Downlink Interference Penalty Algorithm for Power Control, Scheduling, and User Association

Jobin Francis, Suresh Kalyanasundaram, Balamurali Natarajan, Rajeev Agrawal, and Neelesh B. Mehta

***Abstract*—Managing inter-cell interference is one of the main challenges in current and next generation wireless systems that aggressively reuse the frequency. Cooperation between interfering cells has been sought to mitigate interference. In this paper, we address the problem of jointly optimizing the transmit powers, user scheduling, and user association in a cellular network to maximize the weighted sum rate (WSR). To this end, we develop a distributed interference penalty algorithm in which the cells update their transmit powers and user schedule to maximize its utility minus an interference cost. The proposed algorithm involves only limited exchange of information via backhaul and has convergence guarantees. Furthermore, we propose a sub-optimal algorithm with lower computational and backhaul overhead. In it, the users are first associated to the base stations (BSs) based on their signal-to-interference-plus-noise-ratios (SINRs). It is then followed by joint optimization of BS transmit powers and user scheduling, for which we develop an interference penalty algorithm as well. We show that the proposed algorithms outperform the computationally complex weighted minimum mean squared error (WMMSE) algorithm.**

I. Introduction

Managing interference in a wireless network is key to improving its spectral efficiency. This is particularly important in next-generation cellular networks, whose design objective is to provide high data rates throughout the coverage area of the network. Near the edge of the cells, users receive lower signal strength, while experiencing higher inter-cell interference (ICI). Hence, providing high data rates to these users is highly challenging and techniques to manage ICI are needed. Conventionally, fractional frequency reuse in which neighboring cells operate on different frequency bands was employed to mitigate ICI. However, due to the scarcity of the spectrum, current cellular networks have frequency reuse factor close to 1, which necessitates the need for novel ways to manage ICI. One such useful technique is coordinated multi-point (CoMP) operation [1]. In it, interference management occurs through coordination between the cells.

A few of the prominent CoMP techniques are dynamic point blanking (DPB), coordinated scheduling (CS), and dynamic point selection (DPS). In DPB, the base stations (BSs) are either transmitting at the maximum transmit power or are muted. In other words, the coordinating cells employ binary transmit power control. Note that muting the BS results in waste of radio resources. However, if the gains in the neighboring cells, due to reduced interference, offsets this loss then the network as a whole benefits from cell-muting. In this paper, however, we do not enforce the binary restriction on the transmit powers. In CS, as the name implies, the user to be scheduled in each cell is determined in a coordinated manner. In DPS, users can dynamically switch from one BS to another, which in essence is identical to fast hand-off. Other popular CoMP schemes are joint transmission (JT) in which multiple BSs serve the same user, and coordinated beamforming (CB) for cells with multiple antennas. To achieve better performance, combinations of the CoMP schemes mentioned above can be used. In this paper, we focus on optimizing the transmit powers, user scheduling, and user association, and their combinations. Note that power control, scheduling, and association are proxies for DPB, CS, and DPS, respectively.

An important consideration in employing a CoMP scheme is the amount of backhaul signalling involved. Distributed algorithms with minimal exchange of information between BSs over the backhaul are preferred. This is for the following reasons. First, we want to conserve the backhaul bandwidth. Second, the backhaul has an associated delay, which renders the information conveyed over the backhaul outdated.

It is also important to ensure that the network throughput, which is the sum-throughput of all the users in the network, do not suffer too much while helping the cell-edge users. Hence, there must be a proper trade-off between user-fairness and network throughput. This is ensured by using the utility maximization framework. In it, the objective is to maximize the network utility, which is the sum of utilities of the users in the network. Utility of a user is an increasing, concave function of its long-term throughput. The online policy of maximizing the weighted sum rate (WSR) of the network at every instant is shown to maximize the network utility [2]. WSR is the weighted sum of instantaneous rates of the scheduled users in the network, which depends on their instantaneous channel quality. Here, the weight of a user is defined as the gradient of the utility function evaluated at the value of the throughput received by the user so far.

One of the commonly used utility functions is the logarithmic utility function, in which case the scheduler becomes the widely used proportional fair (PF) scheduler. It schedules the user with the highest PF metric for transmission. The PF metric of a user is the ratio of its instantaneous rate to the throughput received by the user so far [3]. We note that

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the algorithms developed in the paper are applicable for any general utility function as the specific utility function only determines the weights assigned to each user.

Our goal is to develop algorithms, which optimize transmit powers, user scheduling and user association among the coordinating BSs to maximize the WSR. Note that the joint optimization is a mixed-integer problem as scheduling/association of each user is represented by binary variables. Thus, joint optimization is challenging. We follow an interference-penalty based strategy in which BSs are penalized for the interference it generates – a strategy first adopted in [4]. This strategy has the advantage of distributed implementation as each BS solves a local optimization problem given its interference cost.

A. Related Literature

We now briefly survey the literature on joint optimization on transmit powers, schedule, and association.

The joint power control and user association problem for sum-rate maximization is considered in [5]–[7]. An additional quality-of-service constraint is incorporated for each user in [5], [6]. In all of these papers, iterative algorithms that alternately optimize transmit powers and user association are developed. Although [6] describes a joint optimization algorithm, it is computationally intensive and can only be used as a benchmark.

WSR maximization objective is considered in [8], [9]. However, [8] focuses on developing schemes that are robust to misrepresentation of downlink channel gains by the users. In [9], weighted minimum mean square error (WMMSE) framework is used for joint association and beamforming. However, this algorithm is computationally intensive. Distributed algorithms to maximize the logarithmic utility function are presented in [10], [11]. These papers assume a static channel model. Consequently, the objective function for optimization is not WSR. In [11], however, the authors describe a low complexity heuristic for joint beamforming and association to maximize the WSR. Proportional fairness utility maximization is also considered in [12], wherein the joint power control and association problem is shown to be NP-hard, and a low complexity greedy algorithm is developed for it. User association with partial muting of the macro-cells is investigated in [13].

Interference Penalty Algorithms: An interference penalty algorithm for a general class of utility functions is developed in [4], [14]. In it, the transmit powers are adjusted in an iterative manner to maximize the utility of a peer-to-peer network. This algorithm has been extended to multiple antenna networks in [15], [16] and to handle non-separable user utility functions in [17]. In [18], an interference penalty algorithm is developed to optimize the uplink transmit powers and the receive beamforming vectors. An interference penalty algorithm to jointly optimize the transmit powers, user schedule, and resource allocation in the uplink is developed in [19].

B. Contributions

In this paper, we develop a downlink interference penalty algorithm (DL IPA) that jointly optimizes the transmit powers, user schedule, and association to maximize WSR. It is a distributed, iterative algorithm in which the BSs exchange interference prices over the backhaul. Each BS computes the interference cost using the interference prices of users scheduled in the neighboring BSs, and adjusts its transmit power and user schedule to maximize the weighted rate minus the interference cost. The algorithm is guaranteed to converge to a stationary point of the optimization problem.

We also develop a simpler, sub-optimal algorithm. In it, a user associates to the BS to which it has the highest signal-to-noise-plus-interference (SINR). Once the user association is fixed, we then jointly optimize the transmit powers and the user scheduling. To this end, we extend the interference penalty algorithm in [14] to handle user scheduling as well. We shall see that this sub-optimal algorithm has a performance very close to the DL IPA for joint power control, scheduling, and association.

C. Organization and Notations

This paper is organized as follows. In Section II we develop the DL IPA for joint optimization of transmit powers and user scheduling. In Section III interference penalty algorithm for joint power, schedule, and user association is discussed. Simulation results are given in Section IV followed by our conclusions in Section V.

We use the notation $[u]_a^b$ to denote $\max\{\min\{u, b\}, a\}$.

II. DL IPA for Joint Power Control and User Scheduling

In this section, we assume that the user association is fixed. It can be determined using the long-term channel statistics of the users. Our objective is to optimize transmit powers and user schedule to maximize the WSR. We consider a single-antenna cellular network in which both the BSs and the users are equipped with one antenna.

A. Problem Formulation

Consider a cellular network with N BSs and K users. Let $\mathcal{K}_n$ denote the set of users associated to cell n.[1] Let P_n denote the transmit power of BS n and $G_{k_n m}$ denote the channel gain to user $k_n \in \mathcal{K}_n$ from BS m. Then, the SINR of user k_n, denoted by γ_{k_n}, is given by

$$\gamma_{k_n} = \frac{G_{k_n n} P_n}{\sigma^2 + \sum_{m=1, m \neq n}^{N} G_{k_n m} P_m}, \tag{1}$$

where σ^2 is the thermal noise power. The rate of transmission to user k_n if scheduled in cell n, denoted by r_{k_n}, is given by the Shannon capacity formula. That is,

$$r_{k_n} = \log_2 \left(1 + \gamma_{k_n}\right). \tag{2}$$

Let w_{k_n} denote the weight of user k_n and let the binary variable x_{k_n} denote scheduling decision for user k_n in cell

[1]If $\mathcal{K}_n$ is a nullset (no user is attached to the cell), we set its transmit power to zero as there are no users to transmit to.

n. Then, the WSR maximization problem is:

$$\mathbf{P}1: \quad \max \sum_{n=1}^{N} \sum_{k_n \in \mathcal{K}_n} x_{k_n} w_{k_n} \log\left(1+\gamma_{k_n}\right), \tag{3}$$

$$\text{s.t.}\ \gamma_{k_n} = \frac{G_{k_n n} P_n}{\sigma^2 + \sum_{m=1, m\neq n}^{N} G_{k_n m} P_m}, \tag{4}$$

$$\sum_{k_n \in \mathcal{K}_n} x_{k_n} = 1, \quad n = 1, \ldots, N, \tag{5}$$

$$x_{k_n} \in \{0,1\}, \forall k_n \in \mathcal{K}_n,\ n = 1, \ldots, N, \tag{6}$$

$$0 \leq P_n \leq P_{\max}, \quad n = 1, \ldots, N. \tag{7}$$

The constraint (6) mandates that exactly one user is scheduled in each cell and (7) constrains the maximum transmit power of any BS. The binary restriction on x_{k_n} in (6) can be relaxed without changing the optimum. This claim is proved in Appendix A. Thus, the constraint (6) can be rewritten as $0 \leq x_{k_n} \leq 1$. We now develop the interference penalty algorithms below.

B. Algorithms

The KKT conditions for the optimization problem $\mathbf{P}1$ with the binary restriction of x_{k_n} relaxed is given by

$$\sum_{n=1}^{N} \sum_{k_n \in \mathcal{K}_n} x_{k_n} \frac{d}{dP_i} w_{k_n} \log\left(1+\gamma_{k_n}\right) + \rho_i - \eta_i = 0, \tag{8}$$

$$w_{k_i} \log\left(1+\gamma_{k_i}\right) + \lambda_{k_i} - \mu_{k_i} + \sigma_i = 0, \tag{9}$$

$$\lambda_{k_i} x_{k_i} = 0, \tag{10}$$

$$\mu_{k_i}\left(x_{k_i} - 1\right) = 0, \tag{11}$$

$$\rho_i P_i = 0, \tag{12}$$

$$\eta_i\left(P_i - P_{\max}\right) = 0, \tag{13}$$

$$\lambda_{k_i}, \mu_{k_i}, \rho_i, \eta_i \geq 0, \quad k_i \in \mathcal{K}_i,\ i = 1, \ldots, N, \tag{14}$$

where $\lambda_{k_i}, \mu_{k_i}, \rho_i, \eta_i$ are the Lagrangian multipliers corresponding to the inequality constraints and σ_i is the Lagrangian multiplier for the equality constraint in (5).

Let

$$\pi_{k_n} = -\frac{d}{dI_{k_n}} w_{k_n} \log\left(1+\gamma_{k_n}\right) \tag{15}$$

$$= w_{k_n} \frac{\gamma_{k_n}}{1+\gamma_{k_n}} \frac{1}{\sigma^2 + I_{k_n}}, \tag{16}$$

where $I_{k_n} = \sum_{m \neq n} G_{k_n m} P_m$ is the interference seen by user k_n in cell n. Note that π_{k_n} denotes the increase in the weighted rate of user k_n with a marginal decrease in interference. We shall refer to π_{k_n} as the *interference price* of user k_n. Now, (8) can be rewritten as

$$\sum_{k_i \in \mathcal{K}_i} x_{k_i} \frac{d}{dP_i} w_{k_i} \log\left(1+\gamma_{k_i}\right) + \sum_{n=1, n\neq i}^{N} \sum_{k_n \in \mathcal{K}_n} x_{k_n} \pi_{k_n} G_{k_n i} + \rho_i - \eta_i = 0. \tag{17}$$

Assuming fixed interference prices and transmit powers of BSs other than i, (17) together with eqs. (9) to (14) are the necessary conditions for the following optimization problem.

$$\mathbf{P}2: \quad \max \sum_{k_i \in \mathcal{K}_i} w_{k_i} x_{k_i} \log\left(1+\gamma_{k_i}\right) - P_i C_i \tag{18}$$

$$\text{s.t.}\ 0 \leq x_{k_i} \leq 1,\ \forall k_i \in \mathcal{K}_i \tag{19}$$

$$\sum_{k_i \in \mathcal{K}_i} x_{k_i} = 1, \tag{20}$$

$$0 \leq P_i \leq P_{\max}, \tag{21}$$

where $C_i = \sum_{n=1, n\neq i}^{N} \sum_{k_n \in \mathcal{K}_n} x_{k_n} \pi_{k_n} G_{k_n i}$ is the interference cost per unit power incurred by BS i. Let P_i^* and $x_{k_i}^*$ denote the optimal power and user schedule solutions of the above problem. The optimal transmit power if user k_i is scheduled in cell i, denoted by $P_{k_i}^*$, is given by

$$P_{k_i}^* = \left[\frac{w_{k_i}}{C_i} - \frac{\sigma^2 + I_{k_i}}{G_{k_i, i}}\right]_0^{P_{\max}}. \tag{22}$$

Thus, we have obtained the optimal transmit power for BS i considering each user as the candidate user for scheduling. We now evaluate the objective function for each user assuming that BS i is transmitting at the optimal power for that user. The user that has the highest objective function value is the optimal user to schedule. It can be stated mathematically as follows: Let $\gamma_{k_i}^*$ denote the SINR of user k_i, when BS i is transmitting at power $P_{k_i}^*$. Then, the scheduled user in cell i is given by

$$k_i^* = \arg\max_{k_i \in \mathcal{K}_i} \left[w_{k_i} \log\left(1+\gamma_{k_i}^*\right) - P_{k_i}^* C_i\right]. \tag{23}$$

The optimal transmit power for BS i is the transmit power corresponding to the scheduled user. That is,

$$P_i^* = P_{k_i^*}^*. \tag{24}$$

Notice that given the interference cost C_i, which can be computed from the interference prices of the scheduled users in other cells and their cross channel gains from cell i, the optimization problem $\mathbf{P}2$ can be solved locally at BS i. This motivates the following distributed algorithm for the joint power and scheduling optimization problem in $\mathbf{P}1$.

1) DL IPA for Joint Power Control and Scheduling (DL IPA JPCS): The steps of the DL IPA for joint power and schedule optimization is given in Algorithm 1.

The convergence of the DL IPA JPCS is proved in Appendix B. The proof is along the lines in [14]. We show that the WSR is non-decreasing in the number of iterations. This guarantees convergence of the algorithm as the WSR of the network is bounded.

2) Alternating Power and Schedule Update (APSU): Notice that DL IPA JPCS involves significant backhaul signalling. This is because power update in any cell results in interference price change in all the cells, and these new interference prices have to be exchanged via backhaul. We now describe a sub-optimal algorithm with reduced backhaul signalling. In it, the power and user schedule are optimized in an alternating

Algorithm 1: DL IPA JPCS

1 Initialize with a feasible transmit power vector and user schedule. Compute the interference prices of scheduled users for the current power profile, and convey it to other cells via backhaul.
2 Select a cell; Update its user schedule and transmit power using equations (23) and (24), respectively. The interference prices of the scheduled users are recomputed using (16) for the new power profile. These are then conveyed to other cells via backhaul.
3 Move to another cell, and repeat step 2.
4 Repeat step 3 until convergence.

manner. Given the user schedule, the optimization problem in $\mathbf{P}1$ reduces to that of optimizing the transmit powers alone. We employ the simultaneous power update algorithm proposed in [14] for it. With simultaneous power update, all the BSs update their transmit powers at the same time. The interference prices are then updated and are exchanged. Thus, the backhaul signalling is reduced. Note, however, that this algorithm does not have any convergence guarantees.

The steps of the algorithm are given below.

Algorithm 2: APSU

1 Initialize with a feasible transmit power vector and user schedule. Compute the interference prices of the scheduled users for the current power profile, and convey it to other cells via backhaul.
2 For the current user schedule and the interference prices, update the transmit power of all the BSs using (22).
3 For the new power profile, update the user schedule in each cell by selecting the user with the highest weighted rate (PF scheduling). Compute the interference prices of the scheduled users using (16), and exchange them via backhaul.
4 Repeat steps 2 and 3 until convergence or a maximum number of iterations is reached.

III. DL IPA for Joint Power Control, Scheduling, and User Association

In this section, we focus on jointly optimizing the transmit powers, user schedule, and user association to maximize the WSR. Here, the user association is not fixed, and a user can dynamically switch from one cell to another. Thus, performance improvements are possible through BS selection-diversity gains and dynamic user load balancing benefits [20].

A. Problem Formulation

As before, let K and N denote the number of users and BSs in the network, respectively. [2] Let G_{kn} denote the channel gain of user k from BS n and w_k denote the weight of user k. Further, let x_{kn} denote the binary variable indicating the association and scheduling decision for user k. That is, user k is scheduled in cell n if x_{kn} is equal to 1. Then, the WSR maximization problem [2] is given by

$$\mathbf{P}3: \quad \max \sum_{n=1}^{N}\sum_{k=1}^{K} x_{kn} w_k \log\left(1+\gamma_{kn}\right) \tag{25}$$

$$\gamma_{kn} = \frac{G_{kn}P_n}{\sigma^2 + \sum_{m=1, m\neq n}^{N} G_{km}P_m} \tag{26}$$

$$\sum_{k=1}^{K} x_{kn} = 1, \quad n = 1,\ldots,N \tag{27}$$

$$\sum_{n=1}^{N} x_{kn} \leq 1, \quad k = 1,\ldots,K \tag{28}$$

$$x_{kn} \in \{0,1\}, \forall k, n, \tag{29}$$

$$0 \leq P_n \leq P_{\max} \quad n = 1,\ldots,N. \tag{30}$$

Notice that (27) mandates that exactly one user is scheduled in each cell, and (28) constrains that user k is served by at most one BS. As in the previous section, we can relax the binary restriction on x_{kn} in (29) without affecting the optimum. Thus, the constraint (29) is replaced by $0 \leq x_{kn} \leq 1$.

B. Algorithms

As before, we first write down the KKT conditions for the optimization problem $\mathbf{P}3$ after relaxing the binary restriction of x_{kn}. They are given by

$$\sum_{n=1}^{N}\sum_{k=1}^{K} x_{kn}\frac{d}{dP_i} w_k \log\left(1+\gamma_{kn}\right) + \rho_i - \eta_i = 0, \tag{31}$$

$$w_k \log\left(1+\gamma_{ki}\right) + \lambda_{ki} - \mu_{ki} + \omega_k - \sigma_i = 0, \tag{32}$$

$$\lambda_{ki} x_{ki} = 0, \tag{33}$$

$$\mu_{ki}\left(x_{ki} - 1\right) = 0, \tag{34}$$

$$\omega_k \left(\sum_{i=1}^{N} x_{ki} - 1\right) = 0, \tag{35}$$

$$\rho_i P_i = 0, \tag{36}$$

$$\eta_i \left(P_i - P_{\max}\right) = 0, \tag{37}$$

$$\lambda_{ki}, \mu_{ki}, \omega_k, \rho_i, \eta_i \geq 0, \tag{38}$$

$$k = 1,\ldots,K,\, i = 1,\ldots,N,$$

where $\lambda_{k_i}, \mu_{k_i}, \omega_k, \sigma_i, \rho_i, \eta_i$ are the Lagrangian multipliers corresponding to the inequality constraints and σ_i is the Lagrangian multiplier for the equality constraint in (27).

Let us define the interference price of user k when associated to cell n as

$$\pi_{kn} = -\frac{d}{dI_{kn}} w_k \log\left(1+\gamma_{kn}\right) \tag{39}$$

$$= w_k \frac{\gamma_{kn}}{1+\gamma_{kn}} \frac{1}{\sigma^2 + I_{kn}}, \tag{40}$$

where $I_{kn} = \sum_{m=1, m\neq n}^{N} G_{km}P_m$ is the interference seen by user k when associated to cell n. As before, we rewrite (31) using the interference prices. This together with the rest

[2] We assume that K is greater than or equal to N.

of the KKT conditions are the necessary conditions for the following optimization problem, assuming fixed interference prices, transmit powers, and user schedule in other cells.

$$\mathbf{P}4: \quad \max \sum_{k=1}^{K} x_{ki} w_k \log\left(1+\gamma_{ki}\right) - P_i C_i, \tag{41}$$

$$\text{s.t.} \sum_{k=1}^{K} x_{ki} = 1, \tag{42}$$

$$x_{ki} \leq 1 - \sum_{n \neq i} x_{kn}, \tag{43}$$

$$0 \leq x_{ki} \leq 1, \quad k = 1, \ldots, K, \tag{44}$$

$$0 \leq P_i \leq P_{\max}, \tag{45}$$

where $C_i = \sum_{n \neq i} \sum_{l=1}^{K} x_{ln} \pi_{ln} G_{li}$ is the interference penalty per unit power at BS i. Observe that the optimization problem above is to maximize the weighted rate of cell i (its utility) minus the penalty for the interference it causes to other cells. Further note that the constraint (43) implies that only users that are not scheduled in any other cell are eligible for scheduling in cell i.

We can obtain closed-form solutions for the above optimization problem as given below. Let P_i^* and $k^*(i)$ denote the optimal transmit power and scheduled user in cell i. The optimal power for BS i if user k is scheduled in it, denoted by P_{ki}^*, is given by

$$P_{ki}^* = \left[\frac{w_k}{C_i} - \frac{\sigma^2 + I_{ki}}{G_{ki}}\right]_0^{P_{\max}}. \tag{46}$$

The optimal user to schedule is determined as follows. For each eligible user, we evaluate the objective function when BS i is transmitting at the optimal power corresponding to that user. The optimal user is the user that has the highest objective function value. Let γ_{ki}^* denote the SINR of user k when served by BS i with power P_{ki}^*. Then, the scheduled user in cell i is given by

$$k^*(i) = \underset{k \in \{1,\ldots,K\}}{\arg\max} \left[\left(1 - \sum_{n=1, n \neq i}^{N} x_{kn}\right) \times \left(w_k \log(1+\gamma_{ki}^*) - P_{ki}^* C_i\right)\right]. \tag{47}$$

Finally, the optimal transmit power of BS i is the optimal power for the scheduled user. That is,

$$P_i^* = P_{k^*(i)}^*. \tag{48}$$

Notice that given the scheduled users in other cells and their interference prices, the optimization problem $\mathbf{P}4$ can be solved locally at BS i. This motivates the following iterative algorithm for solving the optimization problem in $\mathbf{P}3$.

1) DL IPA for Joint Power Control, Scheduling, and Association (DL IPA JPCSA): The steps of the DL IPA for joint optimization of transmit powers, user scheduling, and association are given in Algorithm 3.

Algorithm 3: DL IPA JPCSA

1. Initialize with a feasible transmit power vector and user schedule. Every cell computes the interference price of its scheduled user, which is then conveyed to other cells along with the index of the scheduled user.
2. Select a cell; Recompute its user schedule and transmit power using equations (47) and (48), respectively. The interference prices of all the scheduled users are updated for the new power profile. Cells exchange the user schedule and interference prices via backhaul.
3. Move to another cell, and repeat step 2.
4. Repeat step 3 until convergence.

The convergence of the algorithm can be proved as follows. We show that the WSR is non-decreasing in the number of iterations as in Appendix B. This together with the fact that the WSR of the network is bounded, guarantees convergence.

We note that any arbitrary set of users can be used as the initial user schedule. In each iteration of the algorithm, a cell recomputes its user schedule by selecting the user which has the highest objective function among the users that are not scheduled in any other cell, as given in (47). Suppose that the user schedule of a cell is updated in an iteration. Then, the previously selected user for scheduling in that cell is now eligible for scheduling in other cells. Thus, the algorithm is capable of handling ‘bad’ initial schedules. Eventually, the algorithm pairs users with BSs such that the WSR is maximized.

2) Sub-optimal Algorithms: In the following sub-optimal algorithms, the user association is first fixed. Subsequently, the transmit powers and user schedule are optimized. We employ the Max-SINR criterion for association. That is, a user associates to the cell with the highest SINR among the candidate cells for association, assuming that all the BSs are transmitting at the maximum transmit power. Then, joint power and user scheduling optimization is carried out. We can either use DL IPA JPCS or the sub-optimal APSU for it.

Notice that for the DL IPA JPCSA described in Section III-B1 involves a computationally intensive search over all the users in the network to determine the optimum user to schedule. This is avoided in the sub-optimal algorithms as user association is determined beforehand. Further, significant reductions in backhaul signalling can be achieved by using the sub-optimal APSU algorithm for power and schedule optimization.

IV. Simulation Results

We consider a 7-cell hexagonal cellular layout with wrap-around. The cell radius $R = 1000$m. We drop 70 users randomly in the network area. Users experience lognormal shadowing in addition to Rayleigh fading. The shadowing standard deviation is 8 dB. The pathloss in dB at a distance d is given by $-L_0 - 10\eta \log_{10}(d/d_0)$. Here, η is the pathloss exponent and L_0 is the pathloss at the reference distance d_0, which is given by $L_0 = 20\log_{10}\left(\frac{4\pi d_0}{\lambda}\right)$, where λ is the wavelength. We set $\eta = 3.8$, $d_0 = 50$m, $\lambda = (1/6)$m, and $\frac{\sigma^2}{P_{\max}} = 4.0038812 \times 10^{-15}$.

The performance of an algorithm is evaluated in terms of its sum-utility, which is the sum of the utilities of the users in the network. Specifically, for the sum-logarithmic utility considered in this paper, we report the geometric mean (GM) of the users' throughput, which is an equivalent measure of sum-logarithmic utility. In addition, we also report the average and the fifth percentile (5-percentile) throughput of the users.

We benchmark the proposed algorithms against a baseline scheme that employ no coordination between the BSs (no CoMP). In this scheme, all the BSs are transmitting at full transmit power and PF scheduler is used in each cell. Further, the users associate to the nearest BS.

We first compare the proposed algorithms for joint power control and user scheduling against the baseline scheme. The simulation results are given in Table I. We limit the number of interference price updates in DL IPA JPCS and APSU to 70 and 10, respectively. The algorithm exits upon reaching this number.

TABLE I
COMPARISON OF JOINT POWER CONTROL AND SCHEDULING ALGORITHMS

Algorithm	GM (bits/s/Hz)	Average (bits/s/Hz)	5-percentile (bits/s/Hz))
No CoMP	0.6844	0.8075	0.3059
DL IPA JPCS	0.7765	0.8664	0.4324
Sub-optimal APSU	0.7693	0.8684	0.3929

Note that the DL IPA JPCS improves the GM by 13% over the baseline scheme. This increment translates to 7% and 41% improvements in the average and 5-percentile throughput, respectively. Further, note that the sub-optimal APSU has performance close to DL IPA JPCS. This makes APSU appealing as it enables significant savings in the backhaul signalling, while retaining almost all of the gains.

We now study the algorithms proposed for joint power control, scheduling, and association. The results are tabulated in Table II. The number of interference price updates in DL IPA JPCSA is set to 140.

TABLE II
COMPARISON OF JOINT POWER CONTROL, SCHEDULING, AND ASSOCIATION ALGORITHMS

Algorithm	GM (bits/s/Hz)	Average (bits/s/Hz)	5-percentile (bits/s/Hz))
No CoMP	0.6844	0.8075	0.3059
DL IPA JPCSA	0.8177	0.8842	0.5502
Max-SINR + DL IPA JPCS	0.8174	0.8841	0.5494
Max-SINR + APSU	0.8148	0.8815	0.5452
WMMSE	0.7691	0.8461	0.4865

Note that the DL IPA JPCSA improves the GM by 19% over the baseline scheme, which is significantly higher than the 12% improvement achieved by the WMMSE algorithm proposed in [9]. Interestingly, the performance of ad hoc SINR-based association algorithms is very close to that of the optimal DL IPA JPCSA. Thus, these sub-optimal algorithms are appealing, especially the variant that uses the APSU algorithm as it has lower backhaul signalling overhead. Finally, we note that these performance improvements are considerably higher than those of the joint power control and scheduling algorithms indicating that dynamic user association can significantly improve the performance of CoMP.

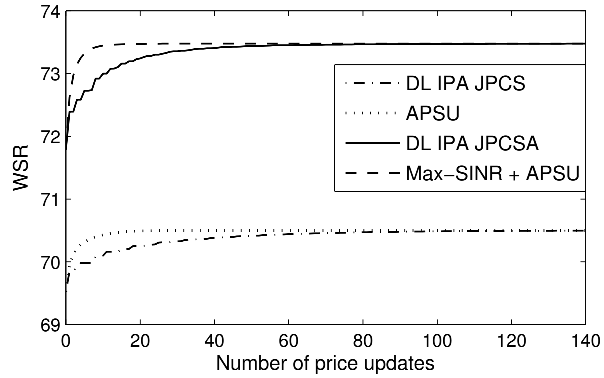

Fig. 1. WSR against the number of price updates for a typical channel realization with unit weight assigned to all the users.

Figure 1 plots the WSR against the number of price updates for a typical channel realization with user weights all set to unity. The curve for Max-SINR + DL IPA JPCS is seen to identical to DL IPA JPCSA and has been skipped to avoid clutter. The sub-optimal APSU algorithms is seen to converge, although it is not always guaranteed. Further, we see that the number of price updates required for them to converge (~ 10) is significantly lower than their DL IPA counterparts (~ 70).

V. CONCLUSIONS

We presented a novel, iterative, interference penalty algorithm to maximize the WSR of a cellular network by jointly optimizing the transmit powers, user schedule, and user association. We also developed sub-optimal algorithms, which have significantly lower computational and backhaul overhead, with very little loss in performance. In doing so, we developed an interference penalty algorithm to jointly optimize the transmit powers and user scheduling, for a fixed user association. We observed that the proposed algorithms outperform the WMMSE algorithm proposed in the literature.

An avenue for future work is to explore ways to extend the proposed algorithms to multiple-antenna networks. Another interesting future direction is to modify the algorithms to handle dynamic spectrum allocation in which multiple links are served simultaneously using orthogonal spectrum. We have not considered the issue of backhaul latency due to which it is possible that the algorithm may not converge within the coherence time of the channel. Further studies are required to evaluate the impact of backhaul latency on the performance of the proposed algorithms.

APPENDIX

A. Relaxing the Optimization Problem

Consider the optimization problem in (3) with the binary constraint in (6) replaced by $0 \leq x_{k_n} \leq 1$. Let the vectors $\mathbf{x}^*$ and $\mathbf{P}^*$ denote the optimal user schedule and transmit powers, respectively. Without loss of generality, assume that users s_n and t_n are scheduled in cell n. That is, $0 < x_{s_n}, x_{t_n} < 1$ and $x_{s_n} + x_{t_n} = 1$. Their weighted rates are $w_{s_n} \log(1+\gamma^*_{s_n})$ and $w_{t_n} \log(1+\gamma^*_{t_n})$, respectively. Here $\gamma^*_{s_n}$ and $\gamma^*_{t_n}$ are their respective SINRs corresponding to the transmit power vector $\mathbf{P}^*$. Note that WSR can be improved by setting $x_k = 1$ for $k = \underset{k\in\{s_n,t_n\}}{\arg\max}\, w_k \log(1+\gamma^*_k)$. That is, we schedule the user with the higher weighted rate among the two. This contradicts the optimality assumption of $\mathbf{x}^*$. Thus, the optimizer $\mathbf{x}^*$ must be binary vector.

B. Convergence of DL IPA JPCS

Let $U_{k_n}(\cdot)$ denote the weighted rate of user k_n in cell n, i.e., $U_{k_n}(\gamma_{k_n}) = w_{k_n} \log(1+\gamma_{k_n})$. It can be shown that

$$\frac{d^2 U(\gamma_{k_n})}{dI^2_{k_n}} \geq 0, \tag{49}$$

which means that $U_{k_n}(\cdot)$ is a convex function of I_{k_n} assuming that all other parameters are fixed. Therefore,

$$U(\gamma_{k_n}) \geq U(\gamma^o_{k_n}) + \left.\frac{dU(\gamma_{k_n})}{dI_{k_n}}\right|_{\mathbf{P}^o} \left(I_{k_n} - I^o_{k_n}\right) \tag{50}$$

$$= U(\gamma^o_{k_n}) - \pi_{k_n}\left(I_{k_n} - I^o_{k_n}\right), \tag{51}$$

where $\gamma^o_{k_n}$ and $I^o_{k_n}$ are the SINR and interference of user k_n for the current power profile $\mathbf{P}^o$.

Now suppose cell i updates its power and user schedule by solving problem $\mathbf{P}2$ given the current power profile $\mathbf{P}^o$. After the update, we have

$$\sum_{k_i\in\mathcal{K}_i} x_{k_i} U_{k_i}(\gamma_{k_i}) - P_i C_i \geq \sum_{k_i\in\mathcal{K}_i} x^o_{k_i} U_{k_i}(\gamma^0_{k_i}) - P^o_i C_i, \tag{52}$$

where $C_i = \sum_{n=1,n\neq i}^N \sum_{k_n\in\mathcal{K}_n} x^o_{k_n} \pi_{k_n} G_{k_n i}$. Here, $x^o_{k_n}$ is the scheduling indicator for user k_n in cell n. Note that it is unchanged during the update in cell i. Further, note that

$$\left(P_i - P^o_i\right) C_i = \sum_{\substack{n=1\\n\neq i}}^N \sum_{k_n\in\mathcal{K}_n} x^o_{k_n} \pi_{k_n} \left(I_{k_n} - I^o_{k_n}\right). \tag{53}$$

From (51), we get

$$\left(P_i - P^o_i\right) C_i \geq \sum_{\substack{n=1\\n\neq i}}^N \sum_{k_n\in\mathcal{K}_n} x^o_{k_n} \pi_{k_n} \left(U(\gamma^o_{k_n}) - U(\gamma_{k_n})\right). \tag{54}$$

Substituting (54) in (52) and rearranging the terms, we get

$$\sum_{n=1}^N \sum_{k_n\in\mathcal{K}_n} x_{k_n} U(\gamma_{k_n}) \geq \sum_{n=1}^N \sum_{k_n\in\mathcal{K}_n} x^o_{k_n} U(\gamma^o_{k_n}). \tag{55}$$

Thus, the WSR of the network in non-decreasing in each iteration. Since the WSR of the network is bounded, the iterative algorithm is guaranteed to converge.

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