Pricing for Distributed Resource Allocation in MAC without SIC under QoS Requirements with Malicious Users

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Abstract—**We develop the noncooperative game with individual pricing for the general multiple access channel (MAC) system without successive interference cancellation (SIC). Each user allocates its own power by optimizing the individual utility function with clever price adaptation. We show that by the proposed prices, the best response (BR) power allocation of each user converges rapidly. The individual prices are proposed such that the Shannon rate-based quality-of-service (QoS) requirement of each user is achieved at the unique Nash equilibrium (NE) point. We analyse different behavior types of the users, especially the malicious behavior and the resulting NE power allocation and achievable rates of all the users with malicious users. We illustrate the convergence of the BR dynamic and the Price of Malice (PoM) by numerical simulations.**

I. Introduction

In future heterogeneous dense wireless networks, the quality-of-service (QoS) requirements can be controlled with resource allocation via distributed pricing. We consider the distributed pricing framework for the general multiple access channel (MAC). In this context, the MAC consists of several mobile transmitters and a base station (BS). With today's high demand of the data communications, the QoS of each user in the wireless system becomes the main concern. In the present work, we develop the physical layer resource allocation, namely the power allocation, in order to satisfy the QoS requirement of each user in the MAC [4]. The system is distributed in the sense that each user allocates its own power by optimizing the individual utility. The distributed pricing is adopted in the individual utility function of each user such that the QoS requirement is achieved at the Nash equilibrium (NE) power allocation.

There are previous works concerning distributed resource allocation for different scenarios. In [1], the authors consider a distributed power control scheme for wireless ad hoc networks, in which each user announces a price that reflects compensation paid by other users for their interference. The MAC game models are discussed in [2] in which each transmitter makes individual decisions regarding their power level or transmission probability. The authors in [3] address the efficient distributed power control via convex pricing of users' transmission power in the uplink of CDMA wireless networks supporting multiple services. The CDMA power control as a noncooperative game is also discussed in [4], where a cost function is introduced as the difference between the pricing and utility functions. A game-theoretic approach is discussed in [5] for power control in ad-hoc networks. In [6], the power control and beamformer design is investigated for interference networks, based on the exchange of interference prices. A set of prices corresponding to all degree of freedoms (DoFs) must be exchanged to achieve the centralized optimal allocation.

The behavior of users on networked systems ranges from altruistic on the one end to malicious (adversarial) on the other end. While altruistic users aim to improve the overall network performance, selfish users develop strategies to maximize their own throughput and obtain a disproportionate share of resources. A malicious user, on the other hand, aims to disrupt the whole network. Malicious behavior may be due to the users inherent maliciousness or in competitive scenarios where the loss of a competing user will likely result in future gains for oneself. Well-known examples of such adversarial behavior include jamming in wireless networks and denial-of-service (DoS) attacks [7]-[8]. In this paper, we model the coexistence of selfish and malicious players by introducing an overarching noncooperative game-theoretic framework. Specifically, we adopt a pricing mechanism approach in which a set of rules and incentives [9] are used to control the outcome of the underlying game between the players. The malicious user submits QoS requirement like the regular users in order not to get detected but computes the best response power strategies for a modified utility function with the goal to harm the other links and cause interference to others.

In our work, we develop the distributed power allocation with individual pricing for the general MAC system without successive interference cancelation (SIC). Each user has a rate-based QoS requirement, which is guaranteed through the noncooperative game with the given prices. We show that by the proposed prices, the best response (BR) power of each

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user converges rapidly. The QoS requirement of each user is achieved at the NE point. The pricing should be given such that the BR power converges to achieve the QoS requirement of each user and the malicious behavior of the users is prevented. The behavior types of the users are analysed, especially the malicious behavior and the resulting NE power allocation and achievable rates of all the users with malicious player. The strategy-proof mechanism is designed with the punishment prices. Numerical results illustrate the sum power needed to support a given number of users and the Price of Malice (PoM) as a function of the number of the malicious users.

The paper outline is as follows. In Sec. II, the system and channel model are discussed. The noncooperative game with individual pricing is analyzed in Sec. III, where the BR and NE power and the pricing to ensure the QoS requirement of each user are proposed in detail. The malicious behavior of the users are investigated in Sec. IV with the private types. The resulting BR and NE power with private types are obtained. The numerical illustrations in Sec. V show that the BR dynamic of the proposed noncooperative game converges rapidly to the NE power, which guarantees the rate requirement of each user.

II. Preliminaries

A. System Model

We study the general MAC with K transmitters and one receiver as the BS. All the transmitters and the BS are equipped with single antenna. In the rest of the paper, we will not differentiate users and transmitters. Each user $i \in K$ has a rate requirement $\underline{u}_i$ to be guaranteed by the MAC system. The linear receiver without SIC is considered, therefore each user suffers from the interference of all the other users. We assume the system guarantees the rate requirement of each user by providing the individual prices β_i. The non-cooperative game is discussed in the system, where each user maximizes its own utility u_i as a function of the price β_i. We introduce the prices $\boldsymbol{\beta}$ such that the feasible rate requirement of each user can be achieved at the NE point of the non-cooperative game with minimum power allocation. The strategy set of each user is their power allocation with single power constraint $p_i < p_i^{max}$.

B. Channel Model

The received signal at the BS is given by $y = \sum_{k=1}^{K} h_k x_k + n$, where x_k is the transmit signal of user k, n is the additive white Gaussian noise with zero-mean and variance of σ_n^2. The channel gain of user k is denoted by $\alpha_k = \mid h_k \mid^2$.

We assume the quasi-static block flat-fading channel gains $\boldsymbol{\alpha} = \alpha_1, \ldots, \alpha_K$ are independent of each other and remain constant for a sufficiently long period of time.

The Shannon rates are considered as the QoS criterion. For the MAC system described above, the achievable rate of each user is

$$r_i(p_i, p_{-i}) = \log\left(1 + \frac{\alpha_i p_i}{1 + \sum_{k\neq i} \alpha_k p_k}\right),$$

where $p_{-i} = [p_1, \cdots, p_{i-1}, p_{i+1}, \cdots, p_K]$ denotes the power allocation of all the other users except user i.

III. System Operation with Truthful Agents

The noncooperative game of the MAC system can be formulated as an economic model, where the consumers are the users. The trading good is the power. And the producer provides the individual prices β_i. Since each user has a rate requirement $\underline{u}_i$ to be guaranteed and the interferences are coupled among all the users, the demand in power of each user is dependent on the other users. In order to better illustrate the properties of the model, we introduce the normalized distributed pricing term $\beta_i(p_{-i})$ as a function of the individual price β_i and the demand of all the other users $F(p_{-i})$, i.e.,

$$\beta_i(p_{-i}) = \frac{\beta_i}{F(p_{-i})}. \tag{1}$$

The utility function of each user is based on the achievable rate $r_i(p_i, p_{-i})$ and the normalized pricing term as follows.

$$u_i(p_i, p_{-i}) = r_i(p_i, p_{-i}) - \beta_i(p_{-i}) p_i. \tag{2}$$

When there is single link, i.e., no interference is presented, the utility function is $u_i(p_i, p_{-i}) = r_i(p_i, p_{-i}) - \beta_i p_i$. In the multiuser case, the interference obviously influences the quality of the good (resource) that user i buys. In order to express the quality loss due to interference, the higher interference, the lower the pricing term, and thus the more power allocation. Therefore, the pricing term $\beta_i(p_{-i})$ is normalized by the noise plus interference caused by all the other users.

$$\begin{aligned} u_i(p_i, p_{-i}) &= r_i(p_i, p_{-i}) - \frac{\beta_i}{1 + \sum_{k\neq i} \alpha_k p_k} p_i \quad (3) \\ &= \log\left(1 + \frac{\alpha_i p_i}{1 + \sum_{k\neq i} \alpha_k p_k}\right) \\ &\quad - \frac{\beta_i}{1 + \sum_{k\neq i} \alpha_k p_k} p_i. \quad (4) \end{aligned}$$

The normalized pricing term denotes the quality of the good. If the interference from other users is high, then the price of the power for user i should be lower in order to achieve the rate requirement.

The game in normal form $\mathcal{G}$ is described by the K players. Their strategy space is $[0, p_i^{max}]$ and their utility function is $u_i(p_i, p_{-i})$. The price controls the trade-off between maximizing the achievable rate and saving as much power as possible. The game best response dynamic (BRD) can be expressed as the K coupled problems

$$\begin{aligned} \max_{p_i} \quad & u_i(p_i, p_{-i}) = r_i(p_i, p_{-i}) - \frac{\beta_i}{1 + \sum_{k\neq i} \alpha_k p_k} p_i \\ s.t. \quad & 0 < p_i < p_i^{max} \qquad \forall i = 1, \ldots, K. \end{aligned} \tag{5}$$

A basic result from game-theory is that each fixed point of the BRD is an NE of the game, although in general, convergence of the BRD is not guaranteed, nor is the existence of the fixed point.

A. Best Response Power Allocation

If the prices $\beta_i = 0$, transmitting with full power p_i^{max} is the BR of each user. Due to the pricing term for positive $\beta_i > 0$, we can conclude the first result as follows.

Proposition 1: For all $i = 1, \dots, K$, define $\underline{p}_i$ as

$$\underline{p}_i(p_{-i}) = \left(\frac{1}{\beta_i} - \frac{1}{\alpha_i}\right)(1 + \sum_{k \neq i} \alpha_k p_k). \tag{6}$$

The i-th user's best-response is given by $p_i^{BR} = \max(0, \min(\underline{p}_i, p_i^{max}))$. Moreover, the noncooperative game $\mathcal{G}$ always admits an NE $\{p_i^{BR}\}_{i=1}^{K}$.

Proof: Solve the first derivative of $u_i(p_i, p_{-i})$ to be zero with respect to p_i.

$$\begin{aligned}\frac{\partial u_i(p_i, p_{-i})}{\partial p_i} &= \frac{\alpha_i}{1 + \sum_{k \neq i} \alpha_k p_k + \alpha_i p_i} - \frac{\beta_i}{1 + \sum_{k \neq i} \alpha_k p_k} \\ &= 0. \end{aligned} \tag{7}$$

The positive result $\underline{p}_i$ is achieved in (6) if $\beta_i < \alpha_i$. Otherwise it is set to zero to avoid negative power.

The second derivative of $u_i(p_i, p_{-i})$ with respect to p_i is

$$\frac{\partial^2 u_i(p_i, p_{-i})}{\partial p_i^2} = \frac{-\alpha_i^2}{(1 + \sum_{k \neq i} \alpha_k p_k + \alpha_i p_i)^2} < 0. \tag{8}$$

By observing that the strategy set of each user is a compact and convex set, $u_i(p_i, p_{-i})$ is a continuous function with respect to the powers of all users, and concave with respect to p_i, which implies the existence of at least one NE. ∎

B. Nash Equilibrium Power Allocation

The noncooperative game $\mathcal{G}$ always admits at least one NE power allocation $\{p_i^{BR}\}_{i=1}^{K}$. In this part, we figure out the NE point and show that it is unique.

Proposition 2: The Nash equilibrium power allocation of each user i in the noncooperative game $\mathcal{G}$ in the general MAC system is $p_i^{NE} = \max(0, \min(\underline{p}_i^{NE}, p_i^{max}))$. With given individual prices β_i,

$$\underline{p}_i^{NE} = \frac{\alpha_i - \beta_i}{\alpha_i^2} \cdot \frac{1}{\sum_{j=1}^{K} \frac{\beta_j}{\alpha_j} - K + 1}. \tag{9}$$

The noncooperative game $\mathcal{G}$ always admits this unique NE point.

Proof: Please refer to Appendix 1. ∎

C. Pricing for QoS Requirements

As shown in [10], the power allocation to achieve the rate requirement $\underline{u}_i$ of each user is

$$p_i^U = \frac{B_K}{\alpha_i} \cdot \frac{2^{\underline{u}_i} - 1}{2^{\underline{u}_i}}, \tag{10}$$

where $B_K = \frac{1}{\sum_{j=1}^{K} \frac{1}{2^{\underline{u}_j}} - K + 1}$ is a constant for given $\underline{u}_j, j = 1, \cdots, K$.

In order to determine the individual prices, $\underline{p}_i^{NE}$ should be equal to p_i^U. Therefore, we solve the universal individual prices for the distributed MAC as follows.

Lemma 1: In the K-user non-cooperative game $\mathcal{G}$ of the general MAC system, the rate requirement $\underline{u}_i$ of each user i is achieved with the NE power allocation p_i^{NE} if the individual price is

$$\beta_i = \frac{\alpha_i}{2^{\underline{u}_i}}. \tag{11}$$

Proof: Solve the equation $\underline{p}_i^{NE} = \frac{\alpha_i - \beta_i}{\alpha_i^2} \cdot \frac{1}{\sum_{j=1}^{K} \frac{\beta_j}{\alpha_j} - K + 1} = p_i^U = \frac{B_K}{\alpha_i} \cdot \frac{2^{\underline{u}_i} - 1}{2^{\underline{u}_i}}$ for β_i. ∎

In order to ensure the positive power allocation and therefore to guarantee the rate requirement of each user, the following conditions regarding the number of users in the wireless system, the individual prices and the channel states should be fulfilled.

Corollary 1: In the general K-user MAC system without SIC, the rate requirement of each user i is achieved by the BRD power allocation if and only if

$$K - 1 < \sum_{i=1}^{K} \frac{\beta_i}{\alpha_i} < K. \tag{12}$$

Proof: The proof is obtained by guaranteeing $\underline{p}_i^{NE} > 0$ in (9). ∎

Remark 1: The region in (12) is equivalent to the feasible utility region in Corollary 1 in [10], if the individual prices β_i are given in (11).

IV. Malicious Behavior

We define V_i to denote the private type [11] of users in the system, where

- Malicious users, $0 < V_i \leq 1$,
- Selfish users, $V_i = 0$,
- Altruistic users, $-1 \leq V_i < 0$.

The private type V_i of each user i is a continuous value between $[-1, 1]$, which denotes the extent of its behavior. For example, if user i's private type is $V_i = 1$, then it is an extreme malicious user and if $V_i = -1$, then it is an extreme altruistic user. Since each user i in the noncooperative game $\mathcal{G}$ has the individual rate requirement $\underline{u}_i$ to be achieved besides maximizing its utility function $u_i(p_i, p_{-i}, V_i)$, altruistic users who benefits the other users are not concerned in the current model. Later on, we focus on considering the malicious behavior with the private types V_i.

Let the normalized noise plus interference to user i caused by all the other users be $I_i(p_{-i}) = 1 + \sum_{k \neq i} \alpha_k p_k$. The utility function of each user with type V_i is defined as

$$u_i = r_i(p_i, p_{-i}) - \frac{\beta_i}{I_i(p_{-i})} p_i - V_i \sum_{j \neq i} r_j(p_j, p_{-j}) \tag{13}$$

$$\begin{aligned} &= r_i(p_i, p_{-i}) - V_i \Big(\sum_{j \neq i} \log(1 + \sum_{l=1}^{K} \alpha_l p_l) \\ &\quad - \log(1 + \sum_{l \neq j} \alpha_l p_l) \Big) - \frac{\beta_i}{I_i(p_{-i})} p_i, \end{aligned} \tag{14}$$

where the first term in (13) is its own achievable rate, the second term is the pricing term and the third term is its influence on the other users. For malicious users, they get benefit from harming all the other users. Since $V_i > 0$, the best response of the malicious user is different from the selfish users. The malicious behavior and its influence on the resulting NE power allocation is interesting and necessary for the mechanism design. In (14), the sum rate term $\log(1+\sum_{l=1}^{K}\alpha_l p_l)$ is not the target of the malicious user because this will also harm its own rate. Therefore, the utility of the malicious user is focused on the other terms. The utility function is modified to

$$\begin{aligned} u_i &= r_i(p_i,p_{-i}) - \frac{\beta_i}{I_i(p_{-i})}p_i \\ &\quad + V_i\left(\sum_{j\neq i}\log(1+\alpha_i p_i + \sum_{l\neq j,i}\alpha_l p_l)\right) \\ &= r_i(p_i,p_{-i}) + V_i\Big(\sum_{j\neq i}\log\Big((1+\sum_{l\neq j,i}\alpha_l p_l) \\ &\quad (1+\frac{\alpha_i p_i}{1+\sum_{l\neq j,i}\alpha_l p_l})\Big)\Big) - \frac{\beta_i p_i}{I_i(p_{-i})}. \end{aligned} \tag{15}$$

Since $1+\sum_{l\neq j,i}\alpha_l p_l$ is independent of the power of the malicious user p_i, it is skipped further on in order to simplify the calculation. The second term in (15) becomes $V_i\left(\sum_{j\neq i}\log(1+\frac{\alpha_i p_i}{1+\sum_{l\neq j,i}\alpha_l p_l})\right)$. Instead of the complicated $\sum_{j\neq i}\log$ term, we linearize the maximization problem as follows. The final utility function of each user with the private types V_i is

$$u_i(p_i,p_{-i},V_i) = r_i(p_i,p_{-i}) - \frac{\beta_i}{I_i(p_{-i})}p_i + \frac{V_i\alpha_i p_i}{I_i(p_{-i})}, \tag{16}$$

where the third term reflects the interference to all the other users after linearization.

A. Best Response Power Allocation with Malicious User

Follow a similar procedure as in Section III, we obtain the distributed BR power allocation of each user with private types V_i and NE point with malicious users in the system.

Lemma 2: For all $i = 1,\ldots,K$ with type V_i from (16), define $\underline{p}_i(V_i)$ as

$$\underline{p}_i(V_i) = \left(\frac{1}{\tilde{\beta}_i(V_i)} - \frac{1}{\alpha_i}\right)(1+\sum_{k\neq i}\alpha_k p_k). \tag{17}$$

Here $\tilde{\beta}_i(V_i) = \beta_i - V_i\alpha_i$ is the individual price with type V_i. The i-th user's best-response with type V_i is given by $p_i^{BR}(V_i) = \max(0,\min(\underline{p}_i(V_i), p_i^{max}))$.

Proof: Solve $\underline{p}_i(V_i)$ for the first derivative $\frac{u_i(p_i,p_{-i},V_i)}{\underline{p}_i} = 0$ from (16). ∎

We observe that the BR power allocation of the malicious user is higher than if it was a regular user, i.e., $\underline{p}_j^m > \underline{p}_j^S$. Since all the selfish users only optimize their utility function in (5), their BR power allocation p_i^S remains as in (10), and it is exactly $\underline{p}_i(V_i)$ in (17) when $V_i = 0$.

B. Nash Equilibrium Power Allocation with Malicious User

Next, we analyse the NE power allocation of the noncooperative game $\mathcal{G}$ with private type V_i. From (9), we can conclude the following result.

Proposition 3: The Nash equilibrium power allocation of each user i in the noncooperative game $\mathcal{G}$ in the general MAC system with private type V_i is $p_i^{NE}(V_i) = \max(0,\min(\underline{p}_i^{NE}(V_i), p_i^{max}))$, where given the individual prices $\tilde{\beta}_i(V_i)$ with type V_i,

$$\underline{p}_i^{NE}(V_i) = \frac{\alpha_i - \tilde{\beta}_i(V_i)}{\alpha_i^2}\cdot\frac{1}{\sum_{j=1}^{K}\frac{\tilde{\beta}_j(V_j)}{\alpha_j} - K + 1}. \tag{18}$$

The noncooperative game always admits this unique NE point.

Proof: The proof follows the same steps as in the Appendix by replacing the individual price β_i with $\tilde{\beta}_i(V_i)$. ∎

In order to understand the influence of the malicious behaviour on the resulting NE power and the rate of both the selfish and malicious users comprehensively, we have the following Proposition.

Proposition 4: With the individual price $\beta_i = \frac{\alpha_i}{2^{\underline{u}_i}}$, the Nash equilibrium power allocation $p_i^{NE}(V_i)$ of each user i in the noncooperative game $\mathcal{G}$ in the general MAC system with private type V_i is higher than or equal to p_i^U in (10), where

$$\underline{p}_i^{NE}(V_i,V_{-i}) = \frac{1+V_i-2^{-\underline{u}_i}}{\alpha_i}\frac{1}{\sum_{j=1}^{K}(2^{-\underline{u}_i}-V_j)-K+1}. \tag{19}$$

The resulting rate $r_i(V_i)$ is

- $r_i(V_i) = \underline{u}_i$, for selfish users with $V_i = 0$
- $r_i(V_i) > \underline{u}_i$, for malicious users with $0 < V_i \leq 1$.

Proof: Insert $\tilde{\beta}_i(V_i) = \beta_i - V_i\alpha_i$ with $\beta_i = \frac{\alpha_i}{2^{\underline{u}_i}}$ into (18), then (19) is proved. It can be observed that the second term in (19) is a constant for all the users with the given type V_j and it is larger if there exists at least one user with $V_i > 0$. If all the users are selfish, $\underline{p}_i^{NE}(V_i,V_{-i}) = p_i^U$, which is the minimum power allocation in order to achieve the rate requirement $\underline{u}_i$ of each user i. The power $\underline{p}_i^{NE}(V_i,V_{-i})$ of malicious users is greater than that of selfish users because the first term is greater if $V_i > 0$.

Finally, we calculate the achievable rate of each user with $p_i^{NE}(V_i)$. The rate requirement $\underline{u}_i$ can be achieved for the selfish users with $V_i = 0$. Since the power allocation of malicious users is larger than that of selfish users, their actual rate is greater than their rate requirements. ∎

V. Numerical Results

In this section, we present some numerical results of our proposed distributed pricing framework in the general MAC system without SIC under individual QoS requirement $\underline{u}_i$.

Define the channel gains $\alpha_i = |h_i|^2 \sim \chi_n^2$ with diversity order n. Fig. 1 shows the system sum power $\sum_{i=1}^{K}p_i^{NE}$ with different diversity order n for different numbers of users in the MAC. The rate requirement $\underline{u}_i = 0.05$.

Fig. 2 shows the convergence rate of the BRD using the chosen price β_i in (11) for the 2-user MAC. It is shown that

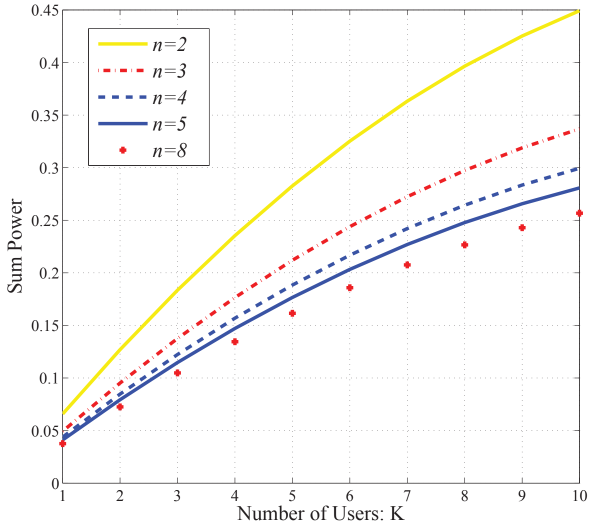

Fig. 1. Utility function with respect to the power

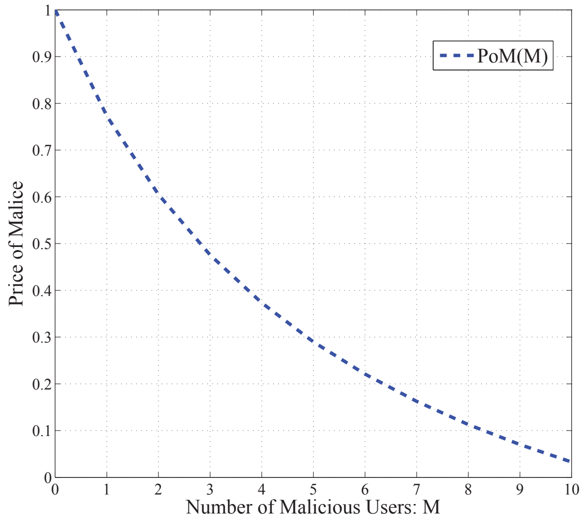

Fig. 3. Price of Malice vs. Number of Malicious Users

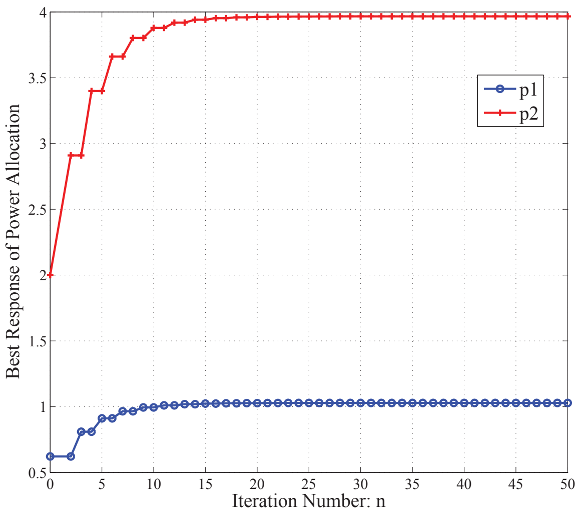

Fig. 2. Convergence of the BR Dynamics

the BRD converges quite fast. The parameters are $\alpha_1 = 2$, $\alpha_2 = 1$. $\underline{u}_1 = 0.5$, $\underline{u}_2 = 1.2$. The convergence points of the power allocation are the same as the NE power $\underline{p}_i^{NE}$ in (9), where $\underline{p}_i^{NE} = p_i^U$.

Fig. 3 shows the Price of Malice (PoM) of the proposed model. PoM is introduced in [12]. The PoM captures the ratio between the Nash Equilibrium in a purely selfish system and the worst NE with M malicious players. Formally, PoM in our case is

$$PoM(M) = \frac{P_{sum}^{NE}(0)}{P_{sum}^{NE}(M)}, \tag{20}$$

where $P_{sum}^{NE}(M)$ denotes the sum power allocation when there are M malicious users. $P_{sum}^{NE}(M) = \sum_{i=1}^{K} \underline{p}_i^{NE}(V_i, V_{-i})$ in which M users are with $V_i > 0$ and $K - M$ selfish users are with $V_i = 0$.

We apply the $PoM(M)$ to evaluate how much loss in the sum power consumption of the whole MAC system with K users when M malicious users exist. In the simulation, the total number of users in the system is $K = 10$. The rate requirement of each user is set to be $\underline{u}_i = 0.05$ in order to satisfy the feasible region. The channel gain is set to be $\alpha_i = 1$. The private type of the M malicious user i is $V_i = 0.06$, while $V_j = 0$ for the $K-M$ selfish users. When there is no malicious user in the system, PoM is one and it is strictly decreasing with the number of malicious users. It is observed that the PoM quickly drops from one if one or two malicious users are added. The PoM decreases more than 20% when there is one malicious user, which indicates the importance of the counter mechanism.

VI. Conclusion

In this paper, we investigate the noncooperative game for the general MAC system without SIC, where each user allocates its own power by maximizing its individual utility function. We propose the individual prices in the utility function such that the Shannon rate-based QoS requirement of each user is satisfied at the NE point power allocation. We provide the BRD power allocation, which converges rapidly to the unique NE point. The different types regarding the user behavior are analysed, especially the malicious behavior. The resulting power allocation and the achievable rates at the NE power allocation for all the users with the different individual types are observed.

APPENDIX

Proof: In order to determine the NE power allocation $\underline{p}_i^{NE}$, we make a trick to jointly solve the set of utility maximization problems in (5). We formulate it as linear equations $\boldsymbol{A} + \boldsymbol{D} \cdot \underline{\boldsymbol{p}} = \underline{\boldsymbol{p}}$. Therefore, $\underline{\boldsymbol{p}}$ is solved by

$$\underline{\boldsymbol{p}} = (\boldsymbol{I} - \boldsymbol{D})^{-1} \cdot \boldsymbol{A}, \tag{21}$$

where the matrix $\boldsymbol{D}$ is formulated as

$$\boldsymbol{D} = \begin{bmatrix} 0 & A_1\alpha_2 & \dots & A_1\alpha_K \\ A_2\alpha_1 & 0 & \dots & A_2\alpha_K \\ \vdots & \vdots & \ddots & \vdots \\ A_K\alpha_1 & A_K\alpha_2 & \dots & 0 \end{bmatrix}, \tag{22}$$

where $A_i = \frac{1}{\beta_i} - \frac{1}{\alpha_i}$.

Using the Crammer's rule, $\underline{\boldsymbol{p}} = \frac{\det(\boldsymbol{B}^i)}{\det(\boldsymbol{B})}$, where $\boldsymbol{B} = \boldsymbol{I} - \boldsymbol{D}$. The matrix $\boldsymbol{B}^i$ is the matrix of $\boldsymbol{B}$ where the ith column is replaced by the vector $\boldsymbol{A}$.

$$\boldsymbol{B} = \begin{bmatrix} 1 & -A_1\alpha_2 & \dots & -A_1\alpha_K \\ -A_2\alpha_1 & 1 & \dots & -A_2\alpha_K \\ \vdots & \vdots & \ddots & \vdots \\ -A_K\alpha_1 & -A_K\alpha_2 & \dots & 1 \end{bmatrix}. \tag{23}$$

Now we solve $\det(\boldsymbol{B}^i)$ and $\det(\boldsymbol{B})$.

$$\begin{aligned} &\det(\boldsymbol{B}^i) \\ =\;& \prod_{i=1}^{K} A_i \prod_{j\neq i} \alpha_j \cdot \det \begin{bmatrix} 1 & -1 & \dots & -1 \\ 1 & \frac{1}{A_2\alpha_2} & \dots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & -1 & \dots & \frac{1}{A_K\alpha_K} \end{bmatrix} \\ =\;& \prod_{i=1}^{K} A_i \prod_{j\neq i} \alpha_j \cdot \det \begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & \frac{1+A_2\alpha_2}{A_2\alpha_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \dots & \frac{1+A_K\alpha_K}{A_K\alpha_K} \end{bmatrix} \\ =\;& \prod_{i=1}^{K} A_i \prod_{j\neq i} \alpha_j \left(1 + \frac{1}{A_j\alpha_j}\right). \end{aligned} \tag{24}$$

$$\begin{aligned} \det(\boldsymbol{B}) &= \prod_{i=1}^{K} A_i\alpha_i(-1)^K \det \begin{bmatrix} \frac{-1}{A_1\alpha_1} & 1 & \dots & 1 \\ 1 & \frac{-1}{A_2\alpha_2} & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & \frac{-1}{A_K\alpha_K} \end{bmatrix} \\ &= \prod_{i=1}^{K} A_i\alpha_i \cdot \det \begin{bmatrix} C & \frac{1+A_1\alpha_1}{A_1\alpha_1} & \dots & \frac{1+A_1\alpha_1}{A_1\alpha_1} \\ 0 & \frac{1+A_2\alpha_2}{A_2\alpha_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1+A_K\alpha_K}{A_K\alpha_K} \end{bmatrix} \\ &= \prod_j (1 + A_j\alpha_j) \left(1 - \sum_{j=1}^{K} \frac{A_j\alpha_j}{1 + A_j\alpha_j}\right), \end{aligned} \tag{25}$$

where $C = \frac{1}{A_1\alpha_1} - \frac{1+A_1\alpha_1}{A_1\alpha_1} \cdot \sum_{j\neq 1} \frac{A_j\alpha_j}{1+A_j\alpha_j}$. Therefore, the BR power $\underline{p}_i = \frac{\det(\boldsymbol{B}^i)}{\det(\boldsymbol{B})}$ is

$$\underline{p}_i = \frac{A_i}{1 + A_i\alpha_i} \cdot \frac{1}{1 - \sum_{j=1}^{K} \frac{A_j\alpha_j}{1+A_j\alpha_j}}. \tag{26}$$

Insert $A_i = \frac{1}{\beta_i} - \frac{1}{\alpha_i}$, The proposition is proved. ∎

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