The Shape of Shapes: An Ontological Exploration

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Abstract: What are shapes? The ancient Greek geometer is perhaps the most familiar with these entities, but shape is general enough to transcend domains of inquiry. In this communication I present a broad examination of *shape* and related notions, such as *form*, *boundary* and *surface.* I aim to explore this question and describe the category of shape by identifying some of its most general features. I contrast geometric (or perfect) shapes with physical or organic shapes (the shape of objects), discuss granularity, artifactuality, the cognitive aspect of shape and introduce a preliminary ontological hierarchy of shape.

Keywords: ontology, shape, form, boundary, physical discontinuity, mathematical entity

1.0 Introduction

What are shapes? The ancient Greek geometer is perhaps the most acquainted with the notion of shape, having made geometry into an abstract discipline, while keeping it “firmly rooted in the real world where it could give him true and reliable information about actual spatial relationships.”[1]. Yet all of us have an implicit idea of shape in our mind’s eye, whether from education in geometry or from our inherent ability to recognize recurrent and similar forms and figures in the world. In this communication I aim to explore this question and describe our notion of *shape*. I identify some general features of shapes, and determine their ontological category. The notions of form, boundary, surface and granularity are introduced in relation to shape.

An important distinction to keep in mind is that between ideal, perfect and abstract geometric shapes on the one hand, and imperfect[I], physical or organic mind-external shapes on the other. Call the former ‘geometric shapes’ and the latter ‘physical shapes’ or ‘organic shapes’. This distinction can be understood as being parallel to types (classes, universals, general entities) and instances (individuals or particulars in the world). Geometric shapes typically have precise mathematical formalizations. Their exact physical manifestations are not, so far as I am aware, observed in mind-external reality, only approximated by entities exhibiting a similar shape. In this sense geometric shapes are idealizations or abstractions. This makes geometric shapes similar to types or universals. Their instances are inexact replicas of the *shape type* in question, but have similar attributes or properties in common, properties characterizing the type.

By contrast, organic or physical shapes are irregular or uneven shapes of mind-external objects or things in the world. A planet is not perfectly spherical, and the branches of

[I] In using the words ‘perfect’ or ‘imperfect’ I do not use them in any normative sense.

a tree are not perfectly cylindrical, for example. ‘Perfectly’ is used here in the sense of coinciding with or physically manifesting the exact mathematical definitions, or precise symmetrical relations, of geometric shapes. Objects and physical phenomena in the world, rarely if ever, manifest or exhibit any concretization of geometric shapes, but this is not to say that it is not possible or that it does not obtain at times. Objects are not precisely symmetrical about a given axis, cube-shaped things do not have faces of exactly the same area, for example, and there is no concretization of a perfect sphere. Note that the terms ‘object’ and ‘material object’ are used in the sense of ‘object’ in [2]. The term ‘entity’ is used in its usual way, as a catch-all for anything that exists, from objects to processes to thoughts.

I begin with a discussion of the dimensionality of shapes in section 2.0, followed by an ontological categorization of shape in 3.0. Descriptions and definitions of ‘form’ and ‘shape’ are introduced in 4.0, boundary and surface are introduced in 5.0, granularity in 6.0, and a simple ontology in 7.0. Section 8.0 ends with my closing remarks and reflections. Throughout I enumerate some plausible features and descriptions of *shape*.

2.0 Dimensionality of Shape

It seems clear that shapes are not to be found in the realm of the zero-dimensional. There are no zero-dimensional shapes, for that which is zero-dimensional, although an entity in the arsenal of the mathematician, is but an extensionless point. From an abstract perspective, points can be understood as parts of (higher-dimensional) shapes. For example, for every line segment there are two points bounding its extent. Although points in space appear natural, they are abstractions which cannot be materialized [3].

Extending to the one-dimensional (1D for short), we can say that shapes come into being. Examples are lines, including straight lines and line segments, rays, sinusoids, parabolas, and so forth. If we understand lines and other geometric entities as being introduced by ancient mathematicians to represent more-or-less straight objects in the world and, thus, as idealizations or abstractions of mind-external objects,[II] then we are better able to view these 1D entities as geometric shapes.

The notion of shape connotes *extension* from both perspectives of physical space (as in mind-external), and mental, conceptual or abstract space (as in mind-internal).[III] The latter is to be understood as the geometer’s abstract realm of perfect shapes that have, or could have, precise formulations. To avoid confusions, we may call this realm ‘geometric space’. It is here that the geometer manipulates her objects of inquiry and it is to the Earth (and to spatiality, among other things) that the geometer applies and projects her tools and domain-specific objects (shapes). Perhaps shape is a *spatial form*, a subtype of the more general type *form*. Note that in using the words ‘abstract’ and ‘realm’ I do not suggest the literal existence of an abstract world divorced from our spatiotemporal universe. I make no commitment to the mind-external reality of mathematical entities, but an inquiry into the ontology of shape will require exploration into the philosophy of mathematics. This will involve inquiry into the

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[II] With regard to *lines*: http://en.wikipedia.org/wiki/Line_%28geometry%29. It is reasonable that geometric entities were used, posited or introduced by the ancients as representations of objects in the world. That geometry has been practically used (art, navigation and surveying, etc.) since antiquity, and that it likely started as a practical enterprise, attests to that.

[III] This is not to say that shapes literally occupy our minds (as if they were physical occupants), only that abstract and perfect geometric shapes are not found outside of the mind.

ontological status of mathematical entities and the relation of mathematics and other formal expressions to the world. At the very least and in some sense, mathematical entities exist in a cognitive fashion (perhaps as representations), in minds as ideas. Although I am skeptical, I leave open the question as to their mind-independent existence. Mathematics, in conjunction with its applicable expression in physics, is the formal language we use to communicate order, structure, pattern, and (nomological) principledness in the universe.

These geometric entities reflect the ideal and perfect shapes we are able to conceive, mathematically formalize, identify in (some say *discover*) and project onto the imperfect (although structured) spatio-temporal world of objects, artifacts, landscapes, astronomical phenomena and geographic entities. Practical examples of projecting geometric shapes onto the mind-external world include reference shapes in global positioning systems, coastal navigation and surveying. It is very likely that the development of geometry began with solving practical problems in mind.

We have, then, the following. (S1a) *Shapes are necessarily extended*. Alternatively, (S1b) *If an entity has or exhibits a shape then that entity is necessarily spatially extended*, and (S2) *Shapes are at least one-dimensional (Shapes have at least one-dimensional extent)*, or alternatively, *If an entity has or exhibits a shape, then that entity is at least one-dimensional.*[IV] In (S1a) I make no mention of the kind of extension—spatial, temporal, or otherwise—because we can understand shapes as spatially and materially extended in mind-external reality, and as extended in a non-physical or immaterial sense in conceptual, cognitive, or mind-internal reality. Representations of shape may fall under one of these types of extension.

Two-dimensionality (2D for short) presents us with some of the most familiar types of shape: the circle, the triangle and its subtypes (equilateral, isosceles, right, scalene), the square, the hexagon, and so forth. Here is where our basic studies in geometry find a home. Here is where we comprehend points, sides (or edges), angles, congruence, and symmetry. As we vary the length of one or more sides of symmetric or equilateral 2D shapes, we form a hodgepodge of asymmetrical shapes. Likewise, as we vary the curvature of curved shapes, all sorts of unique and irregular figures are born. It quickly becomes apparent that there are infinitely many shapes in the geometer's toolbox: (S3) *There are infinitely many geometric shapes*. This applies to two- three- and higher-dimensional shapes alike.

Under a given geometry of space—Euclidean or Non-Euclidean, for example—2D shapes can be formally described. The geometric descriptions inform us of their properties, such as having a certain number (quantity) of sides[V], and the number and measure of angles formed by those sides. These descriptions may vary depending on the adopted geometry. Using spherical geometry, the sum of the angles of a triangle is greater than that of a triangle in Euclidean geometry.[VI] The specific length of the sides, however, is not described. Increasing or decreasing the length of the sides of a square, for example, will not cause it to cease to be a square so long as (a) the lengths of all the sides remain equal, and (b) the dimensionality remains constant. In other words, if the identity of the shape as a square persists, then any decrease in the length of all the four sides does not result in a *dimensional*

[IV] If the reader is very uncomfortable with the idea of lines, sinusoids and other 1D entities being classified as shapes, then two-dimensionality may be considered the minimum dimensionality in which shapes take shape. Such a change has no significant effect on this communication.

[V] Shapes can be classified by the number of sides they have. For the three-dimensional polyhedra, they can be classified according to the number of faces.

[VI] Transcending the specific geometry, the general notion of triangle (or *triangular*) remains.

collapse or *dimensional descent* into a (one-dimensional) point. It may be asserted that any specificity in the measure of the sides of a shape indicates the presence of an individual shape, a shape instance. This is not obvious, however, because abstract geometric shapes can be constructed without their manifestation obtaining (as the particular shape of an object in the world). Each side of a 2D shape can be considered one of its parts.

Having four connected sides of equal length is an *essential property* of the square, as is having four angles of ninety degrees formed by the meeting of the sides. If a shape loses one of its sides, then it also loses its identity as that type of shape. A square that loses one of its four sides is no longer a square. If we consider sides to be parts of shapes, employing either a broad sense of parthood for material and immaterial entities, or a specific (say, immaterial) sense of parthood, then we exclude some ontological categorizations of shape. For example, we rule out *independent continuant* [2] as the ontological category of shape because independent continuants retain their identity while losing or gaining parts.[VII] It follows that *object* is ruled out as well because it is a subtype of independent continuant. Although shapes are a unity that is self-contained and extended, it is not at all clear that they are material entities so *object* is not an appropriate ontological classification. In a similar vein, *individuals* can gain and lose parts [4], but if we are presented with an individual (particular) shape and remove one of its sides it no longer persists as the same individual. No longer is it an instance of its type. To avoid excluding shapes as individuals—essentially excluding shape instances—one can argue that shapes do not have parts. Shapes that lose parts in both geometric space and in physical space seem to lose their identity, however. Finally, it makes little sense to say that the square changes into a line, but it does make sense to speak of the shape of an object changing. This fact is reflected in an account of shapes as properties of objects that undergo change over time.

When we arrive at three-dimensionality (3D for short) we arrive at our world of everyday objects. The objects in this dimensionality are expressed as having length, width, and height. In the abstract three-dimensional realm of the geometer we form and describe entities such as cubes, spheres, cylinders, and rectangular pyramids. These are all three-dimensional geometric shapes. Here, the geometer can describe 3D shapes in terms of lower-dimensional shapes. A cube has six 2D faces for example. In using the term 'side', we say: a 1D side is the shape itself, a 2D side is an edge, and a 3D side is a face (or simply *side*). Comparing these ideal 3D entities with objects in the physical world, we find oranges that have a roughly spherical shape, tree trunks approximating cylinders, and so forth.

In our 3D spatio-temporal universe, however, matters become more complicated because we no longer are solely considering the perfect abstract shapes of the geometer. We are now considering the shape *of* things. We are describing objects in the world as having a shape, and we observe similarity-of-shape among objects. Further, physics, physical geometry, and physical space-time all are related. Objects, organisms, and even aggregate-like entities such as clouds have the shape they do in virtue of a number of factors, all of which work together to present the shapes we observe, recurrently recognize, and (occasionally) formalize. Some of these factors include environmental processes, biological processes, and processes internal to the object. These factors, in turn, have a nomological foundation affording recurrence, similarity of shape and attribute-agreement in general. In this way, I am

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[VII] We also rule out shapes as independent continuants because the latter do not inhere in any entity. As we explore further we see that shapes are likely properties, and properties are said to inhere in other entities.

inclined toward understanding many types or universals[VIII] in terms of nomological factors rather than vice versa.

If universals are our way of explaining similarity, recurrence, repetition, and attribute-agreement in the world, then these latter aspects are, in turn, the result of the principled structure of the world. I am resistant to the idea that all there is to an ontological account of shape (and other categories) is that shape types exist in nature and that shape instances inhere in objects. Rather, the physical shape of an object is due to the complex network of naturally principled and physical influences involved in the existential becoming (or creation, if you will) of the object exhibiting that shape.[IX] Since ontology (applied and philosophical) traditionally makes use of types and individuals, universals and instances, the task is to produce an account of shape in these terms. This is particularly the case if it is true that "the appropriate subdivision of continuous reality is that into specific types of individuals"[3]. It is, nevertheless, accurate to understand many types or universals in terms of, or as describing[X], (i) physical principles or laws of nature underlying (ii) the above-mentioned lower-level elements (phenomena in the world). In other words, the explanation of *shape* as an ontological category involves numerous elements and considerations.

3.0 Shapes and The Shaped: Shapes as Properties and Shapes as Objects

With respect to the mind-external world, notice that if shapes are properties (of things), then we may have a situation in which properties have properties. At first glance this seems true because we predicate *shape* of objects in the world; we say that objects *have* a certain shape. We also describe types of shapes as having specific properties. If a shape is defined as having a particular number of sides (as with polygons), a particular curvature (as with *curved shapes*, such as the circle and the ellipse), specific relations between sides, or otherwise, then it should be apparent that we are describing properties of properties of things. We might be inclined to say that it is the shape that has a certain amount of angles and sides, rather than the object bearing the shape in question, but this is not entirely accurate. Shapes, conceived as objects in their own right (in geometric space), have sides, but in our spatiotemporal world, objects have sides, and surfaces, as well. When we divorce the shape from that which has the shape via abstraction, we use 'side' for the former as much as we do for the latter. The distinction between geometric and physical space, between ideas and ideal or cognitive constructions and material mind-external particulars is significant.

In the abstract geometric space of the geometer, we conceive or speak of shapes *as if* they were objects in their own right and identify (perhaps, prescribe) their properties. We say 'as if' because the term 'object' is intuitively applied to material entities in the world. Relative to the geometer's domain it is the case that shapes have particular properties, but here 'object' implicitly takes on a broader meaning such that there are geometric objects. The geometer's toolbox contains infinitely many of these so called objects. *A* triangle, we may say, is a polygon that has three angles formed by three connected edges at the endpoint of each edge, the sum of the angles being 180° (relative to Euclidean geometry, of course). *A* square is a polygon that has four right angles formed by four connected sides of equal length.

[VIII] If only in the philosophical sense.
[IX] An account of geometric shapes may not be so easily stated, however.
[X] In a unified if compartmentalized manner

The fact that descriptions of geometric shapes do not specify the length of the sides demonstrates the generality and abstract nature of shapes. This is one reason to hold that *shape* is a *type*, an ontological category. Another reason is that the notion of shape appears to be domain-neutral: it is general enough to be applied to astronomy, biology, cognitive science, geography, geology, physics and psychology[XI]. In considering shapes as independently existing entities we predicate properties of them.

The mind-external world does not, however, present us with these geometric shapes—these ideal and often perfectly symmetrical entities—as independently existing. This does not imply that there is no correlation, in some principled sense, between geometric models (or descriptions) and physical phenomena that are explained by those models. Although there are no perfect cylinders or other simple shapes, more complex geometric shapes are found in the world. Fractal geometry has numerous natural manifestations, including trees, leaves, lightning, and sea shells. When we shift from the abstract space of the geometer to the physical space of our universe—to determine whether the former explains, corresponds to, is concretized or manifested in reality—we do not find independently existing circles or squares. There are, in reality, no isolated points, lines or surfaces.[5] If these entities are anything over and above mental constructs, they are *dependent* on other entities for the existence.

Nature presents us with shapes similar to those in the geometer's toolbox, but what we are presented with is objects *with* a shape. We recognize shapes *in* objects, so to speak, and this is how we pretheoretically (and linguistically) understand *shape* as a property. A honeycomb is hexagonal, but not a precisely symmetric hexagon; the orbits of planetary bodies are represented, explained, and predicted using the conic sections, but bodies and their orbits undergo perturbations due to gravitational gradients (among other influences); and planetary bodies exhibit spheroidal and ellipsoidal shapes but are not perfect spheres or ellipsoids. The list goes on. The more that shapes are specialized, in an ontological hierarchy for example, the closer these phenomena can be formally represented and described. The shapes of natural bodies are *irregular*. These physical or organic shapes do not reflect the exact specifications of mathematical descriptions of geometric shapes. They *approximate* geometric shapes. More precisely, (S4) *(the shape of) objects in the world approximate geometric shapes*. Furthermore, category terms such as 'hexagonal' and 'spherical' have a certain degree of vagueness that reflects the imprecision with which individuals exhibit geometric shapes. This vagueness, this generality, also permits a variety of physical shapes to be considered instances, if not types, of the respective term.

We may treat geometric shapes as if they were independently existing entities, but common sense indicates that they are abstractions with no *exact* mind-external physical manifestation, and it would be a mistake to betray that intuition. That which we consider to be *shape* is intimately dependent on that which has the shape. In the mind-external world, shapes, it seems, are properties of things. They [things] must have a shape, i.e. be delineated by a shape.[6, brackets added] We say that a physical object *exhibits* a shape. Thus, (S5) *shapes must always be shapes of something in the mind-external world*. Outside idealized geometric space, it does not make sense to posit the existence of an independently existing shape, a shape with no bearer. The shape cannot exist, but as an idea, without an entity that bears, exhibits, or has that shape.

[XI] With respect to, say visual mental representations.

This does not detract from the theoretical and practical utility of manipulating geometric entities within conceptual space because it is often in that space that the seeds of insight (into how the world works) are planted, and it is to the external world that we apply those insights in order to scientifically progress and bear the fruit of those seeds.

Generally speaking, (S6) *shapes are dependent entities*. [2] classifies this kind of entity as a *dependent continuant*. These dependent entities are said to *inhere in* (the inherence relation) objects. What is it to *have* a shape? Having a shape (a dependent continuant) is to bear a property[XII]. The inherence relation seems to be what is meant by having a shape. With regard to the distinction between types and individuals, we have *shape property types*, and *shape property instances*. We can be more precise in the relationship between the shape property instance and its bearer by explicating the dependence relation between them.

A shape-instance (a physical or organic shape), *a*, is *one-sidedly dependent* on its bearer, *b*. This means that *a* is one-sidedly dependent upon *b* if and only if *a* is dependent on *b* but *b* is independent of *a*.[7] Alternatively, the relation between a shape and the object in which it inheres is that of *specific dependence* for instances. Specific dependence indicates that when the (specific) bearer ceases to exist, the (specifically) borne entity (the dependent entity, the shape instance) ceases to exist as well. As [8] states:

> On the instance level… *a* depends_on *b* =def. *a* is necessarily such that if *b* ceases to exist than *a* ceases to exist.
>
> On the type level… *A specifically_depends_on B* =def. for every instance *a* of *A*, there is some instance *b* of *B* such that *a* depends_on *b.*

For types, this amounts to saying that all shape instances have some entity (a bearer) on which they existentially depend. That bearer is most often a material entity such as an object. Specific dependence is a possible relation between shapes and objects because shape instances are unique to each bearer. The shape of this particular sculpture is not borne by any other sculpture, just as the shape of any organism is unique to that organism while exhibiting attribute-agreement and a similarity of kind. They are similar, but not exact replicas of one another, and their difference is due to variability in the aforementioned factors[XIII].

Shape instances are not isolable from their bearer. The only sense in which they are isolable is (a) cognitively, as an abstraction, and (b) in representational artifacts [9], [10]. Physical shapes are more akin to shape instances. Geometric shapes are more akin to shape types. A star shape, as a geometric shape, has star shape property instances exhibited by, say, particular star fish. The objects we observe in the world approximate geometric shapes, and deviate from those shapes by varying degrees. In the next section I explore the more general notion of *form* and introduce some definitions of both ‘form’ and ‘shape’.

4.0 Form and Shape

In saying that something has the form of a circle, the use of the phrase “form of a circle” is simply a manner of speaking. Uttering this (and similar) phrase(s) can be understood as speaking *as if* the circle is an independently existing entity, having a property: the form. Our commonplace expressions and knowledge of shapes reflect a primarily qualitative,

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XII [2] and [20] uses the term ‘quality’ instead of ‘property’.

XIII See the end of section 2.0.

perhaps purely visual, understanding of these entities. When we say something has the form of a circle, square, triangle, or any other geometric shape, we are speaking as though there is an entity in the world that is *a circle* and its form—a property of this entity—is its shape. We do not sincerely and pretheoretically believe this to be the case, however. We know that this kind of discourse does not make reference to a free-floating shape in space, but involves a turn-of-phrase, an analogy or an abstraction at best. In this context, this is the extent of our prephilosophical considerations of shape. It is arguable, of course, but our commonsense intuitions about geometric entities (perhaps all mathematical entities) may be that mathematical realism does not obtain in the world.

We also utter expressions such as "That cloud has the form of a bird.", where "form", again, is synonymous with "shape". In this context, we use "a bird" as a general phrase, aware of the existence of particular birds. Indeed, our notion of shape is intimately connected with our experience interacting with the world. We most commonly know the form or shape of a bird by observing individual birds, but this is surely not the only avenue into knowledge of bird forms. Part of the regularity and attribute-agreement among species of birds (and other natural or biological entities) is their form or shape. We recognize recurrent shapes that vary slightly from one another in the world. That there are similar bodily forms (shapes) is sufficient to posit corresponding types. Moreover, the object in question—the bird—cognitively takes on a degree of vagueness just as shape terms such as 'spherical' do. That is, we do not necessarily have a particular type of bird in mind when we utter these expressions, but do have an approximate and primarily qualitative[XIV] idea of the body shape of birds. In short, "form of a circle" reads "the shape of a circle", which suggests that circles have a property called 'shape'. What is the shape of a circle? The answer, surely, is that it is circular! We have a shape of a shape, but again, this is a *façon de parler*.

The term 'form', itself, is particularly ambiguous. It can be used in a broader fashion than 'shape', and is used in a number of domains of inquiry, such as art, biology (as in morphology), and philosophy. 'Form' (the Greek *eidos*) has been defined in multiple ways [11]:

F1) The shape and structure of something as distinguished from its material
F2) The essential nature of a thing as distinguished from its matter
F3) One of the different modes of existence, action, or manifestation of a particular thing or substance : KIND
F4) One of the different aspects a word may take as a result of inflection or change of spelling or pronunciation
F5) A mathematical expression of a particular type
F6) Orderly method of arrangement (as in the presentation of ideas) : manner of coordinating elements (as of an artistic production or course of reasoning). Or a particular kind or instance of such arrangement.
F7) A visible and measurable unit defined by a contour: a bounded surface or volume

These definitions, in some form or other, reflect the ideas of the ancient Greeks, namely Plato's Theory of Forms [12]. The idea common to the definitions is that of the distinction between generality, repeatability and recurrence on the one hand, and particularity on the other. In modernity, the former is expressed with the term 'universal' to signify what is

[XIV] We do not, for example, intuitively know precise proportions.

universal or commonly shared, general and similar about particular entities in the world. Generality and similarity among particulars is due to the nomological and structured or patterned character of the world. That pattern is communicated formally, as (F5) suggests, with mathematical and physical (as in physics[XV]) expressions.

Consider, now, the following definitions and descriptions of 'shape', which provide some valuable insights, of which I find the first particularly useful.

1) The shape of an object located in some space is a geometrical description of the part of that space occupied by the object, as determined by its external boundary – abstracting from location and orientation in space, size, and other properties such as colour, content, and material composition.[XVI]
2) External form or contour; that quality of a material object (or geometrical figure) which depends on constant relations of position and proportionate distance among all the points composing its outline or its external surface; a particular variety of this quality.[13]
3) The outward form of an object defined by outline.
   The figure or outline of the body of a person.[14]
4) The particular physical form or appearance of something.[15]

The reference to constant relations and proportionality in (2) reflects properties essential to geometric shapes. For example, that the points on the circumference of a circle are equidistant from the center reflects constant relations and proportions at work. What is common to most of these definitions is an external aspect related to shape, a central consideration when seeking to describe the general features of the category of *shape*. The *shapes of objects* (physical shapes) are identified relative to and based on the external surface, boundary, or form of the object.

5.0 Boundary, Surface and Shape

The world presents us with numerous physical discontinuities, from the exterior boundaries of solid objects to interfaces (a type of boundary) such as the interface between the surface of the ocean and the atmosphere above. Objects, such as an orange, a flower, or a coffee mug, can be clearly identified as having physical boundaries. The crisp edges of a sheet of paper delineate the spatial and material extent of the sheet. These boundaries separate, but do not isolate, objects from one another and from their surrounding environments. The environment, broadly speaking, and the object are, in fact, interacting (if only at certain granular levels). They are not isolated from one another. We identify shapes by these physical discontinuities, and they help us to understand the relationship between material entities and their shape.

Form, shape, boundary and surface are related. *Form* can be asserted to be a superclass of *shape*, and *boundary* a superclass of *surface*. The totality of the surfaces of an object determines, in part, the shape of the object. In other words, the shape of an object is determined by its external boundary(s). The surfaces—the external boundaries—of objects, in

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[XV] It is physics that provides us with the connection to and the comprehension of phenomena in the world, not pure mathematics.

[XVI] The original source is unknown, but can be read at http://en.wikipedia.org/wiki/Shape.

turn, depend on the material constitution of the object, as well as external and internal processes[XVII] affording the formation, and preservation, of the boundary. For a very nice treatment of external and internal processes see [16] and [17].

(S7) *All shapes (geometric and physical) have, are composed of, or are bounded by boundaries*.[XVIII] Shapes are also self-contained. The external boundaries of 1D shapes are the shapes themselves. The external boundaries of 2D shapes are more similar to outlines or blueprints, and are a composite of 1D boundaries. The external boundaries of 3D shapes are surfaces, where surfaces can be described as 2D shapes[XIX]. Objects end at their surfaces. The shape of an object—the physical or organic shape—is identified at the limits of the spatial extent of the object. *The shape of an object depends on the spatial extension of the object.* If many objects share similar limits of spatial extension, we say that those objects have the same shape. In this way, that similarity affords the positing of shape property types. The object, then, is said to *bear* a shape property, an instance of the shape type. That instance—the particular shape—is understood as a property of the object.

Consider a particular vase and the shape of the vase. The shape of the vase is not the vase. The shape of the vase, considered in itself or in isolation from the vase, is if nothing more, an abstraction of an aspect of the vase. That aspect—the shape—is perceived as a gestalt (a whole, a unity or totality) in virtue of the limit of the spatial and material extension of the vase as well as in virtue of our cognitive apparatus.[XX] The vase ends at its surface. We observe discontinuity between the vase and its surroundings. The physical discontinuity between object and the surrounding medium or space makes the shape of the object apparent to our awareness. The presence of physical discontinuities affords the existence and observation of physical shapes.

Thus, we may describe shape by referring to boundaries or physical discontinuities in reality. A 2D shape, for example, can be understood as the figure formed by, or the region enclosed by, (a) self-connected bonafide boundaries, or (b) the projection of fiat boundaries [18] onto an entity or spatial region that is otherwise not delimited. An example of such an *undelineated region* would be the spatial regions used in triangulation. The 2D spatial region traced out from the earth to the moon to the sun is an undelineated or fiat spatial region. With regard to (b), we do not identify physical or organic shapes in the world, but superimpose or project geometric shapes onto physical entities, distances or spatial regions as well as entire phenomena.

In a sense, this is describing *fiat shapes*, shapes that are demarcated by human choice. They are shapes delimited in regions that may or may not have existing physical discontinuities. With the projection of fiat boundaries, we identify fiat shapes. These fiat shapes are often, if not always, based on an awareness of or familiarity with geometric shapes and objects exhibiting shapes. You can trace out a circular shape in the air with your finger, but without the cognitive awareness of the gestalt, the conceptual whole, that is *the circle* (or circularity), you are doing nothing other than moving your finger. There is always a cognitive

[XVII] Examples include molecular motion, and environmental as well as atmospheric pressures.

[XVIII] Perhaps shapes can be characterized as an abstraction of boundaries.

[XIX] I do, however, find it unsettling to posit the literal existence of 2D parts of a 3D entity, specifically the mind-external existence of 2D surfaces in our 3D space-time.

[XX] In describing shape in this way, it should become clearer that there cannot be free-floating shapes in anything but geometric or conceptual space, and shapes (like all properties) cannot be isolated except in the mind. This isolation constitutes some acts of abstraction.

aspect to shapes. Where no physical discontinuities are present (or subsequently created), only fiat shapes can come to be.

What is the case, at minimum, is that we mentally project ideal geometric shapes onto entities in the world, and extract geometric shapes from entities in the world. The projection of a 2D shape may be an abstraction from a spatial region or an abstraction that is superimposed onto a spatial region, rather than the spatial region itself. Depending on what space(-time) is, if anything in itself, the shape is not necessarily the space itself, but is related to it. When we consider 3D geometric or organic shapes there is a sense in which the shape is related, if not identical, to the spatial region that the shape bounds. Manipulating geometric shapes in geometric space presupposes the general notion of space. In observing or thinking about the shape of individual objects we cognitively extract their shape (an act of abstraction) as a bounded volume of the space occupied by the object. This extraction ideally has the exact dimensions of the spatial extension of the object, but this exactness is better suited for computational applications. In transitioning to the geometer's realm, that extraction can become generalized so that the exact dimensions are excluded, leaving behind a shape type. 3D shapes (and *shape* in general) are inextricably related to our notion of space, the space we live and breathe in.

The shape of an object is dependent on the material constitution of the object (among other factors). Boundaries, in general, exist always in consort with, and are determined in their nature by the things they bound.[5] The shape of an object is determined by the outer or exterior boundary of the object. If no physical discontinuity is present, then no (bona fide) physical shape is present. (S8) *The shape of an object is dependent on the object and determined by the exterior boundary or surface of the object*.

A shape is recognizable in the world because we observe similarity in (i) the boundaries of objects in the world or (ii) in artifacts representing shapes. When the geometer cognitively manipulates shapes she also (mentally) observes the totality of the external boundary of the shape. We may also describe shapes in the following manner. To say that there are shape types or universals in reality that are instantiated as physical shapes in objects is to say certain natural principles guide and underlie phenomena, processes and entities in the world. The non-random interaction (or dynamicity) of these entities and the manifestations of natural principles unfolds in a way that consistently and recurrently results in objects exhibiting a certain limit to the spatial extension of the object. The totality of that limit is cognitively perceived as a whole and is considered to be *the shape of the object.* The geometric shape of an object, then, is in some sense the abstraction of the limit of the physical extent of the object. Describing *shape* in this way—with reference to acts of abstraction—does not imply that there are no physical boundaries constituting the whole external surface of an object.

In other words, the shape of material objects is shaped by a number of elements: natural principles underlying physical processes, the entities that participate in processes, and the internal processes of objects and other entities. The limit marks a boundary, a physical discontinuity between the spatial extension of the object and its environment (or other objects). It is conceived and perceived as a unity, a gestalt. As such it can be recognized time and again when similar unities (similar shapes) are presented. When an animal recognizes the silhouette of a predator (or prey) and instinctively flees for safety (or commences the pursuit), the silhouette is cognitively apprehended as a gestalt. (S9) *Shapes are unities, wholes*, and are understood and conceptualized as such.

Finally and in brief, the cognitive element that has been mentioned permits the introduction of art and representation as they relate to shape. How are shapes related to their representations, such as artworks? A drawing, a painting, or a computer-generated graphic of a circle (or any shape-geometric or organic) are each representational artifacts *intended* to represent. Assuming the intent of the artist is to represent a circle, the content of the representation is, surely, a circle but the question remains: what is the ontological status of the circle, and of shape in general? Is the representation that of an ideal cognitive construct that nonetheless has some relation to mind-external reality[XXI], that of the *idea* of the circle or of circularity, that of a shape universal, or a representation of individual circular shaped objects in the world? Do art works represent a particular circle or the general entity *circularity*, that is the *Circle* type? This is complicated territory, involving mental representation, intentionality, artifactual representation, perception, and cognition in general. Although beyond the limitations of this communication, these are relevant considerations for the ontology of shape. Indeed, the philosophical and applied ontology development of top- mid- or lower-level ontologies, necessitates, in my view, further exploration into the human element, specifically intentionality, consciousness and cognition, creativity, artificiality, and the mental domain overall. Ontologies themselves, after all, are artifacts.

6.0 Granular Shape

Shape has a relationship to granularity. At the mesoscopic granular level of tables, chairs, trees and people, we can identify imperfect cases of geometric shapes. When we zoom into the surface of a table we see the cracks, bumps, and imperfections not perceptible with the naked eye.

The shape of objects will physically change with the change in the material constitution and boundaries of the object. The shape of objects will also figuratively change with the granular level at which we analyze the object. At the mesoscopic level of everyday objects, we identify approximate shapes of things. When we perform a *granular descent*— analyzing an entity at a finer level of detail (a finer granular level)— we see that the shape of these objects deviates from their previously identified geometric shape. The edges of the aforementioned sheet of paper are crisp (flat, smooth, etc.) relative to the mesoscopic granular level, but coarse at a finer level. We observe microscopic crevices, indentations, valleys and other irregularities in the surface, in the material of the sheet. Thus, finer granular levels will exhibit the irregularities present in the surface.

The exact shape of objects may, in fact, be indeterminate if only due to the margin of error and finite accuracy of our instruments, including our eyes. These facts seem to mirror approaching a mathematical limit: we can descend to lower granular levels and get a more accurate description of the shape without a full characterization of it. Yet when we perform granular descents there is a sense in which we no longer are describing the shape of the object in question. The precision with which we describe the shape of an entity will likely depend on the need or practical application.

7.0 Towards a Shape Ontology

[XXI] The relation to mind-external reality may very simply (and poetically) be ourselves—human beings—and the manner in which we manifest or act upon mind-internal constructions (such as ideas).

Using the traditional endurant or continuant top-level category, and assuming shapes are, indeed, properties, qualities, or dependent entities, we can formulate an simple ontology of shapes similar to the hierarchy displayed in Figure 1 (both figures are rendered using [19]). Given the foregoing considerations, as well as our commonsense intuitions about shape, this ontological categorization should be fairly comprehensible. Types, classes or universals are represented by ovals, individuals or instances by rectangles, and relations by labeled arrows. Note that some types are not necessarily the direct or immediate subtypes of their related ancestor; I have omitted some potential intermediate types in order to minimize congestion and foster brevity. The *is a* (subsumption) relation holds between types: *A is a B* is tantamount to *A is a subtype of B* and is also understood as *Every instance of A is an instance of B*. The *instance of* (instantiation) relation holds between types and their instances. In addition, the shape types may alternatively be labeled ‘Circular’, ‘Spherical’, ‘Triangular’, and so on, rather than ‘Circle’, ‘Sphere’ and ‘Triangle’ without any lose of meaning.

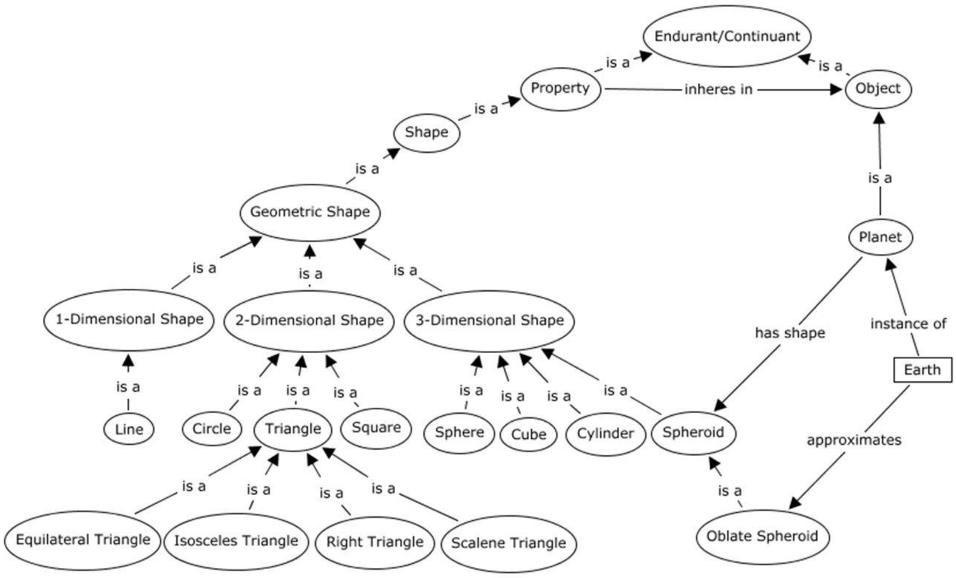

Fig. 1: A simple shape ontology with sibling shape types differentiated by dimensionality. Ovals represent types or classes, squares individuals, and labeled arrows represent relations between types.

Two relations, aside from inherence, between objects and geometric shapes are used in these diagrams, but both may not be necessary. The *has shape* relation (*hasShape* in what follows) has as relata object types (the domain) and geometric shape types (the range). The asymmetric, intransitive and irreflexive *approximation* relation holds between individual objects and geometric shapes; the relata are object instances (the domain) and geometric shape property types (the range). The Earth approximates an oblate spheroid, for example. Alternatively, an object *exhibits* a shape, or an object *exemplifies* [21] a shape. I use ‘approximation’ to emphasize the inexactness of particulars as compared with ideal abstract forms or shapes. The greater the nesting—the more specialized subtypes of shapes—the more accurate the representation of the shape of a particular object. The more specialized the shape types, the closer we come to truthfully saying (for individuals): object O hasShape shape S. The *hasShape* relation in figure 1 can be understood as indicating a principled relation between the type (Planet as a natural kind) and its property type. When formalized[XXII], this can be understood as 'All planets have a spheroidal shape' or 'All planets are spheroidal'. We can develop expressions such as 'Earth approximates Oblate Spheroid', and 'Planet hasShape Spheroid', where 'Oblate Spheroid', 'Planet', and 'Spheroid' are types. Let us consider two specific concerns.

Earlier I mentioned that shape cannot be categorized as an independent continuant in the sense of [2] because identity is not preserved with the loss of a part (a side for shapes[XXIII]) of the entity (the shape). So long as the parent class—*continuant*—continues not to be defined in a similar manner, shapes can remain as *dependent continuants*, and thus fit within the hierarchy. If all continuants preserve their identity in the face of the loss or acquisition of parts, then it is not clear that shapes do so, but [2], displayed in [22], cites shape as a *quality* (a dependent continuant). This will call for inquiry into parthood as it relates to shape. Since geometric shapes may be purely mathematical entities a mid-level ontology of these kinds of entities would also be worth exploring. The second concern is that figure 1 neither represents, nor mentions, shape instances.

If we include shape property instances then we have the following. Appended numbers indicate unique instances.

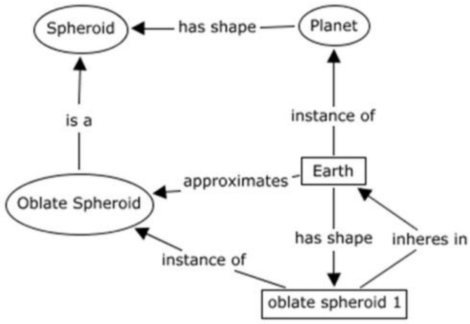

Fig. 2: Shape instances with objects and geometric shapes.

The oblate spheroid property instance is exhibited or borne by the Earth. The Earth's particular shape instance inheres in the Earth. Since *Oblate Spheroid* is subtype of *Spheroid* and all planets have a spheroidal shape, the Earth having a spheroidal shape logically follows. More specifically, the expression 'Earth hasShape oblate spheroid 1' satisfies a constraint expressing the physical principle that all planets (must) have a spheroidal shape. Similarly, additional instance expressions can be added, such as 'Mars hasShape oblate Spheroid 2'. The geometer or, more appropriately, the astrophysicist's geometric descriptions of the particular shape of each planet can then be related to the expression of the shape instance.

Here, geometric shapes are considered properties, but it may very well be that shapes are more appropriately categorized as purely mathematical entities considered as other than properties. This would justify the introduction of a mathematical entity ontology as a potential

XXII Using sorted or typed first-order predicate logic, for example.
XXIII For curved shapes, it is less clear what their parts would be.

lower-level ontology. If geometric shapes are placed within a mathematical entity ontology, then at least two questions still need to be answered: (Q1) What is the ontological status of mathematical entities; are they cognitive or mental constructs, representational artifacts, independent continuants, dependent continuants, immaterial endurants, etc.? How does this lower-level ontology fit into top-level ontologies? (Q2) How do these entities relate to mind-external entities such as material objects and *the shape of objects*? If mathematical entities are categorized as *cognitive* or *mental entities* (perhaps *cognitive artifacts*), then, in the light of (Q2) we would need to answer how these cognitive entities relate to natural phenomena and concrete particulars. In this framework, to say the Earth has the shape of an oblate spheroid is not to say that the Earth has the shape of a particular *thought*, *concept*, or *idea*, but that the Earth exhibits certain physical relations that are represented by content of the thought, if not simply the thought itself. This is a plausible avenue.

Finally, if we wish to include the shape of entities from specific domain such as astronomy, biology or geography we can include lower-level category terms such as 'Biological Shape'[XXIV], or 'Astronomical Shape'. These, however, seem more questionable in that they may be explained in terms of other shapes.

8.0 Closing Remarks

I have presented a broad exploration of *shape*, including its dimensionality and relation to material objects, form, boundary, surface and granularity in an attempt to understand what shapes are. It has been suggested that shape is a property—a dependent entity. As a property, objects can be said to *bear*, *have*, or *exhibit* a shape. More precisely, I suggested that the shape of an object is determined and identified by the exterior boundary of the object. The *exterior boundary* of an object is the limit of the spatial or material extension of the object. With shape considered as the product of an abstraction, the inquiry becomes more complex and necessarily involves a cognitive aspect. I introduced the question of the status of shapes as mathematical entities, and as cognitive devices, discussing artifactuality, and abstraction, specifically projection of shapes and extraction of them.

The ontology of the canonical world of geometry is one of perfect entities: geometric shapes that are often symmetrical and lacking of distortions or irregularities. Their properties are often, if not always, described mathematically. Perfect shapes are only found in the abstract, whereas the mind-external world presents us with irregular shapes, some of which closely approximate the objects in the geometer's toolbox. Our environments never present shapes in isolation from other entities. We do not experience shapes in isolation from what has the shape. The only sense in which shapes can be separated from material objects is in the sense of an abstraction, where the shape is abstracted away from the object. We perform an act of abstraction such that the shape is conceived as though it were isolable.

The distinction between types (classes or universals) and instances (individuals or particulars) paralleled, to a certain degree, the distinction between geometric shapes—ideal, abstract and perfect shapes that are often mathematically formalized—and physical or organic shapes, the shape *of* mind-external entities. Although the shape of objects can be formally expressed[XXV], geometric space presents us with a set of entities (shape types) that are general

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[XXIV] An example would be the shape of a bird.

[XXV] For any physical object it is likely that we can formulate a mathematical description and produce an idealized shape for it. Consider the Earth and the construct of the geoid. An additional characterization enough to have objects in mind-external reality approximate if not bear instances of them. These ideal constructs, so conceived, are similar to types, and shapes of particular objects in the world are more like instances. Shape, as an ontological category, can be conveyed in terms of shape property types with specializations thereof, and shape property instances inhering in individual objects. As more and more specializations of a shape type are discovered or formulated, we come closer to an expression of the actual shape of things.

An overarching consideration and implicit assumption was that of describing types or universals using the nomological aspect of the world. For example, if instances (objects) of a particular kind have a similar shape, they do so in virtue of physical principles underlying the numerous factors (influences, processes, entities and phenomena in general) that go into forming the limit of the spatial extension of the object. Some of those elements partially characterize the object kind.

Finally, I presented a preliminary ontological hierarchy using the *approximation* relation and the *hasShape* relation (section 7). I introduced two potentially overlapping possibilities (among many, no doubt): one, that geometric shapes are cognitive artifacts, and two, that geometric shapes belong in a mathematical entity ontology. With these possibilities in mind I stressed the need to determine the ontological status of shapes as such, as well as the relation of mathematical entities to (a) objects exhibiting a shape and (b) more generally, to the phenomena in the world that mathematical and physical expressions purport to explain.

Continued research into the ontology of shapes will, thus, necessitate entering the domain of the philosophy of mathematics. What is relationship between geometric entities such as shapes (and mathematical entities in general), the human being, cognition and the universe? Our use of geometry and physics appears to indicate that our idealizations in conceptual space are able to touch on (express, explain and discover) aspects of, and phenomena in, the world. Perhaps ideal shapes such as the Platonic Solids do, indeed, have efficacious correlations to patterns and processes in the world. There is, indeed, a harmony of, and in, the world.

Geometry itself will be invaluable to the inquiry, as will development and formalization of shape axioms and formal relations between the relevant categories. Overall, clarifying the distinction between abstract ideal shapes and physical shapes is needed. Additional topics of interest include a mereological (or mereotopological) description of shapes, the ontology of complex natural shapes such as fractals, the relation between shapes and negative entities (such as holes), and the shape of temporally-extended entities.

The development of an ontology of shape must take a multi-disciplinary approach. With the advent of modern science, geometry (and physics) and physical space-time are interrelated. The former is primarily a mind-dependent enterprise with practical applications beyond the mind. The latter is understood as mind-independent reality that has a correlation to the expressions in the former.[XXVI] Given this overlap, domain-experts from various disciplines, including artists, cognitive scientists, geometers, mathematicians, and physicists will positively contribute to inquiry into and understanding of *shape*.

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of shape is this: that two objects have the same shape, or that their particular shapes are instances of the same shape type means that the external boundary marking the limit of the spatial extension of each of the two objects can be similarly geometrically described. These objects will not have identical dimensions, and they will not necessarily match the specifications of the geometric shape that is considered to be the type of which their shape properties are instances. They approximate the shape type.

[XXVI] This too, is an assumption: that modern space-time theories are accurate representations of the world.

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