Utility-Sharing Games: How to Improve the Efficiency with Limited Subsidies (short paper)

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Abstract

We consider the problem of improving the efficiency of utility-sharing games, by resorting to a limited amount of subsidies. Utility-sharing games model scenarios in which strategic and self-interested players interact with each other by selecting resources. Each resource produces a utility that depends on the number of players selecting it, and each of these players receives an equal share of this utility. As the players' selfish behavior may lead to pure Nash equilibria whose total utility is sub-optimal, previous work has resorted to subsidies, incentivizing the use of some resources, to contrast this phenomenon.

In this work, we focus on the case in which the budget used to provide subsidies is bounded. We consider a class of mechanisms, called α-subsidy mechanisms, that allocate the budget in such a way that each player's payoff is re-scaled up to a factor $\alpha \geq 1$. We design a specific sub-class of α-subsidy mechanisms, that can be implemented efficiently and distributedly by each resource, and evaluate their efficiency by providing upper bounds on their price of anarchy. These bounds are parametrized by both α and the underlying utility functions and are shown to be best-possible for α-subsidy mechanisms. Finally, we apply our results to the particular case of monomial utility functions of degree $p \in (0, 1)$, and derive bounds on the price of anarchy that are parametrized by p and α.

Keywords

Utility Games, Resource Allocation, Subsidy Mechanisms, Pure Nash Equilibrium, Price of Anarchy

1. Introduction

In several real-life contexts arising from economics, operation research and computer science, we face the necessity of allocating a set of utility-producing resources to agents, in such a way that the total utility is maximized. For example, we could consider a scenario, connected with management engineering, in which each resource models a project or a task to be completed, and each agent is an employee of a company, or a server in a content delivery network, that can be assigned to one of the tasks. It is reasonable to assume that, the more the number of employees assigned to a task, the more the quality of the completed task (or the lower the

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completion time, or the higher the probability that the task will be correctly completed). Indeed, the employees assigned to the same task can work in team, and it is expected that the resulting quality improves as the working team includes new members.

When completed, each task generates a profit (i.e., a utility) that is proportional to the resulting quality, and this profit (or a percentage of it) is equally shared among the employees who contributed to the task. As the number of employees and tasks could be very high, the presence of a centralized coordinator imposing all the assignments might be impracticable. Therefore, a decentralized implementation of the system, where each worker autonomously decides which task she wants to contribute to, is a more reasonable choice. To describe the effects of decentralization, we consider a game representation of the system in which each worker acts as a player who aims at maximizing the fraction of the profit that she receives (i.e., her payoff). This creates an interplay of strategic behavior, in which players compete with each other by selecting the tasks (i.e., the resources) that maximize their payoff. This may lead to suboptimal outcomes, in which the total utility is lower than the one achievable by a central authority imposing an optimal assignment of players to resources.

Algorithmic game theory [2] offers several tools to describe how the strategic choices of the players may affect the total utility achieved by all resources. First, the notion of *pure Nash equilibrium* [3], that is an outcome in which no player can increase her payoff by unilaterraly deviating to another strategic choice, is used to model stable solutions arising from selfish behavior. Then, the *Price of Anarchy* [4], which compares the total utility of any pure Nash equilibrium against the optimal total utility achievable in a centralized and coordinated environment, is adopted to quantify the lack of cooperation and coordination.

2. Our Contribution

Given the difficulties in coordinating the players' strategic behavior, a reasonable approach to convey them toward better pure Nash equilibria is that of providing subsidies encouraging the use of certain resources. Several works [5, 6, 7, 8, 9, 10, 11, 12] showed the effectiveness of this idea, by designing ad-hoc subsidy allocation mechanisms that are able to improve the price of anarchy. The amount of subsidies that these mechanisms require, however, can be very high, thus limiting their applicability to most real-life contexts, where budgets are usually severely constrained.

In this work, we show how to improve the efficiency of decentralized allocation systems, when the total amount of subsidies available to each resource is somewhat constrained by the total utility that can be generated by the resource itself. We model allocation systems as a class of games, called *utility-sharing games*, which constitutes a subclass of the general framework of *monotone valid utility games* defined in [13], and is similar and/or equivalent to other game classes studied in [14, 15, 16, 17, 9, 18, 19]. In utility-sharing games, we have a finite set of players, a finite set of resources available to the players, and each resource is associated with a certain *utility function* whose value is equally shared among the players selecting it. Each player aims at maximizing her *payoff*, given by the fraction of utility she receives.

The main novelty of this work is the design and the analysis of α*-subsidy mechanisms* (α-SMs), a new class of subsidy allocation mechanisms that, parametrized by a value $\alpha \geq 1$, allocate to each resource an amount of subsidies that is at most $\alpha - 1$ times the utility produced by the resource, so that the players' payoffs can be re-scaled up by a multiplicative factor α.

2.1. Efficiency of α-SMs for General Utility Functions

We provide tight bounds on the price of anarchy guaranteed by α-SMs for several classes of utility-sharing games. In particular, the obtained results hold for *payoff-regular* functions, which are very general payoff functions whose related utility functions satisfy some weak regularity properties (e.g., monotonicity and concavity).

To improve the efficiency of utility-sharing games with payoff-regular functions, we consider a sub-class of α-SMs, called *optimal-congestion-based* α-SMs, that can be computed and executed in polynomial time. In particular, an optimal-congestion-based α-SM $\Pi_\alpha(\boldsymbol{\sigma}^*)$ assigns to each resource r a subsidy that depends on the congestion of r under a given optimal strategy profile $\boldsymbol{\sigma}^*$ and the strategy profile played in the game. In the following theorem, we provide upper bounds on the resulting price of anarchy that depend on α, the number of players n, and the class of utility functions of the game. We first give some preliminary notations. Given $\alpha \geq 1$, $n \in \mathbb{N}$ and a payoff-regular function f, let

$$\beta_{n,\alpha}(f) := \sup_{x,y \in \mathbb{N}_{\geq 0}: n \geq x \geq y \geq 0, x > 0} y \left(f(y) - \frac{1}{\alpha} f(x) \right) (x f(x))^{-1};$$

furthermore, given a class of payoff-regular functions $\mathcal{D}$, let $\beta_{n,\alpha}(\mathcal{D}) := \sup_{f \in \mathcal{D}} \beta_{n,\alpha}(f)$ and $\beta_\alpha(\mathcal{D}) := \sup_{n \in \mathbb{N}} \beta_{n,\alpha}(\mathcal{D})$.

Theorem 1. *Let $\mathcal{D}$ be a class of payoff-regular functions. Given a game SG with at most $n \geq 2$ players and payoff functions in $\mathcal{D}$, and given an optimal-congestion-based α-SM $\Pi_\alpha(\boldsymbol{\sigma}^*)$ for SG, we have that $\mathrm{PoA}(\mathrm{SG}, \Pi_\alpha(\boldsymbol{\sigma}^*)) \leq \dfrac{1}{\alpha} + \beta_{n,\alpha}(\mathcal{D}) \leq \dfrac{1}{\alpha} + \beta_\alpha(\mathcal{D})$.*

We also show that optimal-congestion-based mechanisms achieve the best-possible performances within the general class of α-SMs, that is, no α-SM can further lower our bounds on the price of anarchy.

Theorem 2. *Let $\mathcal{D}$ be a class of payoff-regular functions. For any $\epsilon > 0$ and $n \in \mathbb{N}_{\geq 2}$, there exists a utility-sharing game SG with at most n players and payoff functions in $\mathcal{D}$ such that, for any α-SM Π_α for SG, the price of anarchy of SG under Π_α is higher than $1/\alpha + \beta_{n,\alpha}(\mathcal{D}) - \epsilon$.*

Finally, the following corollary of Theorem 1 and Theorem 2 provides a tight bound on the price of anarchy that depends on the number of players only, under mild assumptions on the considered utility functions.

Corollary 1. *Let SG be a utility-sharing game with at most $n \geq 2$ players and payoff-regular functions. For any optimal-congestion-based α-SM $\Pi_\alpha(\boldsymbol{\sigma}^*)$ for SG, we have that $\mathrm{PoA}(\mathrm{SG}, \Pi_\alpha(\boldsymbol{\sigma}^*)) \leq 1 + \frac{1}{\alpha}\left(1 - \dfrac{1}{n}\right) \leq 1 + \frac{1}{\alpha}$. Furthermore, no α-SM can achieve, for general payoff-regular functions, a better price of anarchy.*

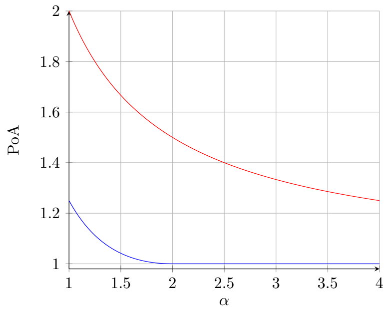

Figure 1: The red line represents, for any $\alpha \geq 1$, the price of anarchy under optimal-based-congestion α-SMs for games when the underlying utility functions are not specified (Corollaries 1), while the blue line represents the price of anarchy of games whose utility functions are monomials of degree $1/2$ (a case of Theorem 3).

We have that, for any utility-sharing game and sufficiently large $\alpha \geq 1$, all pure Nash equilibria induced by optimal-congestion-based α-SMs maximize the total utility. Thus, our approach guarantees the same performance of the subsidy allocation mechanisms studied in [9], that, differently from ours, may fail under some budget limitations. Furthermore, for $\alpha = 1$, we re-obtain the tight bounds on the price of anarchy for utility-sharing games without subsidies, already shown in [9, 18].

2.2. The Case of Monomial Utility Functions

As a case study, we apply our general results to the specific case of utility functions representable as monomials of fixed degree $p \in [0, 1]$.

Theorem 3. . *Given $p \in (0,1)$, a utility-sharing game* SG *with utility functions of type $u(x) = w \cdot x^p$ for some $w > 0$ and an optimal-congestion-based α-SM $\Pi_\alpha(\boldsymbol{\sigma}^*)$ for* SG*, we have $\mathrm{PoA}(\mathrm{SG}, \Pi_\alpha(\boldsymbol{\sigma}^*)) \leq \dfrac{1-t}{\alpha} + t^p$, with $t = \min\left\{(\alpha p)^{\frac{1}{1-p}}, 1\right\}$. Furthermore, no α-SM can achieve, in general, a better price of anarchy.*

As an example, we consider the case of $p = \frac{1}{2}$. By Theorem 3, we have that the price of anarchy under optimal-congestion-based α-SMs of games SG with monomial utility functions of type $u(x) = w \cdot x^{1/2}$ is equal to $\frac{\alpha^2+4}{4\alpha}$ for $\alpha < 2$, and equal to 1 for $\alpha \geq 2$. In Figure 1, we see how the price of anarchy varies over $\alpha \geq 1$, and we compare it with the case of general functions.

3. Further Related Work

An extended version of the results presented in this work appeared in [20].

The first general game-theoretic model for decentralized resource allocation systems with payoff-maximizing players is that of monotone valid utility games [13], where the payoff functions satisfy some mild assumptions, such as monotonicity and submodularity w.r.t. the selected resources. In this seminal paper, a tight bound of 2 on the price of anarchy of monotone valid utility games is provided. Subsequently, several (sub)classes of monotone valid utility games have been introduced and studied. A work that is strictly close to ours is [18], which studies the price of anarchy of (an equivalent model of) utility-sharing games, and provided tight bounds that are parametrized by the number of players and the considered utility functions; tight bounds on the price of anarchy for more general settings in which the set of available resources is player-specific is also provided. Papers [15, 9] model strategic project selection as specific instantiations of monotone valid utility games, and provide more specific bounds on the price of anarchy and other efficiency metrics (such as the price of stability [21]). In [17, 19], the efficiency of specific monotone valid utility games where, differently from our model of utility-sharing games the sharing rules do not necessarily split each resource utility in an equal way among the players selecting it, has been considered.

The problem of determining mechanisms improving the price of anarchy in utility-sharing games (so as for their variants and/or generalizations) has been widely considered in the literature. In [9], it is shown how to assign a credit (i.e., a subsidy) to each project, so as to guarantee that any pure Nash equilibrium is an optimal strategy profile. A considerable amount of work [6, 7, 8, 10, 11, 12] shows how to modify the payoffs of the players participating in utility-sharing games (e.g., via subsidies), with the purpose of improving the efficiency of pure Nash equilibria; in particular, tight bounds on the resulting price of anarchy, that depend on the considered class of utility functions, are provided.

Utility-sharing games are strictly related to the cost-minimization game-theoretic model of *congestion games* [22, 23]. Congestion games are resource selection games with a finite set of cost-minimizing players and a finite set of resources, where each player selects a subset of resources (among a finite collection that is player-specific), and the cost of each selected resource is a function of the number of players selecting it. The problem of measuring the price of anarchy of congestion games has been a hot-topic in algorithm game theory in the last two decades [24, 25, 26, 27], and several works have provided upper and lower bounds depending on the considered cost-functions [28, 29, 30, 31, 32, 33, 34, 35] or the structure of the players' strategies [36, 29, 37, 31, 38, 39, 40, 33, 41]. Furthermore, several works have also focused on the design and analysis of mechanisms to improve the price of anarchy, e.g., *taxation mechanisms* [42, 43, 44, 8, 45, 46, 47], *Stackelberg strategies* [48, 33, 49, 50], *coordination-mechanisms* [51, 52, 53] and *cost-sharing mechanisms* [51, 54, 55].

4. Future Works

Our work leaves several research directions on the problem of improving the efficiency in utility sharing games via limited subsidies.

First of all, our subsidy mechanisms dynamically depend on the actual congestion of each resource. Thus, it would be interesting to show how the efficiency can be improved if subsidies do not depend on the current game configuration. Another research direction could be that of finding subsidy (or taxing) mechanisms with limited budget for more general variants of utility-sharing games, where players can also be cost-minimizers (as in congestion games [23]) and/or have different weights and/or can select different subsets of resources). Finally, still with the aim of improving the efficiency of the considered games, it would be also interesting to consider other mechanisms than subsidy disbursements (e.g., Stackelberg strategies [49]).

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