Computers and Mathematics with Applications 58 (2009) 521–527

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Combination of interval-valued fuzzy set and soft set

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ARTICLE INFO

Article history:
Received 18 August 2008
Received in revised form 4 April 2009
Accepted 8 April 2009

Keywords:
Fuzzy soft set
Interval-valued fuzzy set
Interval-valued fuzzy soft set
Operation
Choice value

ABSTRACT

The soft set theory, proposed by Molodtsov, can be used as a general mathematical tool for dealing with uncertainty. By combining the interval-valued fuzzy set and soft set models, the purpose of this paper is to introduce the concept of the interval-valued fuzzy soft set. The complement, "AND" and "OR" operations are defined on the interval-valued fuzzy soft sets. The DeMorgan's, associative and distribution laws of the interval-valued fuzzy soft sets are then proved. Finally, a decision problem is analyzed by the interval-valued fuzzy soft set. Some numerical examples are employed to substantiate the conceptual arguments.

1. Introduction

Soft set theory [1] was firstly proposed by a Russian researcher—Molodtsov, in 1999. Such theory is a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. Where the soft set theory is different from traditional tools for dealing with uncertainties, such as the theory of probability, the theory of fuzzy sets and the theory of rough sets, is that it is free from the inadequacy of the parametrization tools of those theories. In Ref. [1], Molodtsov has successfully applied the soft set theory in several directions, such as smoothness of functions, game theory, operations research, riemann-integration, perron integration, probability, theory of measurement and so on.

Presently, work on the soft set theory is progressing rapidly. Maji et al. [2] first defined some operations on soft sets. They also introduced the soft set into the decision-making problem [3] that is based on the concept of knowledge reduction in the rough set theory [4]. Chen et al. presented a new definition of soft set parameterization reduction [5], and compared this definition with the related concept of knowledge reduction in the rough set theory. Kong et al. [6] introduced the definition of normal parameter reduction into soft sets and then presented a heuristic algorithm to compute normal parameter reduction of soft sets.

From the discussion above, we can see that all of these works are based on the classical soft set theory. The soft set model, however, can also be combined with other mathematical models. For example, by amalgamating the soft set and algebra, Aktaş et al. proposed the definition of of soft groups [7], Feng et al. proposed the concept of soft semirings, soft subsemirings, soft ideals, idealistic soft semirings and soft semiring homomorphisms [8]. In Ref. [9], the authors applied the notion of soft sets to the theory of BCK/BCI-algebras. Maji et al. presented the concept of the fuzzy soft set [10] which is based on a combination of the fuzzy set and soft set models. Yang et al. defined the operations on fuzzy soft sets [11], which are based on three fuzzy logic operators: negation, triangular norm and triangular conorm. Zou et al. introduced the soft set and fuzzy soft set into the incomplete environment respectively in Ref. [12].

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doi:10.1016/j.camwa.2009.04.019

The purpose of this paper is to combine the interval-valued fuzzy set [13] and soft set, from which we can obtain a new soft set model: Interval-valued Fuzzy Soft Set. To facilitate our discussion, we first present the standard soft set and fuzzy soft set in Section 2. In Section 3, the concept of interval-valued fuzzy soft set is presented. The complement, "AND" and "OR" operations on the interval-valued fuzzy soft sets are then defined. In Section 4, the interval-valued fuzzy soft set is used to analyze a decision making problem. We then conclude the paper with a summary and outlook for further research.

2. Preliminaries

Molodtsov [1] defined the soft set in the following way. Let U be an initial universe set and E be a set of parameters.

Definition 1. A pair $\langle F, E\rangle$ is referred to as a soft set if and only if F is a mapping of E into the set of all subsets of the set U.

In other words, the soft set is not a type of set, but a parameterized family of subsets of the universe U. Let us denote $P(U)$ by set of all subsets of U, then F is a mapping such that

$$F : E \to P(U). \tag{1}$$

Such a mapping reflects the innate character of the concept of a soft set, i.e. a soft set is a mapping from parameters to $P(U)$. $\forall\epsilon \in E, F(\epsilon)$ may be considered as the set of ϵ-approximate elements of the soft set $\langle F, E\rangle$.

By introducing the concept of fuzzy set into the theory of soft sets, Ref. [10] proposed the concept of the fuzzy soft set as follows.

Definition 2. Let $\mathscr{P}(U)$ denotes the set of all fuzzy subsets of U, then a pair $(\widetilde{F}, E)$ is called a fuzzy soft set over $\mathscr{P}(U)$, where $\widetilde{F}$ is a mapping given by

$$\widetilde{F} : E \to \mathscr{P}(U). \tag{2}$$

A fuzzy soft set is a parameterized family of fuzzy subsets of U, thus, its universe is the set of all fuzzy sets of U, i.e. $\mathscr{P}(U)$. A fuzzy soft set is actually a special case of a soft set because it is still a mapping from parameters to a universe. The difference between a soft set and a fuzzy soft set is that in a fuzzy soft set, the universe to be considered is the set of fuzzy subsets of U.

Generally speaking, $\forall\epsilon \in E, \widetilde{F}(\epsilon)$ is a fuzzy subset of U and it is called the *fuzzy value set* of the parameter ϵ. To distinguish it from the standard soft set, let us denote $\mu_{\widetilde{F}(\epsilon)}(x)$ by the membership degree that object x holds the parameter ϵ where $x \in U$ and $\epsilon \in E$. In other words, $\widetilde{F}(\epsilon)$ can be written as a fuzzy set such that

$$\widetilde{F}(\epsilon) = \{\langle x, \mu_{\widetilde{F}(\epsilon)}(x)\rangle : x \in U\}. \tag{3}$$

3. Interval-valued fuzzy soft set

3.1. Concept of interval-valued fuzzy soft set

Obviously, the combined result of fuzzy set and soft set theory is a fuzzy soft set. However, it should be noticed that in many real applications, the membership degree in a fuzzy set cannot be lightly confirmed. It is more reasonable to give an interval-valued data to describe membership degree. From such point of view, Zadeh further proposed the concept of an interval-valued fuzzy set. By combining the interval-valued fuzzy set and soft set, it is natural to define the interval-valued fuzzy soft set model. First, let us briefly introduce the concept of the interval-valued fuzzy set.

Definition 3 (*[14]*). An interval-valued fuzzy set $\hat{X}$ on an universe U is a mapping such that

$$\hat{X} : U \to Int([0, 1]), \tag{4}$$

where $Int([0, 1])$ stands for the set of all closed subintervals of $[0, 1]$, the set of all interval-valued fuzzy sets on U is denoted by $\widetilde{\mathscr{P}}(U)$.

Suppose that $\hat{X} \in \widetilde{\mathscr{P}}(U), \forall x \in U, \mu_{\hat{X}}(x) = [\mu^-_{\hat{X}}(x), \mu^+_{\hat{X}}(x)]$ is called the degree of membership an element x to $\hat{X}$. $\mu^-_{\hat{X}}(x)$ and $\mu^+_{\hat{X}}(x)$ are referred to as the lower and upper degrees of membership x to $\hat{X}$ where $0 \le \mu^-_{\hat{X}}(x) \le \mu^+_{\hat{X}}(x) \le 1$.

The complement, intersection and union of the interval-valued fuzzy sets are defined as follows: let $\hat{X}, \hat{Y} \in \widetilde{\mathscr{P}}(U)$, then

- the complement of $\hat{X}$ is denoted by $\hat{X}^C$ where

$$\mu_{\hat{X}^C}(x) = 1 - \mu_{\hat{X}}(x) = [1 - \mu^+_{\hat{X}}(x), 1 - \mu^-_{\hat{X}}(x)];$$

- the intersection of $\hat{X}$ and $\hat{Y}$ is denoted by $\hat{X} \cap \hat{Y}$ where

$$\mu_{\hat{X}\cap\hat{Y}}(x) = \inf[\mu_{\hat{X}}(x), \mu_{\hat{Y}}(x)] = [\inf(\mu^-_{\hat{X}}(x), \mu^-_{\hat{Y}}(x)), \inf(\mu^+_{\hat{X}}(x), \mu^+_{\hat{Y}}(x))];$$

Table 1
An interval-valued fuzzy soft set $(\widetilde{\mathscr{F}}, A)$.

U	ϵ_1	ϵ_2	ϵ_3	ϵ_4
h_1	[0.7, 0.9]	[0.6, 0.7]	[0.3, 0.5]	[0.5, 0.8]
h_2	[0.6, 0.8]	[0.8, 1.0]	[0.8, 0.9]	[0.9, 1.0]
h_3	[0.5, 0.6]	[0.2, 0.4]	[0.5, 0.7]	[0.7, 0.9]
h_4	[0.6, 0.8]	[0.0, 0.1]	[0.7, 1.0]	[0.6, 0.8]
h_5	[0.8, 0.9]	[0.1, 0.3]	[0.9, 1.0]	[0.2, 0.5]
h_6	[0.8, 1.0]	[0.7, 0.8]	[0.2, 0.5]	[0.7, 1.0]

Table 2
An interval-valued fuzzy soft set $(\widetilde{\mathscr{G}}, B)$.

U	ε_1	ε_2	ε_3
h_1	[0.3, 0.5]	[0.9, 1.0]	[0.6, 0.7]
h_2	[0.7, 0.9]	[0.8, 1.0]	[0.1, 0.3]
h_3	[0.5, 0.7]	[0.2, 0.5]	[0.9, 1.0]
h_4	[0.2, 0.3]	[0.1, 0.3]	[0.2, 0.4]
h_5	[0.8, 0.9]	[0.9, 1.0]	[0.5, 0.7]
h_6	[0.9, 1.0]	[0.5, 0.6]	[0.8, 1.0]

- the union of $\hat{X}$ and $\hat{Y}$ is denoted by $\hat{X} \cup \hat{Y}$ where

$$\mu_{\hat{X}\cup\hat{Y}}(x) = \sup[\mu_{\hat{X}}(x), \mu_{\hat{Y}}(x)] = [\sup(\mu^-_{\hat{X}}(x), \mu^-_{\hat{Y}}(x)), \sup(\mu^+_{\hat{X}}(x), \mu^+_{\hat{Y}}(x))];$$

Definition 4. Let U be an initial universe and E be a set of parameters, a pair $(\widetilde{\mathscr{F}}, E)$ is called an interval-valued fuzzy soft set over $\widetilde{\mathscr{P}}(U)$, where $\widetilde{\mathscr{F}}$ is a mapping given by

$$\widetilde{\mathscr{F}} : E \to \widetilde{\mathscr{P}}(U). \tag{5}$$

An interval-valued fuzzy soft set is a parameterized family of interval-valued fuzzy subsets of U, thus, its universe is the set of all interval-valued fuzzy sets of U, i.e. $\widetilde{\mathscr{P}}(U)$. An interval-valued fuzzy soft set is also a special case of a soft set because it is still a mapping from parameters to $\widetilde{\mathscr{P}}(U)$.

$\forall \epsilon \in E$, $\widetilde{\mathscr{F}}(\epsilon)$ is referred as the interval fuzzy value set of parameter ϵ, it is actually an interval-valued fuzzy set of U where $x \in U$ and $\epsilon \in E$, it can be written as:

$$\widetilde{\mathscr{F}}(\epsilon) = \{\langle x, \mu_{\widetilde{\mathscr{F}}(\epsilon)}(x)\rangle : x \in U\}, \tag{6}$$

here, $\widetilde{\mathscr{F}}(\epsilon)$ is the interval-valued fuzzy membership degree that object x holds on parameter ϵ. If $\forall \epsilon \in E$, $\forall x \in U$, $\mu^-_{\widetilde{\mathscr{F}}(\epsilon)}(x) = \mu^+_{\widetilde{\mathscr{F}}(\epsilon)}(x)$, then $\widetilde{\mathscr{F}}(\epsilon)$ will degenerated to be a standard fuzzy set and then $(\widetilde{\mathscr{F}}, E)$ will be degenerated to be a traditional fuzzy soft set.

Example 1. Suppose the following,

- U is the set the houses under consideration and $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$.
- A is the set of parameters and $A = \{\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4\}$ = {beautiful, wooden, cheap, in the green surroundings}.

The tabular representation of an interval-valued fuzzy soft set $(\widetilde{\mathscr{F}}, A)$ is shown in Table 1. In Table 1, we can see that the precise evaluation for each object on each parameter is unknown while the lower and upper limits of such an evaluation is given. For example, we cannot present the precise degree of how beautiful house h_1 is, however, house h_1 is at least beautiful on the degree of 0.7 and it is at most beautiful on the degree of 0.9.

Definition 5. Suppose that $(\widetilde{\mathscr{F}}, E)$ is an interval-valued fuzzy soft set over $\widetilde{\mathscr{P}}(U)$, $\widetilde{\mathscr{F}}(\epsilon)$ is the interval fuzzy value set of parameter ϵ, then all interval fuzzy value sets in interval-valued fuzzy soft set $(\widetilde{\mathscr{F}}, E)$ are referred to as the interval fuzzy value class of $(\widetilde{\mathscr{F}}, E)$ and is denoted by $C_{(\widetilde{\mathscr{F}},E)}$, then we have

$$C_{(\widetilde{\mathscr{F}},E)} = \{\widetilde{\mathscr{F}}(\epsilon) : \epsilon \in E\}. \tag{7}$$

Example 2. Following Example 1, we have $C_{(\widetilde{\mathscr{F}},A)} = \{\widetilde{\mathscr{F}}(\epsilon_1), \widetilde{\mathscr{F}}(\epsilon_2), \widetilde{\mathscr{F}}(\epsilon_3), \widetilde{\mathscr{F}}(\epsilon_4)\}$ where:

$$\widetilde{\mathscr{F}}(\epsilon_1) = \{\langle h_1, [0.7, 0.9]\rangle, \langle h_2, [0.6, 0.8]\rangle, \langle h_3, [0.5, 0.6]\rangle, \langle h_4, [0.6, 0.8]\rangle, \langle h_5, [0.8, 0.9]\rangle, \langle h_6, [0.8, 1.0]\rangle\}$$
$$\widetilde{\mathscr{F}}(\epsilon_2) = \{\langle h_1, [0.6, 0.7]\rangle, \langle h_2, [0.8, 1.0]\rangle, \langle h_3, [0.2, 0.4]\rangle, \langle h_4, [0.0, 0.1]\rangle, \langle h_5, [0.1, 0.3]\rangle, \langle h_6, [0.7, 0.8]\rangle\}$$
$$\widetilde{\mathscr{F}}(\epsilon_3) = \{\langle h_1, [0.3, 0.5]\rangle, \langle h_2, [0.8, 0.9]\rangle, \langle h_3, [0.5, 0.7]\rangle, \langle h_4, [0.7, 1.0]\rangle, \langle h_5, [0.9, 1.0]\rangle, \langle h_6, [0.2, 0.5]\rangle\}$$
$$\widetilde{\mathscr{F}}(\epsilon_4) = \{\langle h_1, [0.5, 0.8]\rangle, \langle h_2, [0.9, 1.0]\rangle, \langle h_3, [0.7, 0.9]\rangle, \langle h_4, [0.6, 0.8]\rangle, \langle h_5, [0.2, 0.5]\rangle, \langle h_6, [0.7, 1.0]\rangle\}$$

Definition 6. Let U be an initial universe and E be a set of parameters, suppose that $A, B \subset E$, $(\widetilde{\mathscr{F}}, A)$ and $(\widetilde{\mathscr{G}}, B)$ are two interval-valued fuzzy soft sets, we say that $(\widetilde{\mathscr{F}}, A)$ is an interval-valued fuzzy soft subset of $(\widetilde{\mathscr{G}}, B)$ if and only if

(1) $A \subset B$;
(2) $\forall \epsilon \in A$, $\widetilde{\mathscr{F}}(\epsilon)$ is an interval-valued fuzzy subset of $\widetilde{\mathscr{G}}(\epsilon)$;

which can be denoted by $(\widetilde{\mathscr{F}}, A)\widetilde{\subset}(\widetilde{\mathscr{G}}, B)$.

By the above definition, we can see that $(\widetilde{\mathscr{F}}, A)$ is an interval-valued fuzzy soft subset of $(\widetilde{\mathscr{G}}, B)$ if the following two conditions hold: the set of parameters of interval-valued fuzzy soft set $(\widetilde{\mathscr{F}}, A)$ is a subset of the set of parameters of interval-valued fuzzy soft set $(\widetilde{\mathscr{G}}, B)$; interval-valued fuzzy set $\widetilde{\mathscr{F}}(\epsilon)$ is a subset of interval-valued fuzzy set $\widetilde{\mathscr{G}}(\epsilon)$ where ϵ is an arbitrary parameter held by $(\widetilde{\mathscr{F}}, A)$ and $(\widetilde{\mathscr{G}}, B)$.

$(\widetilde{\mathscr{F}}, A)$ is said to be an interval-valued fuzzy soft super set of $(\widetilde{\mathscr{G}}, B)$, if $(\widetilde{\mathscr{G}}, B)$ is an interval-valued fuzzy soft subset of $(\widetilde{\mathscr{F}}, A)$. We denote it by $(\widetilde{\mathscr{F}}, A)\widetilde{\supset}(\widetilde{\mathscr{G}}, B)$.

Example 3. Given two interval-valued fuzzy soft sets $(\widetilde{\mathscr{F}}, A)$ and $(\widetilde{\mathscr{G}}, B)$, $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$. Here U is the set of houses, $A = \{\epsilon_1, \epsilon_2\} = \{\text{beautiful, wooden}\}$, $B = \{\epsilon_1, \epsilon_2, \epsilon_3\} = \{\text{beautiful, wooden, cheap}\}$, and

$$\widetilde{\mathscr{F}}(\epsilon_1) = \{\langle h_1, [0.7, 0.9]\rangle, \langle h_2, [0.6, 0.8]\rangle, \langle h_3, [0.5, 0.6]\rangle, \langle h_4, [0.6, 0.8]\rangle, \langle h_5, [0.8, 0.9]\rangle, \langle h_6, [0.8, 1.0]\rangle\}$$
$$\widetilde{\mathscr{F}}(\epsilon_2) = \{\langle h_1, [0.6, 0.7]\rangle, \langle h_2, [0.8, 1.0]\rangle, \langle h_3, [0.2, 0.4]\rangle, \langle h_4, [0.0, 0.1]\rangle, \langle h_5, [0.1, 0.3]\rangle, \langle h_6, [0.7, 0.8]\rangle\}$$
$$\widetilde{\mathscr{G}}(\epsilon_1) = \{\langle h_1, [0.8, 1.0]\rangle, \langle h_2, [0.6, 0.8]\rangle, \langle h_3, [0.7, 0.9]\rangle, \langle h_4, [0.6, 0.8]\rangle, \langle h_5, [0.8, 1.0]\rangle, \langle h_6, [0.8, 1.0]\rangle\}$$
$$\widetilde{\mathscr{G}}(\epsilon_2) = \{\langle h_1, [0.6, 0.7]\rangle, \langle h_2, [0.9, 1.0]\rangle, \langle h_3, [0.4, 0.5]\rangle, \langle h_4, [0.1, 0.3]\rangle, \langle h_5, [0.1, 0.3]\rangle, \langle h_6, [0.9, 1.0]\rangle\}$$
$$\widetilde{\mathscr{G}}(\epsilon_3) = \{\langle h_1, [0.3, 0.5]\rangle, \langle h_2, [0.8, 0.9]\rangle, \langle h_3, [0.5, 0.7]\rangle, \langle h_4, [0.7, 1.0]\rangle, \langle h_5, [0.9, 1.0]\rangle, \langle h_6, [0.2, 0.5]\rangle\}$$

Clearly, we have $(\widetilde{\mathscr{F}}, A) \widetilde{\subset} (\widetilde{\mathscr{G}}, B)$.

Definition 7. Let $(\widetilde{\mathscr{F}}, A)$ and $(\widetilde{\mathscr{G}}, B)$ be two interval-valued fuzzy soft sets, $(\widetilde{\mathscr{F}}, A)$ and $(\widetilde{\mathscr{G}}, B)$ are said to be interval-valued fuzzy soft equal if and only if

(1) $(\widetilde{\mathscr{F}}, A)$ is an interval-valued fuzzy soft subset of $(\widetilde{\mathscr{G}}, B)$,
(2) $(\widetilde{\mathscr{G}}, B)$ is an interval-valued fuzzy soft subset of $(\widetilde{\mathscr{F}}, A)$;

which can be denoted by $(\widetilde{\mathscr{F}}, A) = (\widetilde{\mathscr{G}}, B)$.

3.2. Operations on interval-valued fuzzy soft sets

Definition 8. The complement of a soft set $(\widetilde{\mathscr{F}}, A)$ is denoted by $(\widetilde{\mathscr{F}}, A)^C$ and is defined by

$$(\widetilde{\mathscr{F}}, A)^C = (\widetilde{\mathscr{F}}^C, \neg A), \tag{8}$$

where $\forall \alpha \in A$, $\neg\alpha =$ not α, is the not set of the parameter α, which holds the opposite meanings of parameter α;

$$\widetilde{\mathscr{F}}^C : \neg A \to \widetilde{\mathscr{P}}(U) \tag{9}$$

is a mapping given by $\widetilde{\mathscr{F}}^C(\beta) = \left(\widetilde{\mathscr{F}}(\neg\beta)\right)^C, \forall \beta \in \neg A$.

Example 4. Consider another interval-valued fuzzy soft set $(\widetilde{\mathscr{G}}, B)$ shown in Table 2. In Table 2, the universe U is same as the universe in Table 1, i.e. $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ is the set of the houses under consideration; $B = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\} = \{\text{convenient traffic, modern style, in good repair}\}$ is the set of the parameters.

Thus, by Definition 9, we have

$$\widetilde{\mathscr{G}}^C(\neg\varepsilon_1) = \{\langle h_1, [0.5, 0.7]\rangle, \langle h_2, [0.1, 0.3]\rangle, \langle h_3, [0.3, 0.4]\rangle, \langle h_4, [0.7, 0.8]\rangle, \langle h_5, [0.1, 0.2]\rangle, \langle h_6, [0.0, 0.1]\rangle\}$$
$$\widetilde{\mathscr{G}}^C(\neg\varepsilon_2) = \{\langle h_1, [0.0, 0.1]\rangle, \langle h_2, [0.0, 0.2]\rangle, \langle h_3, [0.5, 0.8]\rangle, \langle h_4, [0.7, 0.9]\rangle, \langle h_5, [0.0, 0.1]\rangle, \langle h_6, [0.4, 0.5]\rangle\}$$
$$\widetilde{\mathscr{G}}^C(\neg\varepsilon_3) = \{\langle h_1, [0.3, 0.4]\rangle, \langle h_2, [0.7, 0.9]\rangle, \langle h_3, [0.0, 0.1]\rangle, \langle h_4, [0.6, 0.8]\rangle, \langle h_5, [0.3, 0.5]\rangle, \langle h_6, [0.0, 0.2]\rangle\}$$

Definition 9. The "AND" operation on the two interval-valued fuzzy soft sets $(\widetilde{\mathscr{F}}, A)$, $(\widetilde{\mathscr{G}}, B)$ is defined by

$$(\widetilde{\mathscr{F}}, A) \wedge (\widetilde{\mathscr{G}}, B) = (\widetilde{\mathscr{H}}, A \times B), \tag{10}$$

where $\widetilde{\mathscr{H}}(\alpha, \beta) = \widetilde{\mathscr{F}}(\alpha) \cap \widetilde{\mathscr{G}}(\beta), \forall (\alpha, \beta) \in A \times B$.

Example 5. Compute the result of the "AND" operation on the interval-valued fuzzy soft sets $(\widetilde{\mathscr{F}}, A)$ and $(\widetilde{\mathscr{G}}, B)$ showed in Tables 1 and 2 respectively.

Table 3
"AND" operation on Tables 1 and 2.

U	$\epsilon_1, \varepsilon_1$	$\epsilon_1, \varepsilon_2$	$\epsilon_1, \varepsilon_3$	$\epsilon_2, \varepsilon_1$	$\epsilon_2, \varepsilon_2$	$\epsilon_2, \varepsilon_3$
h_1	[0.3, 0.5]	[0.7, 0.9]	[0.6, 0.7]	[0.3, 0.5]	[0.6, 0.7]	[0.6, 0.7]
h_2	[0.6, 0.8]	[0.6, 0.8]	[0.1, 0.3]	[0.7, 0.9]	[0.8, 1.0]	[0.1, 0.3]
h_3	[0.5, 0.6]	[0.2, 0.5]	[0.5, 0.6]	[0.2, 0.4]	[0.2, 0.4]	[0.2, 0.4]
h_4	[0.2, 0.3]	[0.1, 0.3]	[0.2, 0.4]	[0.0, 0.1]	[0.0, 0.1]	[0.0, 0.1]
h_5	[0.8, 0.9]	[0.8, 0.9]	[0.5, 0.7]	[0.1, 0.3]	[0.1, 0.3]	[0.1, 0.3]
h_6	[0.8, 1.0]	[0.5, 0.6]	[0.8, 1.0]	[0.7, 0.8]	[0.5, 0.6]	[0.7, 0.8]
U	$\epsilon_3, \varepsilon_1$	$\epsilon_3, \varepsilon_2$	$\epsilon_3, \varepsilon_3$	$\epsilon_4, \varepsilon_1$	$\epsilon_4, \varepsilon_2$	$\epsilon_4, \varepsilon_3$
h_1	[0.3, 0.5]	[0.3, 0.5]	[0.3, 0.5]	[0.3, 0.5]	[0.5, 0.8]	[0.5, 0.7]
h_2	[0.7, 0.9]	[0.8, 0.9]	[0.1, 0.3]	[0.7, 0.9]	[0.8, 1.0]	[0.1, 0.3]
h_3	[0.5, 0.7]	[0.2, 0.5]	[0.5, 0.7]	[0.5, 0.7]	[0.2, 0.5]	[0.7, 0.9]
h_4	[0.2, 0.3]	[0.1, 0.3]	[0.2, 0.4]	[0.2, 0.3]	[0.1, 0.3]	[0.2, 0.4]
h_5	[0.8, 0.9]	[0.9, 1.0]	[0.5, 0.7]	[0.2, 0.5]	[0.2, 0.5]	[0.2, 0.5]
h_6	[0.2, 0.5]	[0.2, 0.5]	[0.2, 0.5]	[0.7, 1.0]	[0.5, 0.6]	[0.7, 1.0]

Table 4
"OR" operation on Tables 1 and 2.

U	$\epsilon_1, \varepsilon_1$	$\epsilon_1, \varepsilon_2$	$\epsilon_1, \varepsilon_3$	$\epsilon_2, \varepsilon_1$	$\epsilon_2, \varepsilon_2$	$\epsilon_2, \varepsilon_3$
h_1	[0.7, 0.9]	[0.9, 1.0]	[0.7, 0.9]	[0.6, 0.7]	[0.9, 1.0]	[0.6, 0.7]
h_2	[0.7, 0.9]	[0.8, 1.0]	[0.6, 0.8]	[0.8, 1.0]	[0.8, 1.0]	[0.8, 1.0]
h_3	[0.5, 0.7]	[0.5, 0.6]	[0.9, 1.0]	[0.5, 0.7]	[0.2, 0.5]	[0.9, 1.0]
h_4	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.2, 0.3]	[0.1, 0.3]	[0.2, 0.4]
h_5	[0.8, 0.9]	[0.9, 1.0]	[0.8, 0.9]	[0.8, 0.9]	[0.9, 1.0]	[0.5, 0.7]
h_6	[0.9, 1.0]	[0.8, 1.0]	[0.8, 1.0]	[0.9, 1.0]	[0.7, 0.8]	[0.8, 1.0]
U	$\epsilon_3, \varepsilon_1$	$\epsilon_3, \varepsilon_2$	$\epsilon_3, \varepsilon_3$	$\epsilon_4, \varepsilon_1$	$\epsilon_4, \varepsilon_2$	$\epsilon_4, \varepsilon_3$
h_1	[0.3, 0.5]	[0.9, 1.0]	[0.6, 0.7]	[0.5, 0.8]	[0.9, 1.0]	[0.6, 0.8]
h_2	[0.8, 0.9]	[0.8, 1.0]	[0.8, 0.9]	[0.9, 1.0]	[0.9, 1.0]	[0.9, 1.0]
h_3	[0.5, 0.7]	[0.5, 0.7]	[0.9, 1.0]	[0.7, 0.9]	[0.7, 0.9]	[0.9, 1.0]
h_4	[0.7, 1.0]	[0.7, 1.0]	[0.7, 1.0]	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]
h_5	[0.9, 1.0]	[0.9, 1.0]	[0.9, 1.0]	[0.8, 0.9]	[0.9, 1.0]	[0.5, 0.7]
h_6	[0.9, 1.0]	[0.5, 0.6]	[0.8, 1.0]	[0.9, 1.0]	[0.7, 1.0]	[0.8, 1.0]

By Definition 9, we have

$$\begin{aligned}\widetilde{\mathscr{H}}(\epsilon_1, \varepsilon_1) &= \widetilde{\mathscr{F}}(\epsilon_1) \cap \widetilde{\mathscr{G}}(\varepsilon_1)\\ &= \{\langle h_1, [0.3, 0.5]\rangle, \langle h_2, [0.6, 0.8]\rangle, \langle h_3, [0.5, 0.6]\rangle, \langle h_4, [0.2, 0.3]\rangle, \langle h_5, [0.8, 0.9]\rangle, \langle h_6, [0.8, 1.0]\rangle\}\end{aligned}$$

The result of $(\widetilde{\mathscr{F}}, A) \wedge (\widetilde{\mathscr{G}}, B)$ is showed in Table 3.

Definition 10. The "OR" operation on the two interval-valued fuzzy soft sets $(\widetilde{\mathscr{F}}, A)$, $(\widetilde{\mathscr{G}}, B)$ is defined by

$$(\widetilde{\mathscr{F}}, A) \vee (\widetilde{\mathscr{G}}, B) = (\widetilde{\mathscr{H}}, A \times B), \tag{11}$$

where $\widetilde{\mathscr{H}}(\alpha, \beta) = \widetilde{\mathscr{F}}(\alpha) \cup \widetilde{\mathscr{F}}(\beta), \forall(\alpha, \beta) \in A \times B$.

Example 6. The result of the "OR" operation on $(\widetilde{\mathscr{F}}, A)$ and $(\widetilde{\mathscr{G}}, B)$ in Tables 1 and 2 is shown in Table 4.

Theorem 1 (*DeMorgan's Laws of Interval-Valued Fuzzy Soft Sets*). *Let $(\widetilde{\mathscr{F}}, A)$, $(\widetilde{\mathscr{G}}, B)$ are two interval-valued fuzzy soft sets, we have*

$$\left((\widetilde{\mathscr{F}}, A) \wedge (\widetilde{\mathscr{G}}, B)\right)^C = (\widetilde{\mathscr{F}}, A)^C \vee (\widetilde{\mathscr{G}}, B)^C;$$

$$\left((\widetilde{\mathscr{F}}, A) \vee (\widetilde{\mathscr{G}}, B)\right)^C = (\widetilde{\mathscr{F}}, A)^C \wedge (\widetilde{\mathscr{G}}, B)^C.$$

Proof.

$$\begin{aligned}(\widetilde{\mathscr{F}}, A)^C \vee (\widetilde{\mathscr{G}}, B)^C &= (\widetilde{\mathscr{F}}^C, \neg A) \vee (\widetilde{\mathscr{G}}^C, \neg B)\\ &= (\widetilde{\mathscr{I}}, \neg A \times \neg B) \quad \text{where } \widetilde{\mathscr{I}}(\neg\alpha, \neg\beta) = \widetilde{\mathscr{F}}^C(\neg\alpha) \cup \widetilde{\mathscr{G}}^C(\neg\beta)\\ &= (\widetilde{\mathscr{I}}, \neg(A \times B)).\end{aligned}$$

Suppose that $(\widetilde{\mathscr{F}}, A) \wedge (\widetilde{\mathscr{G}}, B) = (\widetilde{\mathscr{H}}, A \times B)$, then we have $\left((\widetilde{\mathscr{F}}, A) \wedge (\widetilde{\mathscr{G}}, B)\right)^C = (\widetilde{\mathscr{H}}, A \times B)^C = (\widetilde{\mathscr{H}}^C, \neg(A \times B))$.

Table 5
Resultant interval-valued fuzzy soft set $(\widetilde{\mathscr{H}}, P)$.

U	$\epsilon_1, \varepsilon_1$	$\epsilon_1, \varepsilon_3$	$\epsilon_2, \varepsilon_2$	$\epsilon_2, \varepsilon_3$	$\epsilon_3, \varepsilon_1$	$\epsilon_4, \varepsilon_2$
h_1	[0.3, 0.5]	[0.6, 0.7]	[0.6, 0.7]	[0.6, 0.7]	[0.3, 0.5]	[0.5, 0.8]
h_2	[0.6, 0.8]	[0.1, 0.3]	[0.8, 1.0]	[0.1, 0.3]	[0.7, 0.9]	[0.8, 1.0]
h_3	[0.5, 0.6]	[0.5, 0.6]	[0.2, 0.4]	[0.2, 0.4]	[0.5, 0.7]	[0.2, 0.5]
h_4	[0.2, 0.3]	[0.2, 0.4]	[0.0, 0.1]	[0.0, 0.1]	[0.2, 0.3]	[0.1, 0.3]
h_5	[0.8, 0.9]	[0.5, 0.7]	[0.1, 0.3]	[0.1, 0.3]	[0.8, 0.9]	[0.2, 0.5]
h_6	[0.8, 1.0]	[0.8, 1.0]	[0.5, 0.6]	[0.7, 0.8]	[0.2, 0.5]	[0.5, 0.6]

$\forall(\alpha, \beta) \in A \times B$, we have

$$\begin{aligned} \widetilde{\mathscr{H}}^C(\neg\alpha, \neg\beta) &= \left(\widetilde{\mathscr{H}}(\alpha, \beta)\right)^C \\ &= \left(\widetilde{\mathscr{F}}(\alpha) \cap \widetilde{\mathscr{G}}(\beta)\right)^C \\ &= (\widetilde{\mathscr{F}}(\alpha))^C \cup (\widetilde{\mathscr{G}}(\beta))^C \\ &= \widetilde{\mathscr{F}}^C(\neg\alpha) \cup \widetilde{\mathscr{G}}^C(\neg\beta). \end{aligned}$$

From the discussion above, we have $\left((\widetilde{\mathscr{F}}, A) \wedge (\widetilde{\mathscr{G}}, B)\right)^C = (\widetilde{\mathscr{F}}, A)^C \vee (\widetilde{\mathscr{G}}, B)^C$.

Similar to the above progress, it is not difficult to prove that $\left((\widetilde{\mathscr{F}}, A) \vee (\widetilde{\mathscr{G}}, B)\right)^C = (\widetilde{\mathscr{F}}, A)^C \wedge (\widetilde{\mathscr{G}}, B)^C$. □

Theorem 2. *Let $(\widetilde{\mathscr{F}}, A)$, $(\widetilde{\mathscr{G}}, B)$ and $(\widetilde{\mathscr{H}}, C)$ be three interval-valued fuzzy soft sets, then we have*

Associative law of interval-valued fuzzy soft sets

$$(\widetilde{\mathscr{F}}, A) \wedge ((\widetilde{\mathscr{G}}, B) \wedge (\widetilde{\mathscr{H}}, C)) = ((\widetilde{\mathscr{F}}, A) \wedge (\widetilde{\mathscr{G}}, B)) \wedge (\widetilde{\mathscr{H}}, C);$$
$$(\widetilde{\mathscr{F}}, A) \vee ((\widetilde{\mathscr{G}}, B) \vee (\widetilde{\mathscr{H}}, C)) = ((\widetilde{\mathscr{F}}, A) \vee (\widetilde{\mathscr{G}}, B)) \vee (\widetilde{\mathscr{H}}, C);$$

Distribution law of interval-valued fuzzy soft sets

$$(\widetilde{\mathscr{F}}, A) \wedge ((\widetilde{\mathscr{G}}, B) \vee (\widetilde{\mathscr{H}}, C)) = ((\widetilde{\mathscr{F}}, A) \wedge (\widetilde{\mathscr{G}}, B)) \vee ((\widetilde{\mathscr{F}}, A) \wedge (\widetilde{\mathscr{H}}, C));$$
$$(\widetilde{\mathscr{F}}, A) \vee ((\widetilde{\mathscr{G}}, B) \wedge (\widetilde{\mathscr{H}}, C)) = ((\widetilde{\mathscr{F}}, A) \vee (\widetilde{\mathscr{G}}, B)) \wedge ((\widetilde{\mathscr{F}}, A) \vee (\widetilde{\mathscr{H}}, C)).$$

Proof. $\forall\alpha \in A, \forall\beta \in B$ and $\forall\gamma \in C$, we have $\widetilde{\mathscr{F}}(\alpha) \cap (\widetilde{\mathscr{G}}(\beta) \cap \widetilde{\mathscr{H}}(\gamma)) = (\widetilde{\mathscr{F}}(\alpha) \cap \widetilde{\mathscr{G}}(\beta)) \cap \widetilde{\mathscr{H}}(\gamma)$, from which we can conclude that $(\widetilde{\mathscr{F}}, A) \wedge ((\widetilde{\mathscr{G}}, B) \wedge (\widetilde{\mathscr{H}}, C)) = ((\widetilde{\mathscr{F}}, A) \wedge (\widetilde{\mathscr{G}}, B)) \wedge (\widetilde{\mathscr{H}}, C)$ holds. Similarity, we also have $(\widetilde{\mathscr{F}}, A) \vee ((\widetilde{\mathscr{G}}, B) \vee (\widetilde{\mathscr{H}}, C)) = ((\widetilde{\mathscr{F}}, A) \vee (\widetilde{\mathscr{G}}, B)) \vee (\widetilde{\mathscr{H}}, C)$.

$\forall\alpha \in A, \forall\beta \in B$ and $\forall\gamma \in C$, by the properties of the interval-valued fuzzy sets, we have $\widetilde{\mathscr{F}}(\alpha) \cap (\widetilde{\mathscr{G}}(\beta) \cup \widetilde{\mathscr{H}}(\gamma)) = (\widetilde{\mathscr{F}}(\alpha) \cap \widetilde{\mathscr{G}}(\beta)) \cup (\widetilde{\mathscr{F}}(\alpha) \cap \widetilde{\mathscr{H}}(\gamma))$, from which we can conclude that $(\widetilde{\mathscr{F}}, A) \wedge ((\widetilde{\mathscr{G}}, B) \vee (\widetilde{\mathscr{H}}, C)) = ((\widetilde{\mathscr{F}}, A) \wedge (\widetilde{\mathscr{G}}, B)) \vee ((\widetilde{\mathscr{F}}, A) \wedge (\widetilde{\mathscr{H}}, C))$ holds. Similarity, we also have $(\widetilde{\mathscr{F}}, A) \vee ((\widetilde{\mathscr{G}}, B) \wedge (\widetilde{\mathscr{H}}, C)) = ((\widetilde{\mathscr{F}}, A) \vee (\widetilde{\mathscr{G}}, B)) \wedge ((\widetilde{\mathscr{F}}, A) \vee (\widetilde{\mathscr{H}}, C))$. □

4. Application of interval-valued fuzzy soft set

In Ref. [15], by using the fuzzy soft set theory, Roy et al. have presented an algorithm for identification of an object, which is based on the comparison of different objects. Later, in Ref. [16], the authors pointed out that Roy's algorithm is incorrect and then they presented a modified algorithm, which is based on the comparison of choice values of different objects, i.e. the higher the choice value, the better the house is. In the following, we will use such a modified algorithm to solve a decision making problem which is based on the concept of the interval-valued fuzzy soft set.

Step 1: Input the set of interval-valued fuzzy soft sets.

In this paper, suppose that the interval-valued fuzzy soft sets $(\widetilde{\mathscr{F}}, A)$ and $(\widetilde{\mathscr{G}}, B)$ showed in Tables 1 and 2 are under consideration.

Step 2: Input the parameter set P as observed by the observer.

Considering the above two interval-valued fuzzy soft sets $(\widetilde{\mathscr{F}}, A)$ and $(\widetilde{\mathscr{G}}, B)$, if we perform an "AND" operation on such interval-valued fuzzy soft sets, then we get the results as Table 3 shows. Suppose that we are considering the set of parameters P such that $P = \{\{\epsilon_1, \varepsilon_1\}, \{\epsilon_1, \varepsilon_3\}, \{\epsilon_2, \varepsilon_2\}, \{\epsilon_2, \varepsilon_3\}, \{\epsilon_3, \varepsilon_1\}, \{\epsilon_4, \varepsilon_2\}\}$.

Step 3: Compute the corresponding resultant interval-valued fuzzy soft set $(\widetilde{\mathscr{H}}, P)$ from $(\widetilde{\mathscr{F}}, A)$ and $(\widetilde{\mathscr{G}}, B)$.

By Table 3 and the set of parameters, we can get the resultant interval-valued fuzzy soft set as Table 5 shows.

Step 4: $\forall h_i \in U$, compute the choice value c_i for each house h_i such that

$$c_i = [c_i^-, c_i^+] = \left[\sum_{p\in P} \mu^-_{\widetilde{\mathscr{H}}(p)}(h_i), \sum_{p\in P} \mu^+_{\widetilde{\mathscr{H}}(p)}(h_i)\right].$$

The result is shown in Table 6.

Table 6
Choice value.

U	$\epsilon_1, \varepsilon_1$	$\epsilon_1, \varepsilon_3$	$\epsilon_2, \varepsilon_2$	$\epsilon_2, \varepsilon_3$	$\epsilon_3, \varepsilon_1$	$\epsilon_4, \varepsilon_2$	c_i
h_1	[0.3, 0.5]	[0.6, 0.7]	[0.6, 0.7]	[0.6, 0.7]	[0.3, 0.5]	[0.5, 0.8]	$c_1 = [2.9, 3.9]$
h_2	[0.6, 0.8]	[0.1, 0.3]	[0.8, 1.0]	[0.1, 0.3]	[0.7, 0.9]	[0.8, 1.0]	$c_2 = [3.1, 4.3]$
h_3	[0.5, 0.6]	[0.5, 0.6]	[0.2, 0.4]	[0.2, 0.4]	[0.5, 0.7]	[0.2, 0.5]	$c_3 = [2.1, 3.2]$
h_4	[0.2, 0.3]	[0.2, 0.4]	[0.0, 0.1]	[0.0, 0.1]	[0.2, 0.3]	[0.1, 0.3]	$c_4 = [0.7, 1.5]$
h_5	[0.8, 0.9]	[0.5, 0.7]	[0.1, 0.3]	[0.1, 0.3]	[0.8, 0.9]	[0.2, 0.5]	$c_5 = [2.5, 3.6]$
h_6	[0.8, 1.0]	[0.8, 1.0]	[0.5, 0.6]	[0.7, 0.8]	[0.2, 0.5]	[0.5, 0.6]	$c_6 = [3.5, 4.5]$

Step 5: $\forall h_i \in U$, compute the score r_i of h_i such that

$$r_i = \sum_{h_j \in U} \left((c_i^- - c_j^-) + (c_i^+ - c_j^+) \right).$$

Thus, we have $r_1 = 5.0, r_2 = 8.6, r_3 = -4.0, r_4 = -22.6, r_5 = 0.8, r_6 = 12.2$.

Step 6: The decision is any one of the elements in S where $S = max_{h_i \in U}\{r_i\}$.

In our example, house h_6 is the best choice because $\max_{h_i \in U}\{r_i\} = \{h_6\}$. This result is reasonable because we can see that $c_6 \geq c_i$ where $i = 1, 2, 3, 4, 5$, i.e. h_6 has the highest choice value.

5. Conclusions

Pioneering work on the soft set has been done by Molodtsov. The soft set theory is a general mathematical tool for dealing with uncertain, fuzzy, or vague objects. In this paper, the concept of the interval-valued fuzzy soft set is proposed. The interval-valued fuzzy soft set is a combination of an interval-valued fuzzy set and a soft set. The complement, "AND" and "OR" operations are then defined on the interval-valued fuzzy soft sets. We also proved the DeMorgan's, associative and distribution laws of the interval-valued fuzzy soft sets. Finally, an illustrative example is used to show the validity of the interval-valued fuzzy soft set in a decision making problem.

In further research, the parameterization reduction of the interval-valued fuzzy soft set is an important and interesting issue to be addressed.

Acknowledgment

This work is supported by the Natural Science Foundation of China (Nos. 60472060, 60572034 and 60632050).

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