Applying Simulation and Reliability to Vehicle Routing Problems with Stochastic Demands

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Abstract

Vehicle Routing Problems (VRPs) cover a wide range of well-known NP-hard problems where the aim is to serve a set of customers with a fleet of vehicles under certain constraints. Literature contains several approaches -coming from different fields like Operations Research, Artificial Intelligence and Computer Science- which try to get good (near-optimal) solutions for small-, mid- and large-size instances. The Vehicle Routing Problem with Stochastic Demands (CVRPSD) is a particular case of VRP where demands made by clients are random, which introduces uncertainty in the problem. Thus, a good aprioristic solution may become unfeasible during the delivery phase if total demand in a route exceeds total vehicle capacity. This paper presents a flexible approach for the CVRPSD, which is based on the combined use of Monte Carlo simulation and reliability indices. Our methodology provides a set of alternative solutions for a given CVRPSD instance. These solutions depend on a parameter which controls the probability of suffering route failures during the actual delivering phase. A numerical example illustrates the methodology and its potential applications.

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1 Introduction

The Vehicle Routing Problem with Stochastic Demand (VRPSD) is a well-known NP-hard problem in which a set of stochastic customers' demands must be served by a fleet of homogeneous vehicles departing from a depot, which initially holds all available resources. There are some tangible costs associated with the distribution of these resources from the depot to the customers. In particular, it is usual for the model to explicitly consider costs due to moving a vehicle from one node -customer or depot- to another. These costs are usually related to the total distance traveled, but can also include other factors such as the number of vehicles employed, service times for each customer, etc. The classical goal here consists of determining the optimal set of routes that minimizes those tangible costs under the following set of constraints:

i All routes begin and end at the depot.

ii Each vehicle has a maximum load capacity, which is considered to be the same for all vehicles.

iii Each customer has a (stochastic) demand that must be satisfied.

iv Each customer is supplied by a single vehicle.

v A vehicle cannot stop twice at the same customer -at least without incurring in a penalty cost.

Even though the deterministic Capacitated Vehicle Routing Problem (CVRP) -the one which assumes well-known customers' demands- has been studied for decades and a large set of efficient optimization methods, heuristics and metaheuristics have been developed [9, 15, 20], the non-deterministic VRPSD is still in its infancy and new efficient methodologies need to be developed [22]. All in all, the number of VRP articles published in refereed journals has grown exponentially in the last 50 years [12], which shows the relevance of the topic for current researchers and practitioners.

The main difference between the CVRP and the VRPSD is that in the former each customer's demand is known beforehand, while in the latter the actual demand of each customer has a stochastic nature -i.e., only its statistical distribution is known beforehand, but its exact value is revealed only when the vehicle reaches the customer. This random behavior of the customers' demands could cause an expected feasible solution to become an unfeasible one if the final demand of any route exceeds the actual vehicle capacity. This situation is referred to as "route failure", and when it occurs corrective actions -i.e., changes in the original routes- must be introduced to obtain a new feasible solution. While some authors have tried to model the costs associated with these route failures [25], the presented approach also focuses on reducing the probability of occurrence of such undesirable situations to a reasonable value -a parameter to be defined by the decision-maker. In other words, our methodology proposes the construction of reliable solutions, i.e., solutions with a low probability of suffering a route failure. This is basically attained by constructing routes in which expected demand will be somewhat lower than the vehicle capacity. Put in another way, the idea is to keep a certain amount of vehicle capacity surplus while designing the routes, so that if final routes' demands exceed their expected values up to a certain limit, they can be satisfied without incurring in a route failure. Of course, a trade-off exists between routes' reliability and costs minimization and, therefore, an optimal balance must be set for these two factors, which is the main goal of the methodology presented here.

2 Related Work

The VRPSD is often modeled as a two-stage problem where the solution from stage one specifies a route for each vehicle, and customer demands are revealed at each stop along the planned routes. When routes fail, recourse actions are implemented to serve any remaining customers, and the solution from stage two specifies the actual route of each vehicle taking into account the recourse actions. Thus, the objective of the VRPSD is to construct a set of planned routes that minimize the sum of the costs of the planned routes and the expected cost of the recourse action.

Historically speaking, Tillman [26] is considered the first to study the VRPSD problem in a multidepot context. He proposed a saving based heuristic for its solution. Later, Golden and Stewart [14] presented a chance constrained model and two recourse models. The first model defined a penalty proportional to vehicle overcapacity, while in the second one, the penalty is proportional to the expected demand in excess of the vehicle capacity. In the following decade, Dror and Trudeau [11] developed further procedures showing that for VRPSD the expected travel cost depends on direction of the travel even in the symmetric case. Nevertheless, the most interesting contributions to this problem are due to Bertsimas [1, 3, 2]. Furthermore, Dror et al. [10] proved that some of the properties established by Jaillet [16, 17] and Jaillet and Odoni [18] extend to the VRPSD.

The VRPSD occurs in many practical situations. Chepuri and Homem de Mello [6] investigate the delivery of petroleum products, industrial gases, and home heating oil. Dessouky et al. [8] present the example of delivering supplies to cities under a state of emergency, Yang et al. [27] consider the delivery of items to hospitals and restaurants, and Markovic et al. [21] suggest that VRPSD can be used to model the delivery and pickup of mail, packages, and recycled material from offices and industrial plants. Gendreau et al. [13] first presented an exact solution for the VRPSD using an integer Lshaped method for solving a stochastic linear program; however, the number of customers is limited. Bertsimas and Simchi-Levi [4] offered an early review of stochastic vehicle routing research and a comprehensive set of references.

Most approaches to the VRPSD can be divided into static and dynamic. The static, or aprioristic approach, designs routes before actual demands become known and the route sequence does not change during real-time execution. The already mentioned Jaillet [17] introduced the aprioristic-solution approach to the probabilistic TSP with stochastic customer requests, later generalized to the VRP with stochastic customers and demand. Other papers related to the static approach are those of Jaillet and Odoni [18] and Bertsimas et al. [3]. The dynamic -or re-optimization- approach does not plan routes in advance, but instead makes routing decisions one step at a time. Information is updated each time a vehicle arrives at a customer and observes demand. The problem is typically modeled as a Markov decision process where a decision is defined as which customer to visit next and whether or not to return to the depot. Examples of re-optimization approaches can be seen in Secomandi [23, 24]. Additionally, the VRPSD -and probabilistic traveling salesman problem- are generally considered to be aprioristic routing even though there is a probability of route failure. Campbell and Thomas [5] provide a comprehensive review of advances and challenges in aprioristic routing.

3 The Logic Behind our Approach

Our approach is based on the fact that, while the VRPSD is yet an emerging research area, extremely efficient metaheuristics do already exist for solving the CVRP -which can be seen as a particular case of VRPSD with constant demands. In fact, techniques based on the use of Genetic Algorithms, Tabu Search, Ant Colony Optimization or Hybrid GRASP are able to provide near-optimal solutions for most known small- and medium-size CVRP benchmarks. Thus, our approach aims at transforming the problem of solving a given VRPSD instance to a problem of solving several associated CVRP instances, each of them characterized by a specific level of route-failures risk.

Consider a VRPSD instance defined by a set of n customer nodes with stochastic demands D_i following a well-known statistical distribution, and a vehicle maximum capacity, VMC. In this scenario, we propose to apply the methodology shown in Fig.1, which basically reduces the problem of solving a VRPSD instance to the problem of solving several "conservative" CVRP instances. The term conservative refers here to the fact that only a certain percentage k, with $0 < k \leq 1$, of the vehicle total capacity will be considered during the routing design phase (in other words, a percentage of the total vehicle capacity will be reserved for attending possible "emergencies" caused

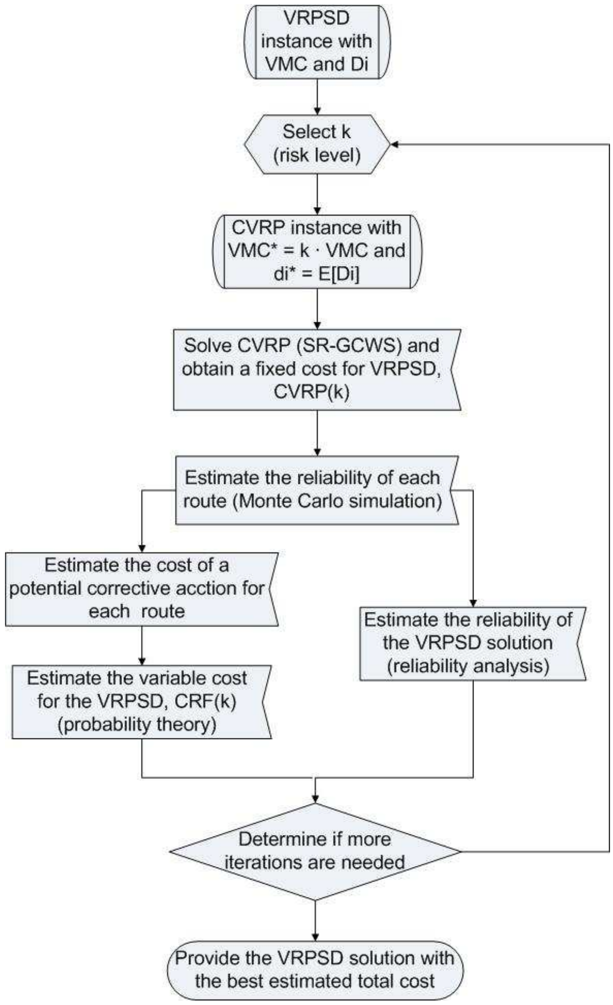

Figure 1: Flow diagram for the proposed methodology

by under-estimated random demands during execution time). This reserve is usually known in inventory theory literature as *safety stocks*.

The details of the methodology are given next:

1. Set a value for k, the percentage of the maximum vehicle capacity that will be used during the routing design stage, giving $VMC^* = k \cdot VMC$.

2. Consider the $CVRP(k)$ defined by a total vehicle capacity of VMC^* and by the deterministic demands $d_i^* = E[D_i]$ (mean or expected value of each random demand).

3. Solve the $CVRP(k)$ by using any efficient metaheuristic. Notice that the solution of this CVRP is also a possible solution for the original

VRPSD. Moreover, it will be a feasible VRPSD solution as far as there will be no route failure, i.e., as far as the extra-demand that might be originated during execution time in each route does not exceed the vehicle reserve capacity $VRC^* = (1-k) \cdot VMC$. Notice also that the cost given by this solution, $C_{CVRP}(k)$, can be considered as the base or fixed cost of the VRPSD solution, i.e., the cost of the VRPSD in case that no route failures occur. Should a route failure occur, corrective actions like vehicle returning to the depot for a reload will need to be considered and, of course, related variable costs, $C_{RF}(k)$, will need to be added for obtaining the total costs of the solution associated to the current k-value, $C_{VRPSD}(k) = C_{CVRP}(k) + C_{RF}(k)$.

4. Obtain an estimate for the reliability of each route, R_j ($1 \leq j \leq m$ for a solution with m routes). In this context, the reliability of a route is defined as the probability that that route does not suffer a failure. This value can be estimated by direct Monte Carlo simulation using the statistical distributions that model the demands of the nodes in each route -observe that in each route over-estimated demands could be sometimes compensated by under-estimated demands. To this end, a number of trials (e.g. one thousand or more) can be randomly generated. Each of these trials will provide a random value for the total demand in a given route. Then, the relative frequency of trials in which that total demand has not exceeded VMC can be used as an estimate of the route's reliability.

5. Estimate the costs of each route failure, $C^j_{RF}(k)$ with $1 \leq j \leq m$. It seems reasonable to do this by calculating the costs associated with a roundtrip from the depot to an imaginary client located at a distance from the depot that equals the average distance from the depot to the clients in the route. In fact, this could be considered even a conservative estimate, since for routes with an appropriate orientation the potential failure is likely to occur in clients closer to the depot than the aforementioned imaginary client. In other words, if the transportation costs are symmetric, route's orientation should be selected so that the last clients being served will be closer to the depot than the first clients being served, since potential route failures are more likely to occur in the final part of the route (See Fig.2).

6. Obtain an estimate for the solution's variable costs, $C_{RF}(k)$, by using the probabilistic expression shown in Eq.1:

$$C_{RF}(k) = \sum_{j=1}^{m} (1 - R_j) \cdot C^j_{RF}(k) \tag{1}$$

7. Obtain an estimate for the solution reliability level. This can be attained by simply multiplying the reliabilities of each route -we are

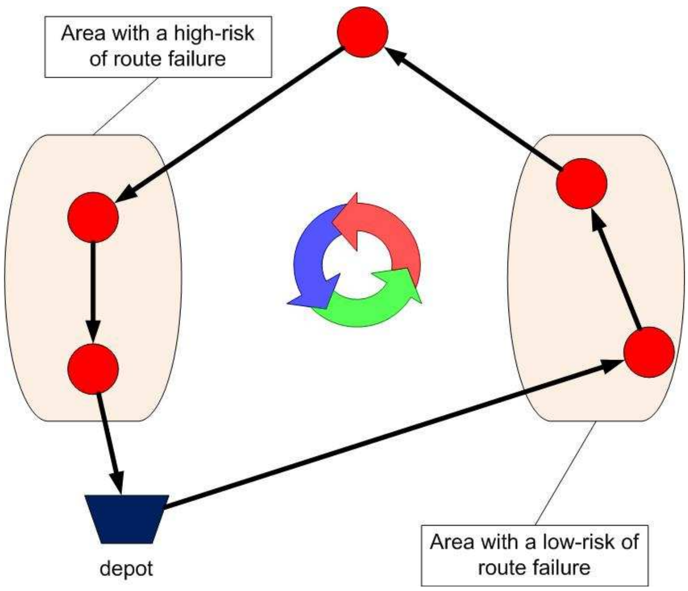

Figure 2: Choosing the correct orientation for a given route

assuming here that nodes' demands are independent among routes, which seems a reasonable assumption to make in most real cases. A solution reliability level can be considered as a measure of the feasibility of that solution in the VRPSD context.

8. Depending on the total costs and the reliability indices associated with the solution already obtained, repeat the process from Step 1 with a new value of the parameter (that is, check how different levels of "being conservative" when reserving some vehicle capacity for emergencies affect to the final VRPSD solution).

9. Finally, provide the best solution found so far, i.e., the one with a lowest total expected cost.

As far as we know, the approach presented here offers some interesting advantages over other existing approaches for solving the VRPSD. Next, some potential benefits offered by this approach are discussed:

- The methodology is valid for any statistical distribution with a known mean, either theoretical (e.g.: Normal, Log-Normal, Weibull, Gamma, etc.) or experimental (in which case bootstrapping techniques can be used to generate the random values). In our opinion, real-life demands should not necessarily follow a Normal or a Poisson distribution (as it is often assumed in the VRPSD literature), but it should be possible to model them by using any theoretical (both discrete and continuous) or experimental distribution offering either non-negative values or asymmetries generated by long right-hand tails (e.g. a Log-Normal, a Weibull, a Gamma, etc.).

- In some sense, the methodology is "reducing" a complex VRPSD -where no efficient metaheuristics have been developed yet-, to a more tractable CVRP where excellent, fast and extensively tested metaheuristics exists. This adds credibility to the quality of the final solution or solutions provided to the decision-maker.
- Moreover, the fact that we can consider different scenarios based on different values for the parameter k, which represents a certain level of risk, makes the methodology flexible.
- Providing the reliability indices for each solution allows the decision-maker to select that solution that best fits his/her utility function, including those with a lower fixed cost but higher variability (risk) or those with a higher fixed cost but lower variability (even if the total expected costs are the same for both solutions). In this sense, our approach could provide a list of selected solutions and their associated properties as shown in Table 1.
- Enterprises dealing with routing problems create (routing) plans very often -e.g. daily or weekly. They usually work with the same clients, and they have deep information on their real time behaviour. That makes these enterprises to have valuable information about possible future changes in demands when applying the current plans. That can be translated into higher or lower risk levels when choosing both the k-values and the solution to apply. Thus, the flexibility of the presented methodology is not used blindly, but with the support of previous experience.

# routes	k	Fixed cost	Expected variable cost	Total expected cost	Estimated reliability
11	0.9	3200	1700	4900	0.76
12	0.8	3520	1200	4720	0.83
13	0.7	4020	950	4970	0.95

Table 1: Example of a possible output offered to the decision-maker

4 Experimental Results

The methodology described in this paper has been implemented as a Java application. Java was used here since, being an object-oriented language, it facilitates the rapid development of a prototype. A standard personal computer, Intel®Core^{TM}2 Duo CPU at 1.6 GHz and 2 GB RAM, was used to perform the experiment described next, which was run directly on the Eclipse IDE for Java. For illustration of our methodology, we constructed a mid-size VRPSD instance based on a well-know CVRP instance, the *A-n80-k10* (available at *http://www.branchandcut.org/*). We used the same nodes and vehicle maximum capacity as in the original *A-n80-k10* ($VMC = 100$) but we substituted the deterministic demands, d_i, for random demands, D_i, with $E[D_i] = d_i$ and $Var[D_i] = 2$ for all $1 \leq i < 80$. We also required that each of these random demands follow a log-normal distribution with location parameter μ_i and scale parameter σ_i which -according to the properties of the log-normal distribution- will be given by the expressions shown in Eq.'s 2 and 3:

$$\mu_i = \ln(E[D_i]) - \frac{1}{2} \cdot \ln\left(1 + \frac{Var[D_i]}{E[D_i]^2}\right) \tag{2}$$

$$\sigma_i = \left|\sqrt{\ln\left(1 + \frac{Var[D_i]}{E[D_i]}\right)}\right| \tag{3}$$

Notice that, by setting $E[D_i] = d_i$ for all $1 \leq i < 80$, we can recover the CVRP *A-n80-k10* instance by just defining $Var[D_i] = 0$ for all $1 \leq i < 80$. In other words, we are just introducing some variance to the CVRP *A-n80-k10* instance to construct our own VRPSD instance based on the log-normal distribution.

Next, we set $k = 0.95$ (i.e., we reserved 5% of total vehicle capacity for attending potential route failures during the current execution of the delivery stage) and solved -using the SR-GCWS algorithm [19]- the CVRP(0.95) instance defined by the following parameters: $d_i^* = E[D_i] = d_i$ and $VMC^* = 0.95 \cdot VMC = 95$. After some seconds of execution, we obtained a 10-route solution with a base cost $C_{CVRP} = 1833.37$, which we considered to be a reasonable good value since it improved in about 2.9% the solution provided by the Clarke & Wright heuristic [7] for this CVRP(0.95) instance, which has an associated cost of 1882.84. Notice that, in the case of the standard *A-n80-k10* instance, the gap between the best-known solution (cost = 1766.50) and the one provided by the Clarke & Wright heuristic (cost = 1860.94) is only about 5.3%. Now, we used Monte Carlo simulation to estimate the reliability of each route in the obtained solution: for each client i in a route r, we used the corresponding statistical distribution to generate a random value for D_i and then we checked whether or not the total demand in r exceeded the real vehicle capacity $VMC = 100$ (notice that we are assuming here independency among different clients' demands). By repeating this process $1,000$ times we obtained an estimate of the probability that each route can be implemented without suffering a route failure, R_j with $1 \leq j \leq 10$. Finally, we estimated the solution's reliability by simply multiplying its routes' reliabilities (since a solution will be safely implemented without route failures if, and only if, each and every route in that solution is implemented without route failures, i.e., a solution can be seen as a series system of routes). Table 2 shows the estimated values obtained for the reliability of each route and for the global reliability. It also shows the estimated cost for a route failure, C_{RF}^{j} with $1 \leq j \leq 10$, the corresponding variable costs associated to each route and also the total estimated variable costs for the whole solution, C_{RF}.

Element	Reliability	Cost of a potential failure	Estimated variable costs
Route 1	0.84	$2 \cdot 101.37$	32.03
Route 2	0.97	$2 \cdot 36.12$	2.53
Route 3	0.96	$2 \cdot 45.73$	3.29
Route 4	0.93	$2 \cdot 32.11$	4.56
Route 5	0.86	$2 \cdot 81.45$	22.97
Route 6	0.88	$2 \cdot 89.65$	20.80
Route 7	0.99	$2 \cdot 60.99$	1.30
Route 8	0.91	$2 \cdot 71.68$	12.90
Route 9	0.94	$2 \cdot 75.72$	8.48
Route 10	0.91	$2 \cdot 70.59$	13.41
Solution	0.43	—	122.32

Table 2: Reliability and variable costs values for each route

In summary, for $k = 0.95$, the total estimated cost for the VRPSD solution is $C_{VRPSD} = C_{CVRP} + C_{RF} = 1955.69$. From a computational point of view, the whole process described before can be completed in just a few minutes, and probably in just a few seconds if we use a multi-threaded C/C++ version of the algorithm and a more powerful computer. Therefore, it seems reasonable to repeat these steps with other values of the parameter k. Table 3 shows different VRPSD solutions that we obtained by using the values $k = 0.90$ and 0.85. For each of the three considered solutions, the following information is provided: number of routes employed, base costs (those from the associated CVRP(k) instance), gap between the CVRP solution and the one provided by the Clarke & Wright heuristic (for the *A-n80-k10*, we consider our CVRP solution to be a reasonable good one when this gap is about 3% or higher), estimates for the variable and total costs, and an estimate of the solution reliability index.

Notice that, as expected by the discussion developed during the methodology explanation, higher values of the parameter k tend to produce solutions with less routes and lower base costs, but they also tend to have lower reliability levels and, therefore, higher variable costs which can significantly affect the expected total costs of the associated VRPSD solution.

k	# Routes	Base cost	Gap (%)	Variable cost	Total costs	Reliability
0.95	10	1833.37	2.9	122.32	1955.69	0.43
0.90	11	1910.06	3.1	11.31	1921.37	0.93
0.85	12	1997.15	4.4	1.47	1998.62	0.99

Table 3: Costs and Reliability levels for different VRPSD solutions

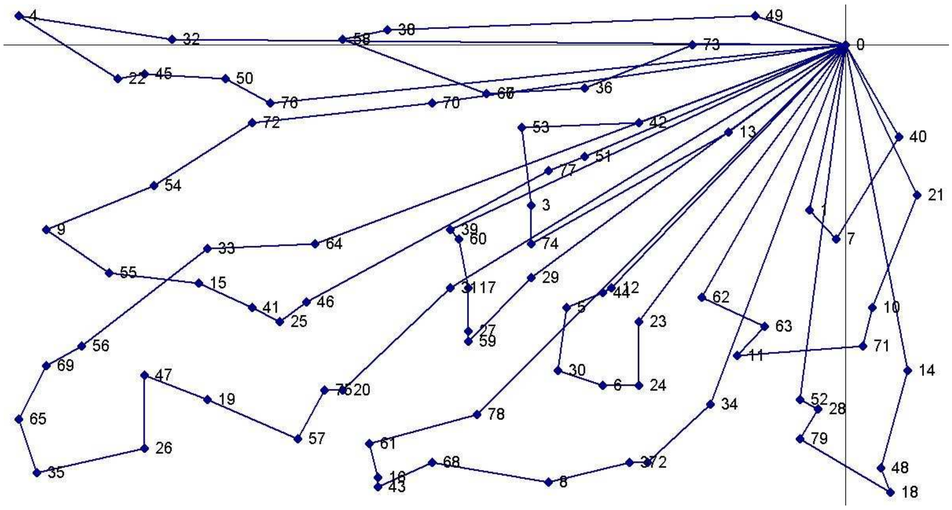

Figure 3: Solution for stochastic *A-n80-k10* with $k = 0.90$ (11 routes)

Among the three options presented in Table 2, the one which would be probably selected for most decision-makers would be the VRPSD solution obtained with $k = 0.90$, since it combines the lowest total costs with a reasonable reliability level. Fig.3 shows a graphical representation of this solution. Observe, however, that other values of the parameter k (e.g. $k = 0.92$ or $k = 0.88$) could also be explored before taking a final decision.

5 Future Work

As a future research work, we plan to compare the efficiency and robustness of the proposed approach against alternative optimization methods lying on mathematical or constraint programming models. More specifically, the major idea is to replace the meta-heuristic used in the step 3 of the proposed methodology by a rigorous two-stage stochastic optimization approach. In this way, the solution generated will simultaneously consider multiple scenarios for the customer demands instead of the one based only on the expected value of each random demand. By considering the uncertain information of demands in a proactive way, we expect being able to generate cost-effective solutions with higher reliability, although at the expense of a potential significant increase of the computational cost. The trade-off between solution quality and computational time will be carefully evaluated. In addition, since routes failures may be reduced but never eliminated, we also plan to developed efficient dynamic optimization methods for quickly updating the original solution after the occurrence of route failures.

6 Conclusions

We have presented a hybrid approach to solving the Vehicle Routing Problem with Stochastic Demands. The approach combines Monte Carlo simulation with reliability indices and a well-tested metaheuristic developed by the authors. The basic idea of our methodology is to consider a vehicle available capacity lower than the actual maximum vehicle capacity when constructing VRP solutions. This way, this capacity surplus can be used when necessary to cover routes failures without having to assume the usually high costs involved in vehicle restock trips. Our approach provides the decision-maker with a set of alternative solutions, each of them characterized by their total estimated costs and their reliability values -the former reflecting the probability of that solution being a feasible one-, leaving to him/her the responsibility of selecting the specific solution to be implemented according to his/her utility function. Some tests have been performed to illustrate the methodology and discuss its potential benefits over previous works.

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