ is not *iid*. However $A^t(l) \leq R^t(l-1)$ for all $t \geq 0$. Thus, if we consider a discrete queue with $\{R^t(l-1)\}$ arrival process and $\{R^t(l), l \geq 0\}$ as the service process, assuming stability, using approximation (16) provides an approximate upper bound. We will see from simulations that this can be a good upper bound. This is especially so since minimizing power ensures that we have some what heavy trafiic situation at each queue. Thus, we will use these bounds in Section VI to obtain policies which guarantee mean end-to-end delays for individual flows.

	$L_{5,7}$	$L_{7,9}$	$L_{9,10}$	$L_{10,12}$	total
$\hat{q}$	30.10	49.58	49.37	49.33	178.38
$\tilde{q}$	30.81	41.38	43.67	41.68	157.54
$\hat{P}$	556.37	556.37	963.08	556.37	2632.19
$\tilde{P}$	487.41	508.56	878.51	507.53	2382.01

TABLE I. SINGLE USER: AVERAGE QUEUE LENGTHS AND AVERAGE POWERS

C. Example

For simulations, we consider a multihop network of 20 nodes. Node i and node j are connected, if $|i-j| \leq 2$, and $i \neq j$; otherwise the channel gains are so bad that we ignore those channels. If $i > j$ then link (i, j) takes channel gains from the set c_{i-j}, and if $i < j$ then the link (i, j) takes channel gain from the set c_{2+j-i}, where, c_1 ={ 1, 2, 3, 4, 5, 6 }, c_2 = { 1, 3.03, 5.79, 9.18, 13.13 , 17.58 }, c_3 = { 1, 2.46, 4.17, 6.06, 8.1, 10.27 }, c_4 = { 1, 3.48, 7.22, 12.12, 18.11, 25.15 }. Channel gains from set c_i hold with the probabilities $\mathcal{P}_1$ = { 0.2, 0.1, 0.15, 0.25, 0.2, 0.1 }, $\mathcal{P}_2$ = { 0.15, 0.1, 0.25, 0.2, 0.1, 0.2 }, $\mathcal{P}_3$ = { 0.1, 0.2, 0.3, 0.2, 0.1, 0.1 }, $\mathcal{P}_4$ = { 0.2, 0.2, 0.1, 0.1, 0.2, 0.2 } respectively.

The power required for given rate r and channel gain h is $\frac{2^{2r}-1}{h}$ for all channels. We consider the cost of link (i, j) as $E[\frac{1}{H_{ij}}]$. By using Dijkstra algorithm, we find the shortest cost path from source node 5 to destination node 12, which is $5 \to 7 \to 9 \to 10 \to 12$. We take the independent sets by half-duplex constraints. The two independent sets are $\{(5, 7), (9, 10)\}$ and $\{(7, 9), (10, 12)\}$. The input process generated at source node 5 has Binomial distribution with mean 3. We have provided service rate $E[R] = 3.2$ on all links. We take $\gamma_1 = \gamma_2 = \frac{1}{2}$ and minimize the total average power on all the links in the network by substituting in (10)-(13) and solving the LP. Also, we have simulated the system for 10^6 slots. In Table 1 we provide the results, where $\hat{q}$ and $\hat{P}$ are the theoretical average queue lengths from (16) and the corresponding powers, while $\tilde{q}$ and $\tilde{P}$ are the respective simulated values. We observe that (16) provides an approximation for the first queue but an upper bound for the other queues. These approximations/bounds are generally quite good. Also, the theortical power is an upper bound on the actual value since these provide service rates which are a little higher than the actual rate $E[A]$.

We have also solved the full nonconvex optimization problem (10)-(13) for local minima starting with 15 different initial conditions. The total simulated average power consumption in the network for the best local minima is 2198 and the theoretical value is 2448. The total simulated mean end-to-end queue lengths of the network via simulation is 162 and via the approximation (16) is 176. We see that solving the much simpler LP for $\gamma_1 = \gamma_2 = \frac{1}{2}$ provides a reasonably good solution for this example.

We also used iterative greedy algorithm to improve the route. Link $(5,7)$ was removed from the network. Then the least cost path obtained was $5 \rightarrow 6 \rightarrow 8 \rightarrow 10 \rightarrow 12$. This path was used with $\gamma_1 = \gamma_2 = \frac{1}{2}$ and the optimization LP reduced the total power by 30. However removing link $(9,10)$ and getting a new lowest cost link from 5 to 12 led to increase in the total power consumed.

V. MULTIUSER, MULTIHOP

In this section, we consider the case where multiple source-destination pairs exist in the multihop network. Our objective is to stabilize all the queues in the network. The channels may experience fading. Let the average arrival rate of flow f be $E[A_f]$. We consider the policies which minimize (3) subject to (4).

Let the routes for the M users be fixed. For stability of the queues we require $E[R_{ij}] \geq \sum_f E[A_f] + \epsilon$, for each link (i,j) where the summation is over the flows passing through the link (i,j).

The notation remains as in Section II. Also, as before γ_k is the probability with which independent set S_k will transmit in a slot and Γ_{ij} is the probability with which link (i,j) will be allowed to transmit. Also, let β_{ij}^{cu} be the probability that node i will transmit c packets on link (i,j) when its channel gain is h_{ij}^u and it is allowed to transmit. We consider the optimization problem,

$$\min \sum_{(i,j)} \sum_u \sum_c \phi(c, h_{ij}^u) p_{ij}^u \beta_{ij}^{cu} \Gamma_{ij} \tag{19}$$

subject to,

$$\sum_u \sum_c c p_{ij}^u \beta_{ij}^{cu} \Gamma_{ij} \geq E[A] + \epsilon, \ \forall (i,j), \tag{20}$$

$$0 \leq \beta_{ij}^{cu} \leq 1, \ \forall (i,j), c, u, \tag{21}$$

$$\sum_c \beta_{ij}^{cu} \leq 1, \forall (i,j), u, \tag{22}$$

$$\sum_{k=1}^{d} \gamma_k = 1, \gamma_k > 0, \ \forall k \ . \tag{23}$$

The objective function and the constraints are non-convex but given all the γ_ks it becomes an LP for which we can easily get the optimal solution. Then we iterate greedily to improve over to better γ_ks. Similarly, if all β_{ij}^{cu} are known, then it is an LP in γ_ks and this takes care of the important example of (7).

For given fixed routes, for each of the source-destination pairs, the above policy provides us with a network of feed-forward queues. For each link (i,j) its queue gets serviced at rate R_{ij}^t, which forms an *iid* sequence. Using results from [16], Theorem 1 extends to this setup also. Let $q^t = (q_{ij}^t, (i,j) \in \mathcal{L})$ and $\mathcal{P}^t = (\mathcal{P}_{ij}^t, (i,j) \in \mathcal{L})$ the corresponding (i,j) distributions.

Theorem 2. *For the given policy, $\{q^t\}$ has a unique stationary distribution π. Starting from any initial distribution, $\mathcal{P}^t$ converges in total variation to its stationary distribution π. Also, if for an $\alpha \geq 1$, $E\left[(A_f)^{\alpha+1}\right]$ for each f then $E_\pi\left[(q_{ij})^\alpha\right] < \infty$ for each (i,j). Furthermore, $\|\mathcal{P}^t - \pi\| \leq \beta_3 \beta_4^{-\alpha-1}$ for some $0 < \beta_4 < 1$ and β_3 a positive constant.* □

Approximations for mean queue lengths $E_\pi[q_{ij}]$ are obtained as in the last section where we need to consider the statistics of the input process A_{ij}^t at link (i,j). To get the upper bound system, this process is replaced by the *iid* input $\sum_{(k,i)\in\mathcal{L}} R_{k,i}^t$.

A. Example

We consider the network example of Section IV with 3 users. We consider Binomial arrivals with mean 1 for the first user, Poisson arrivals with mean 1.05 for the second user and a discrete distribution with values $\{0,1,2,3\}$ taken with probabilities $\{0.35, 0.35, 0.2, 0.1\}$ for the third user. The objective is to provide stability of all queues. Service rate at each link is taken 0.1 more than the respective mean values. We simulated the system for 10^6 slots.

We used Dijkstra's algorithm to get the least cost paths. The route for the first user is $13 \rightarrow 11 \rightarrow 9 \rightarrow 7$, for the second user is $5 \rightarrow 7 \rightarrow 9 \rightarrow 10 \rightarrow 12$ and for the third user is $15 \rightarrow 17 \rightarrow 19$. In this example, we get 4 independent sets $S_1 = \{$(5,7), (10,12), (11,9), (15,17)$\}$, $S_2 = \{$(7,9), (13,11), (17,19), (10,12)$\}$, $S_3 = \{$(9,10), (5,7), (13,11), (15,17)$\}$, $S_4 = \{$(9,7), (10,12), (13,11), (17,19)$\}$. We solved the optimization problem (19)-(23) by taking $\gamma_i = \frac{1}{4}$, for all i. We have tabulated the simulated and theoretical average queue lengths and average powers in Tables II, III, IV for the three flows. In Tables II and III, links $(13,11)$ and $(10,12)$ appear in three of the four independent sets. Thus, their Γ_{ij} are $\frac{3}{4}$, resulting in substantially lower power consumption on these links.

We also solved the optimization problem (19) fully for local optima, starting with 15 different initial conditions. For the best local optimum, the total simulated and theoretical powers consumed by the three flows is 657.41 and 731.56. The end-to-end simulated mean delays for the three flows are $38.17, 614.82, 4.42$ and the respective theoretical end-to-end mean delays are $45.42, 680.57, 5.57$. From Tables II-IV the total simulated and theoretical powers consumed by the three flows is $34.24, 697.52, 3.01$ and $39.37, 768.02, 3.38$ The end-to-end simulated mean delays for the three flows are$58.11, 157.28, 24.12$ and respective theoretical values are $61.40, 185.43, 26.36$. Again we see that the much simpler optimization LP with $\gamma_i = \frac{1}{4}, i = 1, ..., 4$ provides a solution which is very close to the best local optimum obtained. From

	$L_{13,11}$	$L_{11,9}$	$L_{9,7}$	total
$\hat{q}$	8.33	19.05	34.02	61.40
$\tilde{q}$	8.42	22.39	27.30	58.11
$\hat{P}$	0.81	19.27	19.29	39.37
$\tilde{P}$	0.72	16.47	17.05	34.24

TABLE II. MULTIUSER : FLOW 1

	$L_{5,7}$	$L_{7,9}$	$L_{9,10}$	$L_{10,12}$	total
$\hat{q}$	19.95	47.88	72.88	44.72	185.43
$\tilde{q}$	18.32	46.36	57.20	35.40	157.28
$\hat{P}$	6.42	278	481.54	2.06	768.02
$\tilde{P}$	5.82	249	440.77	1.93	697.52

TABLE III. MULTIUSER: FLOW 2

	$L_{15,17}$	$L_{17,19}$	total
$\hat{q}$	11.48	14.88	26.36
$\tilde{q}$	12.14	11.98	24.12
$\hat{P}$	1.69	1.69	3.38
$\tilde{P}$	1.51	1.50	3.01

TABLE IV. MULTIUSER: FLOW 3

Table II-IV, we see that node 9 transmits on 4 links, resulting in 4 independent sets. This is caused by traffic from source nodes 5,13 passing through node 9. Consequently all the links passing through node 9 consuming high power. Thus we removed link (7,9) and recomputed least cost route for source node 5. This resulted in new path $5 \to 6 \to 8 \to 10 \to 12$. Consequently, the routes became disjoint, resulting in drastic reduction in power. The total theoretical powers on the 2 routes is 30.27,5.04.

VI. RATE GUARANTEES, MINIMUM DELAY CONSTRAINTS

In this section, we first consider flows which always have data to transmit, e.g., TCP connections carrying long files. The QoS requirement is to provide certain minimum rate to each flow. Subject to this, we want to minimize the total network average power (3).

If we replace ϵ and δ by zero in Sections III, IV, V, we get optimal polices that guarantee minimum rates which also minimize the total average power consumption in the system. Stability of intermediate queues may still be of concern. This is because, even with TCP connections, we do not want the queueing delays to tend to infinity. Thus, we can make $\epsilon, \delta = 0$ at the source node but keep them positive at all intermediate nodes.

Now we consider flows which require an upper bound on mean end-to-end delay. This can be a first level QoS for real time traffic and can also be useful for data traffic.

First consider a single flow with *iid* input $\{A_f^t, t \geq 0\}$. Suppose its route has been fixed. We desire that its mean end-to-end delay be $\leq \overline{D}$. By Little's law, it provides an upper bound $E[A_f]\overline{D}$ on the mean end-to-end queue length on the fixed route (assuming all other deterministic processing, propagation delays etc. have been taken into account). We split this mean end-to-end queue length bound on the given route into individual mean queue length bounds $\overline{q}_1, ..., \overline{q}_{L_1}$ such that $\sum_{i=1}^{L_1} \overline{q}_i = E[A_f]\overline{D}$ (which we can optimize over). Now we solve the optimization problem (10) subjected to (11)-(13) and the additional constraints corresponding to the $E[q(l)]$ upper bounds (16) corresponding to $\overline{q}_1, ..., \overline{q}_{L_1}$. The resulting problem is non-convex and hence one obtains local minima (since it is a smooth optimization problem) using standard algorithms. Starting from random initial conditions and taking the best local minimum can provide a good solution.

The above scheme can be made to provide an upper bound on end-to-end mean delays on the intermediate nodes for TCP flows also (thus providing minimum rate guarantee as well as an upper bound on the end-to-end mean delay in the network).

The extension to multiuser case is obvious. In fact we can consider the system where some flows have only stability requirements, some minimum rate guarantees and some end-to-end mean delay requirement in the system and formulate a single optimization problem).

We can reduce the total power requirement if we give priority to the delay sensitive traffic on any given link over the other traffic. In that case the mean delay requirement gets relaxed compared to the case when it is not given priority.

A. Example

We demonstrate the above algorithm on the network of 20 nodes with the specifications in Section IV-C. There are three users. The first user requires only the stability of its queue, second user requires its end-to-end mean delay ≤ 30 slots and the third user requires the minimum rate guarantee of 1.05 packets/slot. The respective routes are $5 \to 7 \to 9 \to 10$; $6 \to 7 \to 9 \to 11$ and $19 \to 17 \to 16 \to 19$.

We have 3 independent sets, $S_1 = \{$(7,9), (19,17)$\}$, $S_2 = \{$(5,7), (9,10), (17,16)$\}$ and $S_3 = \{$(6,7), (9,11), (19,17) $\}$.

The arrival distribution for first and the second user are Poisson and Binomial with means $1, 1.5$. While the third user always has data to transmit.

We split the end-to-end mean of 45 into three parts $10, 20, 15$ along the route (optimization over this split should also be done, but is being ignored here; we only want to show that any split is acheivable by our algorithm). Stability for user 1 is attained by serving it at more than the arrival rate at each link. We took $\gamma_i = \frac{1}{3}$ for $i = 1, 2, 3$ and solved the optimization problem. The mean queue lengths for user 2 are provided in Table V. The average powers on the corresponding links are also provided. We see that the simulation end-to-end mean queue lengths satisfy the upper bound specified. From simulations, we saw that user 3 gets the rate 1.05 packets/sec and all the queues are stable.

Required $\hat{q}$	Simulated $\tilde{q}$	Sim. Power	Th. Power
$10, 20, 15$	$11.9, 4.23, 10.5$	$112, 12600, 147$	$156, 56000, 183$
$7, 30, 8$	$8.95, 24.35, 6.1$	$157, 5450, 182$	$247, 6790, 312$
$5, 35, 5$	$7.28, 28.89, 4.3$	$241, 4547, 172$	$442, 5485, 365$
$2.5, 40, 2.5$	$4.65, 27.79, 4.71$	$865, 4353, 835$	$2851, 5177, 2728$
$3.75, 37.5, 3.75$	$5.82, 27.7, 5.96$	$438, 4463, 267$	$972, 5352, 601$

TABLE V. MULTIUSER WITH QOS: FLOW 2

The flows 1 and 2 are passing via the common link $(7, 9)$. If, we give priority to the real time data of user 2 over user 1 on link $(7, 9)$, the simulated power at link $(7, 9)$ reduces to 8.1×10^3 while the theoretical power is 10.2×10^3. The power consumed at the other links stays same.

We observe that link $(7, 9)$ consumes much more power than the other two links. Thus, we increase the mean queue length requirement of link $(7, 9)$ to 30 and that of link $(6, 7)$ and $(9, 11)$ to 7 and 8 respectively. This reduced the power of link $(7, 9)$ substantially. Next we further increased the mean queue length required at link $(7, 9)$ to 35 leading to further reduction in power at $(7, 9)$ and also of the total power. Further increase in mean queue length at $(7, 9)$ to 37.5 led to an increase in the total power. These details are provided in Table V. One can do similar optimization of splitting the end-to-end mean queue length for the case when the dealy sensitive traffic is given priority.

VII. CONCLUSIONS

We have considered the problem of joint routing, scheduling and power control in a multihop network. Our main objective is to minimize the total average power while providing end-to-end QoS to all users. QoS can be the stability of the queues, a minimum rate guarantee per or an upper bound on the end-to-end mean delay for a flow. Power is consumed only during the transmission of data. We have considered the case when the power is a general non-decreasing function of transmission rate. Our (sub)optimal polices are easy to implement and computationally very efficient.

REFERENCES

[1] S. Asmussen, Applied probability and queues, Second edition, Springer-Verlag, 2003.

[2] D. P. Bertsekas and R. G. Gallager, "Data Networks", *Prentice Hall* , Second edition, 1992.

[3] J. Boxma, V. Sharma and D. K. Prasad, "Performace Analysis of a FLuid Queue with Random Service Rate in Discrete Time", *Springer-Verlang* , Berlin Heidelberg, pp. 568-581, 2007.

[4] L. Bui, A. Eryilmaz, Member, R. Srikant, and X. Wu, "Asynchronous Congestion Control in Multi-Hop Wireless Networks With Maximal Matching-Based Scheduling", *IEEE/ACM Transactions on networking*, Vol. 16, No. 4, Aug. 2008.

[5] M. Cao, V. Raghunathan, S. Hanly, V. Sharma and P. R. Kumar, "Power control and Transmission scheduling for network utility maximization in wireless networks", *Proceedings of the 46th IEEE Conference on Decision and Control, New Orleans, LA, USA*, Dec. 2007.

[6] Y. Cui V. K. N. Lau R. W. H. Huang S. Zhang, "A Survey on Delay-Aware Resource Control for Wireless Systems Large Deviation Theory, Stochastic Lyapunov Drift and Distributed Stochastic Learning", *arXiv: 1110.4535v1 [cs.PF]*, Oct. 2011.

[7] L. Georgiadis, M. J. Neely, and L. Tassiulas, "Resource allocation and cross-layer control in wireless networks", *Foundations and Trends in Networking*, Vol. 1, No. 1, pp. 1144, 2006.

[8] V. Harish, M. Rahul, V. Sharma, "Joint Routing Scheduling and Power control for Multihop MIMO Networks", *National Conference on Communications (NCC), India* , Feb. 2012.

[9] L. Huang and M. J. Neely, "Delay reduction via lagrange multipliers in stochastic network optimization", *IEEE Transactions on Automatic Control*, pp. 842857, Apr. 2011.

[10] L. Huang, S. Moeller, M. J. Neely, and B. Krishnamachari, Lifo-backpressure achieves near optimal utility-delay tradeoff", *WiOpt*, pp. 842857, May 2011.

[11] V. Joseph, V. Sharma and U. Mukherji, "Joint power control, scheduling and routing for multihop energy harvesting sensor networks", *Proceedings of the 4th ACM workshop on Performance monitoring etc.,* PM2HW2N, pp. 128-136, 2009.

[12] W. Kramer, M. Langenbach-Beltz , "Approximate formulae for the delay in the queueing systems GI/GI/1", *Eighth International Telemetric Congress*, Melbourne, pp. 235-1-8, 1976.

[13] V. Satya Kumar, V. Sharma, "Low Complexity Power and Scheduling Policies for Wireless Networks", *Information Theory and Applications Workshop (ITA)*, Sandiego, 2014.

[14] V. Satya Kumar, V. Sharma, "Efficient Low Complexity Power Allocation Policies for Wireless Communication Systems Guaranteeing QoS", *tenth International Conference on Signal Processing and Communications (SPCOM 2014)*, Indian Institute of Science, July 2014.

[15] V. Satya Kumar, V. Sharma, "Joint Routing, Scheduling and Power Control Providing QoS for Wireless Multihop Networks", *National Conference on Communications (NCC), India*, Feb. 2015.

[16] V. Sharma, "Some limit theorems for regenerative queues", *Queueing Systems*, Vol. 30, pp. 341-363, 1998.