On the Delay/Cost Tradeoff in Wireless Mobile Delay-Tolerant Networks

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***Abstract*—An infinite number of nodes travel on $\mathbb{R}^2$, along straight lines, with a common speed v_n. Any transmission across a distance d incurs a cost $c(d)$. We devise and study, both by analysis (using stochastic geometry tools) and simulation, forwarding rules that transport packets towards a given direction using combinations of wireless transmissions and physical transfers on node buffers. The forwarding rules are evaluated in terms of two metrics: the packet delivery delay and the aggregate transmission cost, both per unit of distance covered. We explore the tradeoff between these two metrics in terms of the points on the delay-cost plane achieved by our forwarding rules.**

***Index Terms*—Poisson Point Processes, Delay-Tolerant Networks, Mobile Wireless Networks, Delay/Cost Tradeoffs.**

I. Introduction

A number of routing protocols have recently been proposed, in various mobile wireless settings, that combine geographic routing with delay-tolerant routing. Under these protocols, packets are transported towards their destination using combinations of wireless transmissions (to nodes suitably placed closer to their destination) and physical transfers (along nodes suitably moving in the direction of their destination) [1], [2], [3], [4], [5], [6], [7].

Motivated by these works, we develop and study a related abstract setting using tools from stochastic geometry: The network comprises an infinite number of mobile nodes; Nodes move along straight lines, and the mobility model is such that at any time instant they are distributed according to a homogeneous spatial Poisson point process; Each transmission across a distance d comes at the expense of a cost $c(d) = d^2$ and is instantaneous; Packets must travel towards a given direction to a destination located at an infinite distance, through a combination of wireless transmissions and physical transfers on node buffers.

We look for feasible combinations (D_p, C_p) of the normalized delay D_p and normalized cost C_p, which are defined as the time taken and the aggregate transmission cost incurred, respectively, per unit of distance that a packet progresses towards the direction of its destination. It is intuitively clear that a tradeoff must exist between these two metrics: if the packet increases its use of wireless transmissions as opposed to physical transfers, the normalized cost C_p should become higher and the normalized delay should become lower, and vice versa. We derive feasible (D_p, C_p) combinations both analytically, using tools from stochastic geometry coupled with a simplifying approximation, as well as by simulation. These feasible combinations are achieved by novel forwarding rules that carefully balance the two transport mechanisms available to the packet (i.e., wireless transmission and physical transfer), while continuously taking into account the evolving topology in its neighborhood.

The rest of this work is organized as follows: in Section II we discuss related work. In Section III we introduce our network model and performance metrics D_p, C_p. In Section IV we define the two forwarding rules we study, Rules I and II. In Sections V and VI we calculate analytically, using a simplifying approximation, the (D_p, C_p) combinations achieved by Rule I. In Section VII we present simulation results for both Rules I and II. We conclude in Section VIII. Some conceptually straightforward but lengthy derivations are omitted, but appear in [8].

II. Related Work

Tradeoffs involving the packet delivery delay are not limited to the wireless domain. In [5] a very general formulation, applicable to all Delay-Tolerant Networks (DTNs), is introduced for quantifying the tradeoff between the delay in the delivery of a packet and the transport cost, comprised of a term due to the storage of the packet at intermediate buffers and a term due to its transmission from node to node. In contrast to our work, the tradeoff is established for specific instances of the network evolution, using network optimization tools, and without utilizing probabilistic analysis.

Under the hybrid geographic/delay-tolerant protocols of [1], [2], [3], [4], [5], packets are routed using combinations of transmissions to nodes that are closer to the destination than their current holder (as in geographic routing), and sojourns on the buffers of their current holders (as in delay-tolerant routing). Depending more on transmissions and less on physical transfers leads to reduced delays but increased transmission costs (e.g., more bandwidth required, which reduces the end-to-end throughput in the network). Depending more on physical transfers and less on transmissions leads to the opposite effects. The forwarding rules we study in this work follow the same principle, but we use an idealized stochastic geometry framework that is amenable to analysis.

Stochastic geometry has been applied to the analysis of wireless networks with great success, leading to a wealth of insightful results [9], [10]. However, in such works nodes are typically assumed to be immobile, excepting a few notable cases.

For example, in [11], the authors investigate the maximum possible speed with which information can propagate in a wireless DTN with mobile nodes, without placing a constraint on the paths considered by the packet. This last assumption is in contrast to our work, and more akin to calculating the speed with which a packet propagates if it is flooded to the whole network.

Also, in [7] the authors calculate delay/throughput tradeoffs under a geographical forwarding rule termed Constrained Relative Bearing. Differing from our work, the authors derive results on the relation between the delay and the throughput that are asymptotic in terms of the number of nodes, in the sense of [12], [13]. Furthermore, the approach in [7] is more holistic, calculating bounds on the orders of system-level parameters, i.e., the aggregate throughput and the packet delivery delay, whereas our work is more focused on the precise values of the packet-level parameters D_p and C_p.

We also note that stochastic geometry models with mobile nodes have also been introduced in the study of single node isolation times in wireless mobile networks [14], space-time network percolation [15], and wireless sensor network coverage [16]. These topics do not directly involve unicast routing, and so are tangentially related to this work.

Finally, we mention that an effort preliminary to our work here has appeared in [6]. There, however, we considered a very special case of the main forwarding rule we treat here, the analysis was simpler and with stronger approximations, and there was no comparison between analysis and simulation results.

III. Network Model and the Delay-Cost Plane

A. Network Model

At time $t = 0$, an infinite number of nodes are placed on the infinite plane $\mathbb{R}^2$ according to a Poisson distribution with density λ. Beginning at $t = 0$, each node moves with a fixed velocity vector, of magnitude v_n common for all nodes, and a direction chosen uniformly, and independently of the rest. It is straightforward to show that, under these assumptions, nodes are Poisson distributed with density λ for all $t \geq 0$ (cf. Section 1.3.3 of [9]). Furthermore, by a standard thinning argument, the nodes whose direction of travel forms an angle with the x axis within the interval $[\chi, \chi+\Delta\chi]$ are also Poisson distributed with density $\lambda\frac{\Delta\chi}{2\pi}$.

Two nodes separated by a distance d can exchange a packet at a cost $c(d) = d^2$. We select this function of the distance as it reflects accurately the need of the transmission to secure a 'quiet area' of roughly circular shape around the receiver for the correct reception of the packet [13]. Other choices would also be possible with our formulation. For simplicity, we do not specify an upper bound on the distance that a transmission can cover. Finally, packet exchanges are instantaneous, errorless, and not subject to interference and media access constraints.

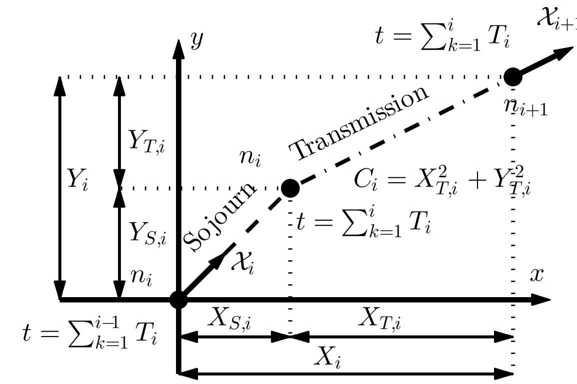

Fig. 1. The i-th stage of a journey.

We focus on a specific packet that must be delivered at a destination located at an infinite distance which, for simplicity, is taken to be in the direction of the positive x-axis. The packet can travel to its destination through a combination of physical transfers (involving only delay) and wireless transmissions (involving only cost). There is a clear tradeoff between delay and cost: the more a packet moves towards its destination using wireless transmissions instead of physical transfers, the higher the cost, but the lower the delay, and vice versa.

Without any loss of generality, we assume that the packet is following a **forwarding rule** under which the resulting **journey** is comprised of **stages**, which we index by $i = 1, 2, \ldots$. Each stage i consists of two parts: a **sojourn** at the buffer of a node n_i, that lasts for a **sojourn time** T_i, and a wireless **transmission**, from node n_i to node n_{i+1}. Observe that stage i lasts from time $\sum_{k=1}^{i-1} T_k$ until time $\sum_{k=1}^{i} T_k$. Let $\mathcal{X}_i$ be the direction of travel of node n_i. Let $X_{S,i}$ and $Y_{S,i}$ be the changes in the x and y coordinates of the packet due to its i-th sojourn. Let $X_{T,i}$ and $Y_{T,i}$ be the changes in the x and y coordinates of the packet due to its i-th transmission. Let $C_i = X_{T,i}^2 + Y_{T,i}^2$ be the **transmission cost** of the i-th stage. We also define the total changes in the x and y coordinates during the i-th stage as $X_i = X_{S,i} + X_{T,i}$ and $Y_i = Y_{S,i} + Y_{T,i}$ respectively. We refer to X_i as the **progress** towards the destination made at stage i. These definitions are summarized in Fig. 1.

Note that some stages will consist only of a transmission, i.e., a node will retransmit a packet the moment it receives it, and therefore for these stages $X_{S,i} = Y_{S,i} = T_i = 0$.

B. Normalized Delay and Cost

We define the **(normalized packet) delay** D_p and the **(normalized packet) cost** C_p of the forwarding rule as the following limits, provided they exist:

$$D_p \triangleq \lim_{n\to\infty} \frac{\sum_{i=1}^{n} T_i}{\sum_{i=1}^{n}(X_{S,i} + X_{T,i})}, \tag{1}$$

$$C_p \triangleq \lim_{n\to\infty} \frac{\sum_{i=1}^{n} C_i}{\sum_{i=1}^{n}(X_{S,i} + X_{T,i})} = \lim_{n\to\infty} \frac{\sum_{i=1}^{n}(X_{T,i}^2 + Y_{T,i}^2)}{\sum_{i=1}^{n}(X_{S,i} + X_{T,i})}. \tag{2}$$

The normalized packet delay D_p is the limit, as $n \to \infty$, of the total time it takes for the first n stages to complete divided by the progress towards the destination during these stages. The normalized packet cost is the limit, as $n \to \infty$, of the aggregate cost incurred during the first n stages divided by the progress made during these stages. They are measured in seconds per meter and units of cost (i.e., square meters) per meter respectively.

A few comments are in order. First of all, the two limits may not exist for some forwarding rules, for example forwarding rules that vary with time. However, we expect that the limits will exist for most simple, time-invariant rules. Establishing general conditions for their existence is outside the scope of this work. Secondly, the y-coordinates $Y_{S,i}$ and $Y_{T,i}$ do not appear in the common denominator of the two fractions that measures progress towards the destination. This is because offsets in the direction of the y-axis do not have a positive effect, as the destination is towards the direction of the positive x-axis. Thirdly, observe that the normalized packet delay is simply the inverse of the average speed, which might perhaps be a more intuitive figure of merit. We opt to use normalized packet delay for reasons of mathematical convenience.

The pair (D_p, C_p) describes the efficiency of the forwarding rule, and in this work we mainly calculate the (D_p, C_p) pairs achieved by two specific forwarding rules, introduced in the following section, for various choices of their parameters. The 'holy grail' problem coming out of this work is finding all Pareto optimal combinations (D_p, C_p) and the forwarding rules that achieve them. Based on the investigation conducted here, and related works [11], [17], we believe that this task is formidable, and we leave it for future work.

IV. Forwarding Rules

A. Rule I

Whenever a packet is created at or relayed to a node A, node A scans for other nodes that could act as relays for the packet within an area termed the **forwarding region** $\mathcal{F}$. $\mathcal{F}$ is a subset of $\mathbb{R}^2$ that is defined relative to node A, and moving with it. We assume that it is closed, bounded, and convex. For the forwarding rule to perform well, we expect that $\mathcal{F}$ must be placed, with respect to node A, towards the destination of the packet, i.e., the positive x-axis. We assume, therefore, that node A is outside $\mathcal{F}$, or at most on its boundary.

Node A surveys the nodes within $\mathcal{F}$ and then:

1) Let there be nodes in $\mathcal{F}$ whose directions of travel (with respect to the positive x-axis) are within $[-\chi_m, \chi_m]$, where $\chi_m \in (0, \pi]$ is a parameter termed the **maximum (angular) deviation**. Then, node A immediately transmits the packet to that node B among them with the smallest quotient C/x of transmission cost (from A to B) C over **instantaneous progress** $x = x_B - x_A$, where x_A and x_B are the current x-coordinates of A and B.
2) Otherwise, A will keep the packet until one such node B appears on the boundary of $\mathcal{F}$. A will then transmit the packet to node B.

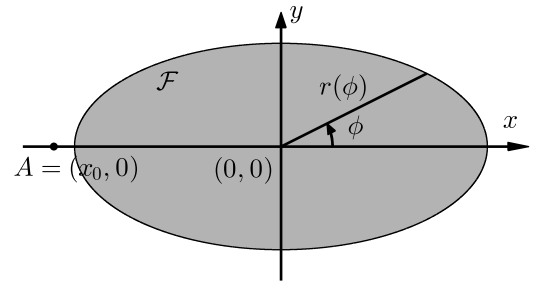

Fig. 2. Description of the forwarding region $\mathcal{F}$.

We refer to this rule as **Rule I**. As the name suggests, the instantaneous progress x is the distance towards the destination instantly covered by A sending the packet to B. Also, the motivation for restraining the direction of travel of a receiving node in the range $[-\chi_m, \chi_m]$ is clear: we want to avoid using nodes whose direction of travel is not sufficiently close to the direction of the destination. Finally, the motivation for selecting the node with the smallest C/x quotient is also clear: as we want to minimize the long term quotient of total cost over total progress, it makes sense to also minimize the cost over progress quotient of each hop.

Different instances of this forwarding rule differ on the choices of $\mathcal{F}$ and χ_m. We expect to be able to trade off D_p with C_p by tuning these two parameters. For example, we expect that as the area of $\mathcal{F}$ becomes larger, D_p becomes smaller but C_p becomes larger.

As shown in Fig. 2, we describe $\mathcal{F}$ in terms of a coordinate system whose origin $(0, 0)$ lies in the interior of $\mathcal{F}$, and such that the current packet holder A is located in $(x_0, 0)$. Let the continuous function $r(\phi)$ be such that $(r(\phi), \phi)$ traces the boundary of $\mathcal{F}$ in polar coordinates as ϕ goes from 0 to 2π.

B. Rule II

Rule I requires that all nodes acting as packet relays have a direction of travel within $[-\chi_m, \chi_m]$. However, within this range some directions are better than others. Also, when deciding on whether or not to relay a packet to a node, it is useful to consider the travel direction of the current holder as well.

Therefore, we create **Rule II** as follows: Firstly, we select as $\mathcal{F}$ a circular disk of diameter R centered at the location $\left(\frac{R}{2}, 0\right)$, with the current holder, i.e., node A, placed at the origin. Secondly, as soon as node A receives the packet, it starts to scan $\mathcal{F}$ until it finds a node B such that

$$\frac{C}{x} \leq \frac{R}{2} \times \left[1 + \frac{bv_{x,B} - av_{x,A}}{2v_n}\right].$$

While no such node exists, node A carries the packet in its buffer. If multiple nodes exist that satisfy this condition, the one with the smallest quotient C/x is selected as the next relay. In the above, $v_{x,B}$ and $v_{x,A}$ are the x-components of the velocities of nodes B and A respectively, and v_n is the node speed. The parameters $b, a \geq 0$ are used to tune the effects that $v_{x,B}$ and $v_{x,A}$ have on the decision, respectively. The condition that the direction of travel of all relays must be in $[-\chi_m, \chi_m]$ is not used.

Under Rule II, the larger $v_{x,A}$ is, the smaller is the right hand side of the new condition, and so the harder it becomes for B to satisfy it. Similarly, the larger $v_{x,B}$ is, the larger is the right hand side of the condition, and so the easier it becomes for B to satisfy it.

Regarding the shape of $\mathcal{F}$, we chose a circular disk due to its simplicity and because in a related, pure geographic, non-DTN routing setting [18] it was shown that the circular disk is the optimal shape for the forwarding region, given that $c(d) = d^2$. (We stress that we do not have a proof of the optimality of this shape in our setting.)

V. Statistics of the First Stage for Rule I

As a preliminary to the approximate calculation of the (D_p, C_p) pairs achievable by Rule I, which will be given in Section VI, in this section we focus on the *first* stage, i.e., the period between the creation of the packet and its first transmission to its first relay. So let a packet be created at time $t = 0$, and let A be its source node. According to our mobility model of Section III, node A, and hence also the forwarding region $\mathcal{F}$, are moving with speed v_n towards a random direction $\mathcal{X}_1$ which is uniformly distributed in $[-\pi, \pi]$.

We will calculate the expected values of the random variables $X_{S,1}, X_{T,1}, C_1, T_1$ conditioned on the event $\mathcal{X}_1 = \theta$, for $\theta \in [-\pi, \pi]$. We will calculate these by further conditioning on the events M and M', where M is the event that the forwarding region $\mathcal{F}$ is initially, i.e., at time $t = 0$, empty of nodes with a direction of travel within the $[-\chi_m, \chi_m]$ interval. We have

$$E(*|\mathcal{X}_1 = \theta) = E(*|\mathcal{X}_1 = \theta, M)P(M) + E(*|\mathcal{X}_1 = \theta, M')(1 - P(M)), \quad (3)$$

where the asterisk $*$ can be any of $X_{S,1}, X_{T,1}, C_1, T_1$. Observe that

$$P(M) = \exp\left(-\lambda'|\mathcal{F}|\right), \quad (4)$$

where we define $\lambda' \triangleq \frac{\chi_m}{\pi}\lambda$ and where $|\mathcal{F}|$ is the area of the forwarding region $\mathcal{F}$.

We will also calculate the conditional pdf $f_{\mathcal{X}_2|\mathcal{X}_1=\theta}(\chi)$ of the direction $\mathcal{X}_2$ of the first relay, i.e., the *second* node to hold the packet, also using conditioning on the event M, as follows:

$$f_{\mathcal{X}_2|\mathcal{X}_1=\theta}(\chi) = f_{\mathcal{X}_2|\mathcal{X}_1=\theta,M}(\chi)P(M) + f_{\mathcal{X}_2|\mathcal{X}_1=\theta,M'}(\chi)(1 - P(M)). \quad (5)$$

The aforementioned quantities that are conditional on the event M' are derived in Section V-A, and those conditional on the event M are derived in Section V-B.

A. $\mathcal{F}$ Is Initially Not Empty

In this case, we first note that the direction of travel of the first relay (i.e., the node that receives the packet from its holder) is uniformly distributed in the range $[-\chi_m, \chi_m]$, and independent of the value of $\mathcal{X}_1$, therefore

$$f_{\mathcal{X}_2|\mathcal{X}_1=\theta,M'}(\chi) = \begin{cases} \frac{1}{2\chi_m}, & |\chi| \le |\chi_m|, \\ 0, & |\chi_m| < |\chi| \le \pi. \end{cases} \quad (6)$$

Furthermore, in the event M', $X_{S,1} = T_1 = 0$, therefore we have

$$E(X_{S,1}|\mathcal{X}_1 = \theta, M') = E(T_1|\mathcal{X}_1 = \theta, M') = 0. \quad (7)$$

To calculate the remaining two expectations needed, i.e., $E(X_{T,1}|\mathcal{X}_1 = \theta, M')$ and $E(C_1|\mathcal{X}_1 = \theta, M')$, we first observe that $X_{T,1}$ and $Y_{T,1}$, and hence also $C_1 = X_{T,1}^2 + Y_{T,1}^2$, are independent of $\mathcal{X}_1 = \theta$. This significantly simplifies the calculations.

We will first consider the statistics of the quotient

$$Q = \frac{C_1}{X_{T,1}} = \frac{X_{T,1}^2 + Y_{T,1}^2}{X_{T,1}}.$$

Let $A(q)$ be the area of the subset of the forwarding region $\mathcal{F}$ for which $Q \le q$. Note that the locus of the points on the plane for which $Q \le q$ is a disk with center at $\left(\frac{q}{2}, 0\right)$ and radius $\frac{q}{2}$. Therefore, $A(q)$ is increasing with q, with $A(0) = 0$ and $A(\infty) = \lim_{q\to\infty} A(q) = |\mathcal{F}|$. Note that

$$P\left(q \le Q \le q + dq|M'\right) = \frac{P\left(q \le Q \le q + dq, M'\right)}{P(M')} = \frac{\exp\left(-\lambda' A(q)\right)\lambda' A'(q)dq}{1 - \exp\left(-\lambda'|\mathcal{F}|\right)},$$

where $A'(q)$ is the derivative of $A(q)$. It follows that the conditional distribution of Q is

$$f_Q(q) = \frac{\lambda' A'(q)}{1 - \exp\left(-\lambda'|\mathcal{F}|\right)} \exp\left(-\lambda' A(q)\right).$$

To find the expectations $E[X_{T,1}|\mathcal{X}_1 = \theta, M', Q = q]$ and $E[C_{T,1}|\mathcal{X}_1 = \theta, M', Q = q]$ we consider the locus of the points within $\mathcal{F}$ for which we have $\frac{x^2+y^2}{x} = q$, which is the intersection of $\mathcal{F}$ with a circle of radius $\frac{q}{2}$ centered at $\left(\frac{q}{2}, 0\right)$. Let $L(q)$ be the length of the locus, and let $x(s)$ and $y(s)$ be the parametrized coordinates of the locus where s expresses distance along the locus, with $s \in [0, L(q)]$. Therefore,

$$E[X_{T,1}|\mathcal{X}_1 = \theta, M', Q = q] = \int_0^{L(q)} \frac{x(s)}{L(q)}\, ds,$$

$$E[C_1|\mathcal{X}_1 = \theta, M', Q = q] = \int_0^{L(q)} \frac{x^2(s) + y^2(s)}{L(q)}\, ds.$$

The required conditional expectations can then be calculated using

$$E[X_{T,1}|\mathcal{X}_1 = \theta, M'] = \int_0^{\infty} E[X_{T,1}|\mathcal{X}_1 = \theta, M', Q = q] f_Q(q)\, dq, \quad (8)$$

$$E[C_1|\mathcal{X}_1 = \theta, M'] = \int_0^{\infty} E[C_1|\mathcal{X}_1 = \theta, M', Q = q] f_Q(q)\, dq. \quad (9)$$

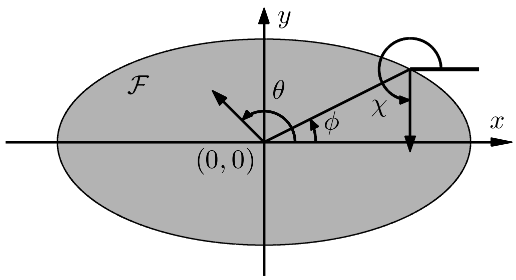

Fig. 3. An empty forwarding region $\mathcal{F}$ traveling towards direction $\mathcal{X}_1 = \theta$.

B. $\mathcal{F}$ is Initially Empty

Next, we calculate the expected values of $X_{S,1}$, $X_{T,1}$, C_1, T_1 and the distribution of $\mathcal{X}_2$, subject to $\mathcal{X}_1 = \theta$ and to the event M that $\mathcal{F}$ is initially empty.

Consider the following counting process $\{N_{\chi,\phi;\theta}(t),\ t \geq 0\}$: there is an arrival whenever a node enters $\mathcal{F}$ through the part of its boundary described by the range of angles $[\phi, \phi + d\phi]$ and that node has a direction of travel within the range $[\chi, \chi + d\chi]$, where $\phi, \chi \in [-\pi, \pi]$. All angles appear in Fig. 3.

Observe that arrivals of this process at non-overlapping time intervals are independent, as they are caused by nodes that at $t = 0$ existed in non-overlapping regions of $\mathbb{R}^2$. Furthermore, the number of arrivals in a time interval $[t_0, t_1]$ is Poisson distributed, with a rate proportional to the duration of the time interval $[t_1 - t_0]$. Indeed, the number of arrivals equals the number of nodes that existed at time t_0 in a region of space adjacent to $\mathcal{F}$ whose area is proportional to $t_1 - t_0$.) The number of nodes in that region is Poisson distributed, with a parameter proportional to the area of the region, and therefore proportional to the duration of the interval $t_1 - t_0$.

To conclude, *(i)* the number of arrivals in a time interval is Poisson distributed with a rate proportional to the duration of the time interval, and *(ii)* arrivals at non-overlapping time intervals are independent. It follows that the counting process $\{N_{\chi,\phi;\theta}(t), t \geq 0\}$ is Poisson [19]. Observe that the (yet unknown) arrival rate is incremental, due to the fact that we consider an incremental part of the boundary $[\phi, \phi + d\phi]$, and an incremental part of the node directions $[\chi, \chi + d\chi]$. Let $\gamma(\chi, \phi; \theta) d\chi d\phi$ be this rate.

Next, consider the counting process $\{N_{\chi;\theta}(t),\ t \geq 0\}$ of all nodes arriving with a direction within $[\chi, \chi + d\chi]$ at *any* part of the boundary of $\mathcal{F}$. By the additive property of Poisson processes (i.e., the summation of independent Poisson processes is also a Poisson process, with a rate equal to the sum of the rates of the constituent Poisson processes [19]) it follows that this process is also Poisson, with a rate $\gamma(\chi; \theta) d\chi$ such that

$$\gamma(\chi; \theta) = \int_{-\pi}^{\pi} \gamma(\chi, \phi; \theta)\, d\phi. \tag{10}$$

Also consider the counting process $\{N_{\phi;\theta}(t),\ t \geq 0\}$ of all nodes arriving at the boundary $[\phi, \phi + d\phi]$, but with *any* direction in $[-\chi_m, \chi_m]$. By the additive property of Poisson processes, this process is also Poisson. Let its rate be $\gamma(\phi; \theta) d\phi$. We must have

$$\gamma(\phi; \theta) = \int_{-\chi_m}^{\chi_m} \gamma(\chi, \phi; \theta)\, d\chi. \tag{11}$$

Finally, consider the counting process $\{N_\theta(t),\ t \geq 0\}$ of all nodes arriving at *any* point of the boundary with *any* direction in $[-\chi_m, \chi_m]$. Again, the new process is also Poisson, with some rate $\gamma(\theta)$. We must have

$$\gamma(\theta) = \int_{-\chi_m}^{\chi_m} \gamma(\chi; \theta)\, d\chi = \int_{-\pi}^{\pi} \gamma(\phi; \theta)\, d\phi. \tag{12}$$

We will refer to all rates $\gamma(\chi, \phi; \theta)$, $\gamma(\chi; \theta)$, $\gamma(\phi; \theta)$, and $\gamma(\theta)$ as **incidence rates**. Observe that, in order to keep the notation simple, we have used the same symbol, i.e., γ, for all of them. We will differentiate them by their arguments. The incidence rates are calculated in [8], and in the following we will assume they are known.

Having the incidence rates, we can calculate the conditional expectations $E(*|\mathcal{X}_1 = \theta, M)$ (where the asterisk $*$ is any of the random variables $X_{S,1}$, $X_{T,1}$, C_1, and T_1) as well as $f_{\mathcal{X}_2|\mathcal{X}_1=\theta,M}(\chi)$.

Indeed, let the random angle Φ be defined so that the location on the boundary of $\mathcal{F}$ where the first relay appears is $(r(\Phi), \Phi)$. Observe that the density of Φ conditional on $\mathcal{X}_1 = \theta$ is equal to

$$f_{\Phi|\mathcal{X}_1=\theta,M}(\phi) = \frac{\gamma(\phi; \theta)}{\gamma(\theta)}. \tag{13}$$

This is because the minimum of a number of exponential random variables is equal to one of them with probability equal to its rate over the sum of all rates [19]. Therefore,

$$E(X_{T,1}|\mathcal{X}_1 = \theta, M) = \int_0^{2\pi} \left[r(\phi) \cos\phi - x_0 \right] \frac{\gamma(\phi; \theta)}{\gamma(\theta)}\, d\phi, \tag{14}$$

$$E(C_1|\mathcal{X}_1 = \theta, M) = \int_0^{2\pi} \left[(r(\phi)\cos\phi - x_0)^2 + (r(\phi)\sin\phi)^2 \right] \frac{\gamma(\phi; \theta)}{\gamma(\theta)}\, d\phi. \tag{15}$$

Regarding the time T_1, observe that, as the counting process $\{N_\theta(t),\ t \geq 0\}$ is Poisson with rate $\gamma(\theta)$, T_1 is exponentially distributed with expectation

$$E(T_1|\mathcal{X}_1 = \theta, M) = \frac{1}{\gamma(\theta)}. \tag{16}$$

Furthermore, since $X_{S,1} = v_n T_1 \cos \mathcal{X}_1$, we have

$$E(X_{S,1}|\mathcal{X}_1 = \theta, M) = E(v_n T_1 \cos \mathcal{X}_1 | \mathcal{X}_1 = \theta, M) = v_n \cos\theta E(T_1|\mathcal{X}_1 = \theta, M) = \frac{v_n \cos\theta}{\gamma(\theta)}. \tag{17}$$

Finally, by an argument similar to that applied for deriving (13), observe that

$$f_{\mathcal{X}_2|\mathcal{X}_1=\theta,M}(\chi) = \frac{\gamma(\chi; \theta)}{\gamma(\theta)}. \tag{18}$$

C. Wrap-up

We can now calculate numerically the conditional expectations $E(*|\mathcal{X}_1 = \theta)$ of the random variables $X_{S,1}$, $X_{T,1}$, C_1, T_1, using the Eqns. (3) and (4) and the conditional expectations calculated in Sections V-A (Eqns. (7), (8), (9)) and V-B (Eqns. (14), (15), (16), (17)).

We can also calculate the conditional pdf $f_{\mathcal{X}_2|\mathcal{X}_1=\theta}(\chi)$ using Eqns. (4) and (5) together with Eqns. (6) and (18).

VI. Approximate (D_p, C_p) Calculations for Rule I

If the random vectors $(X_{S,i}, X_{T,i}, C_i, T_i)$ describing each stage i were independent and identically distributed, then it would be straightforward to calculate the normalized packet delay and cost using their definitions (1) and (2) along with the Strong Law of Large Numbers (SLLN). Indeed, by the SLLN it would follow that

$$\lim_{n\to\infty} \frac{\sum_{i=1}^n T_i}{n} = E(T_1), \qquad \lim_{n\to\infty} \frac{\sum_{i=1}^n C_i}{n} = E(C_1),$$
$$\lim_{n\to\infty} \frac{\sum_{i=1}^n (X_{S,i} + X_{T,i})}{n} = E(X_{S,1}) + E(X_{T,1}),$$

from which we would have

$$D_p = \frac{E(T_1)}{E(X_{S,1}) + E(X_{T,1})}, \; C_p = \frac{E(C_1)}{E(X_{S,1}) + E(X_{T,1})}. \tag{19}$$

However, for our forwarding rule, we do not expect these random vectors to be either independent or identically distributed. For example, whereas $\mathcal{X}_1$ is uniformly distributed in $[-\pi, \pi]$, the distribution of $\mathcal{X}_2$ (given by integrating the conditional distribution (5) over the uniform distribution of $\mathcal{X}_1$) is zero outside $[-\chi_m, \chi_m]$. Likewise, the distribution of $\mathcal{X}_3$ will not in general be the same as the distribution of $\mathcal{X}_2$, and so on. As the statistics of $X_{S,i}$, $X_{T,i}$, C_i and T_i all depend strongly on the distribution of $\mathcal{X}_{i-1}$, we expect that the vectors $(X_{S,i}, X_{T,i}, C_i, T_i)$ are not identically distributed. These vectors are also not independent; for example, a long sequence of zero sojourn times, i.e., $T_k = T_{k+1} = \cdots = T_{k+m} = 0$ for some $k, m > 0$ with m large, suggests that there are many nodes in the vicinity of node n_{k+m+1}, which means that the conditional expected time $E[T_{k+m+1}|T_k = T_{k+1} = \cdots = T_{k+m} = 0]$ will be smaller than the unconditional expected time $E[T_{k+m+1}]$.

The fact that the vectors $(X_{S,i}, X_{T,i}, C_i, T_i)$ are neither independent nor identically distributed significantly complicates the analysis. Intuitively, we expect that for most reasonable selections of the forwarding region and maximum deviation χ_m, the correlation across the stages diminishes fast enough so that the SLLN approximately holds. Even if this is indeed the case, we need to find the expected values of the components $(X_{S,i}$, $X_{T,i}$, C_i, $T_i)$, which is also not a simple task.

Motivated by these observations, we introduce the following approximation:

First Order Approximation: *We assume the vectors $(X_{S,i}, X_{T,i}, C_i, T_i)$ to be iid, and so we use Eqns. (19) to calculate D_p and C_p. However, these vectors are now distributed according to the vector $(X_{S,1}, X_{T,1}, C_1, T_1)$ when the direction $\mathcal{X}_1$ that the first node has is distributed according to the limiting distribution $g(\chi) = \lim_{i\to\infty} f_{\mathcal{X}_{i+1}}(\chi)$ arrived at by the following iteration, provided the limit exists:*

$$f_{\mathcal{X}_{i+1}}(\chi) = \int_{-\chi_m}^{\chi_m} f_{\mathcal{X}_2|\mathcal{X}_1=\theta}(\chi) f_{\mathcal{X}_i}(\theta)\, d\theta. \tag{20}$$

Intuitively, this approximation can be explained as follows: assume that whenever a hop is made, the process describing the movement of the nodes restarts, with the exception of a single piece of information (hence the name of the approximation), which is the direction of travel of the node that received the packet. Under this approximation, the distributions $f_{\mathcal{X}_i}(\chi)$ of the directions of travel $\mathcal{X}_i$ will evolve according to the given formula (20). We use their limit, $g(\chi) = \lim_{i\to\infty} f_{\mathcal{X}_{i+1}}(\chi)$, to find the statistics of all other random variables of interest.

As the simulations of the next section show, the First Order Approximation leads to numerical results for the normalized packet delay and cost that closely match those results found by simulating the network. In any case, finding a better approximation, or avoiding approximations altogether, is clearly important and is the subject of future work.

VII. Results

In this section we calculate the (D_p, C_p) pairs achieved by our forwarding rules, using the analysis of Sections V and VI for Rule I as well as simulations, for both Rules I and II. It is possible to arrive at analytical results for Rule II, but the derivations are lengthy, and we omit them due to space constraints.

For Rule I, we limit ourselves to the case where the forwarding region $\mathcal{F}$ is the one used by Rule II, i.e., a circular disk of radius R with the current holder of the packet lying on its circumference opposite to the direction of the packet destination. Therefore, we have two parameters available for trading off cost with delay: the radius R of the disk $\mathcal{F}$ and the maximum deviation χ_m. For each pair of values (R, χ_m), there is a corresponding pair (D_p, C_p).

In Fig. 4 the parameters chosen are $v_n = 1$ and $\lambda = 1$. The radius of the forwarding region ranges from $R = 0$ to $R = 5$, and the maximum deviation ranges from $\chi_m = 0$ to $\chi_m = \pi$. In the plot, we have drawn a total of 30 dotted lines, each line showing the evolution of the delay-cost pair as χ_m is fixed but R increases from $R = 0$ to $R = 5$. The values of χ_m used are $\chi_m = \frac{i\pi}{30}$, $i = 1, \ldots, 30$.

As expected, increasing the radius R leads to a decrease of the normalized packet delay (as sojourns become shorter), and an increase of the normalized packet cost (as transmissions become more frequent). The effects of increasing the maximum deviation χ_m are mixed. When the radius is large, it is best to use large values of χ_m. The intuitive explanation is that, since the packets mostly rely on wireless transmissions, the direction a node is moving is not so important, and so it makes sense to use all nodes available, so that the transmission costs are minimized. On the other hand, when the radius R is small, it is best that small values of χ_m are used. Indeed, when R

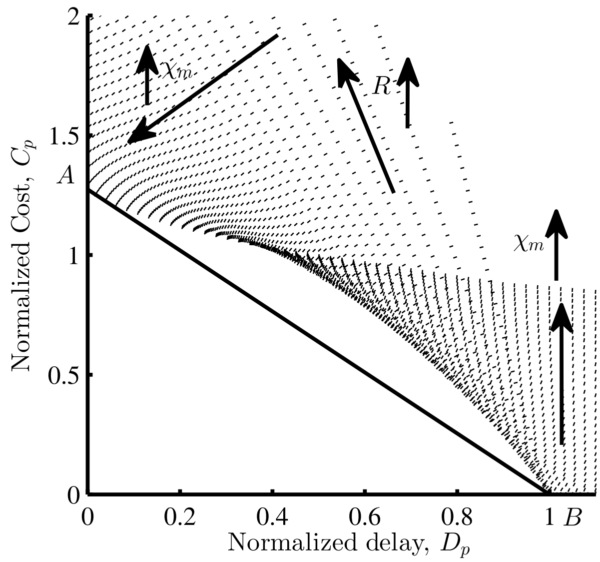

Fig. 4. Delay-Cost plots of Rule I for the case of the circular disk.

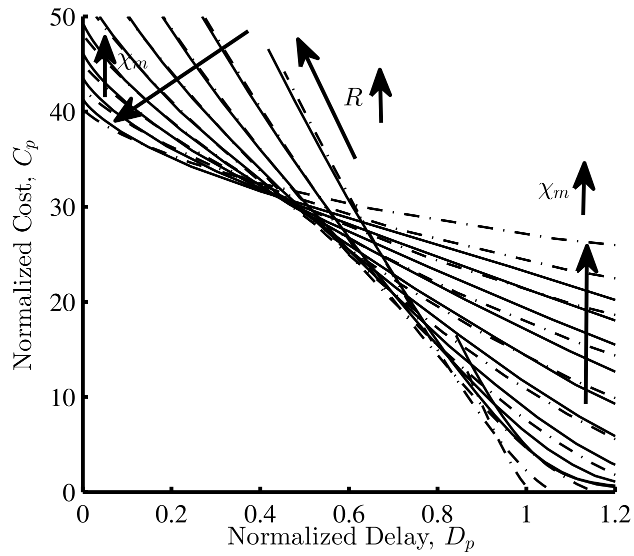

Fig. 5. Simulation (continuous lines) vs. analysis (dash-dotted lines), Rule I.

is small, packets travel to their destination mostly by physical transfer, spending (on the average) a lot of time at each relay, so it is best to avoid relays traveling in the wrong direction.

Two points on the delay-cost plane are of particular interest. The first is the point $B = (1, 0)$ achieved when $R \to 0$ and $\chi_m \to 0$. In this limiting case, our forwarding rule dictates that the packet should find a node moving in the exact direction of the destination, and stay with that node forever, thus traveling with a cost $C_p = 0$ and a delay $D_p = \frac{1}{v_n} = 1$. The second point is the one to which the pair (D_p, C_p) converges as the radius $R \to \infty$, for $\chi_m = \pi$. It is numerically established that this point $A \simeq (0, 1.2732)$. Intuitively, in this limit the packet aggressively gets transmitted from node to node, without being physically transferred at all.

An interesting result is that no combination of parameters R and χ_m leads to a delay-cost pair below the line connecting points A and B. This line is also plotted in Fig. 4, and describes the set of delay-cost pairs achievable by a time division between the two extreme strategies associated with each of these points. Therefore, if we limit ourselves to Rule I, and are only free to select R and χ_m, it is optimal to use time division between the two extreme strategies.

In order to measure the effects of the First Order Approximation, we also evaluated Rule I by simulation. We placed N nodes in a square torus of side L, moving along straight lines with speed v_n, for time T. At the start of the simulation each node has a single packet that must travel towards a distinct direction, different for each packet. We execute Rule I for all packets, and calculate the resulting average (over all packets) normalized packet delay and cost.

In Fig. 5 we plot 10 resulting delay-cost curves. We set $N = 1000$ and $L = 1000$, so that $\lambda = 0.001$, and also $v_n = 1$, $T = 1000$. Each curve is created by keeping χ_m fixed and increasing R from $R = 5$ to $R = 100$. The values of χ_m used are $\frac{\pi}{10}$, $\frac{2\pi}{10}, \ldots, \pi$. We also plot the delay-cost curves derived using the analysis, for the same parameters. The two sets of curves match particularly well in the low-delay regime. In the high-delay regime, the match is not as good, but we attribute this mostly to the fact that the simulation results are based on smaller numbers of hops, and so are not as accurate. However, the overall match between simulation and analysis is encouraging.

Finally, we present simulation results for Rule II, and for the same set of parameters. In Fig. 6 we plot the following, for the environment of Fig. 5: *(i)* the Pareto optimal curve of Rule I, over all combinations of the parameters R, χ_m using analysis, *(ii)* the same Pareto optimal curve using simulations, and finally *(iii)* the Pareto optimal curve of Rule II, over all combinations of the parameters a, b, R, using simulations. The Pareto optimal curves of each protocol describe all delay-cost pairs that are achievable for some combination of parameters, and for which there is no achievable delay-cost pair with strictly better delay *and* cost. The Pareto optimal curves are calculated numerically, by an exhaustive search over the complete parameter space of each case.

Rule II performs better than Rule I, due to the more refined manner in which it selects relays. In fact, part of the Pareto optimal curve of Rule II lies below the time division curve connecting the points A and B. Therefore, there are choices for the parameters of Rule II such that this rule performs better than any time division between the two strategies corresponding to points A and B.

VIII. Conclusions and Future Work

Our contribution is twofold: Firstly, we introduce a novel, abstract stochastic geometry setting that is amenable to analysis and captures the fundamental tradeoff that exists in all mobile wireless delay-tolerant networks between the packet

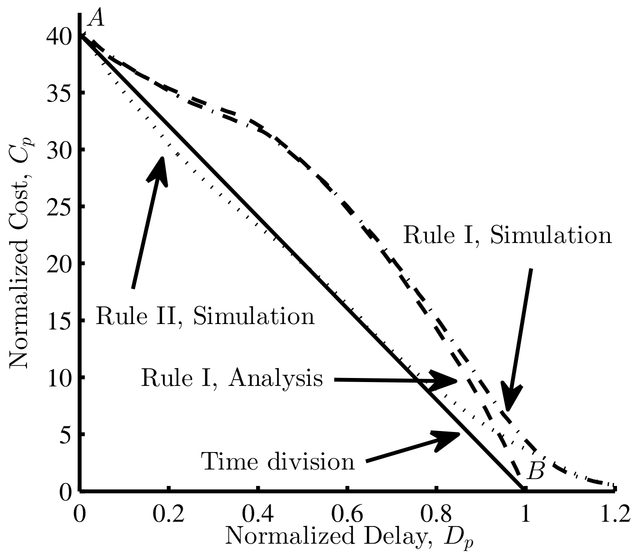

Fig. 6. Pareto optimal curves of Rule I (analysis and simulation) and Rule II (simulation). The points achieved by time divisions between the two extreme limits ($R \to \infty$, $\chi_m \to \pi$) and (R, $\chi_m \to 0$) are also shown.

delivery delay (expressed by the normalized packet delay D_p) and the aggregate transmission cost (expressed by the normalized packet cost C_p). This tradeoff has already been identified in the study of numerous protocols proposed recently that combine geographic with delay-tolerant routing [1], [2], [3], [4], [5], [6], [7].

Secondly, we apply this framework to calculate D_p and C_p for Rule I, by both analysis and simulation, and Rule II, by simulation. Due to the high complexity of the problem, an approximation is used in the analysis, but simulations show that this approximation affects our analytical results only modestly. As expected, Rule II, which takes into account the directions of travel of the current packet holder and the relay candidates, is superior. It is interesting to note, however, that, although this rule is quite elaborate, it fails to achieve delay-cost pairs that are significantly better than pairs that are achieved by a rudimentary time division between two simplistic strategies, one based only on physical transport and one based only on wireless transmissions. In light of this result, designers of related practical protocols should verify that any elaborate rule they propose will indeed perform better than time divisions similar to the aforementioned.

Our future work has two, to some extent conflicting directions. The first one is towards bringing our abstract stochastic setting closer to reality. For example, it is possible to perform analysis using *(i)* alternative cost models that better capture reality and *(ii)* more realistic mobility models under which nodes occasionally change their speed and direction of travel, selecting them from a (possibly non-uniform) distribution that is derived from the underlying environment.

The second direction is towards strengthening our analytical results. For example, calculating analytically, for a wider range of forwarding rules, the (D_p, C_p) pairs they achieve, either without approximations (which appears to be a formidable task unless we consider very specific cases of forwarding rules), or approximations less strong than the one adopted here. The ultimate goal is to find all (D_p, C_p) combinations that are Pareto optimal over *all* forwarding rules.

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