Energy Efficiency Analysis of Relay-Assisted Cellular Networks using Stochastic Geometry

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Abstract—**As energy consumption reduction has been an important concern for the wireless industry, energy-efficient communications is of prime interest for future broadband networks. In this paper, we study the energy efficiency of relay-assisted cellular networks using tools from stochastic geometry. We first derive the coverage probability for the macro base station (MBS) to user (UE), the MBS to relay station (RS), and the RS to UE links, and then we model the power consumption of MBSs and RSs. Based on the analytical model and expressions, the energy efficiency of relay-assisted cellular networks is then evaluated and is shown to be strictly quasi-concave on the transmit power for the MBS to UE link or the RS to UE link. Numerical results also show that the energy efficiency first improves while it hits a ceiling as the MBS density increases.**

I. Introduction

With the exponential growth of wireless traffic, future cellular networks face huge challenges for catering higher data rate and transmission reliability. Deploying relay stations (RSs) is considered to be a promising way to solve the capacity crunch in conventional cellular networks, increasing coverage, throughput and reliability [1]. Meanwhile, reducing the energy consumption has become a pressing requirement, hence energy-efficient design in wireless networks has recently attracted significant attention [2].

There have been some works in the literature about energy efficient relay-assisted networks [3]–[7]. In [3]–[5], energy-efficient relay links are studied, including multi-carrier one-way relay links and two-way relay links. In [6], [7] energy-efficient multi-relay networks are investigated, in which the source node sends messages to the destination node via the relay nodes and the best one or several relay nodes according to a certain criterion are selected to forward the message in order to maximize the network energy efficiency.

In traditional studies on the performance of cellular networks, the cell shape is often supposed to be hexagonal or square, which may not be the case in realistic scenarios (e.g. due to shadowing) or in heterogeneous and uncoordinated networks. Besides, complex time-consuming system-level simulations must be used to evaluate the performance. An approach to derive the coverage and rate in cellular networks based on stochastic geometry has been proposed in [8], which is more realistic and tractable compared to traditional grid models. Under this approach, in [9], the design of energy-efficient heterogeneous cellular networks through the employment of base station sleeping mode strategies as well as small cells are studied. In [10], the relationship between the energy efficiency and the intensity of non-cooperative UEs and cooperative UEs is analyzed based on a stochastic geometry approach. In [11], the effect of base station density on the energy efficiency of relay-assisted cellular networks is investigated using stochastic geometry. However, the paper only considers the special case where the pathloss exponent $\alpha = 4$, while in our work we derive general expressions that are valid for any $\alpha > 2$.

In our work, we analyze the energy efficiency in downlink relay-assisted cellular networks using models based on stochastic geometry [8]. The coverage probability of the macro base station (MBS)-relay stations(RS), the MBS-user (UE) and the RS-UE links are derived and the power consumption for MBSs as well as RSs is modeled. The energy efficiency for relay-assisted cellular networks is then analytically deduced for any $\alpha > 2$. Based on the energy efficiency expression, we show the quasi-concavity of the energy efficiency metric as well as the relationship between energy efficiency and MBS density.

The rest of this paper is organized as follows: in section II, the system model is described. In section III, the energy efficiency for downlink relay-assisted cellular networks is derived. Simulation results are provided in section IV, and concluding remarks are given in section V.

II. System model

We consider a relay-assisted cellular network where the MBSs and RSs are located according to independent homogeneous Poisson point processes (HPPPs) Θ_M and Θ_R with densities λ_M and λ_R, respectively, in the Euclidean plane. In the network, each RS connects to the geographically closest MBS and has coverage of a disk of radius R, i.e. the RSs deployed in the Voronoi cell of an MBS are connected with it (cf. Fig. 1). Half-duplex relay nodes using decode-and-forward (DF) strategy are considered in this paper. UEs are divided into two types: (i) UEs that communicate directly with the MBS (M-UE), which are distributed according to some independent stationary point process and connect with the closest MBS,

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(ii) UEs that send messages to the MBS via the help of RS (R-UE) and are arranged with reference to some independent stationary point process within each RS's circular area. Note that these two different types of UEs are not distinguished in Fig. 1.

As Θ_M is a HPPP with density λ_M, if we denote the distance between M-UE (or RS) and the target MBS as r, the probability density function (pdf) of r is $f_r(r) = 2\pi\lambda_M r \exp\left(-\pi\lambda_M r^2\right)$ [8]. Furthermore, since the distribution of RSs follows a HPPP and R-UEs are distributed according to some independent stationary point process within each relay's circular area, the distance between the R-UE and its corresponding RS l follows a distribution with pdf $f_l(l) = 2l/R^2$.

As shown in Fig. 2, in each cell, the MBS-UE link and the MBS-RS-UE link work in non overlapping frequency bands with bandwidth ω_M and ω_R, respectively. For the MBS-RS-UE link with half-duplex relays, the transmission is divided into two phases and the time duration of both phases is equal. In the first phrase, the MBS sends messages to the RS, while in the second phase, the RS decodes the messages and forwards them to the user. Each MBS is assumed to serve at most one M-UE and RS at any time. If more than one M-UEs (RSs) is located in the MBS's cell, orthogonal resource sharing such as time division multiple access (TDMA) is performed. Similarly, each RS serves one R-UE at any time and TDMA may be used if multiple R-UEs are distributed in the RS's serving area.

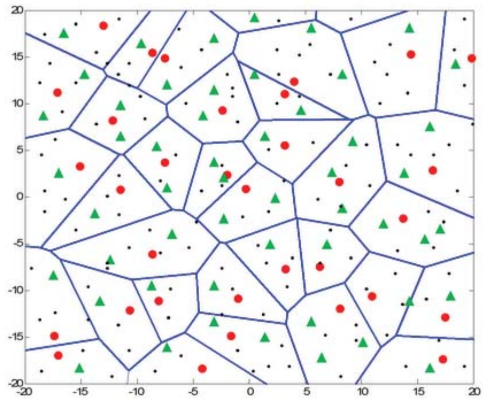

Fig. 1. A Relay-assisted cellular network topology, where the big dots represent MBSs, triangles represent RSs and little dots represent UEs.

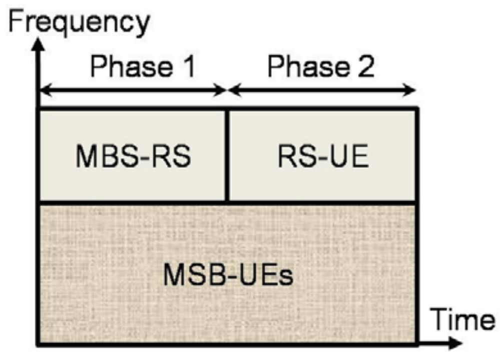

Fig. 2. Relay-assisted Cellular Network frame structure

III. Energy Efficiency Analysis

A. Coverage probability

The signal-to-interference-plus-noise ratio (SINR) in the downlink from the transmitter (MBS or RS) s to the receiver (RS or UE) d is given by

$$\mathrm{SINR}(s \to d) = \frac{P h r^{-\alpha}}{I + \sigma^2} \tag{1}$$

where P is the transmit power for node s, h is the channel power gain due to the small-scale fading, r is the distance between s and d, α is the pathloss exponent, I is the aggregate interference from all the other active transmitters operating in the same frequency band, and σ^2 is the variance of the additive white Gaussian background noise. All channels are assumed to be subject to Rayleigh fading, i.e. $h \sim \exp(1)$.

The coverage probability is defined as the probability that the receive SINR is above a certain threshold Γ and can be written as

$$p_c = \mathbb{P}(\mathrm{SINR} \geq \Gamma) \tag{2}$$

We assume that the transmit power from MBS to M-UE is P_{MU}, the receive threshold is denoted as Γ_{MU}, and $I_{\mathrm{MU}} = \sum_{i \in \Theta_M \setminus \{m_0\}} P_{\mathrm{MU}} h_i r_i^{-\alpha}$ is the aggregate interference from all the other active MBSs except the target MBS m_0. Then the coverage probability for a typical MBS-UE link is

$$\begin{aligned} p_{c\_\mathrm{MU}} &= \mathbb{P}(\mathrm{SINR}_{\mathrm{MU}} \geq \Gamma_{\mathrm{MU}}) \\ &= \int_{r>0} \mathbb{P}(h_0 \geq \frac{\Gamma_{\mathrm{MU}} r^{\alpha}\left(I_{\mathrm{MU}} + \sigma^2\right)}{P_{\mathrm{MU}}} | r) f_r(r) \mathrm{d}r \\ &= \int_{r>0} \exp\left(-\frac{\Gamma_{\mathrm{MU}} r^{\alpha} \sigma^2}{P_{\mathrm{MU}}}\right) L_{I_{\mathrm{MU}}}\left(\frac{\Gamma_{\mathrm{MU}} r^{\alpha}}{P_{\mathrm{MU}}}\right) f_r(r) \mathrm{d}r \end{aligned} \tag{3}$$

where $L_{I_{\mathrm{MU}}}(\cdot)$ is the Laplace transform of I_{MU}.

Considering the definition of Laplace transform and the property of the probability generating functional (PGFL) for PPP [12], we can get

$$L_{I_{\mathrm{MU}}}\left(\frac{\Gamma_{\mathrm{MU}} r^{\alpha}}{P_{\mathrm{MU}}}\right) = \exp\left(-\pi r^2 \lambda_M \rho(\Gamma_{\mathrm{MU}}, \alpha)\right) \tag{4}$$

where $\rho(\Gamma_{\mathrm{MU}}, \alpha) = \Gamma_{\mathrm{MU}}^{2/\alpha} \int_{T_{\mathrm{MU}}^{-2/\alpha}}^{\infty} \frac{1}{1+v^{\alpha/2}} \mathrm{d}v$ and $v = \left(\frac{u}{r \Gamma_{\mathrm{MU}}^{1/\alpha}}\right)^2$.

Combining (3), (4) and the expression of $f_r(r)$, p_{c_MU} can be written as

$$p_{c\_\mathrm{MU}} = 2\pi\lambda_M \int_{r>0} \exp\left(-\pi\lambda_M r^2 \left(1+\rho\left(\Gamma_{\mathrm{MU}},\alpha\right)\right)\right) \times \exp\left(-\Gamma_{\mathrm{MU}} r^\alpha \sigma^2 / P_{\mathrm{MU}}\right) r\mathrm{d}r. \tag{5}$$

Suppose the relay can successfully decode the signals from the MBS if the received SINR is larger than or equal to Γ_{MR} and the transmit power from MBS to RS is P_{MR}, then the coverage probability for the MBS-RS link p_{c_MR} is

$$p_{c\_\mathrm{MR}} = 2\pi\lambda_M \int_{r>0} \exp\left(-\pi r^2 \left(\lambda_M + \lambda\rho\left(\Gamma_{\mathrm{MR}},\alpha\right)\right)\right) \times \exp\left(-\Gamma_{\mathrm{MR}} r^\alpha \sigma^2 / P_{\mathrm{MR}}\right) r\mathrm{d}r, \tag{6}$$

where $\lambda = \min\{\lambda_M, \lambda_R\}$ and $\rho\left(\Gamma_{\mathrm{MR}},\alpha\right) = \Gamma_{\mathrm{MR}}^{2/\alpha} \int_{\Gamma_{\mathrm{MR}}^{-2/\alpha}}^{\infty} \frac{1}{1+v^{\alpha/2}} \mathrm{d}v$.

Note that at any time, there are only at most λ MBSs per square meter that transmit signals to RSs in the network, so the density used for the derivation of the Laplace transform of I_{MR} is λ instead of λ_M. Beyond that, the derivation of the coverage probability p_{c_MR} is almost the same as for p_{c_MU}, hence its proof is omitted here.

Considering the coverage probability for a typical RS-UE link p_{c_RU}, we denote P_{RU} as the transmit power of the RS, Γ_{RU} as the receive threshold, p_{c_RU} can be then expressed as

$$p_{c\_\mathrm{RU}} = \frac{2}{R^2} \int_0^R \exp\left(-\pi\lambda r^2 p_{c\_\mathrm{MR}} \mu\left(\Gamma_{\mathrm{RU}},\alpha\right)\right) \times \exp\left(-\frac{\Gamma_{\mathrm{RU}} r^\alpha \sigma^2}{\left(1+\Gamma_{\mathrm{RU}}\right) P_{\mathrm{RU}}}\right) r\mathrm{d}r, \tag{7}$$

where $\lambda = \min\{\lambda_M, \lambda_R\}$ and $\mu\left(\Gamma_{\mathrm{RU}},\alpha\right) = \left(\frac{\Gamma_{\mathrm{RU}}}{1+\Gamma_{\mathrm{RU}}}\right)^{2/\alpha} \int_0^\infty \frac{1}{1+v^{\alpha/2}} \mathrm{d}v$.

Note that the distance between RS and UE follows a different distribution, and the density used in the above formula is $\lambda p_{c\_\mathrm{MR}}$ because at any time there is at most only one RS communicating with the target MBS and only the RSs with received SINR larger than the target SINR Γ_{MR} forward messages to the R-UEs. As a result, the form of p_{c_RU} is not the same as that of p_{c_MR} or p_{c_MU}.

B. Power consumption model

In this section, we model the power consumption of MBSs and RSs in downlink relay-assisted cellular networks. The relation between total power consumption P_{tot} and transmit radiated power P_T is modeled as [13] [14] $P_{tot} = \beta P_T + P_0$, where $1/\beta$ is the efficiency of the power amplifier, and P_0 is the static power consumption, which includes signal processing overhead, battery backup, cooling power consumption, etc. As a result, for MBS and RS, the total power consumption P_{M_tot}, P_{R_tot} can be given by

$$P_{M\_tot} = \beta_M P_M + P_{M0} \tag{8}$$

$$P_{R\_tot} = \beta_R P_R + P_{R0}, \tag{9}$$

where $1/\beta_M$, $1/\beta_R$ denote the efficiency of the power amplifier for MBS and RS, P_M and P_R account for the total transmit power for MBS and RS, P_{M0} and P_{R0} are the static power consumption for MBS and RS, respectively.

C. Performance metrics

We define the energy efficiency for the relay-assisted cellular networks as

$$\eta_{\mathrm{EE}} = \frac{Area \quad Spectral \quad Efficiency}{Average \quad Network \quad Power \quad Consumption} = \frac{\tau_M + \tau_R}{P_{M\_ave} + P_{R\_ave}} \; (bps/Hz/W), \tag{10}$$

where τ_M and τ_R are the area spectral efficiency over all the MBS-UE links and MBS-RS-UE links, respectively. P_{M_ave} and P_{R_ave} denote the average network power consumption for MBSs and RSs.

The throughput attained at the MBS-UE link is given by $p_{c\_\mathrm{MU}} \frac{w_M}{w_M + w_R} \log_2\left(1+\Gamma_{\mathrm{MU}}\right)$ and the area spectrum efficiency over all MBS-UE links is

$$\tau_M = \lambda_M p_{c\_MU} \frac{w_M}{w_M + w_R} \log_2\left(1+\Gamma_{\mathrm{MU}}\right). \tag{11}$$

The MBS sends messages to the M-UE with transmit power P_{MU}, and to the RS with transmit power P_{MR} only in the one of the two phrases, so the average macrocell network power consumption is

$$P_{M\_ave} = \beta_M \left(\lambda_M P_{\mathrm{MU}} + \lambda P_{\mathrm{MR}}/2\right) + \lambda_M P_{\mathrm{M0}}. \tag{12}$$

As at most only $\lambda = \min\left(\lambda_M, \lambda_R\right)$ MBSs per square meter transmit signals to the RSs, the density used for P_{MR} is λ.

As a DF strategy is adopted by the RS, then the MBS-RS-UE link operates only if both the relay and the user can decode the messages received successfully, namely, the SINR attained at the RS and the R-UE has to be larger than Γ_{MR} and Γ_{RU}, respectively. Thus, the area spectrum efficiency for all the MBS-RS-UE links is given by

$$\tau_R = \frac{1}{2} \lambda p_{c\_\mathrm{MR}} \times p_{c\_\mathrm{RU}} \frac{w_R}{w_R + w_H} \times \min\left\{\log_2\left(1+\Gamma_{\mathrm{MR}}\right), \log_2\left(1+\Gamma_{\mathrm{RU}}\right)\right\}. \tag{13}$$

Note that the 1/2 factor is due to half-duplex operation of the relay nodes.

At any time, the number of RSs taking part in the communication is at most the same as the number of MBSs (λ relays per square meters) and only the relays decoding signals for MBSs successfully forward signals to corresponding R-UEs ($\lambda p_{c\_\mathrm{MR}}$ relays per square meter). The other relays ($\lambda_R - \lambda p_{c\_\mathrm{MR}}$ relays per square meter) do not send signals but only consume fixed power (static part). Thus, the average network power consumption for the RSs is

$$P_{R\_\mathrm{ave}} = \lambda p_{c\_\mathrm{MR}} \left(\beta_R P_{RU}/2 + P_{R0}\right) + \left(\lambda_R - \lambda p_{c\_\mathrm{MR}}\right) P_{\mathrm{R0}}. \tag{14}$$

Combining (10) - (14), the expression of energy efficiency for the relay-assisted cellular networks is acquired.

The following theorem demonstrates the quasi-concavity of energy efficiency function.

Theorem 1. *The energy efficiency function η_{EE} is strictly quasi-concave on P_{MU} and P_{RU} .*

TABLE I
THE ALGORITHM TO FIND OPTIMAL η_{EE} BASED ON THE BISECTION METHOD

Input: the upper bound m and the lower bound n $(m \geq n \geq 0)$, the tolerance ε.
Output: the optimal η_{EE} and the corresponding P_{MU}

1) **While:** $(m - n \geq \varepsilon)$
2) $\quad t = \frac{m-n}{2}$
3) $\quad$ Solve the feasibility problem $\eta_{\mathrm{EE}} \geq t$
4) $\quad$ **if** the problem is feasible
5) $\quad\quad$ **then** $n = t$
6) $\quad$ **else**
7) $\quad\quad$ $m = t$
8) **if** $p_{c\_\mathrm{MU}}\left(P_{\mathrm{MU\_opt}}\right) \geq \theta$ $(\eta_{\mathrm{EE}}\left(P_{\mathrm{MU\_opt}}\right) = t)$
9) $\quad$ **return** t and $P_{\mathrm{MU\_opt}}$
10) **else** solve the equation $p_{c\_\mathrm{MU}}\left(\bar{P}_{\mathrm{MU\_opt}}\right) = \theta$
11) $\quad$ **return** $\eta_{\mathrm{EE}}\left(\bar{P}_{\mathrm{MU\_opt}}\right)$ and $\bar{P}_{\mathrm{MU\_opt}}$

Proof: We only prove η_{EE} is strictly quasi-concave on P_{MU} here, and the process of the proof that η_{EE} is strictly quasi-concave on P_{RU} is similar.

A function $f : \mathbf{R}^n \rightarrow \mathbf{R}$ is said to be strictly quasi-concave if its sublevel set $S_\alpha = \{\mathbf{x} | \mathbf{x} \in \mathbf{dom} f, f(\mathbf{x}) \geq \alpha\}$ is strictly convex for every α [15]. When $\alpha \leq 0$, S_α is obviously convex on P_{MU}. When $\alpha > 0$, S_α can be written as $S_\alpha = \{P_{\mathrm{MU}} | \alpha (P_{M\_ave} + P_{R\_ave}) - \tau_M - \tau_R \leq 0\}$. It can be proved that p_{c_MU} is strictly concave on P_{MU}, so $-\tau_M$ is strictly convex on P_{MU}. Besides, P_{M_ave} linearly increases with P_{MU}, P_{R_ave} and τ_R are not relevant with P_{MU}. As a result, S_α is also strictly convex on P_{MU} and Theorem 1 follows. ■

Due to the strict quasi-concavity of η_{EE} on P_{MU} or P_{RU}, the optimal energy efficiency exists for various values of P_{MU} or P_{RU}.

We now study the following optimization problem:

$$\begin{aligned} \min_{P_{\mathrm{MU}}} \quad & \eta_{\mathrm{EE}} \\ s.t. \quad & p_{c\_\mathrm{MU}} \geq \theta \end{aligned} \tag{15}$$

The above problem aims at finding the optimal energy efficiency under the constraint that the coverage probability for the MBS-UE link is larger than or equal to a certain threshold θ. As η_{EE} is strictly quasi-concave on P_{MU}, the bisection method can be used to find the optimal η_{EE} and the corresponding P_{MU} (denoted $P_{\mathrm{MU\_opt}}$). If $p_{c\_\mathrm{MU}}\left(P_{\mathrm{MU\_opt}}\right) \geq \theta$, then $\eta_{\mathrm{EE}}\left(P_{\mathrm{MU\_opt}}\right)$ is the optimal value. Else, as p_{c_MU} strictly increases monotonically on P_{MU}, $P_{\mathrm{MU\_opt}}$ is less than the power $\bar{P}_{\mathrm{MU\_opt}}$ which makes $p_{c\_\mathrm{MU}}\left(\bar{P}_{\mathrm{MU\_opt}}\right) = \theta$ and $\eta_{\mathrm{EE}}\left(\bar{P}_{\mathrm{MU\_opt}}\right)$ is the optimal value according to the quasi-concavity of η_{EE}. The detailed algorithm to find the optimal η_{EE} based on bisection method is described in Table I.

IV. SIMULATION RESULTS

In this section, we evaluate the derived analytical results through simulation. We use the default values of the system model in Table II unless otherwise stated.

Fig. 3 compares the analytical expressions versus simulated results for coverage probability of the MBS-UE, MBS-RS, RS-UE links. The curves verify that the simulation results match the analytical expressions well and we can use the analytical expressions instead of time-consuming system-level simulations in our work.

TABLE II
PARAMETER VALUES USED IN THE SIMULATIONS

Parameter	Value
λ_M, λ_R	$10^{-5}/m^2$, $10^{-4}/m^2$,
P_{MU}, P_{RU}, P_{MR}	43dBm, 30dBm, 33dBm
R	40m
α	4
w_M, w_R	40MHz, 4MHz
Γ_{MR}, Γ_{RU}, Γ_{MR}	-10dB, -10dB, -10dB
σ^2	-60dBm
β_M, β_R	5.32, 4.8
P_{M0}, P_{R0}	118.7W, 7.5W

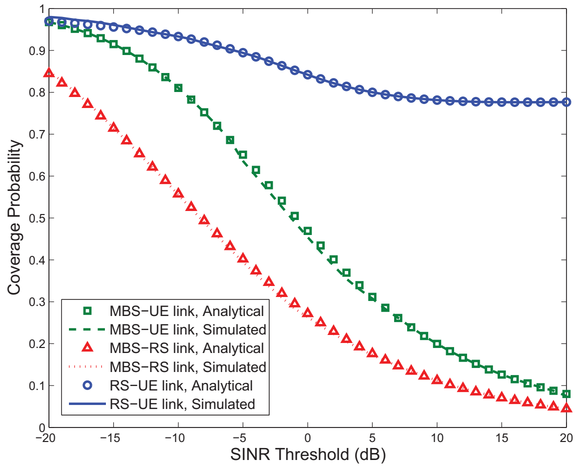

Fig. 3. Comparison of analytical expressions vs. simulated results for coverage probability of links.

Fig. 4 displays the energy efficiency with respect to the density of MBSs for different densities of RSs. It can be seen that the energy efficiency increases with the density of MBSs and saturates when the density goes to infinity. In other words, the growth of area spectral efficiency is faster than the growth of average network power consumption as the density of MBSs increases. But when the density of MBSs is relatively large, continuously increasing MBSs cannot further increase the energy efficiency. In addition, higher density of RSs brings about lower energy efficiency in the case of all other parameters constant.

Fig. 5 shows how the energy efficiency varies with transmit power of MBS P_{MU} for different values of RS densities. The energy efficiency curves in the figure first increase and then decrease, showing a quasi-convex trend as the transmit power varies, as described in Theorem 1. The optimal energy efficiency can be found using the bisection algorithm shown in Table II. If the threshold θ is set to zero and the tolerance ε is 10^{-5}, the optimal energy efficiency is 6.40×10^{-4} bps/Hz/W,

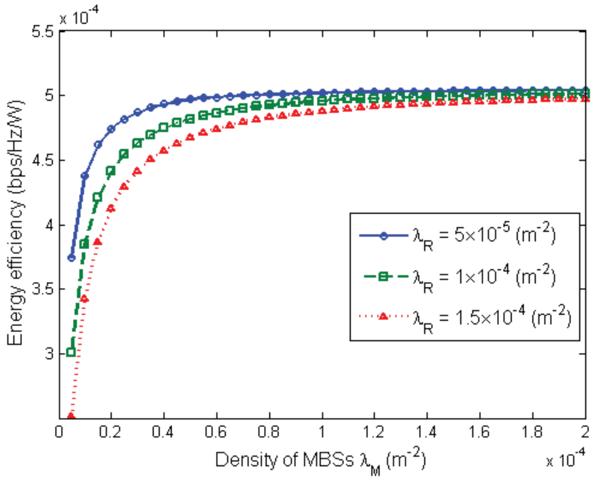

Fig. 4. Energy Efficiency vs. Density of MBSs for different density of RSs

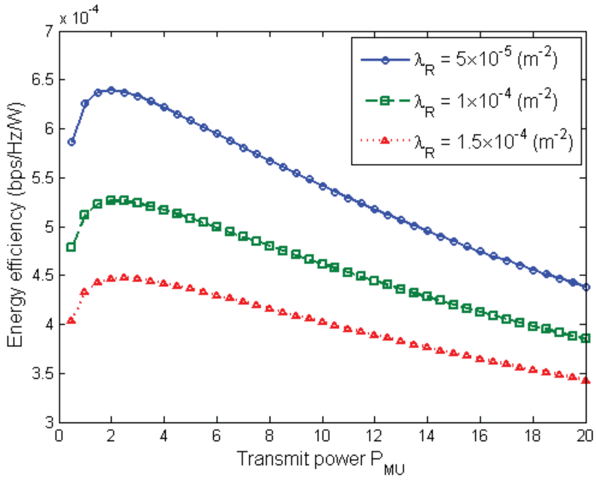

Fig. 5. Energy Efficiency vs. Transmit Power of MBS for different density of RSs

5.26×10^{-4} bps/Hz/W and 4.48×10^{-4} bps/Hz/W corresponding to the three curves when $\lambda_R = 5 \times 10^{-5} m^{-2}, 1 \times 10^{-4} m^{-2}, 1.5 \times 10^{-4} m^{-2}$, respectively. Besides, the figure also reveals that while the density of RSs grows, the energy efficiency drops, which means that the growth of area spectrum efficiency by increasing the density of RSs cannot compensate for the growth of average network power consumption.

V. Conclusion

In this paper, we analyze the energy efficiency of downlink relay-assisted cellular networks where MBSs and RSs are distributed according to independent HPPPs. We first build on a tractable stochastic geometry based model to derive the coverage probabilities for MBS-UE, MBS-RS, and RS-UE links. Then we model the power consumption for both MBS and RS. After that, we obtain expressions of energy efficiency for the whole network. Our results reveal that there is a fundamental tradeoff between energy efficiency and transmit power of MBS, and the main takeaway is that deploying more MBSs generally triggers higher energy efficiency, however this gain hits a ceiling as the density of MBSs increases.

Acknowledgment

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