Routing of Delivery Drones Considering Load and Wind Effects

Satoshi Ito[1], Keishi Akaiwa[1], Yusuke Funabashi[1], Hiroki Nishikawa[1,2], Xiangbo Kong[1], Ittetsu Taniguchi[3], and Hiroyuki Tomiyama[1]

[1] Ritsumeikan University
[2] JSPS Research Fellow
[3] Osaka University

Abstract

Delivery drones have been attracting attention as a promising means of package delivery. However, drones are subject to various factors that affect their flight speed compared to traditional trucks. In past researches, Flight Speed-aware Vehicle Routing Problem (FSVRP) was studied. The FSVRP addresses the problem of minimizing the total flight distance. It was studied considering the effect of the flight speed due to the weight of the cargo. However, the flight speed of drones varies with the wind as well as the load. In this paper, we define Flight Speed-aware Vehicle Routing Problem with the Wind (FSVRPW) for flight distance minimization. We also propose an algorithm to solve the FSVRPW accurately and quickly. The metrics of flight distance, flight time, and running time are compared with the conventional FSVRP.

1 Introduction

In recent years, drones have been used for a variety of purposes due to their lightweight frames and improved battery capacity. Among them, drone-based delivery services have been attracting attention. Drones are considered a promising means of delivery service because they are not affected by road traffic conditions. It also has the advantage of lower CO_2 emissions and labor costs compared to truck-based package delivery. Drones deliver packages automatically, so there is no human work involved in the delivery process. On the other hand, drones cannot fly long distances since they are battery-powered. Therefore, it is necessary to consider what is the best order to increase the number of packages carried by the drone. The truck delivery routing problem has been the subject of much research. However, it is difficult to apply these researches to drones because the considerations for delivery are different between trucks and drones. Therefore, it is important to study delivery routing problems that take into account the characteristics of drones.

The delivery drones routing problems have been the subject of much research [1][2]. There is research on minimizing the delivery completion time when delivering by drone [3]. This research is defined as a constant flight speed of delivery. However, this study does not simulate an exact delivery. Because, as we know the flight speed of a drone varies depending on the weight of the load it is carrying. Therefore, incorporates the change in flight speed due to the weight of the load, has been proposed Flight Speed-aware Vehicle Routing Problem(FSVRP) [4]. In this study, it is assumed that the heavier the total weight of the cargo carried by the drone, the slower the flight speed. Therefore, by prioritizing the delivery of heavier packages, it is realized that the total flight distance will be longer but the total flight time will be shorter. However, the flight speed of drones varies not only with the load but also with the wind. Therefore, it is necessary to consider the effect of wind on flight speed.

In this paper, we address the Vehicle Routing Problem (VRP) for package delivery drones and address the delivery routing problem of minimizing the flight distance considering the change in flight speed due to load and wind. This paper is the first delivery drone routing problem that considers the change in flight speed due to load and wind. In this paper, given a set of package to deliver and wind velocity, the problem tries to find an optimal route that starts from a depot, delivers all of the packages to customers, and comes back to the depot with the effect of wind in mind. To solve FSVRPW, the paper presents a dynamic programming algorithm. Dynamic programming algorithm efficiently finds exactly optimal routes in terms of total flight distance.

The rest of this paper is organized as follows. Section II surveys related work. Section III describes the routing problem for delivery drones. Section IV proposes a dynamic programming algorithm for the FSVRPW. Section V presents evaluation and finally, Section VI concludes this paper.

2 Related Work

Delivery drone routing problems is studied a lot [2][5][6][7]. Vehicle Routing Problems (VRP) is an extension of the traveling salesman problem (TSP) for vehicle delivery. A given package is delivered to a customer using a vehicle. There are various problems in VRP with different constraints and objectives. Examples of constraints include the number of vehicles, the capacity of the cargo, and the battery capacity. Also, examples of objectives include minimizing energy consumption, minimizing flight time, and minimizing the flight distance.

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There are several studies aimed at minimizing the energy consumption[8][5][6][7]. Energy minimizing vehicle routing problems (EMVRP) is a further extension of VRP to minimize energy. Kara et al., assume that the energy consumption is proportional to the product of flight distance and loaded[8]. In [5], an algorithm is proposed that prioritizes the lightest loads to be delivered to the EMVRP. However, the results are worse than the other algorithm. In [6], a dynamic programming method for EMVRP is proposed. The results show that the dynamic programming method is effective. However, this does not take into account the change in flight speed due to the wind and load. In these studies, only drones are considered to deliver packages, but delivery plans combining trucks and drones are also proposed in [7]. Also, in these studies, calculations are done with a constant flight speed. However, the flight speed of drones varies depending on the load and the natural environment. Therefore, it is necessary to consider those effects in realistic delivery planning problems. A delivery planning problem that takes into account the effect of flight speed due to load is FSVRP proposed by Funabashi et al., [4]. FSVRP defines that the larger the load is, the slower the flight speed becomes, and the smaller the load is, the faster the flight speed becomes. It is also proposed by Ito et al., as a delivery planning problem that takes into account the effect of wind[9].

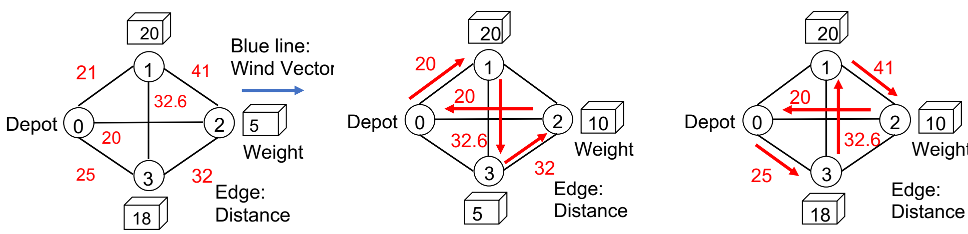
3 The Routing Problem for Delivery Drones

This section describes FSVRP as proposed by Funabashi et al.[4], and defines FSVRPW addressed in this paper.

3.1 A Motivation Example

A variety of methods have been studied for drone delivery planning. However, most of the studies did not consider the effect of wind when determining the order of delivery. Therefore, it is necessary to consider wind in the delivery planning stage. In this paper, we extend the FSVRP proposed by Funabashi as a base. FSVRP is working on a delivery plan that minimizes the flight time when the flight speed is affected by the weight of the load.

We explain the difference between FSVRP and FSVRPW using a concrete example. The map used in the concrete example is shown in Figure 1a. It shows the distance between each customer and the weight of the package to be delivered to each customer when the number of customers is 3.



(a) Map of concrete example (b) Routing of FSVRP (c) Routing of FSVRPW

Figure 1: Routing of concrete example

The distance between each customer is shown in red, and the weight of the package to be delivered to each customer is shown in the square box. The example also assumes that the wind is blowing from customer 2 towards the delivery base. The FSVRP changes its flight speed depending on the weight of the load. Specifically, when the payload is heavy, the speed becomes slower, and when the payload is small, the speed becomes faster. In FSVRPW, the flight speed of drones is changed due to load and wind.

The concrete examples of FSVRP and FSVRPW are shown in Figure 1. FSVRP route is shown in Figure 1b. The delivery order written in the red line is the delivery order in which the flight time calculated by FSVRP is minimized. The flight time of the drone depends not only on the flight distance but also on the total weight of the payload. The heavier the load, the slower the flight speed and the longer the flight time. This implies that heavier loads should be given priority for delivery. Therefore, the optimal flight distance for FSVRP is 104.6 $(20 + 32.6 + 32 + 20)$, and the flight time is 606.2 $(431.5 + 56.3 + 48.0 + 70.4)$. However, this is not the optimal delivery order in FSVRPW. FSVRPW route is shown in Figure 1c. It's not only the load that affects the flight speed of the drone, but also the wind. When delivering a heavy load in a headwind, the flight speed is most affected. The slower flight speed will also increase the flight time. This means that priority should be given to delivering heavy loads with a tailwind. The overall flight time can also be minimized by giving priority to lighter loads if they are in a tailwind. Therefore, the optimal flight distance for FSVRPW is 118.6

(25+32.6+41+20), and the flight time is 417.9 (144.9+152.9+49.7+70.4). As can be seen from this result, it is necessary to consider not only the total weight of the payload but also the wind in the delivery routing.

3.2 Description and Formulation of FSVRP and FSVRPW

This section is about the description and formulation of FSVRP and FSVRPW. The FSVRP dealt with in this paper is based on [4].

We are given N items to deliver. To avoid lost generality, we do not deliver more than two packages to the same customer. In other words, multiple packages to the same customer are combined in advance. Also, the number of customers is N, and packages are numbered from 1 to N. The customer to whom package i $(1 \leq i \leq N)$ is delivered is called customer i or point i, and the depot is numbered 0, as shown in the example in Figure 1. In this paper, we assume that all packages are delivered at one time. Also, all the packages are loaded onto the drone at the depot, and the delivery is started. If the total weight of the packages exceeds the carrying capacity of the drone, the packages need to be divided, which is out of the scope of this paper. Let $d(i1, i2)$ denote the distance between customers $i1$ and $i2$. Also, let $x(j)$ denote the j-th visited customer, which is the decision variable of the routing problem. Since a route starts and ends at the depot, we define:

$$x(0) = x(N+1) = 0 \tag{1}$$

Also, all of the customers are visited once, which is formally defined as follows:

$$1 \leq x(j) \leq N \qquad (1 \leq j \leq N) \tag{2}$$

$$x(j1) \neq x(j2) \qquad (1 \leq j1, j2 \leq N, j1 \neq j2) \tag{3}$$

Let $w(i)$ denote the weight of package i and W_d is the weight of the drone itself. Let $W(j)$ the total payload when the drone departs the j-th customer. When the drone starts delivery, all the packages are loaded. Therefore, the following equation is valid.

$$W(0) = \sum_{i=1}^{N} w(i) \tag{4}$$

When the drone makes the j-th stop $(1 \leq j \leq N)$ at customer $x(j)$, a package of weight $w(x(j))$ is unloaded. Therefore, the total payload when the drone leaves customer $x(j)$ is defined as:

$$W(j) = W(j-1) - w(x(j)) \tag{5}$$

Let $t(i1, i2)$ denote the flight time between customer $i1$ and $i2$. The drone flight time from $i1$ to $i2$ is given by the distance between $i1$ and $i2$ divided by the drone flight speed v. Note that v is a function of payload. Therefore, the flight time between customers j-th visited and $(j+1)$-th visited is defined as:

$$t(x(j), x(j+1)) = d(x(j), x(j+1))/v(W(j)) \tag{6}$$

The FSVRP asks the shortest flight time in all flight routes, and its objective function is defined as:

$$\textbf{minimize} \quad T = \sum_{j=0}^{N} d(x(j), x(j+1))/v(W(j)) \tag{7}$$

The FSVRP addressed in this paper is formally defined as follows. Given w, d and v, find x which minimizes the objective function (7) while meeting the constraints (1)-(6).

In FSVRPW, the effect of wind is added to the velocity used in FSVRP. If the wind speed is v_w, equation (6) is defined as:

$$t(x(j), x(j+1)) = d(x(j), x(j+1))/v_d(W(j), v_w) \tag{8}$$

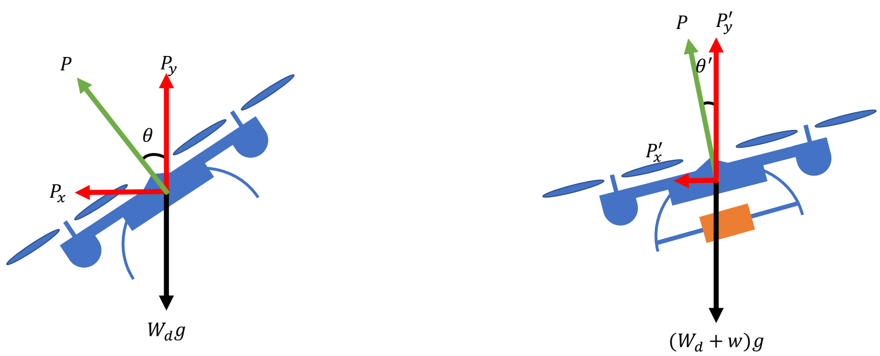
Therefore The FSVRPW asks the shortest flight time in all flight routes, and its objective function is defined as:

$$\textbf{minimize} \quad T = \sum_{j=0}^{N} d(x(j), x(j+1))/v_d(W(j), v_w) \tag{9}$$

The FSVRP addressed in this paper is formally defined as follows. Given w, d, v and v_w find x which minimizes the objective function (9) while meeting the constraints (1)-(5),(8).

3.3 The Influence of Load and Wind on Drone Flight Speed

In the problem defined above, the flight speed function v is specific to the drone. We assume that v is assumed to be given, and how to accurately obtain v is out of the scope of this paper. Instead, the rest of this section only provides a simple approximation of how the load and wind of the drone affect the flight speed v.



(a) Drone without load

(b) Drone with load w

Figure 2: Forces on a drone

Figure 2a shows a drone which flies horizontally. P denotes the driving force of the drone, and W_d denotes a weight of the drone itself. In order not to fall down, the vertical component of P, i.e., P_y, must be equal to the gravity force $W_d \cdot g$. Let θ denotes the pitch angle where P_y is equal to the gravity.

$$P_y = Pcos(\theta) = W_d \cdot g \tag{10}$$

The horizontal component of P, i.e., P_x, is the power towards the destination, and is equal to the air resistance $k \cdot v(0)$ where k is a drone-specific coefficient.

$$P_x = Psin(\theta) = k \cdot v(0) \tag{11}$$

Figure 2b shows a flying drone with load w. In order not to fall down, the pitch angle θ' must be smaller than θ. Then, we derive the following formulas.

$$P'_y = Pcos(\theta') = (W_d + w) \cdot g \tag{12}$$

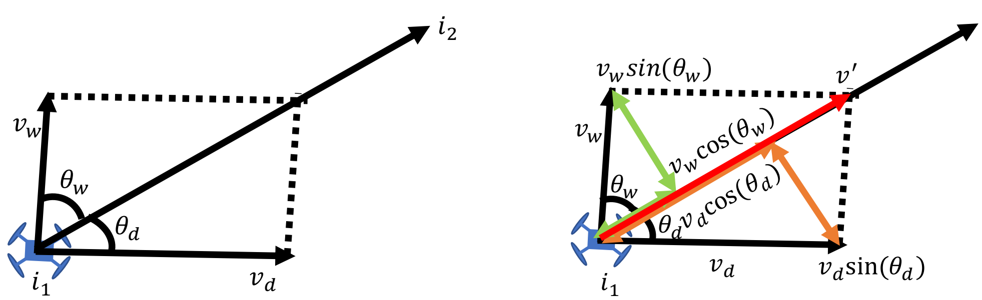
$$P'_x = Psin(\theta') = k \cdot v(w) \tag{13}$$

Hence, we derive:

$$v(w) = \frac{P}{k} sin(\theta') = \frac{sin(\theta')}{sin(\theta)} v(0) \tag{14}$$

$$\theta' = arccos(\frac{(W_d + w) \cdot g}{P}) \tag{15}$$

$$\theta = arccos(\frac{W_d \cdot g}{P}) \tag{16}$$



(a) Situation settings

(b) Speed after effect with the wind

Figure 3: The effect of wind

Next, we consider the effect of the wind in the direction of travel of the drone. Let $\boldsymbol{v_w}$ denote the wind velocity and $\boldsymbol{v_d}$ denote the drone velocity. From the angle formed by the wind and the customer, we define as:

$$|\boldsymbol{v_w}| \sin(\theta_w) = |\boldsymbol{v_d}| \sin(\theta_d) \tag{17}$$

The norm of the synthetic vector $\boldsymbol{v'}$ of $\boldsymbol{v_w}$ and $\boldsymbol{v_d}$ can be expressed as follows:

$$|\boldsymbol{v'}| = |\boldsymbol{v'}| \cos\theta_w + |\boldsymbol{v_d}| \cos\theta_d \tag{18}$$

According to Formula (17) and (18), the scalar of the vector $\boldsymbol{v'}$ is led in the following formula:

$$|\boldsymbol{v'}| = |\boldsymbol{v_w}| \cos(\theta_w) + |\boldsymbol{v_d}| \cos(\arcsin(\frac{|\boldsymbol{v_w}| \sin(\theta_w)}{|\boldsymbol{v_d}|})) \tag{19}$$

Algorithm 1 Dynamic Programming for FSVRPW

```
W_init ← ∑ W
for next_customer ∈ 𝕍 do
    dp[1 << (next_customer − 1)][next_customer] ← flight_time(depot to next_customer with W_init)
    Weight[1 << (next_customer − 1)] ← (W_init − W_next_customer)
end for

for state in [0, 1, 2, ..., (2^N − 1)] do
    for next_customer ∈ 𝕍 do
        if next_customer has not been visited yet then
            for prev_customer ∈ 𝕍 do
                if prev_customer has been already visited then
                    dp[state|(1 << (next_customer − 1))][next_customer] ←
                    min(dp[state][prev_customer]+
                    flight_time(prev_customer to next_customer with Weight[state] and wind velocity),
                            dp[state|(1 << (next_customer − 1))][next_customer])
                    Weight[state|(1 << (next_customer − 1))] ← Weight[state] − W_next_customer
                end if
            end for
        end if
    end for
end for

min_cost ← INFINITE
for prev_customer ∈ 𝕍 do
    min_cost ← min( dp[2^N −1][prev_customer]+flight_time(pre_customer to depot without payload), min_cost)
end for
```

4 Dynamic Programing Algorithm

This section proposes an exact algorithm for the FSVRPW defined in the previous section. The proposed algorithm is based on dynamic programming [10].

Let $\mathbb{S}$ denote a set of customers who are already visited, and let i be the last-visited customer in $\mathbb{S}$. We call a pair $(\mathbb{S}, i)$ as a *state*. Obviously, the initial state is $(\{0\}, 0)$. Then, we define a problem asking the minimum flight time $T(\mathbb{S}, i)$ for delivery from the initial state to state $(\mathbb{S}, i)$. Now, we can derive a recurrence formula to calculate $T(\mathbb{S}, i)$ as follows.

$$T(\mathbb{S}, i) = \min\{\ T(\mathbb{S}\backslash i, i') + t(W'(\bar{\mathbb{S}}) + w(i), d(i', i), v_w) |\ i' \in \mathbb{S}\backslash i\ \} \tag{20}$$

Recall that i is the latest customer in $\mathbb{S}$. In the formula, i' denotes the second latest customer. $T(\mathbb{S}\backslash i, i')$ is the minimum flight time for flying from the depot to i', and $t(W'(\bar{\mathbb{S}}) + w(i), d(i', i))$ is the flight time for flying from i' to i. $W'(\bar{\mathbb{S}})$ denotes the total weight of items which are not yet delivered, which is formulated as:

$$W'(\bar{\mathbb{S}}) = \sum_{k \notin \mathbb{S}} w(k) \tag{21}$$

In Formula (20), it should be noted that, when departing from i', item i is still loaded on the drone. Therefore, $w(i)$ is added to $W'(\bar{\mathbb{S}})$. Also, it is obvious that the flight time at the initial state, i.e., before leaving the depot, is zero.

$$T(\{0\}, 0) = 0 \tag{22}$$

The original routing problem asks the minimum flight time when the drone departs from the depot, visit all of N destinations, and comes back to the depot. Formally, the original problem asks:

$$T(\{0, 1, 2, ..., N, 0\}, 0) \tag{23}$$

The problem in Expression (23) is recursively partitioned into sub-problems according to Formula (20), reaching Formula (22), and then, the optimal route with the shortest delivery time is obtained.

W_{init} is the total weight of all items to be delivered, and $\mathbb{V}$ is a set of customers. *state* represents a set of customers who are already visited. Actually, *state* is a bit-vector of length N, where N is the number of customers. If customer i is visited, the $(i-1)$-th bit is set. $dp[state][customer]$ is a two-dimensional array which stores the flight time, and it corresponds to $T(\mathbb{S}, i)$ in the previous section. For example, $dp[3][2]$ means that the drone already visited customers 1 and 2, and the drone is now at 2. Lines 2-5 calculate the flight time from the depot to the first customer. Then, Lines 7-21 travel all of the remaining customers, and finally in Lines 24-26, the drone comes back to the depot and the minimum flight time is calculated. Lines 7-21 are the main part of the DP algorithm. Instead of recursive procedure calls, the algorithm calculates the flight time with three-level nested loops. The computational complexity of our DP algorithm is $O(2^N \times N^2)$, which is much faster than an exhaustive search of $O(N!)$.

5 Evaluation

The effectiveness of our proposed FSVRP is evaluated through experiments. We implemented several routing algorithms in Python, and are compared in terms of the runtime of the algorithms, total flight time, and the total flight distance.

5.1 Experimental Setup

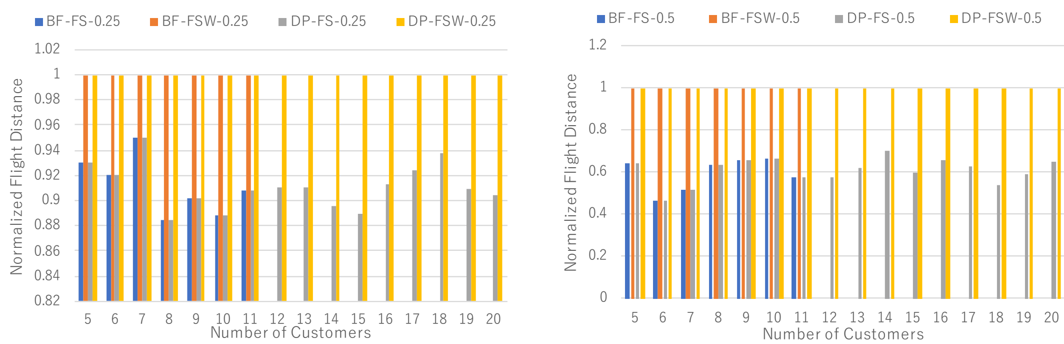
Four routing algorithms shown below are compared in the experiments.

- *FS-BF*: A brute-force algorithm for FSVRP. It exhaustively explores all possible routes (i.e., N! routes) to find the one with minimum flight time.
- *FS-DP*: Dynamic programing algorithm for FSVRP [4].
- *FSW-BF*: A brute-force algorithm for FSVRPW.
- *FSW-DP*: Our DP algorithm proposed in Section 4 for FSVRPW.

In our experiments, we have prepared 20 test cases for each method with 5 20 customers each. The flight distance, flight time, and execution time are calculated from the average of the 20 patterns. The wind strength is tested at 25% and 50% of the drone's speed. We also set the running time limit to 3600 seconds.

5.2 Experimental Results

Figure 4 shows the flight distance for each method at 25% and 50% wind, normalized by DP-FS. The experimental results show that the proposed method has a long flight distance for all customer numbers compared to DP-FS. When comparing DP-FSW-0.25 and DP-FS-0.25, the flight time was reduced by 8.89% on average. The average flight distance increased by 8.89% when compared to DP-FSW-0.25 and DP-FS-0.25, and by 39.37% when compared to DP-FSW-0.5 and DP-FS-0.5. Next, Figure 5 shows the flight time for each method at 25% and 50% wind, normalized by DP-fs. The experimental results show that the proposed method has a short flight time for all customers compared to DP-fs. When comparing DP-FSW-0.25 and DP-FS-0.25, the flight time was reduced by 0.56% on average. The average flight time

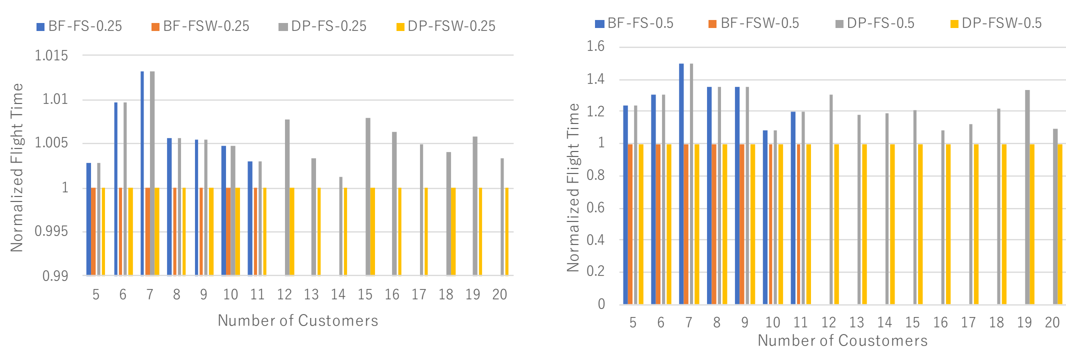


(a) Comparison of flight distance wind is 25%

(b) Comparison of flight distance wind is 50%

Figure 4: Comparison of flight distance

was 0.56% shorter when comparing DP-FSW-0.25 and DP-FS-0.25, and 23.45% shorter when comparing DP-FSW-0.5 and DP-FS-0.5. These results show that even if the flight distance is longer, delivery can be made in a shorter time by using the wind. Next, Figure 6 shows the



(a) Comparison of flight time wind is 25%

(b) Comparison of flight time wind is 50%

Figure 5: Comparison of flight time

computation time for each method. From the results of the experiments, the computation time for BF-FSW-0.25 and BF-FSW-0.5 exceeds one hour when the number of customers exceeds 12, making it impossible to determine the delivery route. On the other hand, DP-FSW-0.25 and DP-FSW-0.5 were able to perform the calculation for all customers within one hour. From the above results, our proposed algorithm can show its usefulness when the number of customers is 20.

6 Conclusions

Drones are expected to be popular vehicles for delivery services. In this paper, a dynamic programming algorithm is proposed defining vehicle routing problems that take into account

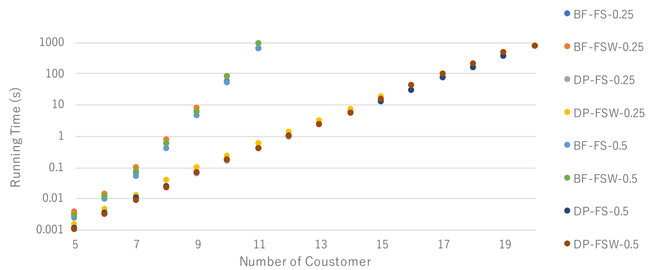


Figure 6: Runtime of routeing algorithms

the changes in flight speed due to load and wind. Experimental results show the effectiveness of the proposed algorithm. In future work, we are considering the effect of dynamically varying wind on the flight speed.

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