Optimal Online Strategies for an Energy Harvesting System with Bernoulli Energy Recharges

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***Abstract*—We consider an energy harvesting system where the fixed size battery of the transmitter is recharged with certain probability at each channel use. For this setup, we explicitly characterize the optimal online energy management strategy for maximizing the long-term throughput under different assumptions on the availability of channel state information. We show that in the case of no fading, the amount of energy allocated to each channel use decreases exponentially with time since the last battery recharge and in the case of fading, the optimal solution has a discounted water-filling structure. Our results reveal that the structure of the optimal energy management strategy in the online case with finite battery is significantly different from the infinite battery and offline cases characterized in the literature.**

I. Introduction

Recent advances in energy harvesting technologies enable wireless devices to harvest the energy they need from the natural resources in their environment. This development opens the exciting new possibility to build wireless networks that are self-powered, self-sustainable and which have lifetimes limited by their hardware and not the size of their batteries.

However, communication with energy-harvesting devices has a number of aspects which make it fundamentally different from conventional battery-powered communication. In conventional systems, energy (or power) is a deterministic quantity continuously available to the transmitter and the receiver, and communication is typically constrained only in terms of average power. However, in harvesting systems energy is not generated at all times and the rate of energy generation can fluctuate significantly over time. In such systems, energy available for communication can be modeled as a stochastic rather than a deterministic process.

The key to effective communication with such wireless devices is the design of optimal energy management strategies. This is a non-trivial problem. If available energy is consumed too fast, transmission can be interrupted in the future due to energy outage. On the other hand, if the energy consumption is very slow, it can result in the wasting of the harvested energy and missed recharging opportunities in the future due to an overflow in the battery capacity. The problem is further complicated when there is randomness in the wireless channel due to fading. In this case, the energy management strategy needs to decide whether or not to wait for a better channel state in the future at the risk of wasting the energy in the meantime due to an overflow in the battery capacity.

The simplest setting which allows to gain insights on this problem is a wireless point-to-point link with a rechargeable transmitter, which we consider here. Optimal energy management for maximizing throughput on this channel has been of significant interest in the recent years. The problem is well-understood in the offline case where energy arrival instants and amounts are assumed to be known uncausally at the transmitter [1–4]. The optimal strategy keeps energy consumption as constant as possible over time while ensuring no energy wasting due to an overflow in the battery capacity. For example, in the case of fading, the optimal strategy is a directional water filling algorithm [3], which implies transmission with piecewise constant energy.

The solution for the more realistic online case, when the energy arrival process is known only causally at the transmitter, is much less understood. Many works propose heuristics inspired by the optimal offline solution, that of keeping the energy consumption as constant as possible over time. Such strategies become optimal in the limit when the battery size tends to infinity [5–10]. For example, [8] shows that an energy allocation strategy which tries to fix the energy allocated at each time slot as close as possible to the average energy arrival rate (long term mean of the energy process) becomes optimal in the limit of infinite battery. However, not much is known about the structure of the optimal online strategies in the more realistic case of finite battery. Indeed, the problem can be casted as a Markov Decision Process and the optimal solution can be computed using dynamic programming [11–14]. However the curse of dimensionality inherent in the dynamic programming solution makes this approach computationally intensive, which may not be suitable for sensor nodes with limited computational capabilities. It also leads to little insight on the structure of the optimal solution and its dependence on major system parameters.

A. Overview of Our Results

In this paper, we focus on a simple model for the energy harvesting process and derive explicit solutions for the online problem. We assume that the transmitter is equipped with a finite size battery, and this battery is recharged over time with Bernoulli energy arrivals, i.e. at any time step the battery is either charged to full with certain probability or no energy is harvested at all. This simple model captures the small battery regime which is relevant for many sensor networks. Tiny sensors are often physically limited in the size of the

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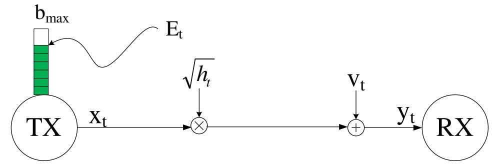

Figure 1: Point to point energy harvesting system with fading channel and limited battery of size b_{max}.

battery they can accommodate and if the energy harvesting process is rich enough, the battery may be fully charged each time there is an energy harvest. We derive the optimal online energy management strategy for this channel and show that it has an exponential structure; the allocated energy decreases exponentially with the time since the last energy arrival. This solution reveals that in the finite battery case, the optimal online solution is structurally different from the solution for the infinite battery and the offline cases, which rather aim to keep the energy expenditure constant.

We next extend our result to the case when there is fading in the wireless channel and the fading states are known ahead of time (but not the energy arrivals). This scenario is motivated by the cognitive radio setup where the variation of the channel states of the secondary users can be due to interference from the primary users. The (cognitive) secondary users can predict the activity of the primary users and hence the interference levels. We show that the optimal solution in this case is a discounted waterfilling algorithm, where the water level decreases exponentially with the time since the last energy arrival. Building on these insights, we also propose a heuristic algorithm for the fully online case, where neither the energy arrivals nor the fading states are known ahead of time. Our numerical evaluations show that the performance of this heuristic algorithm is very close not only to the optimal solution, but also to what is achievable with channel states known uncausally.

II. System Model and Problem Formulation

A. System Model

We consider a point to point discrete-time communication system with an energy harvesting transmitter as depicted in Fig. 1. We assume that the channel is an additive white Gaussian Noise channel (AWGN) potentially with fading. At any time step t, the received signal at the receiver is given by

$$y_t = \sqrt{h_t} x_t + v_t,$$

where x_t is the transmitted signal at time t, h_t is the channel coefficient at time t. We assume h_t to be nonnegative(w.l.o.g.) and i.i.d. of finite mean. Also, v_t is the white Gaussian noise which we assume to be of zero-mean and unit-variance.

The transmitter is equipped with a rechargeable battery of capacity b_{max}, which can be recharged by the energy harvested from the surrounding environment. Let E_t be the amount of energy harvested at time step t. It is a discrete time random process dictated by the randomness in the availability of the exogenous energy. We assume that this energy arrival process is known *causally* at the transmitter, i.e. at time t, the transmitter knows the realization of the random variables $\{E_i\}_{i=0}^{t}$. We assume that E_t is an i.i.d. Bernoulli random process,

$$E_t = \begin{cases} b_{max} & \text{with prob. } q \\ 0 & \text{with prob. } 1-q, \end{cases} \tag{1}$$

where q is the energy harvesting probability. In other words, in each time step either the battery is fully charged or no energy is harvested at all. We assume without loss of generality that transmission starts with the first energy arrival, i.e. $E_1 = b_{max}$.

Let b_t be the amount of energy available in the battery in the beginning of time step t and p_t be the amount of energy allocated for transmission in this time step. Then, we necessarily have $0 \leq p_t \leq b_t$. Moreover, the battery level in the beginning of the next time step is given by

$$b_{t+1} = \begin{cases} b_{max} & \text{if } E_{t+1} = b_{max} \\ b_t - p_t & \text{if } E_{t+1} = 0. \end{cases} \tag{2}$$

Note that this means that if there is remaining energy in the battery at the time of a new harvest, the remaining energy is discarded (or equivalently, only part of the harvested energy, dictated by the available space in the battery, can be stored.)

We assume that a rate function r_t is associated with this channel such that allocating p_t amount of energy to the channel with fade level h_t at time t results in r_t bits of information transfer. We assume that

$$r_t = \frac{1}{2}\log_2(1 + h_t p_t). \tag{3}$$

Note that this corresponds to the capacity function of a Gaussian channel with channel gain $\sqrt{h_t}$ and power p_t. With appropriate scaling, this corresponds to assuming that each time step is long enough or the bandwidth is large enough, so that we can do capacity achieving coding and approach the capacity of the AWGN channel.

B. Problem Formulation

We next define the energy management problem for this channel. The problem depends on the information available at the transmitter regarding the harvesting and the fading processes. We assume that the energy arrival times are only casually known at the transmitter and consider different assumptions on the availability of channel state information regarding the fading process.

1. *Static channel*: We assume that the channel is static and there is no fading, $\forall t\ h_t = 1$. In this case, the only randomness in the system comes from the energy harvesting process. In general, the transmission energy p_t at time step t, can be a function of all the realizations of the energy harvesting process E_t up to time t or equivalently the previous battery states $\{b_i\}_{i=1}^{t}$, i.e. $p_t = g(t, \{b_i\}_{i=1}^{t})$. The goal is to maximize the long-term average throughput

$$\Theta = \liminf_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[r_t], \tag{4}$$

over all possible strategies g that are admissible, i.e. satisfy for any t, b_t:

$$0 \leq g(t, \{b_i\}_{i=1}^{t}) \leq b_t.$$

Here, the expectation is over the energy harvesting process.

Since the process E_t is i.i.d., it is easy to see that $\{b_t\}_{t=1}^{\infty}$ form a Markov chain according to (2). Then, the above maximization problem is equivalent to a Markov Decision Process with instantaneous reward r_t and average reward criterion as defined in (4). Therefore, the optimal energy management strategy is markovian [15]; i.e., at time step t, the transmission energy p_t is only a function of the current state b_t. This means without loss of optimality we can focus only on markovian strategies of the form $p_t = g(t, b_t)$.

2. *Fading channel with channel coefficients known ahead of time*: Here, we assume a fading channel where the realization of fading coefficients is known ahead of time. In this case the energy management strategy at the transmitter can depend not only on the battery level but also on the sequence of past and future channel realizations. Given the sequence of all fading coefficients $\mathbf{h} = \{h_k\}_{k=1}^{\infty}$, the above argument for the optimality of markovian strategies applies by still taking the state to be b_t. A markovian strategy in this case is of the form $p_t = g(t, b_t; \mathbf{h})$.[1] The goal is still to maximize the long-term average throughput in (4) over the set of all admissible strategies that satisfy

$$0 \leq g(t, b_t; \mathbf{h}) \leq b_t,$$

for all t, b_t and $\mathbf{h}$. However, the expectation in (4) is now over the energy harvesting process and conditioned on the fading realization $\mathbf{h}$.Note that even though the channel states are known uncausally, we are still in the online scenario where the energy harvesting process E_t is only causally known.

3. *Online fading channel*: Here, we assume a fading channel and that the fading coefficients are only causally available at the transmitter. In this case, while the energy allocation strategy can a priori depend on $\{(b_i, h_i)\}_{i=1}^{t}$, due to the fact that $\{(b_t, h_t)\}_{t=1}^{\infty}$ forms a Markov chain, markovian strategies are again sufficient to achieve optimality. A markovian energy allocation strategy in this case is of the form $p_t = g(t, b_t, h_t)$. The goal is again to maximize the long-term throughput averaged over the energy harvesting and the fading processes. We call a strategy admissible if the following holds:

$$0 \leq g(t, b_t, h_t) \leq b_t.$$

III. Main Results

The main result of our paper is to provide a closed form expression for the optimal energy allocation strategy for the static channel and the fading channel with channel coefficients known ahead of time (Scenario 1 and 2 in Section II-B respectively). Also, based on the optimal strategy for the second case, we propose a heuristic strategy for the fully online fading scenario (Scenario 3 in Section II-B).

[1]Note that $\mathbf{h}$ serves as a parameter of the function g and is known ahead of time.

We define an epoch as the time interval between two successive energy arrivals. Thus, the time horizon is partitioned into a set of epochs and each time step belongs to a unique epoch. For the time step t, let s_t denote the time of the most recent energy arrival, i.e;

$$s_t = \max\{n | n \leq t,\ E_n = b_{max}\}. \tag{5}$$

Also, let i_t be the index of the time step t in its corresponding epoch, i.e., the number of time steps since the last energy arrival, which can be obtained as

$$i_t = t - s_t + 1. \tag{6}$$

The following theorem characterizes the optimal energy allocation for the static channel and is proved in Section V.

Theorem 1. (Optimal Strategy for the Static Channel) *Consider the system described in Section II-A in the case of a static AWGN channel. The optimal energy allocation strategy is given by*

$$g^*(t, b_t) = \pi^*(i_t), \tag{7}$$

where[2]

$$\pi^*(i) = \begin{cases} \frac{(1-q)^{i-1}}{\mu} - 1 & \text{if } 1 \leq i \leq N \\ 0 & \text{if } N < i, \end{cases} \tag{8}$$

where N is the smallest positive integer satisfying

$$\big(q(b_{max} + N) + 1\big)(1 - q)^N < 1, \tag{9}$$

and μ is given by

$$\mu = \frac{1 - (1 - q)^N}{q(N + b_{max})}. \tag{10}$$

The above theorem states two things of importance. First, the energy allocated to a given time step depends only on the number of time steps since the last energy arrival. Therefore, the same structure is repeated in every epoch. Intuitively, this is not difficult to see since due to our assumption of Bernoulli recharges, the system is reset each time an energy packet arrives (i.e., in the beginning of each epoch) and the epochs are statistically equivalent to each other. Second, it characterizes the optimal energy allocation in each epoch. The optimal strategy allocates non-negative energy only to the first N time steps in the epoch where N is determined by b_{max} and q and moreover the allocated energy decreases exponentially with the time since the last energy arrival (and shifted by 1 due to the subtractive term). Fig. 2 illustrates this strategy through an example. Note how different this structure is from the optimal strategies developed in the literature for the offline and $b_{max} = \infty$ cases, where the optimal strategy typically tries to keep the energy allocation as uniform as possible over time.

We now state our main result for the fading case with channel coefficients known ahead of time, which is proved in Section VI.

Theorem 2. (Optimal Strategy for the Fading Channel with Channel Coefficients Known Ahead of Time) *Consider the*

[2]Note that knowing t and b_t allows to determine i_t.

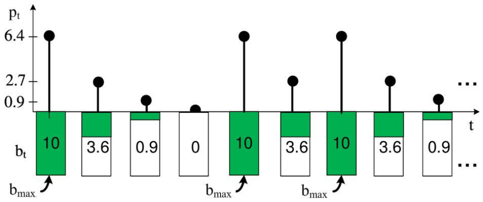

Figure 2: Optimal strategy in AWGN channel. The battery level b_t (bottom boxes) and transmission energy p_t (top stems) are represented for 9 consecutive time steps for a realization of energy arrival process. Here, $b_{max} = 10$ and $q = 0.5$, and hence, μ is determined as 7.43 and $N = 3$.

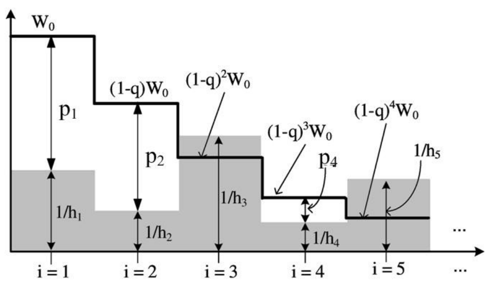

Figure 3: Discounted Water Filling in an epoch of length greater than 5. Initial water level at the beginning of the epoch is represented by W_0 . Also, h_i and p_i are the fading coefficient and transmission energy at time step i, respectively. Here, we have $p_3 = p_5 = 0$.

system described in Section II-A with the fading channel and with channel coefficients known ahead of time. The optimal energy allocation strategy for this case is given by

$$g^*(t, b_t; \{h_k\}_{k=1}^{\infty}) = \pi^*(i_t; \{h_n\}_{n=s_t}^{\infty}), \tag{11}$$

where s_t, i_t *are given by* (5) *and* (6) *respectively, and*

$$\pi^*(i; \{h_n\}_{n=1}^{\infty}) = \left(\frac{(1-q)^{i-1}}{\mu} - \frac{1}{h_i} \right)^+, \tag{12}$$

where $(x)^+ = \max(x, 0)$ *is the hinge function and* μ *is given as the solution of*

$$\sum_{n=1}^{\infty} \left(\frac{(1-q)^{n-1}}{\mu} - \frac{1}{h_n} \right)^+ = b_{max}. \tag{13}$$

As in the earlier case, the theorem states that the allocated energy to each time step depends on t only through i_t. Therefore, the optimal strategy again has the same structure in every epoch. On the other hand, the optimal policy in (12) for each epoch is a water filling expression in which the water level is discounted at each step by a factor of $(1-q)$, and hence we call it a *Discounted Water Filling* algorithm (DWF). Fig. 3 demonstrates the concept of discounted water filling. Note that the height of the water at each time step determines the energy allocated to this time step.

In addition to the above theorems which we prove in the next three sections, we propose a heuristic *Adaptive Discounted Water Filling* (ADWF) strategy for the online fading scenario in Section VII which builds on the insights from these theorems. Finally, in Section VIII, we provide numerical examples of the optimal and heuristic strategies.

IV. Optimality of Epoch Policies

As a step towards proving our main theorems, in this section we argue that the structure of the optimal policy should be the same in each epoch and moreover we can concentrate on maximizing the expected number of bits transmitted in each epoch.

Consider the system in Scenario 2 where the fading coefficients are known ahead of time. Recall that an epoch is the interval between two successive energy arrival. Let L_T be the number of epochs in a time horizon of length T which is a random variable itself. Then, by the strong law of large number we have

$$\lim_{T\to\infty} \frac{T}{L_T} = \lim_{T\to\infty} \frac{T}{\frac{1}{b_{max}} \sum_{t=1}^{T} E_t} = \frac{b_{max}}{\mathbb{E}[E_t]} = \frac{b_{max}}{q b_{max}} = \frac{1}{q}, \tag{14}$$

almost surely.

Now, let R_j be the number of bits transmitted in epoch j; i.e.,

$$R_j = \sum_{t \in ep(j)} r_t, \tag{15}$$

where $ep(j)$ is the set of all time steps contained in epoch j. The definition of the throughput in (4) can be rewritten as

$$\begin{aligned} \Theta &= \liminf_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[r_t] \\ &= \liminf_{T\to\infty} \frac{\sum_{j=1}^{L_T} \mathbb{E}[R_j]/L_T}{T/L_T} \\ &= q \liminf_{T\to\infty} \frac{\sum_{j=1}^{L_T} \mathbb{E}[R_j]}{L_T}, \end{aligned} \tag{16}$$

where the last equation follows from (14). Note that $L_T \to \infty$ almost surely as $T \to \infty$.[3]

According to the equivalent representation of Θ in (16), in order to maximize the throughput it suffices to individually maximize the expected number of transmitted bits in each epoch (i.e., $\mathbb{E}[R_j]$s). Note that the battery is fully charged at the beginning of each epoch and hence, maximizing $\mathbb{E}[R_j]$ and $\mathbb{E}[R_k]$ are independent of each other when $k \neq j$. This fact reveals an important point about the structure of the optimal energy management strategy: it should maximize the expected number of transmitted bits in each epoch. More precisely once an epoch has started, the optimal strategy, based on the information about the future fading coefficients, determines the transmission energies in a way that the expected number of transmitted bits up to the next energy arrival time is maximized. Upon the arrival of the next energy packet, a new epoch has started and the optimal strategy follows the same procedure (probably with different set of future fading coefficients). Thus, the optimal strategy consists of performing the optimal *epoch policy* in each epoch.

In order to define the optimal epoch policy, we first define the epoch policy. Consider an epoch whose future fading

[3]Note that in (16) we are dealing with $\liminf$, but the denominator can still be replaced with its limit because (14) holds in $\lim$.

coefficients are given as $\{h_n\}_{n=1}^{\infty}$. An epoch policy for this sequence of fading coefficients (treated as a parameter) is a function $\pi : \mathbb{N} \to \mathbb{R}$, where $\pi(i; \{h_n\}_{n=1}^{\infty})$ specifies the amount of energy to be allocated to the i^{th} time step in the epoch. Note that even though each epoch will be eventually of some finite length (hence not all of $\{h_n\}_{n=1}^{\infty}$ will fall in the epoch), the epoch policy needs to specify the energy allocation for each $i \geq 0$. We call an epoch policy π admissible if the following conditions hold for any $\{h_n\}_{n=1}^{\infty}$:

$$\begin{aligned} &1.\ 0 \leq \pi(i; \{h_n\}_{n=1}^{\infty}); \quad i = 1, 2, \cdots ; \\ &2. \sum_{i=1}^{\infty} \pi(i; \{h_n\}_{n=1}^{\infty}) \leq b_{max}. \end{aligned} \tag{17}$$

The second condition follows from the fact that the amount of energy we can spend in each epoch can not exceed the available energy in the battery in the beginning of the epoch. We let Π denote the set of all admissible epoch policies.

Given $\{h_n\}_{n=1}^{\infty}$ as the sequence of future fading coefficients of an epoch, we call an epoch policy $\pi(.; \{h_n\}_{n=1}^{\infty})$ optimal if it maximizes the expected number of transmitted bits in this epoch. In other words, the optimal epoch policy for this epoch is given as

$$\pi^* = \underset{\pi \in \Pi}{\text{argmax}}\ \mathbb{E}_{\pi}\big[R\big],$$

where R is the random variable denoting the number of transmitted bits in the epoch and the expectation is taken over the random length of the epoch. Note that when policy π is adopted, the expected number of transmitted bits in the epoch can be explicitly written as

$$\mathbb{E}_{\pi}[R] = \frac{1}{2} \sum_{m=1}^{\infty} \sum_{i=1}^{m} \log_2 \left(1 + h_i \pi(i; \{h_n\}_{n=1}^{\infty})\right) q(1-q)^{m-1}, \tag{18}$$

since the epoch length is a geometrically distributed random variable with mean $1/q$.

Note that according to the definition of the optimal epoch policy, if $\pi^*(.; \{h_n\}_{n=1}^{\infty}))$ is the optimal epoch policy for an epoch whose future fading coefficients are given by $\{h_n\}_{n=1}^{\infty}$ then, $\pi^*(.; \{h'_n\}_{n=1}^{\infty}))$ is also the optimal epoch policy for an epoch whose future fading coefficients are given by $\{h'_n\}_{n=1}^{\infty}$. This together with our previous discussion about the structure of the optimal energy management strategy allows us to conclude that the optimal strategy g is composed of repeating the optimal epoch policy π^* in every epoch parametrized with the future fading coefficients of that epoch. This observation is summarized in the following lemma.

Lemma 3. (Optimality of epoch policy in fading channel) *Consider the system described in Section II-A and Scenario 2 of the fading channel with channel coefficients known ahead of time. If π^* is the optimal epoch policy for this system, then the optimal strategy g is given by the following:*

$$g^*(t, b_t; \{h_n\}_{n=1}^{\infty}) = \pi^*(i_t; \{h_n\}_{n=s_t}^{\infty}), \tag{19}$$

where s_t, i_t are given by (5) *and* (6)*, respectively.*

So far in this section, we were considering Scenario 2 in which there is a fading channel and fade levels are known ahead of time. Scenario 1 (static channel) on the other hand can be regarded as a special case of the second scenario where all the fading coefficient are equal to 1. In this case, by dropping the dependency on the fade levels, we can take $\pi(i)$ to be the transmission energy at the i^{th} time step in an epoch when policy π is adopted. Following from Lemma 3, we immediately have the following corollary for this case.

Corollary 4. (Optimality of epoch policy in static channel) *Consider the system described in Section II-A in the static channel scenario. If π^* is the optimal epoch policy for this system then, the optimal energy allocation strategy is given by the following:*

$$g^*(t, b_t) = \pi^*(i_t), \tag{20}$$

where i_t is given by (6).

In the two following sections, we use Lemma 3 and Corollary 4 to derive the optimal energy allocation strategies and prove Theorems 1 and 2.

V. Optimal Strategy for AWGN Channel

In this section, we consider the described system in Section II-A in the static channel scenario and prove Theorem 1.

Proof of Theorem 1: According to Corollary 4, the optimal energy allocation strategy in this scenario is determined by the optimal epoch policy. According to (18) (when the fading coefficients are equal to 1) and the definition of admissible epoch policy in (17), the optimal epoch policy in the static channel is the solution of the following problem:

$$\begin{aligned} \underset{\pi}{\text{maximize}} \quad & \frac{1}{2} \sum_{m=1}^{\infty} \sum_{i=1}^{m} \log_2(1 + \pi(i)) q(1-q)^{m-1} \\ \text{subject to} \quad & 0 \leq \pi(i); \quad i = 1, 2, \cdots , \\ & \sum_{i=1}^{\infty} \pi(i) = b_{max}. \end{aligned}$$

In the following, we show that the solution to the above problem is given by (8). After removing a $\frac{1}{2 \log 2}$ factor of the objective function (w.l.o.g.), the Lagrangian of the above problem is

$$\begin{aligned} \mathcal{L}(\{\pi(i)\}_{i=1}^{\infty}, &\{\lambda_i\}_{i=1}^{\infty}, \mu) = \\ & - \sum_{m=1}^{\infty} \sum_{i=1}^{m} \log(1 + \pi(i)) q(1-q)^{m-1} \\ & - \sum_{i=1}^{\infty} \lambda_i \pi(i) + \mu \bigg(\sum_{i=1}^{\infty} \pi(i) - b_{max} \bigg), \end{aligned} \tag{21}$$

and then, the KKT conditions are

$$\forall i \geq 1 : \frac{\partial \mathcal{L}}{\partial \pi(i)} = 0,\ \lambda_i \pi(i) = 0,\ \lambda_i \geq 0,\ \pi(i) \geq 0$$

$$\text{and} \quad \sum_{i=1}^{\infty} \pi(i) = b_{max}.$$

Substituting $\mathcal{L}$ from (21) and applying the KKT conditions gives

$$\pi(i) = \begin{cases} \frac{(1-q)^{i-1}}{\mu} - 1 & \text{if } \mu \leq (1-q)^{i-1} \\ 0 & \text{Otherwise.} \end{cases}$$

Since $(1-q)^{i-1} \to 0$ as i gets larger, then e_i is non-zero only for a finite number of indices $i = 1, 2, \cdots, N$, where N is the smallest integer satisfying

$$\mu > (1-q)^N. \tag{22}$$

Now, applying the last KKT condition gives

$$b_{max} = \sum_{i=1}^{N} \left(\frac{(1-q)^{i-1}}{\mu} - 1 \right) = \frac{1-(1-q)^N}{q\mu} - N,$$

and therefore, μ can be written as

$$\mu = \frac{1-(1-q)^N}{q(N + b_{max})}. \tag{23}$$

Substituting (23) into (22) determines N as the smallest integer that satisfies

$$\big(q(b_{max} + N) + 1\big)(1-q)^N < 1,$$

and also gives

$$\pi(i) = \begin{cases} \frac{(1-q)^{i-1}}{\mu} - 1 & \text{if } 1 \leq i \leq N \\ 0 & \text{if } N < i, \end{cases}$$

which together with Corollary 4 completes the proof of Theorem 1. ■

VI. Optimal Strategy for Offline Fading Channel

In this section, we consider the energy harvesting system in Scenario 2, fading channel with fading coefficients known ahead of time. We show that the optimal strategy in this case is a DWF strategy and prove Theorem 2.

Proof of Theorem 2: . According to Lemma 3, the optimal energy allocation in this scenario is specified by the optimal epoch policy. Also, according to (18) and the definition of admissible epoch policies in (17), the optimal epoch policy for an epoch with future fading coefficients given by $\{h_n\}_{n=1}^{\infty}$ is the solution of the following problem:

$$\begin{aligned} \underset{\pi}{\text{maximize}} \quad & \frac{1}{2} \sum_{m=1}^{\infty} \sum_{i=1}^{m} \log_2(1 + h_i \pi(i; \{h_n\}_{n=1}^{\infty})) q(1-q)^{m-1} \\ \text{subject to} \quad & 0 \leq \pi(i; \{h_n\}_{n=1}^{\infty}); \quad i = 1, 2, \cdots, \\ & \sum_{i=1}^{\infty} \pi(i; \{h_n\}_{n=1}^{\infty}) = b_{max}. \end{aligned}$$

Similar to the approach in the proof of Theorem 1 in Section V, after removing a $\frac{1}{2\log 2}$ factor of the objective function (w.l.o.g.) the Lagrangian is

$$\begin{aligned} &\mathcal{L}(\{\pi(i; \{h_i\}_{n=1}^{\infty})\}, \{\lambda_i\}, \mu) = \\ &\quad - \sum_{m=1}^{\infty} \sum_{i=1}^{m} \log(1 + h_i \pi(i; \{h_n\}_{n=1}^{\infty})) q(1-q)^{m-1} \\ &\quad - \sum_{i=1}^{\infty} \lambda_i \pi(i; \{h_n\}_{n=1}^{\infty}) \\ &\quad + \mu \Big(\sum_{i=1}^{\infty} \pi(i; \{h_n\}_{n=1}^{\infty}) - b_{max} \Big), \end{aligned} \tag{24}$$

and the KKT conditions are

$$\begin{aligned} \forall i \geq 1: & \frac{\partial \mathcal{L}}{\partial \pi(i; \{h_n\}_{n=1}^{\infty})} = 0, \ \lambda_i \pi(i; \{h_n\}_{n=1}^{\infty}) = 0, \\ & \lambda_i \geq 0, \quad \pi(i, \{h_n\}_{n=1}^{\infty}) \geq 0, \\ \text{and} \quad & \sum_{i=1}^{\infty} \pi(i; \{h_n\}_{n=1}^{\infty}) = b_{max}. \end{aligned}$$

Substituting $\mathcal{L}$ from (24) into the first 4 KKT conditions gives

$$\pi(i; \{h_n\}_{n=1}^{\infty}) = \left(\frac{(1-q)^{i-1}}{\mu} - \frac{1}{h_i} \right)^+, \tag{25}$$

and applying the last KKT condition determines μ as the solution of the following equation:

$$\sum_{i=1}^{\infty} \left(\frac{(1-q)^{i-1}}{\mu} - \frac{1}{h_i} \right)^+ = b_{max}. \tag{26}$$

Consequently, the optimal epoch policy in the fading channel with offline knowledge of fade levels is given by (25), and this together with Lemma 3 completes the proof of Theorem 2. ■

VII. Sub-Optimal Strategy for the Fading Channel with Online Knowledge of Fade Levels

Consider the energy harvesting system described in Section II in the online fading scenario where the fade levels are only causally known at the transmitter. For this scenario, we propose a suboptimal online strategy for energy allocation which is based on the structure of DWF algorithm introduced in the previous section.

According to Lemma 3, when fading coefficients are known ahead of time, the optimal strategy can be obtained by first finding the optimal epoch policy and then repeating it after each energy arrival. On the other hand, according to Theorem 2, when the fading coefficients are known ahead of time, the optimal epoch policy is given by (12). We follow the same steps as in the previous section to derive a heuristic strategy for the online fading scenario.

In the previous case, the offline knowledge of fading coefficients $\{h_n\}_{n=1}^{\infty}$ is only utilized to determine the water level parameter μ in (13). However, if the fading coefficients are only causally available at the transmitter, at time step j, all the future coefficients $\{h_i\}_{i=j+1}^{\infty}$ are random variables. Therefore, each term in front of the sum on the LHS of (13) would also be a random variable. More precisely, for a given μ and for any $i = 1, 2, \cdots$, define Z_i as

$$Z_i(\mu) = \left(\frac{(1-q)^{i-1}}{\mu} - \frac{1}{h_i}\right)^+, \tag{27}$$

which generally is a random variable due to the randomness in h_i. Once the realization of fading coefficient h_i is known, Z_i is also determined and is not random anymore. Since the realization of all Z_is are not known ahead of time, the optimal water level parameter μ cannot be determined according to (13). In order to tackle this problem, we, at each time step, replace the future (un-realized) Z_is with their expected values and obtain an updated value for μ. In other word, at the time step j, the transmitted energies in the previous time steps (i.e.; $p_1, p_2, \cdots, p_{j-1}$) are known and given the current fading coefficient h_j, the transmission energy for the current time step is determined as

$$p_j = \left(\frac{(1-q)^{j-1}}{\mu_j} - \frac{1}{h_j}\right)^+, \tag{28}$$

where μ_j is the solution of the following equation:

$$b_{max} = p_1 + p_2 + \cdots + p_{j-1} + \left(\frac{(1-q)^{j-1}}{\mu_j} - \frac{1}{h_j}\right)^+ + \sum_{i=j+1}^{\infty} \mathbb{E}\left[\left(\frac{(1-q)^{i-1}}{\mu_j} - \frac{1}{h_i}\right)^+\right].$$

Note that when the fading coefficient are known ahead of time the above equation is equivalent to (13) and all μ_js will be the same. By moving $p_1, p_2, \cdots, p_{j-1}$ to the LHS and substituting Z_i from (27), the above equation is simplified to

$$b_j = \left(\frac{(1-q)^{j-1}}{\mu_j} - \frac{1}{h_j}\right)^+ + \sum_{i=j+1}^{\infty} \mathbb{E}\big[Z_i(\mu_j)\big], \tag{29}$$

where

$$b_j = b_{max} - p_1 - p_2 - \cdots - p_{j-1}$$

is the available energy in the battery at the beginning of time step j, and the expectation in (29) is taken over the fading coefficients.

Now assume a Rayleigh fading channel where h_is are distributed according to the following pdf:

$$f(h) = \alpha e^{-\alpha h}, \quad h \geq 0,$$

for some $\alpha > 0$. Consequently, it is easy to show that for any $\mu > 0$

$$\mathbb{E}[Z_i(\mu)] = \frac{(1-q)^{i-1}}{\mu} e^{-\frac{\alpha\mu}{(1-q)^{i-1}}} - \text{expint}\left(\frac{\alpha\mu}{(1-q)^{i-1}}\right), \tag{30}$$

where the function *expint* is given by

$$\text{expint}(x) = \int_x^{\infty} \frac{e^{-t}}{t}\, dt.$$

Algorithm 1 Adaptive Discounted Water Filling Strategy

1: **Parameters:** $q \in (0,1)$, $b_{max} > 0$
2: **Initialize:** $j \leftarrow 1$ and $b_j \leftarrow b_{max}$
3: **Find** μ_j**:** by solving the following for μ

$$\left(\frac{(1-q)^{j-1}}{\mu} - \frac{1}{h_j}\right)^+ + \phi_j(\mu) = b_j$$

4: **Transmit:** with the following energy:

$$p_j = \left(\frac{(1-q)^{j-1}}{\mu_j} - \frac{1}{h_j}\right)^+$$

5: **Update the state:**
  If Energy packet arrives at next time step
    Go to 2
  Else
    $b_j \leftarrow b_j - p_j$
    $j \leftarrow j + 1$
    Go to 3

Now, for any $j = 1, 2, \cdots$, we define

$$\begin{aligned}\phi_j(\mu) &= \sum_{i=j+1}^{\infty} \mathbb{E}\big[Z_i(\mu)\big] \\ &= \sum_{i=j+1}^{\infty} \left[\frac{(1-q)^{i-1}}{\mu} e^{-\frac{\alpha\mu}{(1-q)^{i-1}}} - \text{expint}\left(\frac{\alpha\mu}{(1-q)^{i-1}}\right)\right],\end{aligned} \tag{31}$$

which indicates the expected value of total energy that is going to be transmitted after the j^{th} time step in the epoch. Using this definition, (29) can be represented as

$$\left(\frac{(1-q)^{j-1}}{\mu_j} - \frac{1}{h_j}\right)^+ + \phi_j(\mu_j) = b_j, \tag{32}$$

which can be numerically solved for μ_j.

Now, we introduce an epoch policy as follows: at any time step j, h_j and b_j are known and μ_j can be numerically computed as the solution of (32). Then, the transmission energy for time step j is determined as in (28). Based on this epoch policy, we introduce the *Adaptive Discounted Water Filling* (ADWF) strategy which basically is to repeat the above policy after each energy arrival. Details and successive steps of ADWF strategy are summarized in Algorithm 1.

It worth mentioning that because of the recurrence structure of ϕ_js, these functions can efficiently be stored in the memory and called at step 3 in the algorithm and hence, the above algorithm can be easily implemented on low-memory systems. Furthermore, for the same set of parameters, the performance of DWF strategy is an upper bound on the performance of ADWF strategy, because the earlier one utilizes the information about future fading coefficients while the later does not. In the following section, we compare the performance of ADWF strategy against DWF which utilizes the offline knowledge of fade levels and show that in contrast to its simplicity, ADWF strategy has near-optimal performance.

VIII. Numerical Evaluations

In this section, we provide numerical examples and demonstrate the performance of optimal and sub-optimal strategies

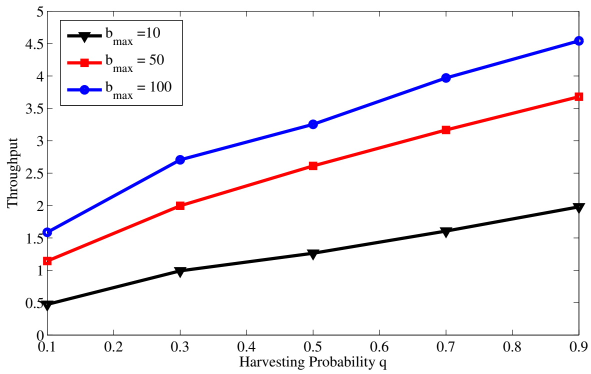

Figure 4: Throughput achieved by the optimal strategy (Theorem 1) in AWGN channel vs. harvesting probability.

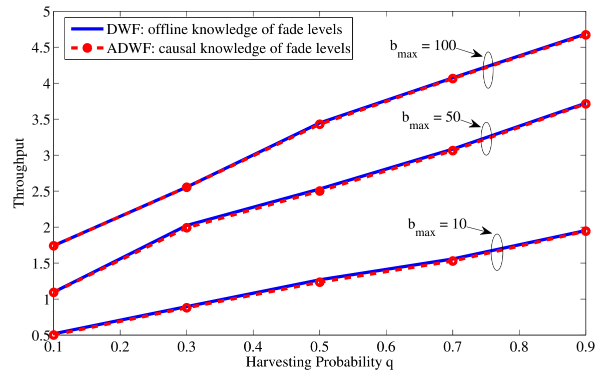

Figure 5: Throughput vs. harvesting probability. The solid plots correspond to the case of offline knowledge of fade levels and when DWF strategy is adopted. Broken plots correspond to the case of online knowledge of fade levels when ADWF strategy is adopted. The solid plots also serve as upper-bounds for bit-per-energy in the online case.

in different scenarios. In order to find the throughput of the system in a specific scenario, we run a simulation of the system over $T = 10000$ time slots where the energy arrival process and fading coefficients are drawn randomly according to their distribution. Then, for the given strategy, Θ is computed as

$$\Theta \approx \frac{\frac{1}{2}\sum_{t=1}^{T}\log_2(1+h_t p_t)}{T}.$$

First, consider the system in the static (AWGN) channel where the optimal strategy is given by Theorem 1. Fig. 4 plots the achieved throughput(Θ) of the system as a function of harvesting probability q. The performance is determined for different battery sizes $b_{max} = 100, 50, 10$. As depicted in this figure, the throughput of the system is increasing both in the harvesting probability and battery size. This is because in both case more energy will be available to use for transmission.

Now, we consider the system over a Rayleigh fading channel and take two cases into account. First, we assume that the realization of the fading coefficients are known ahead of time (second scenario in Section II-B), and the transmission energies are determined according to DWF strategy in Theorem 2. Next, we assume that the fading coefficients are only causally available at the transmitter (third scenario in Section II-B), and the transmission energy at each time step is determined according to ADWF strategy in Algorithm 1. Fig. 5 plots the achieved throughput for these two scenarios as a function of the harvesting probability and for different battery sizes, $b_{max} = 100, 50, 10$. Here, the fading parameter is set to $\alpha = 2$ which means an average fade level of 0.5. As depicted in this figure, the achieved throughput of ADWF strategy (in the online fading case) is very close to the throughput achieved by DWF strategy (in the offline fading case) which serves as an upperbound. Thus, in contrast to its simplicity, ADWF strategy has a performance near the optimal strategy.

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