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On some new operations in soft set theory

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ABSTRACT

Molodtsov introduced the theory of soft sets, which can be seen as a new mathematical approach to vagueness. In this paper, we first point out that several assertions (Proposition 2.3 (iv)–(vi), Proposition 2.4 and Proposition 2.6 (iii), (iv)) in a previous paper by Maji et al. [P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562] are not true in general, by counterexamples. Furthermore, based on the analysis of several operations on soft sets introduced in the same paper, we give some new notions such as the restricted intersection, the restricted union, the restricted difference and the extended intersection of two soft sets. Moreover, we improve the notion of complement of a soft set, and prove that certain De Morgan's laws hold in soft set theory with respect to these new definitions.

1. Preliminaries

In this section, we recall some basic notions in soft set theory. Let U be an initial universe set and E_U be the set of all possible parameters under consideration with respect to U. The power set of U (i.e., the set of all subsets of U) is denoted by $P(U)$ and A is a subset of E. Usually, parameters are attributes, characteristics, or properties of objects in U. In what follows, E_U (simply denoted by E) always means the universe set of parameters with respect to U, unless otherwise specified. Molodtsov [1] defined the notion of a soft set in the following way:

Definition 1.1 (*[1]*). A pair (F, A) is called a *soft set* over U, where F is a mapping given by $F : A \to P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\epsilon \in A$, $F(\epsilon)$ may be considered as the set of ϵ-approximate elements of the soft set (F, A). For illustration, Molodtsov considered several examples in [1]. Similar examples were also discussed in [2,3]. Maji et al. [2] made a theoretical study on the theory of soft sets in more detail. They introduced and investigated the following notions relevant to soft sets.

Definition 1.2 (*[2]*). Let $E = \{e_1, e_2, \ldots, e_n\}$ be a set of parameters. The *NOT set* of E denoted by $\rceil E$ is defined by $\rceil E = \{\rceil e_1, \rceil e_2, \ldots, \rceil e_n\}$ where $\rceil e_i = not\ e_i$, $\forall i \in \{1, 2, \ldots, n\}$.

From the above definition, it is easy to verify the following.

Proposition 1.3 (*[2]*). *Let E be a set of parameters and $A, B \subseteq E$. Then*

1. $\rceil(\rceil A) = A$.
2. $\rceil(A \cup B) = \rceil A \cup \rceil B$.
3. $\rceil(A \cap B) = \rceil A \cap \rceil B$.

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Definition 1.4 (*[2]*). The *complement* of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \rceil A)$, where $F^c : \rceil A \to P(U)$ is a mapping given by $F^c(\alpha) = U - F(\rceil \alpha)$, $\forall \alpha \in \rceil A$.

Definition 1.5 (*[2]*). A soft set (F, A) over U is said to be a *null soft set* denoted by Φ, if $\forall e \in A$, $F(e) = \emptyset$ (null-set).

Definition 1.6 (*[2]*). A soft set (F, A) over U is said to be an *absolute soft set* denoted by $\tilde{A}$, if $\forall e \in A$, $F(e) = U$.

Definition 1.7 (*[2]*). The *union* of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and $\forall e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A) \mathbin{\tilde{\cup}} (G, B) = (H, C)$.

Definition 1.8 (*[2]*). The *intersection* of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cap B$, and $\forall e \in C$, $H(e) = F(e)$ or $G(e)$ (as both are same set). We write $(F, A) \mathbin{\tilde{\cap}} (G, B) = (H, C)$.

Definition 1.9 (*[2]*). Let (F, A) and (G, B) be soft sets over a common universe set U. Then

(a) $(F, A) \wedge (G, B)$ is a soft set defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, $\forall (\alpha, \beta) \in A \times B$, where $\cap$ is the intersection operation of sets.

(b) $(F, A) \vee (G, B)$ is a soft set defined by $(F, A) \vee (G, B) = (K, A \times B)$, where $K(\alpha, \beta) = F(\alpha) \cup G(\beta)$, $\forall (\alpha, \beta) \in A \times B$, where $\cup$ is the union operation of sets.

2. Counterexamples

We begin this section with a result given by Maji et al. in [2].

Theorem 2.1 (*Proposition 2.4 [2]*).

(i) $((F, A) \mathbin{\tilde{\cup}} (G, B))^c = (F, A)^c \mathbin{\tilde{\cup}} (G, B)^c$.
(ii) $((F, A) \mathbin{\tilde{\cap}} (G, B))^c = (F, A)^c \mathbin{\tilde{\cap}} (G, B)^c$.

The following example shows that the assertion (i) in Theorem 2.1 above is incorrect.

Example 2.2. Suppose that there are five houses in the universe U given by

$$U = \{h_1, h_2, h_3, h_4, h_5\}.$$

Let $A = \{\text{cheap, beautiful}\}$ and $B = \{\text{comfortable, beautiful}\}$. The soft sets (F, A) and (G, B) over the common universe U describe the *attractiveness of the houses* which Mr. X and Mr. Y are going to buy, respectively.

According to [2], we can view the soft sets (F, A) and (G, B) as the following collections of approximations:

$(F, A) = \{\text{cheap houses} = \{h_1, h_3, h_5\}, \text{beautiful houses} = \{h_1, h_2, h_4\}\}$, and $(G, B) = \{\text{comfortable houses} = \{h_2, h_5\}, \text{beautiful houses} = \{h_2, h_4\}\}$. Then $(F, A) \mathbin{\tilde{\cup}} (G, B) = \{\text{cheap houses} = \{h_1, h_3, h_5\}, \text{comfortable houses} = \{h_2, h_5\}, \text{beautiful houses} = \{h_1, h_2, h_4\}\}$, and so by Definition 1.4, we have $((F, A) \mathbin{\tilde{\cup}} (G, B))^c = \{\text{not cheap houses} = \{h_2, h_4\}, \text{not comfortable houses} = \{h_1, h_3, h_4\}, \text{not beautiful houses} = \{h_3, h_5\}\}$.

But on the other hand, we have $(F, A)^c = \{\text{not cheap houses} = \{h_2, h_4\}, \text{not beautiful houses} = \{h_3, h_5\}\}$, and $(G, B)^c = \{\text{not comfortable houses} = \{h_1, h_3, h_4\}, \text{not beautiful houses} = \{h_1, h_3, h_5\}\}$. Hence it follows that $(F, A)^c \mathbin{\tilde{\cup}} (G, B)^c = \{\text{not cheap houses} = \{h_2, h_4\}, \text{not comfortable houses} = \{h_1, h_3, h_4\}, \text{not beautiful houses} = \{h_1, h_3, h_5\}\}$.

Therefore $((F, A) \mathbin{\tilde{\cup}} (G, B))^c \neq (F, A)^c \mathbin{\tilde{\cup}} (G, B)^c$.

Another example given below can also be used to illuminate the incorrectness of Theorem 2.1(i). Moreover, it indicates that (ii) of Theorem 2.1 is also an ambiguous statement.

Example 2.3. Let E be the universe set of parameters and $A = \{e_1, e_2, e_3\}$, $B = \{e_3, e_4, e_5\}$ be subsets of E. Let (F, A) and (G, B) be two soft sets over the same universe $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ such that

$$F(e_1) = \{h_1, h_2\},$$
$$F(e_2) = \{h_3, h_4, \},$$
$$F(e_3) = \{h_2, h_3, h_6\},$$
$$G(e_4) = \{h_5\},$$
$$G(e_5) = \{h_6\},$$
$$G(e_3) = \{h_3, h_4, h_5, h_6\}.$$

Let $(F, A)\widetilde{\cup}(G, B) = (H, A \cup B)$, where

$$H(e) = \begin{cases} F(e), & e \in A - B, \\ G(e), & e \in B - A, \\ F(e) \cup G(e) & e \in A \cap B. \end{cases}$$

Then

$$H(e_1) = F(e_1) = \{h_1, h_2\},$$
$$H(e_2) = F(e_2) = \{h_3, h_4\},$$
$$H(e_4) = G(e_4) = \{h_5\},$$
$$H(e_5) = G(e_5) = \{h_6\},$$
$$H(e_3) = F(e_3) \cup G(e_3) = \{h_2, h_3, h_4, h_5, h_6\}.$$

Thus for $\rceil e_3 \in \rceil A \cap \rceil B$, by Definition 1.4, we have

$$H^c(\rceil e_3) = U - H(e_3) = \{h_1\}.$$

On the other hand, let $(F^c, \rceil A)\widetilde{\cup}(G^c, \rceil B) = (K, \rceil A \cup \rceil B)$, where

$$K(\rceil e) = \begin{cases} F^c(\rceil e), & \rceil e \in \rceil A - \rceil B, \\ G^c(\rceil e), & \rceil e \in \rceil B - \rceil A, \\ F^c(\rceil e) \cup G^c(\rceil e), & \rceil e \in \rceil A \cap \rceil B. \end{cases}$$

Then for $\rceil e_3 \in \rceil A \cap \rceil B$,

$$\begin{aligned} K(\rceil e_3) &= F^c(\rceil e_3) \cup G^c(\rceil e_3) \\ &= (U - F(e_3)) \cup (U - G(e_3)) \\ &= \{h_1, h_4, h_5\} \cup \{h_1, h_2\}, \\ &= \{h_1, h_2, h_4, h_5\}. \end{aligned}$$

Clearly, we have $K(\rceil e_3) \neq H^c(\rceil e_3)$ for $\rceil e_3 \in \rceil A \cap \rceil B$. Consequently, we deduce that $((F, A)\,\widetilde{\cup}\,(G, B))^c \neq (F, A)^c\,\widetilde{\cup}\,(G, B)^c$, showing that the assertion (i) in Theorem 2.1 is incorrect.

In addition, let us consider the statement (ii) of Theorem 2.1. Since $A \cap B = \{e_3\}$, $F(e_3) = \{h_2, h_3, h_6\}$ and $G(e_3) = \{h_3, h_4, h_5, h_6\}$, we immediately have that $F(e_3) \neq G(e_3)$, and so by Definition 1.8 the intersection $(F, A)\widetilde{\cap}(G, B)$ simply does not exist. It follows that $((F, A)\,\widetilde{\cap}\,(G, B))^c$ also does not exist, which makes it impossible to check the validity of the equality in (ii). Therefore we conclude that the second statement in Theorem 2.1 is an ambiguous statement.

Remark 2.4. In fact, the incorrectness of Theorem 2.1 (i) can be found by checking the proof of it (see [2], pp. 561). Suppose that $(F, A)\widetilde{\cup}(G, B) = (H, A \cup B)$ and $(F, A)^c\widetilde{\cup}(G, B)^c = (K, \rceil A \cup \rceil B)$. The authors in [2] claim that $H^c(\rceil\alpha) = F^c(\rceil\alpha) \cup G^c(\rceil\alpha)$, $\forall \rceil\alpha \in \rceil A \cap \rceil B$.

However, for any $\rceil\alpha \in \rceil A \cap \rceil B$, we have that $H^c(\rceil\alpha) = U - H(\alpha) = U - (F(\alpha) \cup G(\alpha)) = (U - F(\alpha)) \cap (U - G(\alpha)) = F^c(\rceil\alpha) \cap G^c(\rceil\alpha)$. Hence H^c should be as follows:

$$H^c(\rceil\alpha) = \begin{cases} F^c(\rceil\alpha), & \text{if } \rceil\alpha \in \rceil A - \rceil B, \\ G^c(\rceil\alpha), & \text{if } \rceil\alpha \in \rceil B - \rceil A, \\ F^c(\rceil\alpha) \cap G^c(\rceil\alpha), & \text{if } \rceil\alpha \in \rceil A \cap \rceil B. \end{cases}$$

But as pointed out in [2],

$$K(\rceil\alpha) = \begin{cases} F^c(\rceil\alpha), & \text{if } \rceil\alpha \in \rceil A - \rceil B, \\ G^c(\rceil\alpha), & \text{if } \rceil\alpha \in \rceil B - \rceil A, \\ F^c(\rceil\alpha) \cup G^c(\rceil\alpha), & \text{if } \rceil\alpha \in \rceil A \cap \rceil B. \end{cases}$$

Consequently, we conclude that H^c and K are different in general. This shows that Theorem 2.1 (i) is actually not true.

Now we highlight the errors in Proposition 2.3 of [2] which is stated as follows.

Proposition 2.5 (*Proposition 2.3 [2]*).

(1) $(F, A)\widetilde{\cup}(F, A) = (F, A)$.
(2) $(F, A)\widetilde{\cap}(F, A) = (F, A)$.
(3) $(F, A)\widetilde{\cup}\Phi = \Phi$, *where* Φ *is the null soft set.*
(4) $(F, A)\widetilde{\cap}\Phi = \Phi$.
(5) $(F, A)\,\widetilde{\cup}\,\widetilde{A} = \widetilde{A}$, *where* $\widetilde{A}$ *is the absolute soft set.*
(6) $(F, A)\,\widetilde{\cap}\,\widetilde{A} = (F, A)$.

Yang [4] pointed out that the third assertion $(F,A)\widetilde{\cup}\Phi=\Phi$ in the above proposition is incorrect by a counterexample. Actually it is easy to see that assertions (4), (5) and (6) in Proposition 2.5 are not correct too. The following example illuminates this fact.

Example 2.6. Suppose that there are five *wooden houses* in the universe U given by

$$U=\{h_1,h_2,h_3,h_4,h_5\}.$$

Let $A=$ {brick, muddy, steal, stone} be the set of parameters showing the building material of the houses. Let B be the NOT set of the parameter set A. That is,

$$B=\rceil A=\{\text{not brick, not muddy, not steal, not stone}\}.$$

Let (F,A) and (G,B) be two soft sets over the same universe U, which describe the *construction of the houses*. As in Example 2.2, the soft sets (F,A) and (G,B) can be regarded as collections of approximations. That is,

$$(F,A)=\{\text{brick houses}=\emptyset,\ \text{muddy houses}=\emptyset,\ \text{steal houses}=\emptyset,\ \text{stone houses}=\emptyset\},$$

and

$$\begin{aligned}(G,B)=\{&\text{not the brick houses}=U,\ \text{not the muddy houses}=U,\\ &\text{not the steal houses}=U,\ \text{not the stone houses}=U\}.\end{aligned}$$

By Definitions 1.5 and 1.6 introduced by Maji et al. [2], we have that (F,A) and (G,B) are null soft set and absolute soft set, respectively.

Now by Definition 1.7, the union of (F,A) and (G,B) is a soft set as follows:

$$\begin{aligned}(F,A)\,\widetilde{\cup}\,(G,B)=\{&\text{brick houses}=\emptyset,\ \text{muddy houses}=\emptyset,\\ &\text{steal houses}=\emptyset,\ \text{stone houses}=\emptyset,\\ &\text{not the brick houses}=U,\ \text{not the muddy houses}=U,\\ &\text{not the steal houses}=U,\ \text{not the stone houses}=U\}.\end{aligned}$$

Clearly, $(F,A)\,\widetilde{\cup}\,(G,B)\neq(G,B)$, which indicates that the assertion (5) is not true in general.

Moreover, since $A\cap B=\emptyset$, $(F,A)\widetilde{\cap}(G,B)$ is neither the null soft set (F,A), nor the absolute soft set (G,B). Hence we deduce that the assertions (4) and (6) are incorrect in general.

Theorem 2.7 (*Proposition 2.6 [2]*). *If* (F,A), (G,B) *and* (H,C) *are three soft sets over* U, *then*

(i) $(F,A)\vee((G,B)\vee(H,C))=((F,A)\vee(G,B))\vee(H,C)$,
(ii) $(F,A)\wedge((G,B)\wedge(H,C))=((F,A)\wedge(G,B))\wedge(H,C)$,
(iii) $(F,A)\vee((G,B)\wedge(H,C))=((F,A)\vee(G,B))\wedge((F,A)\vee(H,C))$,
(iv) $(F,A)\wedge((G,B)\vee(H,C))=((F,A)\wedge(G,B))\vee((F,A)\wedge(H,C))$.

The following remark shows that the parameter sets on both sides of the above assertions (iii) and (iv) are inconsistent in general.

Remark 2.8. Let (F,A), (G,B) and (H,C) be soft sets over a common universe U. By Definition 1.9, the soft set $(F,A)\vee((G,B)\wedge(H,C))$ on left side of (iii) has the parameter set $A\times(B\times C)$ and the soft set $((F,A)\vee(G,B))\wedge((F,A)\vee(H,C))$ on right side of (iii) has a set of parameters as $(A\times B)\times(A\times C)$. But in [2] we can not find any notion which ensure $A\times(B\times C)=(A\times B)\times(A\times C)$. Hence in Proposition 2.6 [2], two statements

(iii) $(F,A)\vee((G,B)\wedge(H,C))=((F,A)\vee(G,B))\wedge((F,A)\vee(H,C))$,
(iv) $(F,A)\wedge((G,B)\vee(H,C))=((F,A)\wedge(G,B))\vee((F,A)\wedge(H,C))$

are not true.

3. Some new operations in soft set theory

We point out here that the intersection of two soft sets introduced in [2] (see Definition 1.8) is not a clearly defined notion, which suffers from many problems.

Note first that if (F,A) and (G,B) are two different soft sets, then it is not necessary for these two soft sets to have the same subset of U for a particular common parameter say $c\in A\cap B$, i.e., $F(c)\neq G(c)$ in general. Hence the intersection of two soft sets as defined in [2] may only be a partial operation. That is to say, not any two soft sets result in a new soft set when we calculate such an intersection operation on them.

On the contrary, if this kind of intersection must be regarded as a binary operation, then we can deduce from the definition that any two soft sets over a common universe must have the same approximation value-set for any common parameter, but this is surely not the case. For a parameter which represents a vague concept such as *beautiful houses*, each person has his/her own opinion and the approximation value-sets given by different persons may be extremely different. There does not exist an objective criteria to fix a standard approximation value-set for a given parameter. In fact, it is also worth noting that the union of two soft sets introduced in [2] (see Definition 1.7) implies that two approximation value-sets of a common parameter could be different, while Definition 1.8 implicitly turns to the contrary of this observation. As illustration, one may consider the following example.

Example 3.1. Let $X = \{0, a, b, c, d\}$ be a universe set and $A = B = X$ be parameter sets. Let (L, A) be a soft set over X given by $L(0) = \{0\}$, $L(a) = \{0, a\}$, $L(b) = \{0, b\}$, $L(c) = \{0, a, b, c\}$ and $L(d) = \{0, a, d\}$. Let (R, B) be another soft set over X given by $R(0) = \{0, a, b, c, d\}$, $R(a) = \{0, a, b\}$, $R(b) = \{0, a, d\}$, $R(c) = \{0, a\}$ and $R(d) = \{0, b\}$. For any $c \in A \cap B$, $L(c) \neq R(c)$. It follows that the intersection of (L, A) and (R, B) as defined in [2] does not exist.

To ensure that the intersection of two soft sets free from the above problems, we introduce the following new definition of intersection.

Definition 3.2. The *extended intersection* of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and $\forall e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cap G(e), & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A) \sqcap_{\mathscr{E}} (G, B) = (H, C)$.

In addition, we may sometimes adopt a different definition of intersection given as follows.

Definition 3.3. Let (F, A) and (G, B) be two soft sets over a common universe U such that $A \cap B \neq \emptyset$. The *restricted intersection*[1] of (F, A) and (G, B) is denoted by $(F, A) \Cap (G, B)$, and is defined as $(F, A) \Cap (G, B) = (H, C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cap G(c)$.

Definition 3.4. Let (F, A) and (G, B) be two soft sets over the same universe U such that $A \cap B \neq \emptyset$. The *restricted difference* of (F, A) and (G, B) is denoted by $(F, A) \smile_{\mathscr{R}} (G, B)$, and is defined as $(F, A) \smile_{\mathscr{R}} (G, B) = (H, C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) - G(c)$, the difference of the sets $F(c)$ and $G(c)$.

As mentioned in the last section, several assertions in relation to the null soft sets and the absolute soft sets in [2] are incorrect. These difficulties may be due to the definitions of the related notions in [2]. Next, we shall endeavor to make those definitions more rational.

Definition 3.5. Let U be an initial universe set, E be the universe set of parameters, and $A \subset E$.

(a) (F, A) is called a *relative null soft set* (with respect to the parameter set A), denoted by Φ_A, if $F(e) = \emptyset$ for all $e \in A$.
(b) (G, A) is called a *relative whole soft set* (with respect to the parameter set A), denoted by $\mathfrak{U}_A$, if $F(e) = U$ for all $e \in A$.

The relative whole soft set $\mathfrak{U}_E$ with respect to the universe set of parameters E is called the *absolute soft set* over U.

Definition 3.6. The *relative complement* of a soft set (F, A) is denoted by $(F, A)^r$ and is defined by $(F, A)^r = (F^r, A)$ where $F^r : A \to P(U)$ is a mapping given by $F^r(\alpha) = U - F(\alpha)$ for all $\alpha \in A$.

Clearly, $(F, A)^r = \mathfrak{U}_E \smile_{\mathscr{R}} (F, A)$ and $\left((F, A)^r\right)^r = (F, A)$. It is worth noting that in the above new definition of complement, the parameter set of the complement $(F, A)^r$ is still the original parameter set A, instead of $\rceil A$ as in Definition 1.4. To emphasize this difference, the complement given by Definition 1.4 will be called *neg-complement* (or *pseudo-complement*) in what follows.

Definition 3.7. Let (F, A) and (G, B) be two soft sets over the same universe U such that $A \cap B \neq \emptyset$. The *restricted union* of (F, A) and (G, B) is denoted by $(F, A) \cup_{\mathscr{R}} (G, B)$, and is defined as $(F, A) \cup_{\mathscr{R}} (G, B) = (H, C)$ where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cup G(c)$.

4. De Morgan's laws in soft set theory

In this section, we first show that the following De Morgan's type of results hold in soft set theory for the newly defined relative complement, restricted union and restricted intersection.

Theorem 4.1. *Let (F, A) and (G, B) be two soft sets over the same universe U such that $A \cap B \neq \emptyset$. Then*

(1) $((F, A) \cup_{\mathscr{R}} (G, B))^r = (F, A)^r \Cap (G, B)^r$,
(2) $((F, A) \Cap (G, B))^r = (F, A)^r \cup_{\mathscr{R}} (G, B)^r$.

[1] Note that restricted intersection is also known as bi-intersection in [5].

Proof. (1) Let $(F,A)\cup_{\mathscr{R}}(G,B)=(H,C)$ where

$$H(c)=F(c)\cup G(c)\quad \text{for all } c\in C=A\cap B\neq\emptyset.$$

Since $((F,A)\cup_{\mathscr{R}}(G,B))^r=(H,C)^r$, by definition

$$H^r(c)=U-[F(c)\cup G(c)]=[U-F(c)]\cap[U-G(c)]$$

for all $c\in C$.

Now

$$(F,A)^r\Cap(G,B)^r=\left(F^r,A\right)\Cap\left(G^r,B\right)=(K,C)\quad\text{where } C=A\cap B.$$

So by definition, we have

$$\begin{aligned}K(c)&=F^r(c)\cap G^r(c)\\&=(U-F(c))\cap(U-G(c))\\&=H^r(c),\end{aligned}$$

for all $c\in C$. Hence $((F,A)\cup_{\mathscr{R}}(G,B))^r=(F,A)^r\Cap(G,B)^r$.

(2) Let $(F,A)\Cap(G,B)=(H,C)$ where

$$H(c)=F(c)\cap G(c)\quad \text{for all } c\in C=A\cap B\neq\emptyset.$$

Since $((F,A)\Cap(G,B))^r=(H,C)^r$, by definition

$$H^r(c)=U-(F(c)\cap G(c))=[U-F(c)]\cup[U-G(c)]$$

for all $c\in C$.

Now

$$(F,A)^r\cup_{\mathscr{R}}(G,B)^r=\left(F^r,A\right)\cup_{\mathscr{R}}\left(G^r,B\right)=(K,C)\quad\text{where } C=A\cap B.$$

So by definition, we have

$$\begin{aligned}K(c)&=F^r(c)\cup G^r(c)\\&=(U-F(c))\cup(U-G(c))\\&=H^r(c),\end{aligned}$$

for all $c\in C$. Hence $((F,A)\Cap(G,B))^r=(F,A)^r\cup_{\mathscr{R}}(G,B)^r$. □

By using similar techniques, we can prove that the following De Morgan's laws hold in soft set theory for the extended intersection, the union and the neg-complement.

Theorem 4.2. *Let (F,A) and (G,B) be two soft sets over a common universe U. Then we have the following:*

(1) $((F,A)\,\widetilde{\cup}\,(G,B))^c=(F,A)^c\sqcap_{\mathscr{E}}(G,B)^c$.
(2) $((F,A)\sqcap_{\mathscr{E}}(G,B))^c=(F,A)^c\,\widetilde{\cup}\,(G,B)^c$.

Proof. (1) This can be deduced from Remark 2.4 and Definition 3.2.

(2) Suppose that $(F,A)\sqcap_{\mathscr{E}}(G,B)=(H,A\cup B)$. Then $((F,A)\sqcap_{\mathscr{E}}(G,B))^c=(H,A\cup B)^c=(H^c,\rceil(A\cup B))=(H^c,\rceil A\cup\rceil B)$, where $H^c(\rceil e)=U-H(e)$ for all $\rceil e\in\rceil A\cup\rceil B$. By definition,

$$H(e)=\begin{cases}F(e), & \text{if } e\in A-B,\\ G(e), & \text{if } e\in B-A,\\ F(e)\cap G(e), & \text{if } e\in A\cap B.\end{cases}$$

Thus we have

$$H^c(\rceil e)=\begin{cases}U-F(e)=F^c(\rceil e), & \text{if } \rceil e\in\rceil A-\rceil B,\\ U-G(e)=G^c(\rceil e), & \text{if } \rceil e\in\rceil B-\rceil A,\\ U-(F(e)\cap G(e))=F^c(\rceil e)\cup G^c(\rceil e), & \text{if } \rceil e\in\rceil A\cap\rceil B.\end{cases}$$

Moreover, let $(F,A)^c\,\widetilde{\cup}\,(G,B)^c=(F^c,\rceil A)\widetilde{\cup}(G^c,\rceil B)=(K,\rceil A\cup\rceil B)$. Then

$$K(\rceil e)=\begin{cases}F^c(\rceil e), & \text{if } \rceil e\in\rceil A-\rceil B,\\ G^c(\rceil e), & \text{if } \rceil e\in\rceil B-\rceil A,\\ F^c(\rceil e)\cup G^c(\rceil e), & \text{if } \rceil e\in\rceil A\cap\rceil B.\end{cases}$$

Since H^c and K are indeed the same set-valued mapping, we conclude that $((F,A)\sqcap_{\mathscr{E}}(G,B))^c=(F,A)^c\,\widetilde{\cup}\,(G,B)^c$ as required. □

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References

[1] D. Molodtsov, Soft set theory–First results, Comput. Math. Appl. 37 (1999) 19–31.
[2] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.
[3] H. Aktaş, N. Çağman, Soft sets and soft groups, Inform. Sci. 177 (2007) 2726–2735.
[4] C.F. Yang, A note on soft set theory, Comput. Math. Appl. 56 (2008) 1899–1900. [Comput. Math. Appl. 45 (4–5) (2003) 555–562].
[5] F. Feng, Y.B. Jun, X.Z. Zhao, Soft semirings, Comput. Math. Appl. 56 (2008) 2621–2628.