Power Allocation Games on Interference Channels with Complete and Partial Information

Krishna Chaitanya A, Utpal Mukherji, and Vinod Sharma
Department of ECE, Indian Institute of Science, Bangalore-560012
Email: {akc, utpal, vinod} @ece.iisc.ernet.in

***Abstract*—We consider a Gaussian interference channel with independent direct and cross link channel gains, each of which is independent and identically distributed across time. Each transmitter-receiver user pair aims to maximize its long-term average transmission rate subject to an average power constraint. We formulate a stochastic game for this system in three different scenarios. First, we assume that each user knows all direct and cross link channel gains. Later, we assume that each user knows channel gains of only the links that are incident on its receiver. Lastly, we assume that each user knows only its own direct link channel gain. In all cases, we formulate the problem of finding the Nash equilibrium as a variational inequality (VI) problem and present a novel heuristic for solving the VI. We also present a lower bound on the utility for each user at any Nash equilibrium in the case of the games with partial information. We obtain this lower bound using a water-filling like power allocation that requires only knowledge of the distribution of a user's own channel gains and average power constraints of all users.**

Keywords*—*Interference channel, stochastic game, Nash equilibrium, distributed algorithms, variational inequality.

I. Introduction

We consider a wireless channel which is being shared by multiple users to transmit their data to their respective receivers. The transmissions of different users may cause interference to other receivers. This is a typical scenario in many wireless networks. In particular, this can represent inter-cell interference on a particular wireless channel in a cellular network. The different users want to maximize their transmission rates. This system can be modeled in the game theoretic framework and has been widely studied [1] - [6].

Parallel Gaussian interference channels are considered in [1], [3], and [4] when the channel gains are fixed and known to all users. In [1], this setup is modeled as a strategic form game and existence and uniqueness of a Nash equilibrium (NE) is studied. Conditions under which the water-filling function is a contraction, and thus conditions for uniqueness of NE and for convergence of iterative water-filling, are provided. These results are extended to a multi-antenna system in [5], and an asynchronous version of iterative water-filling is considered in [6]. In [4], some conditions are described under which parallel Gaussian interference channels have multiple Nash equilibria. Using variational inequalities, an algorithm is presented that converges to a Nash equilibrium which minimizes the overall weighted interference.

An online algorithm to reach a NE for parallel Gaussian channels is presented in [2] when the channel gains are fixed but not known to the users. Its convergence is also proved.

In [7], parallel Gaussian interference channels are considered, where each user minimizes total power across the parallel channels subject to a lower bound on signal to interference plus noise ratio. The channel gains are fixed and known to all users. This setup is modeled as a strategic form game and decentralized algorithms are presented based on trial and error that converge to Nash and satisfaction equilibrium points under certain sufficient conditions. In [8], under a very similar setup, sequential and simultaneous iterative water-filling algorithms are presented to find a NE. Further, sufficient conditions for convergence of these algorithms and uniqueness of the NE are studied.

In [9] we consider a Gaussian interference channel with fast fading channel gains whose distributions are known to all the users. We consider power allocation in a non-game-theoretic framework, and provide other references for such a setup. In [9], we have proposed a centralized algorithm for finding the Pareto points that maximize the average sum rate, when the receivers have knowledge of all the channel gains and decode the messages from strong and very strong interferers instead of treating them as noise.

In the present paper, we consider a stochastic game over Gaussian interference channels, where users want to maximize their long term average rates and have long term average power constraints (potential advantages of this over one shot optimization are discussed in [10], [11]). For this system, we obtain existence of a NE and develop algorithms to obtain a NE via variational inequalities. In [12], we had considered a power allocation game with complete channel knowledge at all transmitters and presented an algorithm that converges to a Nash equilibrium under certain conditions on the channel gains. In this paper also, we first consider the complete information game of [12]. We provide a heuristic algorithm to find a NE under general channel conditions. A NE is a fixed point of a given mapping defined later in the paper. We write the problem of finding a fixed point of the mapping as an optimization problem whose global optimal solution is a fixed point. The existing distributed optimization algorithms are known to converge only to local optima, whereas our heuristic algorithm aims to find a global optimum. For this, the heuristic algorithm is split into two phases. In Phase 1, a Picard iteration is applied on the mapping for a fixed number of iterations. In Phase 2, a steepest descent algorithm initialized with the power allocation resulting from Phase 1 is applied to solve for a global optimal solution of the optimization problem.

978-3-9018-8273-9/15/ ©2015 IFIP

Later, in Section VI, we illustrate the advantage that Phase 1 provides in solving for the global optimum. We also note that, in some cases, Phase 1 itself can provide a close approximation of the global optimum.

Furthermore, we also consider the much more realistic situation when a user knows only its own channel gains, whereas the above mentioned literature considers the problem when each user knows all the channel gains in the system. We consider two different partial information games. In the first partial information game, each transmitter is assumed to have knowledge of the channel gains of the links that are incident on its corresponding receiver from all the transmitters. Later, in the other game, we assume that each transmitter has knowledge of its direct link channel gain only. For both the partial information games, we find a NE using the heuristic algorithm.

Finally, in each partial information game, we present a lower bound on the average rate of each user at any Nash equilibrium. This lower bound can be obtained by a user using a water-filling like, easy to find power allocation, that can be evaluated with the knowledge of the distribution of its own channel gains and of the average power constraints of all the users. The distributed heuristic algorithm requires power variables to be communicated among the users during the computation of the Nash equilibrium. If suppose, a transmitter fails to receive power variables from the other transmitters, it can still attain at least the lower bound rate with its water-filling like power allocation.

The paper is organized as follows. In Section II, we present the system model and the three stochastic game formulations. Section III reformulates the complete information stochastic game as an affine variational inequality problem. In Section IV, we propose the heuristic algorithm to solve the formulated variational inequality under general conditions. In Section V we use this algorithm to obtain NE when users have only partial information about the channel gains. We present numerical examples in Section VI. Section VII concludes the paper.

II. System model and Stochastic Game Formulations

We consider a Gaussian wireless channel being shared by N transmitter-receiver pairs. The time axis is slotted and all users' slots are synchronized. The channel gains of each transmit-receive pair are constant during a slot and change independently from slot to slot. These assumptions are usually made for this system [1], [11]. Although not addressed in this paper, our results extend to positive recurrent ergodic Markovian channel state processes.

Let $H_{ij}(k)$ be the random variable that represents channel gain from transmitter j to receiver i (for transmitter i, receiver i is the intended receiver) in slot k. The direct channel power gains $|H_{ii}(k)|^2 \in \mathcal{H}_d = \{g_1^{(d)}, g_2^{(d)}, \ldots, g_{n_1}^{(d)}\}$ and the cross channel power gains $|H_{ij}(k)|^2 \in \mathcal{H}_c = \{g_1^{(c)}, g_2^{(c)}, \ldots, g_{n_2}^{(c)}\}$ where n_1, and n_2 are arbitrary positive integers. Let π_d and π_c be the probability distributions on $\mathcal{H}_d$ and $\mathcal{H}_c$ respectively. We assume that, $\{H_{ij}(k), k \geq 0\}$ is an $i.i.d$ sequence with distribution π_{ij} where $\pi_{ij} = \pi_d$ if $i = j$ and $\pi_{ij} = \pi_c$ if $i \neq j$. We also assume that these sequences are independent of each other.

We denote $(H_{ij}(k), i, j = 1, \ldots, N)$ by $\mathbf{H}(k)$ and its realization vector by $h(k)$ which takes values in $\mathcal{H}$, the set of all possible channel states. The distribution of $\mathbf{H}(k)$ is denoted by π. We call the channel gains $(H_{ij}(k), j = 1, \ldots, N)$ from all the transmitters to the receiver i an incident gain of user i and denote by $\mathbf{H}_i(k)$ and its realization vector by $h_i(k)$ which takes values in $\mathcal{I}$, the set of all possible incident channel gains. The distribution of $\mathbf{H}_i(k)$ is denoted by π_I.

Each user aims to operate at a power allocation that maximizes its long term average rate under an average power constraint. Since their transmissions interfere with each other, affecting their transmission rates, we model this scenario as a stochastic game.

We first assume complete channel knowledge at all transmitters and receivers. If user i uses power $P_i(\mathbf{H}(k))$ in slot k, it gets rate $\log\left(1 + \Gamma_i\left(P\left(\mathbf{H}(k)\right)\right)\right)$, where

$$\Gamma_i(P(\mathbf{H}(k))) = \frac{\alpha_i |H_{ii}(k)|^2 P_i(\mathbf{H}(k))}{1 + \sum_{j \neq i} |H_{ij}(k)|^2 P_j(\mathbf{H}(k))}, \quad (1)$$

$P(\mathbf{H}(k)) = (P_1(\mathbf{H}(k)), \ldots, P_N(\mathbf{H}(k)))$ and α_i is a constant that depends on the modulation and coding used by transmitter i and we assume $\alpha_i = 1$ for all i. The aim of each user i is to choose a power policy to maximize its long term average rate

$$r_i(\mathbf{P}_i, \mathbf{P}_{-i}) \triangleq \limsup_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} \mathbb{E}[\log\left(1 + \Gamma_i\left(P\left(\mathbf{H}(k)\right)\right)\right)], \quad (2)$$

subject to average power constraint

$$\limsup_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} \mathbb{E}[P_i(\mathbf{H}(k))] \leq \overline{P}_i, \text{ for each } i, \quad (3)$$

where $\mathbf{P}_{-i}$ denotes the power policies of all users except i. We denote this game by $\mathcal{G}_A$.

We next assume that the ith transmitter-receiver pair has knowledge of its incident gains $\mathbf{H}_i$ only. Then the rate of user i is

$$r_i(\mathbf{P}_i, \mathbf{P}_{-i}) \triangleq \limsup_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} \mathbb{E}_{\mathbf{H}_i(k)} \left[\mathbb{E}_{\mathbf{H}_{-i}(k)}[\log\left(1 + \Gamma_i(P(\mathbf{H}_i(k), \mathbf{H}_{-i}(k)))\right)]\right], \quad (4)$$

where $P_i(\mathbf{H}(k))$ depends only on $\mathbf{H}_i(k)$ and $\mathbb{E}_X$ denotes expectation with respect to the distribution of X. Each user maximizes its rate subject to (3), we denote this game by $\mathcal{G}_I$.

We also consider a game assuming that each transmitter-receiver pair knows only its direct link gain H_{ii}. This is the most realistic assumption since each receiver i can estimate H_{ii} and feed it back to transmitter i. In this case, the rate of user i is given by

$$r_i(\mathbf{P}_i, \mathbf{P}_{-i}) \triangleq \limsup_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} \mathbb{E}_{\mathbf{H}_{ii}(k)} \left[\mathbb{E}_{\mathbf{H}_{-ii}(k)} [\log\left(1 + \Gamma_i(P(H_{ii}(k), H_{-ii}(k)))\right)]\right], \quad (5)$$

where $P_i(\mathbf{H}(k))$ is a function of $H_{ii}(k)$ only. Here, H_{-ii} denotes the channel gains of all other links in the interference channel except H_{ii}. In this game, each user maximizes its rate (5) under the average power constraint (3). We denote this game by $\mathcal{G}_D$.

We address these problems as stochastic games with the set of feasible power policies of user i denoted by $\mathcal{A}_i$ and its utility by r_i. Let $\mathcal{A} = \Pi_{i=1}^N \mathcal{A}_i$.

We limit ourselves to stationary policies, i.e., the power policy for every user in slot k depends only on the channel state $H(k)$ and not on k. In fact now we can rewrite the optimization problem in $\mathcal{G}$ to find policy $P(\mathbf{H})$ such that $r_i = \mathbb{E}_{\mathbf{H}}[\log(1 + \Gamma_i(P(\mathbf{H})))]$ is maximized subject to $\mathbb{E}_{\mathbf{H}}[P_i(\mathbf{H})] \leq \overline{P}_i$ for all i. Similarly, we can rewrite the optimization problems in games $\mathcal{G}_I$ and $\mathcal{G}_D$. We express power policy of user i by $\mathbf{P}_i = (P_i(h), h \in \mathcal{H})$, where transmitter i transmits in channel state h with power $P_i(h)$. We denote the power profile of all users by $\mathbf{P} = (\mathbf{P}_1, \ldots, \mathbf{P}_N)$.

In the rest of the paper, we prove existence of a Nash equilibrium for each of these games and provide a heuristic algorithm to compute it.

III. Variational Inequality Formulation

The theory of variational inequalities offers various algorithms to find NE of a given game [16]. A variational inequality problem denoted by $VI(K, F)$ is defined as follows.

Definition 1. *Let $K \subset \mathbb{R}^n$ be a closed and convex set, and $F : K \to K$. The variational inequality problem $VI(K, F)$ is defined as the problem of finding $x \in K$ such that*

$$F(x)^T(y - x) \geq 0 \textit{ for all } y \in K.$$

We reformulate the Nash equilibrium problem at hand to an affine variational inequality problem. We denote our game by $\mathcal{G}_A = \left((\mathcal{A}_i)_{i=1}^N, (r_i)_{i=1}^N\right)$, where $r_i(\mathbf{P}_i, \mathbf{P}_{-i}) = \mathbb{E}_{\mathbf{H}}[\log(1 + \Gamma_i(P(\mathbf{H})))]$ and $\mathcal{A}_i = \{\mathbf{P}_i \in \mathbb{R}^N : \mathbb{E}_{\mathbf{H}}[P_i(\mathbf{H})] \leq \overline{P}_i, P_i(h) \geq 0 \text{ for all } h \in \mathcal{H}\}$.

Definition 2. *A point $\mathbf{P}^*$ is a Nash Equilibrium (NE) of game $\mathcal{G}_A = \left((\mathcal{A}_i)_{i=1}^N, (r_i)_{i=1}^N\right)$ if for each user i*

$$r_i(\mathbf{P}_i^*, \mathbf{P}_{-i}^*) \geq r_i(\mathbf{P}_i, \mathbf{P}_{-i}^*) \textit{ for all } \mathbf{P}_i \in \mathcal{A}_i.$$

We now state a theorem on the existence of a pure strategy NE for a non-cooperative game known as Debreu-Glicksberg-Fan theorem ([13], page no. 69).

Theorem 1. *Given a non-cooperative game, if every strategy set $\mathcal{A}_i$ is compact and convex, $r_i(a_i, a_{-i})$ is a continuous function in the profile of strategies $\mathbf{a} = (a_i, a_{-i}) \in \mathcal{A}$ and quasi-concave in a_i, then the game has atleast one pure-strategy Nash equilibrium.* □

Existence of a pure NE for the strategic games $\mathcal{G}_A, \mathcal{G}_I$ and $\mathcal{G}_D$ follows from Theorem 1, since in our game $r_i(\mathbf{P}_i, \mathbf{P}_{-i})$ is a continuous function in the profile of strategies $\mathbf{P} = (\mathbf{P}_i, \mathbf{P}_{-i}) \in \mathcal{A}$ and concave in $\mathbf{P}_i$ for $\mathcal{G}_A, \mathcal{G}_I$ and $\mathcal{G}_D$.

Definition 3. *The best-response of user i is a function $BR_i : \mathcal{A}_{-i} \to \mathcal{A}_i$ such that $BR_i(\mathbf{P}_{-i})$ maximizes $r_i(\mathbf{P}_i, \mathbf{P}_{-i})$, subject to $\mathbf{P}_i \in \mathcal{A}_i$.*

We see that the Nash equilibrium is a fixed point of the best-response function. In the following we provide algorithms to obtain this fixed point for $\mathcal{G}_A$. In Section V we will consider $\mathcal{G}_I$ and $\mathcal{G}_D$. Given other users' power profile $\mathbf{P}_{-i}$, we use Lagrange method to evaluate the best response of user i. The Lagrangian function is defined by

$$\mathcal{L}_i(\mathbf{P}_i, \mathbf{P}_{-i}) = r_i(\mathbf{P}_i, \mathbf{P}_{-i}) + \mu_i(\overline{P}_i - \mathbb{E}_{\mathbf{H}}[P_i(\mathbf{H})]).$$

To maximize $\mathcal{L}_i(\mathbf{P}_i, \mathbf{P}_{-i})$, we solve for $\mathbf{P}_i$ such that $\frac{\partial \mathcal{L}_i}{\partial \mathbf{P}_i(h)} = 0$ for each $h \in \mathcal{H}$. Thus, the component of the best response of user i, $\mathbf{BR}_i(\mathbf{P}_{-i})$ corresponding to channel state h is given by

$$BR_i(\mathbf{P}_{-i}; h) = \max\left\{0, \lambda_i(\mathbf{P}_{-i}) - \frac{(1 + \sum_{j \neq i} |h_{ij}|^2 P_j(h))}{|h_{ii}|^2}\right\}, \quad (6)$$

where $\lambda_i(\mathbf{P}_{-i}) = \frac{1}{\mu_i(\mathbf{P}_{-i})}$ is chosen such that the average power constraint is satisfied.

It is easy to observe that the best-response of user i to a given strategy of other users is water-filling on $\mathbf{f}_i(\mathbf{P}_{-i}) = (f_i(\mathbf{P}_{-i}; h), h \in \mathcal{H})$ where

$$f_i(\mathbf{P}_{-i}; h) = \frac{(1 + \sum_{j \neq i} |h_{ij}|^2 P_j(h))}{|h_{ii}|^2}. \quad (7)$$

For this reason, we represent the best-response of user i by $\mathbf{WF}_i(\mathbf{P}_{-i})$. The notation used for the overall best-response $\mathbf{WF}(\mathbf{P}) = (\mathbf{WF}(P(h)), h \in \mathcal{H})$, where $\mathbf{WF}(P(h)) = (WF_1(\mathbf{P}_{-1}; h), \ldots, WF_N(\mathbf{P}_{-N}; h))$ and $WF_i(\mathbf{P}_{-i}; h)$ is as defined in (6). We use $\mathbf{WF}_i(\mathbf{P}_{-i}) = (WF_i(\mathbf{P}_{-i}; h), h \in \mathcal{H})$.

It is observed in [1] that the best-response $\mathbf{WF}_i(\mathbf{P}_{-i})$ is also the solution of the optimization problem

$$\begin{aligned} \text{minimize} \quad & \|\mathbf{P}_i + \mathbf{f}_i(\mathbf{P}_{-i})\|^2, \\ \text{subject to} \quad & \mathbf{P}_i \in \mathcal{A}_i. \end{aligned} \quad (8)$$

As a result we can interpret the best-response as the projection of $(-f_{i,1}(\mathbf{P}_{-i}), \ldots, -f_{i,N}(\mathbf{P}_{-i}))$ on to $\mathcal{A}_i$. We denote the projection of x on to $\mathcal{A}_i$ by $\Pi_{\mathcal{A}_i}(x)$. We consider (8), as a game in which every user minimizes its cost function $\|\mathbf{P}_i + \mathbf{f}_i(\mathbf{P}_{-i})\|^2$ with strategy set of user i being $\mathcal{A}_i$. We denote this game by $\mathcal{G}'_A$. This game has the same set of NEs as $\mathcal{G}_A$ because the best responses of these two games are equal. We now formulate the variational inequality problem corresponding to the game $\mathcal{G}'_A$.

We note that (8) is a convex optimization problem. Necessary and sufficient optimality conditions for a convex optimization problem ([14], page 210) can be stated as

Theorem 2. *Consider the convex optimization problem,*

$$\begin{aligned} \textit{minimize} \quad & g(x), \\ \textit{subject to} \quad & x \in X, \end{aligned}$$

where $g(x)$ is a convex function and X is a convex set. The necessary and sufficient conditions for x^* to be a solution of *the optimization problem are*

$$\nabla g(x^*)(y - x^*) \geq 0 \text{ for all } y \in X. \quad \square$$

Given $\mathbf{P}_{-i}$, a necessary and sufficient condition for $\mathbf{P}_i^*$ to be a solution of the convex optimization problem of user i is given by

$$\sum_{h \in \mathcal{H}} \left(P_i^*(h) + f_i(\mathbf{P}_{-i}; h)\right)\left(x_i(h) - P_i^*(h)\right) \geq 0, \quad (9)$$

for all $\mathbf{x}_i \in \mathcal{A}_i$. Thus, $\mathbf{P}^*$ is a NE of the game $\mathcal{G}'_A$ if (9) holds for each user i. We can rewrite the N inequalities in (9) in compact form as

$$\left(\mathbf{P}^* + \hat{h} + \hat{H}\mathbf{P}^*\right)^T (x - \mathbf{P}^*) \geq 0 \text{ for all } x \in \mathcal{A}, \quad (10)$$

where $\hat{h}$ is a N_1-length block vector with $N_1 = |\mathcal{H}|$, the cardinality of $\mathcal{H}$, each block $\hat{h}(h), h \in \mathcal{H}$, is of length N and is defined by $\hat{h}(h) = \left(\frac{1}{|h_{11}|^2}, \ldots, \frac{1}{|h_{NN}|^2}\right)$ and $\hat{H}$ is the block diagonal matrix $\hat{H} = \text{diag}\left\{\hat{H}(h), h \in \mathcal{H}\right\}$ with each block $\hat{H}(h)$ defined by

$$[\hat{H}(h)]_{ij} = \begin{cases} 0 & \text{if } i = j, \\ \frac{|h_{ij}|^2}{|h_{ii}|^2}, & \text{else.} \end{cases}$$

The characterization of Nash equilibrium in (10) corresponds to solving for $\mathbf{P}$ in the affine variational inequality problem $VI(\mathcal{A}, F)$,

$$F(\mathbf{P})^T (x - \mathbf{P}) \geq 0 \text{ for all } x \in \mathcal{A}, \quad (11)$$

where $F(\mathbf{P}) = (I + \hat{H})\mathbf{P} + \hat{h}$.

IV. Algorithm to Solve Variational Inequality

In [12], we proved that if $\tilde{H} = (I + \hat{H})$ is positive semidefinite, then the fixed point iteration

$$\mathbf{P}^{(n)} = \Pi_{\mathcal{A}}(\mathbf{P}^{(n-1)} - \tau F_\epsilon(\mathbf{P}^{(n-1)})), \quad (12)$$

converges to a solution of $VI(\mathcal{A}, F_\epsilon)$ for sufficiently small values of τ ([16]), where $F_\epsilon = (I + \hat{H})\mathbf{P} + \hat{h} + \epsilon\mathbf{P}$. As ϵ converges to zero the solution of $VI(\mathcal{A}, F_\epsilon)$ converges to a NE. This condition is much weaker than one would obtain by using the methods in [1]. In the current paper we aim to find a NE even if $\tilde{H}$ is not positive semidefinite. For this, we present a heuristic to solve the $VI(\mathcal{A}, F)$ in general.

We base our heuristic algorithm on the fact that a power allocation $\mathbf{P}^*$ is a solution of $VI(\mathcal{A}, F)$ if and only if

$$\mathbf{P}^* = \Pi_{\mathcal{A}}\left(\mathbf{P}^* - \tau F(\mathbf{P}^*)\right), \quad (13)$$

for any $\tau > 0$. We prove this fact using a property of projection on a convex set that can be stated as follows ([16]):

Lemma 3. *Let $X \subset \mathbb{R}^n$ be a convex set. The projection of $y \in \mathbb{R}^n$, $\Pi(y)$ is the unique element in X such that the following holds.*

$$(\Pi(y) - y)^T (x - \Pi(y)) \geq 0, \text{ for all } x \in X. \quad (14)$$

$\square$

Let $\mathbf{P}^*$ satisfy (13) for some $\tau > 0$. By the property of projection (14), we have

$$\left(\Pi_{\mathcal{A}}\left(\mathbf{P}^* - \tau F(\mathbf{P}^*)\right) - \left(\mathbf{P}^* - \tau F(\mathbf{P}^*)\right)\right)^T \left(\mathbf{Q} - \Pi_{\mathcal{A}}\left(\mathbf{P}^* - \tau F(\mathbf{P}^*)\right)\right) \geq 0 \quad (15)$$

for all $\mathbf{Q} \in \mathcal{A}$. Using (13) in (15), we have

$$\left(\mathbf{P}^* - \left(\mathbf{P}^* - \tau F(\mathbf{P}^*)\right)\right)^T (\mathbf{Q} - \mathbf{P}^*) \geq 0,$$

$$\text{i.e., } \left(\tau F(\mathbf{P}^*)\right)^T (\mathbf{Q} - \mathbf{P}^*) \geq 0.$$

Since $\tau > 0$, we have

$$F(\mathbf{P}^*)^T (\mathbf{Q} - \mathbf{P}^*) \geq 0, \text{ for all } \mathbf{Q} \in \mathcal{A}. \quad (16)$$

Thus $\mathbf{P}^*$ solves the $VI(\mathcal{A}, F)$. Conversely, let $\mathbf{P}^*$ be a solution of the $VI(\mathcal{A}, F)$. Then we have relation (16), which can be rewritten as

$$\left(\mathbf{P}^* - \left(\mathbf{P}^* - \tau F(\mathbf{P}^*)\right)\right)^T (\mathbf{Q} - \mathbf{P}^*) \geq 0, \text{ for all } \mathbf{Q} \in \mathcal{A},$$

for any $\tau > 0$. Comparing with (14), from Lemma 3 we have that (13) holds. Thus, $\mathbf{P}^*$ is a fixed point of the mapping $T(\mathbf{P}) = \Pi_{\mathcal{A}}\left(\mathbf{P} - \tau F(\mathbf{P})\right)$ for any $\tau > 0$.

We can interpret the mapping $T(\mathbf{P})$ as a better response mapping for the optimization (8). Consider a fixed point $\mathbf{P}^*$ of the better response $T(\mathbf{P})$. Then $\mathbf{P}^*$ is a solution of the variational inequality (11). This implies that, given $\mathbf{P}_{-i}^*$, $\mathbf{P}_i^*$ is a local optimum of (8) for all i. Since the optimization (8) is convex, $\mathbf{P}_i^*$ is also a global optimum. Thus given $\mathbf{P}_{-i}^*$, $\mathbf{P}_i^*$ is best response for all i, and hence a fixed point of the better response function $T(\mathbf{P})$ is also a NE.

To find a fixed point of $T(\mathbf{P})$, we reformulate the variational inequality problem as a non-convex optimization problem

$$\begin{aligned} &\text{minimize} \quad \|\mathbf{P} - \Pi_{\mathcal{A}}\left(\mathbf{P} - \tau F(\mathbf{P})\right)\|^2, \\ &\text{subject to} \quad \mathbf{P} \in \mathcal{A}. \end{aligned} \quad (17)$$

The feasible region $\mathcal{A}$ of $\mathbf{P}$, can be written as a Cartesian product of $\mathcal{A}_i$, for each i, as the constraints of each user are decoupled in power variables. As a result, we can split the projection $\Pi_{\mathcal{A}}(.)$ into multiple projections $\Pi_{\mathcal{A}_i}(.)$ for each i, i.e., $\Pi_{\mathcal{A}}(\mathbf{x}) = (\Pi_{\mathcal{A}_1}(\mathbf{x}_1), \ldots, \Pi_{\mathcal{A}_N}(\mathbf{x}_N))$. For each user i, the projection operation $\Pi_{\mathcal{A}_i}(\mathbf{x}_i)$ takes the form

$$\Pi_{\mathcal{A}_i}(\mathbf{x}_i) = \left(\max\left(0, x_i(h) - \lambda_i\right), h \in \mathcal{H}\right), \quad (18)$$

where λ_i is chosen such that the average power constraint is satisfied. Using (18), we rewrite the objective function in (17) with $\tau = 1$ as

$$\begin{aligned} &\|\mathbf{P} - \Pi_{\mathcal{A}}\left(\mathbf{P} - F(\mathbf{P})\right)\|^2 = \\ &\qquad \sum_{h \in \mathcal{H}, i} \left(P_i(h) - \max\left\{0, -f_i(\mathbf{P}_{-i}; h) - \lambda_i\right\}\right)^2 \\ &= \sum_{h \in \mathcal{H}, i} \left(\min\left\{P_i(h), \frac{1 + \sum_j |h_{ij}|^2 P_j(h)}{|h_{ii}|^2} + \lambda_i\right\}\right)^2 \\ &= \sum_{h \in \mathcal{H}, i} \left(\min\left\{P_i(h), P_i(h) + f_i(\mathbf{P}_{-i}; h) + \lambda_i\right\}\right)^2. \end{aligned} \quad (19)$$

At a NE, the left side of equation (19) is zero and hence each minimum term on the right side of the equation must be zero as well. This happens, only if

$$P_i(h) = \begin{cases} 0, \text{ if } \frac{1+\sum_{j\neq i}|h_{ij}|^2 P_j(h)}{|h_{ii}|^2} + \lambda_i > 0, \\ -\frac{1+\sum_{j\neq i}|h_{ij}|^2 P_j(h)}{|h_{ii}|^2} - \lambda_i, \text{ otherwise.} \end{cases}$$

Here, the Lagrange multiplier λ_i can be negative, as the projection satisfies the average power constraint with equality. At a NE user i will not transmit if the ratio of total interference plus noise to the direct link gain is more than some threshold.

We now propose a heuristic algorithm to find an optimizer of (17). This algorithm consists of two phases. Phase 1 attempts to find a better power allocation, using Picard iterations with the mapping $T(\mathbf{P})$, that is close to a NE. For $\mathcal{G}_A$ this is algorithm (12) with $\epsilon = 0$. We use Phase 1 in Algorithm 1 to get a good initial point for the steepest descent algorithm of Phase 2. We will show in Section VI that it indeed provides a good initial point for Phase 2. In Phase 2, using the estimate obtained from Phase 1 as the initial point, the algorithm runs the steepest descent method to find a NE. It is possible that the steepest descent algorithm may stop at a local minimum which is not a NE. This is because of the non-convex nature of the optimization problem. If the steepest descent method in Phase 2 terminates at a local minimum which is not a NE, we again invoke Phase 1 with this local minimum as the initial point and then go over to Phase 2. We present the complete algorithm below as Algorithm 1.

In Section VI we first present an example where $\tilde{H}$ is positive semidefinite. Next we provide examples where $\tilde{H}$ is not positive semidefinite, and the algorithm in [12] does not converge, whereas Algorithm 1 converges to the NEs in just a few iterations of Phase 1 and Phase 2.

Algorithm 1 Heuristic algorithm to find a Nash equilibrium

Fix $\epsilon > 0, \delta > 0$ and a positive integer MAX
Phase 1 : Initialization phase

Initialize $\mathbf{P}_i^{(0)}$ for all $i = 1, \ldots, N$.
for $n = 1 \rightarrow$ MAX **do**
  $\mathbf{P}^{(n)} = T(\mathbf{P}^{(n-1)})$
end for
go to Phase 2.

Phase 2 : Optimization phase

Initialize $t = 1, \mathbf{P}^{(t)} = \mathbf{P}^{(MAX)}$,
loop
  For each i, $\mathbf{P}_i^{(t+1)}$ = Steepest_Descent($\tilde{\mathbf{P}}_i^{(t)}, i$)
  where $\tilde{\mathbf{P}}_i^{(t)} = (\mathbf{P}_1^{(t+1)}, \ldots, \mathbf{P}_{i-1}^{(t+1)}, \mathbf{P}_i^{(t)}, \ldots, \mathbf{P}_N^{(t)})$,
  $\mathbf{P}^{(t+1)} = (\mathbf{P}_1^{(t+1)}, \ldots, \mathbf{P}_N^{(t+1)})$,
  Till $\|\mathbf{P}^{(t+1)} - T(\mathbf{P}^{(t+1)})\| < \epsilon$
  if $\|\mathbf{P}^{(t)} - \mathbf{P}^{(t+1)}\| < \delta$ and $\|\mathbf{P}^{(t+1)} - T(\mathbf{P}^{(t+1)})\| > \epsilon$ **then**
    Go to Phase 1 with $\mathbf{P}^{(0)} = \mathbf{P}^{(t+1)}$
  end if
  $t = t + 1$.
end loop
function STEEPEST_DESCENT($\mathbf{P}^{(t)}, i$)

  $\nabla f(\mathbf{P}^{(t)}) = (\frac{\partial f(\mathbf{P})}{\partial P_i(h)}|_{\mathbf{P}=\mathbf{P}^{(t)}}, h \in \mathcal{H})$
  where $f(\mathbf{P}) = \|\mathbf{P} - T(\mathbf{P})\|^2$
  for $h \in \mathcal{H}$ **do**
    evaluate $\frac{\partial f(\mathbf{P})}{\partial P_i(h)}|_{\mathbf{P}=\mathbf{P}^{(t)}}$ using derivative approximation
  end for
  $\mathbf{P}_i^{(t+1)} = \Pi_{\mathcal{A}_i}(\mathbf{P}_i^{(t)} - \gamma_t \nabla f(\mathbf{P}^{(t)}))$
  return $\mathbf{P}_i^{(t+1)}$
end function

V. Partial Information games

In the partial information games, we can not write the problem of finding a NE as an affine variational inequality, because the best response is not water-filling and should be evaluated numerically. In this section, we show that we can use Algorithm 1 to find a NE even for these games.

A. Game $\mathcal{G}_I$

We first consider the game $\mathcal{G}_I$ and find its NE using Algorithm 1. We follow on similar lines as in Sections III and IV. We write the variational inequality formulation of the NE problem. For user i, the optimization at hand is

$$\begin{aligned} &\text{maximize} && r_i^{(I)}, \\ &\text{subject to} && \mathbf{P}_i \in \mathcal{A}_i, \end{aligned} \tag{20}$$

where $r_i^{(I)} = \sum_{h_i \in \mathcal{I}} \pi(h_i) \mathbb{E}\left[\log\left(1 + \frac{|h_{ii}|^2 P_i(\mathbf{h}_i)}{1+\sum_{j\neq i}|h_{ij}|^2 P_j(H_j)}\right)\right]$. The necessary and sufficient optimality conditions for the convex optimization problem (20) are

$$(\mathbf{x}_i - \mathbf{P}_i^*)^T(-\nabla_i r_i^{(I)}(\mathbf{P}_i^*, \mathbf{P}_{-i})) \geq 0, \text{ for all } \mathbf{x}_i \in \mathcal{A}_i, \tag{21}$$

where $\nabla_i r_i^{(I)}(\mathbf{P}_i^*, \mathbf{P}_{-i})$ is the gradient of $r_i^{(I)}$ with respect to power variables of user i. Then $\mathbf{P}^*$ is a NE if and only if (21) is satisfied for all $i = 1, \ldots, N$. We can write the N inequalities in (21) as

$$(\mathbf{x} - \mathbf{P}^*)^T F(\mathbf{P}^*) \geq 0, \text{ for all } \mathbf{x} \in \mathcal{A}, \tag{22}$$

where $F(\mathbf{P}) = (-\nabla_1 r_1^{(I)}(\mathbf{P}), \ldots, -\nabla_N r_N^{(I)}(\mathbf{P}))^T$. Equation (22) is the required variational inequality characterization. A solution of the variational inequality is a fixed point of the mapping $T_I(\mathbf{P}) = \Pi_{\mathcal{A}}(\mathbf{P} - \tau F(\mathbf{P}))$, for any $\tau > 0$. We use Algorithm 1, to find a fixed point of $T_I(\mathbf{P})$ by replacing $T(\mathbf{P})$ in Algorithm 1 with $T_I(\mathbf{P})$.

B. Better response iteration

In this subsection, we interpret $T_I(\mathbf{P})$ as a better response for each user. For this, consider the optimization problem (20). For this, using the gradient projection method, the update rule for power variables of user i is

$$\mathbf{P}_i^{(n+1)} = \Pi_{\mathcal{A}_i}(\mathbf{P}_i^{(n)} + \tau \nabla_i r_i^{(I)}(\mathbf{P}^{(n)})). \tag{23}$$

The gradient projection method ensures that for a given $\mathbf{P}_{-i}^{(n)}$, $r_i^{(I)}(\mathbf{P}_i^{(n+1)}, \mathbf{P}_{-i}^{(n)}) \geq r_i^{(I)}(\mathbf{P}_i^{(n)}, \mathbf{P}_{-i}^{(n)})$. Therefore, we can interpret $\mathbf{P}_i^{(n+1)}$ as a better response to $\mathbf{P}_{-i}^{(n)}$ than $\mathbf{P}_i^{(n)}$. As the feasible space $\mathcal{A} = \Pi_{i=1}^N \mathcal{A}_i$, we can combine the update rules of all users and write

$$\mathbf{P}^{(n+1)} = \Pi_{\mathcal{A}}(\mathbf{P}^{(n)} - \tau F(\mathbf{P}^{(n)})) = T_I(\mathbf{P}^{(n)}).$$

Thus, the Phase 1 of Algorithm 1 is the iterated better response algorithm.

Consider a fixed point $\mathbf{P}^*$ of the better response $T_I(\mathbf{P})$. Then $\mathbf{P}^*$ is a solution of the variational inequality 22. This implies that, given $\mathbf{P}_{-i}^*$, $\mathbf{P}_i^*$ is a local optimum of (20) for all i. Since the optimization (20) is convex, $\mathbf{P}_i^*$ is also a global optimum. Thus given $\mathbf{P}_{-i}^*$, $\mathbf{P}_i^*$ is best response for all i, and hence a fixed point of the better response function is also a NE. This gives further justification for Phase 1 of Algorithm 1. Indeed we will show in the next section that in such a case Phase 1 often provides a NE for $\mathcal{G}_I$ and $\mathcal{G}_D$ (for which also Phase 1 provides a better response dynamics; see Section V-C below).

C. Game $\mathcal{G}_D$

We now consider the game $\mathcal{G}_D$ where each user i has knowledge of only the corresponding direct link gain H_{ii}. In this case also we can formulate the variational inequality characterization. The variational inequality becomes

$$(\mathbf{x} - \mathbf{P}^*)^T F_D(\mathbf{P}^*) \geq 0, \text{ for all } \mathbf{x} \in \mathcal{A}, \tag{24}$$

where $F_D(\mathbf{P}) = (-\nabla_1 r_1^{(D)}(\mathbf{P}), \ldots, -\nabla_N r_N^{(D)}(\mathbf{P}))^T$,

$$r_i^{(D)} = \sum_{h_{ii}} \pi(h_{ii}) \mathbb{E}\left[\log\left(1 + \frac{|h_{ii}|^2 P_i(h_{ii})}{1 + \sum_{j\neq i} |H_{ij}|^2 P_j(H_j)}\right)\right].$$

We use Algorithm 1 to solve the variational inequality (24) by finding fixed points of $T_D(\mathbf{P}) = \Pi_{\mathcal{A}}(\mathbf{P} - \tau F_D(\mathbf{P}))$. Also, one can show that as for T_I, T_D provides a better response strategy.

D. Lower bound

In this subsection, we derive a lower bound on the average rate of each user at any NE. This lower bound can be achieved at a water-filling like power allocation that can be computed with knowledge of only its own channel gain distribution and average power constraint of all the users.

To compute a NE using Algorithm 1, each user needs to communicate its power variables to the other users in every iteration and should have knowledge of the distribution of the channel gains of all the users. If any transmitter fails to receive power variables from other users, it can operate at the water-filling like power allocation that attains at least the lower bound derived in this section. Other users can compute the NE of the game that is obtained by removing the user that failed to receive the power variables, but treating the interference from this user as a constant, fixed by its water-filling like power allocation. We now derive the lower bound.

1) For $\mathcal{G}_I$: In the computation of NE, each user i is required to know the power profile $\mathbf{P}_{-i}$ of all other users. We now give a lower bound on the utility $r_i^{(I)}$ of user i that does not depend on other users' power profiles.

We can easily prove that the function inside the expectation in $r_i^{(I)}$ is a convex function of $\mathbf{P}_j(h_j)$ for fixed $\mathbf{P}_i(h_i)$ using the fact that ([18]) a function $f : \mathcal{K} \subseteq \mathbb{R}^n \to \mathbb{R}$ is convex if and only if

$$\frac{d^2 f(\mathbf{x} + t\mathbf{y})}{dt^2} \geq 0,$$

for all $\mathbf{x}, \mathbf{y} \in \mathcal{K}$ and $t \in \mathbb{R}$ is such that $\mathbf{x} + t\mathbf{y} \in \mathcal{K}$. Then by Jensen's inequality to the inner expectation in $r_i^{(I)}$,

$$\begin{aligned} r_i^{(I)} &= \sum_{h_i \in \mathcal{I}} \pi(h_i) \mathbb{E}\left[\log\left(1 + \frac{|h_{ii}|^2 P_i(\mathbf{h}_i)}{1 + \sum_{j\neq i} |h_{ij}|^2 P_j(H_j)}\right)\right] \\ &\geq \sum_{h_i \in \mathcal{I}} \pi(h_i) \log\left(1 + \frac{|h_{ii}|^2 P_i(\mathbf{h}_i)}{1 + \sum_{j\neq i} |h_{ij}|^2 \mathbb{E}[P_j(H_j)]}\right) \\ &= \sum_{h_i \in \mathcal{I}} \pi(h_i) \log\left(1 + \frac{|h_{ii}|^2 P_i(\mathbf{h}_i)}{1 + \sum_{j\neq i} |h_{ij}|^2 \overline{P_j}}\right). \end{aligned} \tag{25}$$

The above lower bound $r_{i,LB}^{(I)}(\mathbf{P}_i)$ of $r_i^{(I)}(\mathbf{P}_i, \mathbf{P}_{-i})$ does not depend on the power profile of users other than i. We can choose a power allocation $\mathbf{P}_i$ of user i that maximizes $r_{i,LB}^{(I)}(\mathbf{P}_i)$. It is the water-filling solution given by

$$P_i(h_i) = \max\left\{0, \lambda_i - \frac{1 + \sum_{j\neq i} |h_{ij}|^2 \overline{P_j}}{|h_{ii}|^2}\right\}.$$

Let $\mathbf{P}^* = (\mathbf{P}_i^*, \mathbf{P}_{-i}^*)$ be a NE, and let $\mathbf{P}_i^\dagger$ be the maximizer for the lower bound $r_{i,LB}^{(I)}(\mathbf{P}_i)$. Then, $r_i^{(I)}(\mathbf{P}_i^*, \mathbf{P}_{-i}^*) \geq r_i^{(I)}(\mathbf{P}_i, \mathbf{P}_{-i}^*)$ for all $\mathbf{P}_i \in \mathcal{A}_i$, in particular for $\mathbf{P}_i = \mathbf{P}_i^\dagger$. Thus, $r_i^{(I)}(\mathbf{P}_i^*, \mathbf{P}_{-i}^*) \geq r_i^{(I)}(\mathbf{P}_i^\dagger, \mathbf{P}_{-i}^*)$. But, $r_i^{(I)}(\mathbf{P}_i^\dagger, \mathbf{P}_{-i}^*) \geq r_{i,LB}^{(I)}(\mathbf{P}_i^\dagger)$. Therefore, $r_i^{(I)}(\mathbf{P}_i^*, \mathbf{P}_{-i}^*) \geq r_{i,LB}^{(I)}(\mathbf{P}_i^\dagger)$. But, in general it may not hold that $r_i^{(I)}(\mathbf{P}_i^*, \mathbf{P}_{-i}^*) \geq r_i^{(I)}(\mathbf{P}_i^\dagger, \mathbf{P}_{-i}^\dagger)$.

2) For $\mathcal{G}_D$: We can also derive a lower bound on $r_i^{(D)}$ using convexity and Jensen's inequality as in (25). In the case of $\mathcal{G}_D$, we have

$$r_i^{(D)} \geq \sum_{h_{ii}} \pi(h_{ii}) \log\left(1 + \frac{|h_{ii}|^2 P_i(h_{ii})}{1 + \sum_{j\neq i} \mathbb{E}[|H_{ij}|^2] \overline{P_j}}\right).$$

The optimal solution for maximizing the lower bound is the water-filling solution

$$P_i(h_{ii}) = \max\left\{0, \lambda_i - \frac{1 + \sum_{j\neq i} \mathbb{E}[|H_{ij}|^2] \overline{P_j}}{|h_{ii}|^2}\right\}.$$

VI. Numerical Examples

In this section we compare the sum rate achieved at a NE under the different assumptions on channel gain knowledge, obtained using the algorithms provided above. In all the numerical examples, we have chosen $\tau = 0.1$ with the step size in the steepest descent method $\gamma_t = 0.5$ for $t = 1$ and updated after 10 iterations as $\gamma_{t+10} = \frac{\gamma_t}{1+\gamma_t}$. We choose a 3-user interference channel for Examples 1 and 2 below.

For Example 1, we take $\mathcal{H}_d = \{0.3, 1\}$ and $\mathcal{H}_c = \{0.2, 0.1\}$. We assume that all elements of $\mathcal{H}_d, \mathcal{H}_c$ occur with equal

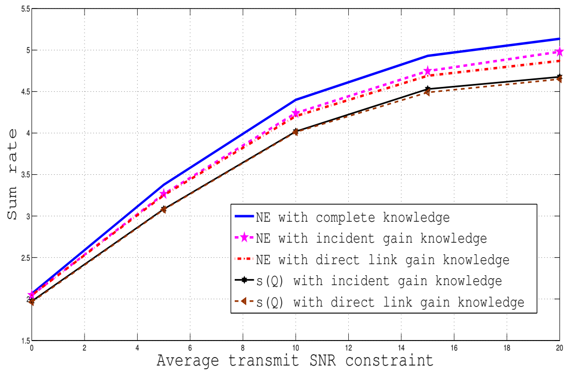

Fig. 1. Sum rate comparison at Nash equilibrium points for Example 1.

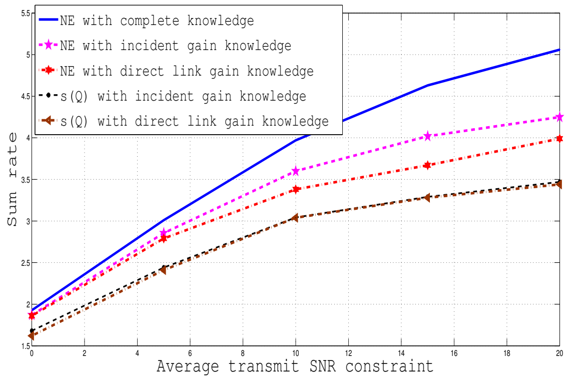

Fig. 2. Sum rate comparison at Nash equilibrium points for Example 2.

probability, i.e., with probability 0.5. Now, the $\tilde{H}$ matrix is positive definite and there exists a unique NE. Thus, the fixed point iteration (12) converges to the unique NE for $\mathcal{G}_A$. Algorithm 1 also converges to a NE not only for $\mathcal{G}_A$ but also for $\mathcal{G}_I$ and $\mathcal{G}_D$.

We compare the sum rates for the NE under different assumptions in Figure 1. We have also computed $\mathbf{Q} = \mathbf{P}^{\dagger}$ that maximizes the corresponding lower bounds (25), evaluated the sum rate $s(\mathbf{Q})$ and compared to the sum rate at a NE. The sum rates at Nash equilibria for $\mathcal{G}_I$ and $\mathcal{G}_D$ are close. This is because the values of the cross link channel gains are close and hence knowing the cross link channel gains has less impact.

We now present two examples in which $\tilde{H}$ is not positive semidefinite but Algorithm 1 converges to a NE for $\mathcal{G}_A$, $\mathcal{G}_I$ and $\mathcal{G}_D$.

In Example 2, we take $\mathcal{H}_d = \{0.3, 1\}$ and $\mathcal{H}_c = \{0.1, 0.5\}$. We assume that all elements of $\mathcal{H}_d, \mathcal{H}_c$ occur with equal probability. We compare the sum rates for the NE obtained by Algorithm 1 in Figure 2. Now we see significant differences in the sum rates.

We consider a 2-user interference channel in Example 3. We take $\mathcal{H}_d = \{0.1, 0.5, 1\}$ and $\mathcal{H}_c = \{0.25, 0.5, 0.75\}$. We assume that all elements of $\mathcal{H}_d, \mathcal{H}_c$ occur with equal probability for user 1, and that the distributions of direct and cross link channel gains are identical for user 2 and are given by $\{0.1, 0.4, 0.5\}$. In this example also, we use Algorithm 1 to find NE for the different cases, and also obtain the lower bound for the partial information cases. We compare the sum rates for the NE in Figure 3.

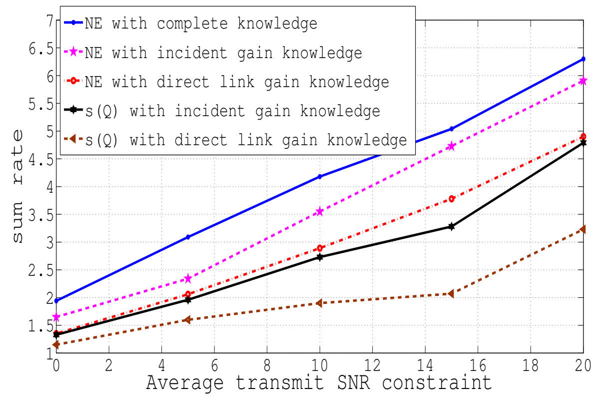

Fig. 3. Sum rate comparison at Nash equilibrium points for Example 3.

We further elaborate on the usefulness of Phase 1 in Algorithm 1. We quantify the closeness of $\mathbf{P}$ to a NE by $g(\mathbf{P}) = \|\mathbf{P} - T(\mathbf{P})\|$. If $\mathbf{P}$ is a NE then $g(\mathbf{P}) = 0$, and for two different power allocations $\mathbf{P}$ and $\mathbf{Q}$ we say that $\mathbf{P}$ is closer to a NE than $\mathbf{Q}$ if $g(\mathbf{P}) < g(\mathbf{Q})$. We now verify that the fixed point iterations in Phase 1 of Algorithm 1 take us closer to a NE starting from any randomly chosen feasible power allocation. For this, we have randomly generated 100 feasible initial power allocations and run Phase 1 for $MAX = 100$ iterations for each randomly chosen initial power allocation, and compared the values of $g(\mathbf{P})$. In the following, we compare the mean, over the 100 initial points chosen, of the values of $g(\mathbf{P})$ immediately after random generation of feasible power allocations, to those after running Phase 1.

We summarize the comparison of mean value of $g(\mathbf{P})$ before and after Phase 1 of Algorithm 1, in Tables I, II and III for Examples 1, 2 and 3 respectively. The first column of the table indicates the constrained average transmit SNR in dB. The second and the third columns correspond to the power allocation game with complete channel knowledge, $\mathcal{G}_A$. The fourth and the fifth columns correspond to the power allocation game with knowledge of the incident channel gains, $\mathcal{G}_I$. The sixth and the seventh columns correspond to the power allocation game with direct link channel knowledge, $\mathcal{G}_D$. The second, fourth and sixth columns indicate the mean of $g(\mathbf{P})$ before running Phase 1, where $\mathbf{P}$ is a randomly generated feasible power allocation. The mean value is evaluated over 100 samples of different random feasible power allocations. The third, fifth and seventh columns indicate the mean value of $g(\mathbf{P})$ after running Phase 1 in Algorithm 1 for the same random feasible power allocations.

It can be seen from the tables that running Phase 1 prior to Phase 2 reduces the value of $g(\mathbf{P})$ when compared with a randomly generated feasible power allocation. Thus, the power allocation after running Phase 1 will be a good choice of power allocation to start the steepest descent in Phase 2. It can also be seen that for all the three examples, for $\mathcal{G}_I$ and $\mathcal{G}_D$, Phase 1 itself converges to the NE, whereas for $\mathcal{G}_A$ Phase 1 may not converge.

At SNR of 20dB, for $\mathcal{G}_A$, Algorithm 1 converged in one iteration of Phase 1 and Phase 2 for Examples 1 and 3. For Example 2, Algorithm 1 converged after Phase 1 in the second iteration of Phase 1 and Phase 2. Phase 2 converged to a local optimum in about 200 iterations in Example 1, about 400 iterations for Example 2 and about 250 iterations in Example 3.

	$g(\mathbf{P})$ for $\mathcal{G}_A$		$g(\mathbf{P})$ for $\mathcal{G}_I$		$g(\mathbf{P})$ for $\mathcal{G}_D$	
SNR(dB)	Before Ph 1	After Ph 1	Before Ph 1	After Ph 1	Before Ph 1	After Ph 1
0	40.82	8.00×10^{-4}	5.01	0.17×10^{-4}	2.48	0.59×10^{-14}
1	51.39	0.027	6.42	0.0005	3.12	0.13×10^{-13}
5	96.5	0.15	11.73	0.0014	5.71	0.54×10^{-3}
10	229.9	0.62	25.45	0.005	12.95	0.0023
15	657.3	2.02	60.6	0.0026	21.69	0.0027
20	2010.7	6.51	80.0	0.0029	31.8	0.0028

TABLE I. COMPARISON OF $g(\mathbf{P})$ IN GAMES $\mathcal{G}_A, \mathcal{G}_I$ AND $\mathcal{G}_D$ BEFORE PHASE 1 AND AFTER PHASE 1 FOR EXAMPLE 1.

	$g(\mathbf{P})$ for $\mathcal{G}_A$		$g(\mathbf{P})$ for $\mathcal{G}_I$		$g(\mathbf{P})$ for $\mathcal{G}_D$	
SNR(dB)	Before Ph 1	After Ph 1	Before Ph 1	After Ph 1	Before Ph 1	After Ph 1
0	41.68	0.12	5.14	0.35×10^{-4}	2.47	0.4×10^{-15}
1	51.43	0.48	6.40	0.13×10^{-3}	3.17	0.18×10^{-14}
5	107.9	2.52	13.4	0.068×10^{-3}	7.1	0.28×10^{-3}
10	309.65	9.76	37.62	0.89×10^{-3}	20.76	0.0016
15	948.37	31.68	98.44	0.0015	29.22	0.0018
20	2974.4	98.85	174.57	0.0027	65.15	0.0033

TABLE II. COMPARISON OF $g(\mathbf{P})$ IN GAMES $\mathcal{G}_A, \mathcal{G}_I$ AND $\mathcal{G}_D$ BEFORE PHASE 1 AND AFTER PHASE 1 FOR EXAMPLE 2.

	$g(\mathbf{P})$ for $\mathcal{G}_A$		$g(\mathbf{P})$ for $\mathcal{G}_I$		$g(\mathbf{P})$ for $\mathcal{G}_D$	
SNR(dB)	Before Ph 1	After Ph 1	Before Ph 1	After Ph 1	Before Ph 1	After Ph 1
0	12.30	0.04	4.07	0.95×10^{-5}	2.30	0.42×10^{-4}
1	14.82	0.05	4.81	0.22×10^{-4}	2.80	0.93×10^{-4}
5	34.21	0.28	10.90	0.47×10^{-3}	5.71	0.89×10^{-3}
10	104.74	0.89	32.34	0.0014	16.82	0.0007
15	325.75	2.43	103.72	0.0016	44.72	0.001
20	1010.10	9.27	271.46	0.0017	107.96	0.002

TABLE III. COMPARISON OF $g(\mathbf{P})$ IN GAMES $\mathcal{G}_A, \mathcal{G}_I$ AND $\mathcal{G}_D$ BEFORE PHASE 1 AND AFTER PHASE 1 FOR EXAMPLE 3.

We have run Algorithm 1 on many more examples and found that it computed the NE, and that for $\mathcal{G}_I$ and $\mathcal{G}_D$ Phase 1 itself converged to the NE.

VII. CONCLUSIONS

We have considered a channel shared by multiple transmitter-receiver pairs causing interference to one another. We formulated stochastic games for this system in which transmitter-receiver pairs may or may not have information about other pairs' channel gains. Exploiting variational inequalities, we presented a heuristic algorithm that obtains a NE in the various examples studied, quite efficiently.

In the games with partial information, we presented a lower bound on the utility of each user at any NE. A utility of at least this lower bound can be attained by a user using a water-filling like power allocation, that can be computed with the knowledge of the distribution of its own channel gains and of the average power constraints of all the users. This power allocation is especially useful when any transmitter fails to receive the power variables from the other transmitters that are required for it to compute its NE power allocation.

ACKNOWLEDGEMENT

This work is partially supported by funding from ANRC.

REFERENCES

[1] G. Scutari, D. P. Palomar, S. Barbarossa, "Optimal Linear Precoding Strategies for Wideband Non-Cooperative Systems Based on Game Theory-Part II: Algorithms," *IEEE Trans on Signal Processing*, Vol.56, no.3, pp. 1250-1267, March 2008.

[2] X. Lin, Tat-Ming Lok, "Learning Equilibrium Play for Stochastic Parallel Gaussian Interference Channels," available at http://arxiv.org/abs/1103.3782.

[3] K. W. Shum, K.-K. Leung, C. W. Sung, "Convergence of Iterative Waterfilling Algorithm for Gaussian Interference Channels," *IEEE Journal on Selected Areas in Comm.,* Vol.25, no.6, pp. 1091-1100, August 2007.

[4] G. Scutari, F. Facchinei, J. S. Pang, L. Lampariello, "Equilibrium Selection in Power Control games on the Interference Channel," *Proceedings of IEEE INFOCOM*, pp 675-683, March 2012.

[5] G. Scutari, D. P. Palomar, S. Barbarossa, "The MIMO Iterative Waterfilling Algorithm," *IEEE Trans on Signal Processing*, Vol. 57, No.5, May 2009.

[6] G. Scutari, D. P. Palomar, S. Barbarossa, "Asynchronous Iterative Water-Filling for Gaussian Frequency-Selective Interference Channels", *IEEE Trans on Information Theory*, Vol.54, No.7, July 2008.

[7] L. Rose, S. M. Perlaza, M. Debbah, and C. J. Le Martret, "Distributed Power Allocation with SINR Constraints Using Trial and Error Learning," *Proc. of IEEE Wireless Communications and Networking Conference*, Paris, France, April 2012.

[8] J. S. Pang, G. Scutari, F. Facchinei, and C. Wang, "Distributed Power Allocation with Rate Constraints in Gaussian Parallel Interference Channels," *IEEE Trans on Information Theory*, Vol. 54, No. 8, pp. 3471-3489, August 2008.

[9] K. A. Chaitanya, U. Mukherji and V. Sharma, "Power allocation for Interference Channel," *Proc. of National Conference on Communications*, New Delhi, 2013.

[10] A. J. Goldsmith and Pravin P. Varaiya, "Capacity of Fading Channels with Channel Side Information," *IEEE Trans on Information Theory*, Vol.43, pp.1986-1992, November 1997.

[11] H. N. Raghava and V. Sharma, "Diversity-Multiplexing Trade-off for Channels with Feedback," *Proc. of 43rd Annual Allerton conference on Communications, Control, and Computing*, 2005.

[12] K. A. Chaitanya, U. Mukherji, and V. Sharma, "Algorithms for Stochastic Games on Interference Channels," *Proc. of National Conference on Communications*, Mumbai, 2015.

[13] Z. Han, D. Niyato, W. Saad, T. Basar and A. Hjorungnes, "Game Theory in Wireless and Communication Networks," *Cambridge University Press*, 2012.

[14] D. P. Bertsekas and J. N. Tsitsiklis, "Parallel and Distributed Computation: Numerical methods," *Athena Scientific*, 1997.

[15] H. Minc, "Nonnegative Matrices," John Wiley & Sons, New York, 1988.

[16] F. Facchinei and J. S. Pang, "Finite-Dimensional Variational Inequalities and Complementarity Problems," Springer, 2003.

[17] D. Conforti and R. Musmanno, "Parallel Algorithm for Unconstrained Optimization Based on Decomposition Techniques," *Journal of Optimization Theory and Applications*, Vol.95, No.3, December 1997.

[18] S. Boyd and L. Vandenberghe, "Convex Optimization," Cambridge University Press, 2004.