# Valid attacks in Argumentation Frameworks with Recursive Attacks

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## Abstract

The purpose of this work is to study a generalisation of Dung's abstract argumentation frameworks that allows representing *recursive attacks*, that is, a class of attacks whose targets are other attacks. We do this by developing a theory of argumentation where the classic role of *attacks* in defeating arguments is replaced by a subset of them, which is extension dependent and which, intuitively, represents a set of "valid attacks" with respect to the extension. The studied theory displays a conservative generalisation of Dung's semantics (complete, preferred and stable) and also of its principles (conflict-freeness, acceptability and admissibility). Furthermore, despite its conceptual differences, we are also able to show that our theory agrees with the AFRA interpretation of recursive attacks for the complete, preferred and stable semantics.

## 1 Introduction

Argumentation has become an essential paradigm for Knowledge Representation and, especially, for reasoning from contradictory information [1; 11] and for formalizing the exchange of arguments between agents in, *e.g.*, negotiation [2]. Formal abstract frameworks have greatly eased the modelling and study of argumentation. For instance, a Dung's argumentation framework (AF) [11] consists of a collection of arguments interacting with each other through an attack relation, enabling to determine "acceptable" sets of arguments called *extensions*.

A natural generalisation of Dung's argumentation frameworks consists in allowing high-order attacks that target other attacks. Here is an example in the legal field, borrowed from [3].

**Example 1.** *The lawyer says that the defendant did not have intention to kill the victim (Argument b). The prosecutor says that the defendant threw a sharp knife towards the victim (Argument a). So, there is an attack from a to b. And the intention to kill should be inferred. Then the lawyer says that the defendant was in a habit of throwing the knife at his wife's foot once drunk. This latter argument (Argument c) is better considered attacking the attack from a to b, than argument a itself. Now the prosecutor's argumentation seems no longer sufficient for proving the intention to kill. This example is represented as a recursive framework in Fig. 1.*

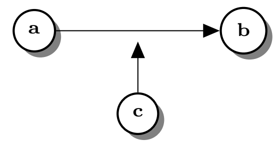


Figure 1: An acyclic recursive framework

Another example, borrowed from [4; 9], will be taken as a running example.

**Example 2.** *Suppose Bob is making decisions about his Christmas holidays, and is willing to buy cheap last minute offers. He knows there are deals for travelling to Gstaad (g) or Cuba (c). Suppose that Bob has a preference for skiing (p) and knows that Gstaad is a renowned ski resort. However, the weather service reports that it has not snowed in Gstaad (n). So it might not be possible to ski there. Suppose finally that Bob is informed that the ski resort in Gstaad has a good amount of artificial snow, that makes it anyway possible to ski there (a). The different attacks are represented on Fig. 2.* □

A semantics for these classes of *recursive frameworks* has been first introduced in [14] in the context of preferences in argumentation frameworks, and latter studied in [4] under the name of AFRA (Argumentation Framework with Recursive Attacks). This version describes abstract argumentation frameworks in which the interactions can be either attacks between arguments or attacks from an argument to another attack. A translation of an AFRA into an AF is defined by the addition of some new arguments and the attacks they produce or they receive. Note that AFRA have been extended in order to handle recursive support interactions together

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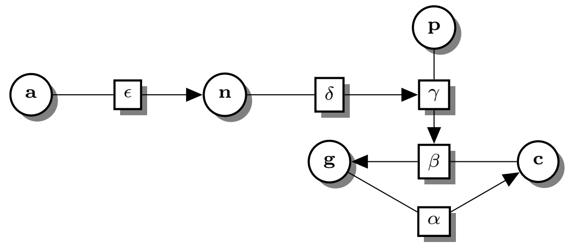


Figure 2: Bob's dilemma: arguments are in circle and attacks in square.

with recursive attacks [9; 10]. However, when supports are removed, these approaches go back to AFRA.

A similar work is described in [7] using the addition of meta-arguments that enable to encode the notions of "grounded attack" and "valid attack". The notion of grounded attack is about the source of the attack and the notion of valid attack is about the link between the source and the target of the attack (*i.e.* the role of the interaction itself). Despite the intuitive results obtained by these approaches regarding complete, stable or grounded extensions, it somehow changes the role that attacks play in Dung's frameworks.

**Example 3.** *Consider the argumentation framework corresponding to Fig. 3. According to Dung's theory, this*

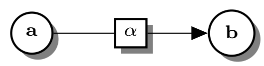


Figure 3: A simple Dung's framework

*framework has three conflict-free sets:* $\varnothing$, $\{a\}$ *and* $\{b\}$*. On the other hand,* $\{a, b\}$ *is a conflict-free set according to AFRA because the attack* $\alpha$ *is not in the set. In fact, in AFRA, such an argumentation framework can be turned into an AF by converting* $\alpha$ *into a new argument as in Fig. 4. In this new framework, it is easy to*

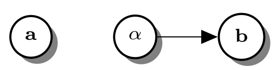


Figure 4: AF framework for AFRA of Fig. 3

*observe that* $\{a, b\}$ *is considered conflict-free in AFRA because there is no attack between* $a$ *and* $b$*. In some sense, the connection between an attack and its source has been lost. As another example of this behaviour, the set* $\{\alpha, b\}$ *is not AFRA-conflict-free despite the fact that the source of* $\alpha$*, the argument* $a$*, is not in the set.* □

In this paper, we study an alternative semantics for argumentation frameworks with recursive attacks based on the following intuitive principles:

**P1** The role played in Dung's argumentation frameworks by attacks in defeating arguments is now played by a subset of these attacks, which is extension dependent and represents the "valid attacks" with respect to that extension.

**P2** It is a conservative generalisation of Dung's framework for the definitions of conflict-free, admissible, complete, preferred, and stable extensions.

For instance, in the proposed semantics, the conflict-free extensions of the framework of Fig. 3 are precisely Dung's conflict-free extensions: $\varnothing$, $\{a\}$ and $\{b\}$. Besides, as we will see later, the attack $\alpha$ is valid with respect to all three extensions because it is not the target of any attack. It is worth noting that, despite its conceptual difference with respect to AFRA, we are able to prove an one-to-one correspondence between our complete, preferred and stable extensions and the corresponding AFRA extensions, in which the set of "acceptable" arguments are the same. This offers an alternative view for the semantics of recursive attacks that we believe to be closer to Dung's intuitive understanding.

## 2 Background

**Definition 1.** *A* Dung's abstract argumentation framework (D-framework *for short*) *is a pair* $\mathbf{AF} = \langle \mathbf{A}, \mathbf{R} \rangle$ *where* $\mathbf{A}$ *is a set of arguments and* $\mathbf{R} \subseteq \mathbf{A} \times \mathbf{A}$ *is a relation representing attacks over arguments.* □

For instance, the graph depicted in Fig. 3 corresponds to the D-framework $\mathbf{AF}_3 = \langle \mathbf{A}_3, \mathbf{R}_3 \rangle$ with the set of arguments $\mathbf{A}_3 = \{a, b\}$ and the attack relation $\mathbf{R}_3 = \{(a, b)\}$.

**Definition 2.** *Given some D-framework* $\mathbf{AF} = \langle \mathbf{A}, \mathbf{R} \rangle$ *and some set of arguments* $S \subseteq \mathbf{A}$*, an argument* $a \in \mathbf{A}$ *is said to be*

*i)* defeated w.r.t. $S$ *iff* $\exists b \in S$ *such that* $(b, a) \in \mathbf{R}$*, and*

*ii)* acceptable w.r.t. $S$ *iff for every argument* $b \in \mathbf{A}$ *with* $(b, a) \in \mathbf{R}$*, there is* $c \in S$ *such that* $(c, b) \in \mathbf{R}$*.* □

To obtain shorter definitions we will also use the following notations:

$$Def(S) \stackrel{\text{def}}{=} \{\ a \in \mathbf{A} \ \big|\ \exists b \in S \text{ s.t. } (b, a) \in \mathbf{R}\ \}$$

$$Acc(S) \stackrel{\text{def}}{=} \{\ a \in \mathbf{A} \ \big|\ \forall b \in \mathbf{A},\ (b, a) \in \mathbf{R} \text{ implies } b \in Def(S)\ \}$$

respectively denote the set of all defeated and acceptable arguments w.r.t. $S$.

**Definition 3.** *Given a D-framework* $\mathbf{AF} = \langle \mathbf{A}, \mathbf{R} \rangle$*, a set of arguments* $S \subseteq \mathbf{A}$ *is said to be*

*i)* conflict-free *iff* $S \cap Def(S) = \varnothing$*,*

*ii)* admissible *iff it is conflict-free and* $S \subseteq Acc(S)$*,*

*iii)* complete *iff it is conflict-free and* $S = Acc(S)$*,*

*iv)* preferred *iff it is* $\subseteq$*-maximal*[1] *admissible,*

*v)* stable *iff it is conflict-free and* $S \cup Def(S) = \mathbf{A}$*.* □

**Theorem 1** ([11])**.** *Given a D-framework* $\mathbf{AF} = \langle \mathbf{A}, \mathbf{R} \rangle$*, the following assertions hold:*

*i) every complete set is also admissible,*

*ii) every preferred set is also complete, and*

*iii) every stable set is also preferred.* □

For instance, in Example 3, the argument $a$ is accepted w.r.t. any set $S$ because there is no argument $x \in \mathbf{A}$ such

[1] With $\subseteq$ denoting the standard set inclusion relation.

that $(x,a) \in \mathbf{R}$. Furthermore, $b$ is defeated and non-acceptable w.r.t. the set $\{a\}$. Then, it is easy to check that $\{a\}$ is stable (and, thus, conflict-free, admissible, complete and preferred). The empty set $\varnothing$ is admissible, but not complete; and the set $\{b\}$ is conflict-free, but not admissible.

## 3 Semantics for recursive attacks

**Definition 4.** *A* recursive argumentation framework $\mathbf{RAF} = \langle \mathbf{A},\mathbf{K},\mathbf{s},\mathbf{t}\rangle$ *is a quadruple where* $\mathbf{A}$ *and* $\mathbf{K}$ *are (possibly infinite) disjunct sets respectively representing arguments and attack names, and where* $\mathbf{s} : \mathbf{K} \longrightarrow \mathbf{A}$ *and* $\mathbf{t} : \mathbf{K} \longrightarrow \mathbf{A} \cup \mathbf{K}$ *are functions respectively mapping each attack to its source and its target.* □

For instance, the argumentation framework of Example 3 corresponds to $\mathbf{RAF}_3 = \langle \mathbf{A}_3, \mathbf{K}_3, \mathbf{s}_3, \mathbf{t}_3\rangle$ where $\mathbf{A}_3 = \{a, b\}$, $\mathbf{K}_3 = \{\alpha\}$, $\mathbf{s}_3(\alpha) = a$ and $\mathbf{t}_3(\alpha) = b$. In general, given any D-framework $\mathbf{AF} = \langle \mathbf{A},\mathbf{R}\rangle$, we may obtain its corresponding argumentation framework $\mathbf{RAF} = \langle \mathbf{A},\mathbf{K},\mathbf{s},\mathbf{t}\rangle$ by defining a set of attack names $\mathbf{K} = \{\, \alpha_{(a,b)} \mid (a,b) \in \mathbf{R}\,\}$. Functions $\mathbf{s}$ and $\mathbf{t}$ are straightforwardly defined by mapping each attack $(a,b) \in \mathbf{R}$ as follows: $\mathbf{s}(\alpha_{(a,b)}) = a$ and $\mathbf{t}(\alpha_{(a,b)}) = b$.

It is worth noting that our definition allows the existence of several attacks between the same arguments. Though this does not make any difference for frameworks without recursive attacks, for recursive ones, it allows representing attacks between the same arguments that are valid in different contexts. For instance, in the example of Figure 5, there are two attacks between $a$ and $b$,

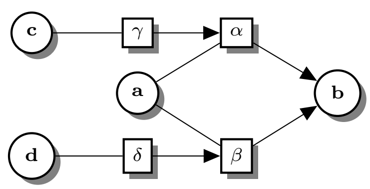


Figure 5: A recursive framework representing attacks in different contexts

namely $\alpha$ and $\beta$, which represent different contexts as they are attacked by different arguments.

**Definition 5** (Structure)**.** *A pair* $\mathfrak{A} = \langle S, \Gamma\rangle$ *is said to be a* structure *of some* $\mathbf{RAF} = \langle \mathbf{A},\mathbf{K},\mathbf{s},\mathbf{t}\rangle$ *iff it satisfies:* $S \subseteq \mathbf{A}$ *and* $\Gamma \subseteq \mathbf{K}$. □

Intuitively, the set $S$ represents the set of "acceptable" arguments w.r.t. the structure $\mathfrak{A}$, while $\Gamma$ represents the set of "valid attacks" w.r.t. $\mathfrak{A}$. Any attack[2] $\alpha \in \overline{\Gamma}$ is understood as non-valid and, in this sense, it cannot defeat the argument or attack that it is targeting.

For the rest of this section we assume that all definitions and results are relative to some given framework $\mathbf{RAF} = \langle \mathbf{A},\mathbf{K},\mathbf{s},\mathbf{t}\rangle$. We extend now the definition of defeated arguments (Definition 2) using the set $\Gamma$ instead of the attack relation $\mathbf{R}$: given a structure of the form $\mathfrak{A} = \langle S, \Gamma\rangle$, we define:

$$Def(\mathfrak{A}) \overset{\text{def}}{=} \{\, a \in \mathbf{A} \mid \exists \alpha \in \Gamma,\ \mathbf{s}(\alpha) \in S \text{ and } \mathbf{t}(\alpha) = a \,\} \quad (1)$$

In other words, an argument $a \in \mathbf{A}$ is defeated w.r.t. $\mathfrak{A}$ iff there is a "valid attack" w.r.t. $\mathfrak{A}$ that targets $a$ and whose source is "acceptable" w.r.t. $\mathfrak{A}$. It is interesting to observe that we may define the *attack relation* associated with some structure $\mathfrak{A} = \langle S, \Gamma\rangle$ as follows:

$$\mathbf{R}_{\mathfrak{A}} \overset{\text{def}}{=} \{\ (\mathbf{s}(\alpha), \mathbf{t}(\alpha)) \mid \alpha \in \Gamma\ \} \quad (2)$$

and that, using this relation, we can rewrite (1) as:

$$Def(\mathfrak{A}) \overset{\text{def}}{=} \{\ a \in \mathbf{A} \mid \exists b \in S \text{ s.t. } (b,a) \in \mathbf{R}_{\mathfrak{A}}\ \} \quad (3)$$

Now, it is easy to see that our definition can be obtained from Dung's definition of defeat (Definition 2) just by replacing the attack relation $\mathbf{R}$ by the attack relation $\mathbf{R}_{\mathfrak{A}}$ associated with the structure $\mathfrak{A}$, or in other words, by replacing the set of all attacks in the argumentation framework by the set of the "valid attacks" w.r.t. the structure $\mathfrak{A}$, as stated in **P1**. Analogously, by

$$Inh(\mathfrak{A}) \overset{\text{def}}{=} \{\ \alpha \in \mathbf{K} \mid \exists b \in S \text{ s.t. } (b,\alpha) \in \mathbf{R}_{\mathfrak{A}}\ \} \quad (4)$$

we denote a set of attacks that, intuitively, represents the "inhibited attacks[3]" w.r.t. $\mathfrak{A}$.

We are now ready to extend the definition of acceptable argument w.r.t. a set (Definition 2):

**Definition 6** (Acceptability)**.** *An element* $x \in (\mathbf{A} \cup \mathbf{K})$ *is said to be* acceptable *w.r.t. some structure* $\mathfrak{A}$ *iff every attack* $\alpha \in \mathbf{K}$ *with* $\mathbf{t}(\alpha) = x$ *satisfies one of the following conditions: (i)* $\mathbf{s}(\alpha) \in Def(\mathfrak{A})$ *or (ii)* $\alpha \in Inh(\mathfrak{A})$. □

By $Acc(\mathfrak{A})$, we denote the set containing all acceptable arguments and attacks with respect to $\mathfrak{A}$. We also define the following order relations that will help us defining preferred structures: for any pair of structures $\mathfrak{A} = \langle S, \Gamma\rangle$ and $\mathfrak{A}' = \langle S', \Gamma'\rangle$, we write $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ iff $(S \cup \Gamma) \subseteq (S' \cup \Gamma')$ and we write $\mathfrak{A} \sqsubseteq_{ar} \mathfrak{A}'$ iff $S \subseteq S'$. As usual, we say that a structure $\mathfrak{A}$ is $\sqsubseteq$-maximal (resp. $\sqsubseteq_{ar}$-maximal) iff every $\mathfrak{A}'$ that satisfies $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ (resp. $\mathfrak{A} \sqsubseteq_{ar} \mathfrak{A}'$) also satisfies $\mathfrak{A}' \sqsubseteq \mathfrak{A}$ (resp. $\mathfrak{A}' \sqsubseteq_{ar} \mathfrak{A}$).

**Definition 7.** *A structure* $\mathfrak{A} = \langle S, \Gamma\rangle$ *is said to be:*

*i)* conflict-free *iff* $S \cap Def(\mathfrak{A}) = \varnothing$ *and* $\Gamma \cap Inh(\mathfrak{A}) = \varnothing$,
*ii)* admissible *iff it is conflict-free and* $(S \cup \Gamma) \subseteq Acc(\mathfrak{A})$,
*iii)* complete *iff it is conflict-free and* $Acc(\mathfrak{A}) = (S \cup \Gamma)$,
*iv)* preferred *iff it is a* $\sqsubseteq$*-maximal admissible structure,*
*v)* arg-preferred *iff it is a* $\sqsubseteq_{ar}$*-maximal preferred structure,*
*vi)* stable[4] *iff* $S = \overline{Def(\mathfrak{A})}$ *and* $\Gamma = \overline{Inh(\mathfrak{A})}$. □

**Example 1 (cont'd)** *Let* **RAF** *be the recursive argumentation framework corresponding to Fig. 6 (Fig. 6 is*

[2] By $\overline{\Gamma} \overset{\text{def}}{=} \mathbf{K} \backslash \Gamma$ we denote the set complement of $\Gamma$.

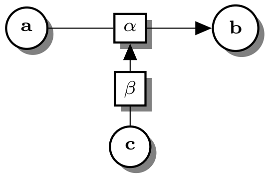


Figure 6: An acyclic recursive framework

*Fig. 1 completed with the attack names). It is easy to check that this framework has a unique complete, preferred and stable structure* $\mathfrak{A} = \langle\{a, b, c\}, \{\beta\}\rangle$*. Furthermore, there are nine admissible structures that are not complete:* $\langle\{a, c\}, \{\beta\}\rangle$, $\langle\{b, c\}, \{\beta\}\rangle$, $\langle\{a\}, \{\beta\}\rangle$, $\langle\{c\}, \{\beta\}\rangle$, $\langle\varnothing, \{\beta\}\rangle$, $\langle\{a, c\}, \varnothing\rangle$, $\langle\{a\}, \varnothing\rangle$, $\langle\{c\}, \varnothing\rangle$ *and* $\langle\varnothing, \varnothing\rangle$*. There are also other conflict-free structures that are not admissible:* $\langle\varnothing, \{\alpha, \beta\}\rangle$, $\langle\{a\}, \{\alpha, \beta\}\rangle$, $\langle\{b\}, \{\alpha, \beta\}\rangle$, $\langle\{a, b\}, \{\beta\}\rangle$, $\langle\{b\}, \{\beta\}\rangle$, $\langle\{a, c\}, \{\alpha\}\rangle$, $\langle\{b, c\}, \{\alpha\}\rangle$, $\langle\{a\}, \{\alpha\}\rangle$, $\langle\{b\}, \{\alpha\}\rangle$, $\langle\{c\}, \{\alpha\}\rangle$, $\langle\varnothing, \{\alpha\}\rangle$, $\langle\{a, b\}, \varnothing\rangle$, $\langle\{a, b, c\}, \varnothing\rangle$, $\langle\{b, c\}, \varnothing\rangle$ *and* $\langle\{b\}, \varnothing\rangle$. □

It is worth to mention that preferred and arg-preferred structures do not necessarily coincide, since there exist preferred structures that do not contain a maximal set of arguments as shown by the following example:

**Example 4.** *Let* **RAF** *be the argumentation framework corresponding to the the graph depicted in Figure 7. Both*

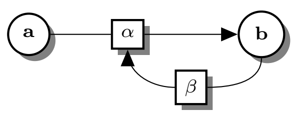


Figure 7: A RAF in which preferred and arg-preferred structures do not coincide

$\mathfrak{A} = \langle\{a, b\}, \{\beta\}\rangle$ *and* $\mathfrak{A}' = \langle\{a\}, \{\alpha, \beta\}\rangle$ *are preferred structures of* **RAF***, but only the former contains a maximal set of arguments and thus* $\mathfrak{A}$ *is the unique arg-preferred structure.* □

We show now[5] that, as in Dung's argumentation theory, there is also a kind of Fundamental Lemma for argumentation frameworks with recursive attacks. For the sake of compactness, we will adopt the following notations: Given a structure $\mathfrak{A} = \langle S, \Gamma\rangle$ and a set $T \subseteq (\mathbf{A} \cup \mathbf{K})$ containing arguments and attacks, by $\mathfrak{A} \cup T \stackrel{\text{def}}{=} \langle S \cup (T \cap \mathbf{A}), \Gamma \cup (T \cap \mathbf{K})\rangle$ we denote the result of extending $\mathfrak{A}$ with the elements in $T$.

**Lemma 1** (Fundamental Lemma)**.** *Let* $\mathfrak{A} = \langle S, \Gamma\rangle$ *be an admissible structure and* $x, y \in Acc(\mathfrak{A})$ *be any pair of acceptable elements. Then, (i)* $\mathfrak{A}' = \mathfrak{A} \cup \{x\}$ *is an admissible structure, and (ii)* $y \in Acc(\mathfrak{A}')$*.* □

Moreover, admissible structures form a complete partial order with preferred structures as maximal elements:

**Proposition 1.** *The set of all admissible structures forms a complete partial order with respect to* $\sqsubseteq$*. Furthermore, for every admissible structure* $\mathfrak{A}$*, there exists an (arg-)preferred one* $\mathfrak{A}'$ *such that* $\mathfrak{A} \sqsubseteq \mathfrak{A}'$*.* □

The following result shows that the usual relation between extensions also holds for structures.

**Theorem 2.** *The following assertions hold:*

*i) every complete structure is also admissible,*
*ii) every preferred structure is also complete, and*
*iii) every stable structure is also preferred.* □

**Example 5.** *As a further example, consider the framework* **RAF** *corresponding to Fig. 8. This framework*

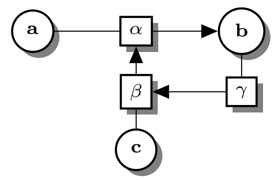


Figure 8: A cyclic recursive framework

*has a unique complete and (arg-)preferred structure* $\mathfrak{A} = \langle\{a, c\}, \{\gamma\}\rangle$*, but no stable one. Note that* $\alpha$ *and* $\beta$ *are neither acceptable nor inhibited w.r.t.* $\mathfrak{A}$*:* $\beta$ *is not inhibited because* $b$ *is not in the structure, so* $\alpha$ *is not acceptable.* $\alpha$ *is not inhibited because* $\beta$ *is not in the structure. And* $\beta$ *is not acceptable because* $b$ *is not defeated (as* $\alpha$ *is not in the structure).* □

**Example 2 (cont'd)** *Consider the framework RAF represented in Fig. 2. This framework has a unique complete, preferred and stable structure:* $\mathfrak{A}_0 = \langle\{a, g, p\}, \{\alpha, \epsilon, \gamma, \delta\}\rangle$*. Among the 63 admissible structures, we find* $\mathfrak{A}_1 = \langle\{a\}, \{\epsilon\}\rangle$, $\mathfrak{A}_2 = \langle\{a\}, \{\epsilon, \delta\}\rangle$*, and* $\mathfrak{A}_3 = \langle\{a\}, \{\alpha, \epsilon, \gamma, \delta\}\rangle$*.* □

## 4 Relation with AFRA

In this section, we establish correspondences between our semantics for recursive attacks and the semantics for AFRA. In [4] a recursive framework is turned into a Dung's framework by adding new arguments and attacks using the following notion of defeat:

**Definition 8** (Defeat)**.** *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t}\rangle$*. An attack* $\alpha \in \mathbf{K}$ *is said to* directly defeat $x \in \mathbf{A} \cup \mathbf{K}$ *iff* $\mathbf{t}(\alpha) = x$*. It is said to* indirectly defeat $\beta \in \mathbf{K}$ *iff* $\alpha$ *directly defeats* $\mathbf{s}(\beta)$*. Then,* $\alpha$ *is said to* defeat $x \in \mathbf{A} \cup \mathbf{K}$ *iff* $\alpha$ *directly defeats* $x$ *or* $\alpha$ *indirectly defeats* $x$*.* □

For instance, in Example 5, it is easy to see that $\alpha$ directly defeats $b$ and indirectly defeats $\gamma$. Hence, $\alpha$ defeats both $b$ and $\gamma$. Attacks $\beta$ and $\gamma$ directly defeat $\alpha$ and $\beta$, respectively. It has been shown in [4] that AFRA extensions can be characterized as the extensions of a Dung's

[3] We will deepen about the intuition of inhibited attacks in Section 6.

[4] By $\overline{Def(\mathfrak{A})} \stackrel{\text{def}}{=} \mathbf{A} \backslash Def(\mathfrak{A})$ we denote the non-defeated arguments. Similarly, by $\overline{Inh(\mathfrak{A})} \stackrel{\text{def}}{=} \mathbf{K} \backslash Inh(\mathfrak{A})$ we denote the non-inhibited attacks. Note also that $S = \overline{Def(\mathfrak{A})}$ and $\Gamma = \overline{Inh(\mathfrak{A})}$ already implies conflict-freeness.

[5] The proofs of propositions, lemmas, theorems given in this paper can be found in [8].

framework whose new set of arguments contains both arguments and attacks and whose new attack relation is the defeat relation of Definition 8. In this sense, under

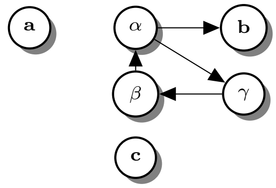


Figure 9: AF framework for AFRA framework of Ex. 5

AFRA, the argumentation framework of Example 5 is turned into the one in Fig. 9 and it can be checked that it has a unique complete and preferred extension $\{a, c\}$ and no stable one. We recall next the formal definitions of AFRA from [4]:

**Definition 9.** *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$*. Then, an element* $x \in (\mathbf{A} \cup \mathbf{K})$ *is said to be* AFRA-acceptable *w.r.t.* $\mathcal{E}$ *iff for every* $\alpha \in \mathbf{K}$ *that defeats* $x$*, there is* $\beta \in \mathcal{E}$ *that defeats* $\alpha$*.* □

**Definition 10** (AFRA-extensions)**.** *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and a set* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$*,* $\mathcal{E}$ *is said to be:*

*i)* AFRA-conflict-free *iff* $\nexists \alpha, x \in \mathcal{E}$ *s.t.* $\alpha$ *defeats* $x$*,*

*ii)* AFRA-admissible *iff* $\mathcal{E}$ *is AFRA-conflict-free and each element of* $\mathcal{E}$ *is AFRA-acceptable w.r.t.* $\mathcal{E}$*,*

*iii)* AFRA-complete *iff it is AFRA-admissible and every* $x \in (\mathbf{A} \cup \mathbf{K})$ *which is AFRA-acceptable w.r.t.* $\mathcal{E}$ *belongs to* $\mathcal{E}$*,*

*iv)* AFRA-preferred *iff it is a* $\subseteq$*-maximal AFRA-admissible extension,*

*v)* AFRA-stable *iff it is AFRA-conflict-free and, for every* $x \in (\mathbf{A} \cup \mathbf{K})$*,* $x \notin \mathcal{E}$ *implies that* $x$ *is defeated by some* $\alpha \in \mathcal{E}$*.* □

As illustrated by Example 3, AFRA does not preserve Dung's notion of conflict-freeness.

**Observation 1.** *AFRA is not a conservative generalisation of Dung's approach.* □

In order to characterize the relation between our approach and AFRA, we will need the following notation. Given some structure $\mathfrak{A} = \langle S, \Gamma \rangle$, by

$$\mathcal{E}_{\mathfrak{A}} \stackrel{\text{def}}{=} S \cup \{ \alpha \in \Gamma \mid \mathbf{s}(\alpha) \in S \}$$

we denote its corresponding AFRA-extension.

Note that the AFRA-extension corresponding to a given structure only contains the attacks of the structure whose source belongs to the structure. The other attacks of the structure do not appear in the AFRA-extension. Intuitively, this selection is motivated by the fact that any attack in an AFRA-extension directly carries a conflict against its target, even if its source is not accepted, something which we avoid in our framework.

**Proposition 2.** *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and a structure* $\mathfrak{A} = \langle S, \Gamma \rangle$*, the following assertions hold:*

*i) if* $\mathfrak{A}$ *is conflict-free, then* $\mathcal{E}_{\mathfrak{A}}$ *is AFRA-conflict-free,*

*ii) if* $\mathfrak{A}$ *is admissible, then* $\mathcal{E}_{\mathfrak{A}}$ *is AFRA-admissible,*

*iii) if* $\mathfrak{A}$ *is complete, then* $\mathcal{E}_{\mathfrak{A}}$ *is AFRA-complete,*

*iv) if* $\mathfrak{A}$ *is preferred, then* $\mathcal{E}_{\mathfrak{A}}$ *is AFRA-preferred,*

*v) if* $\mathfrak{A}$ *is stable, then* $\mathcal{E}_{\mathfrak{A}}$ *is AFRA-stable.* □

For the converse of Prop. 2, we need to introduce some extra notation: Given some set $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$, by $S_{\mathcal{E}} \stackrel{\text{def}}{=} (\mathcal{E} \cap \mathbf{A})$, we denote the set of arguments of $\mathcal{E}$. Then, considering the structure $\mathfrak{A}' = \langle S_{\mathcal{E}}, (\mathcal{E} \cap \mathbf{K}) \rangle$, by

$$\Gamma_{\mathcal{E}} \stackrel{\text{def}}{=} (\mathcal{E} \cap \mathbf{K}) \cup \{ \alpha \in (Acc(\mathfrak{A}') \cap \mathbf{K}) \mid \mathbf{s}(\alpha) \notin \mathcal{E} \} \quad (5)$$

we denote the set of "valid attacks" with respect to $\mathcal{E}$. Finally, by $\mathfrak{A}_{\mathcal{E}} \stackrel{\text{def}}{=} \langle S_{\mathcal{E}}, \Gamma_{\mathcal{E}} \rangle$, we denote the structure corresponding to some AFRA-extension $\mathcal{E}$. Here, we have to add attacks that do not belong to the AFRA-extension. Intuitively, this is due to the fact that, in AFRA, an attack is not acceptable whenever its source is not acceptable [4, Lemma 1]. Hence, we need to add to the structure all those attacks that are non-AFRA-acceptable only because of attacks towards their source.

**Proposition 3.** *Given a* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and a set* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$*, the following assertions hold:*

*i) if* $\mathcal{E}$ *is AFRA-conflict-free, then* $\mathfrak{A}_{\mathcal{E}}$ *is conflict-free,*

*ii) if* $\mathcal{E}$ *is AFRA-admissible, then* $\mathfrak{A}_{\mathcal{E}}$ *is conflict-free,*

*iii) if* $\mathcal{E}$ *is AFRA-complete,* $\mathfrak{A}_{\mathcal{E}}$ *is a complete structure,*

*iv) if* $\mathcal{E}$ *is AFRA-preferred,* $\mathfrak{A}_{\mathcal{E}}$ *is a preferred structure,*

*v) if* $\mathcal{E}$ *is AFRA-stable,* $\mathfrak{A}_{\mathcal{E}}$ *is a stable structure.* □

It is worth to emphasise that for an AFRA-admissible extension, Proposition 3 only ensures that the corresponding structure $\mathfrak{A}_{\mathcal{E}}$ is a conflict-free structure. In fact, there exist AFRA-admissible extensions, whose corresponding structures are not admissible. For instance, consider-

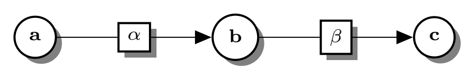


Figure 10: A Dung's framework with two attacks

ing the argumentation framework of Fig. 10, the set $\{\alpha, c\}$ is AFRA-admissible, but the corresponding structure $\langle \{c\}, \{\alpha, \beta\} \rangle$ is not an admissible structure (since $a$ is not in the structure). This discrepancy follows from the fact that, in AFRA, $\alpha$ defeats $\beta$ despite of the absence of $a$ while in our approach attacks whose source is not accepted cannot defeat other arguments or attacks. This difference disappears if we consider what we call *closed* sets. We say that $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ is *closed* iff every attack $\alpha \in (\mathcal{E} \cap \mathbf{K})$ satisfies $\mathbf{s}(\alpha) \in \mathcal{E}$.

**Proposition 4.** *Let* $\mathcal{E}$ *be a closed AFRA-admissible extension. Then,* $\mathfrak{A}_{\mathcal{E}}$ *is an admissible structure.* □

Note that for conflict-freeness and admissibility, the correspondence is not necessarily one-to-one. For instance, $\mathfrak{A} = \langle \{a, c\}, \{\alpha\} \rangle$ and $\mathfrak{A}' = \langle \{a, c\}, \{\alpha, \beta\} \rangle$ are both admissible structures of the framework of Fig. 10 and both of them correspond to the same AFRA-admissible extension $\mathcal{E}_{\mathfrak{A}} = \mathcal{E}_{\mathfrak{A}'} = \{a, c, \alpha\}$. Recall that $\beta$ is acceptable w.r.t. $\mathfrak{A}'$ because it is not attacked. However, it is not AFRA-acceptable w.r.t. $\{a, c, \alpha, \beta\}$ because, in AFRA, $\alpha$ defeats $\beta$ and $\alpha$ is not itself defeated (in fact, $\{a, c, \alpha, \beta\}$ is not even AFRA-conflict-free). On the other hand, note that only $\mathfrak{A}'$ is a complete structure. In fact, for complete structures the correspondence is one-to-one.

Let us denote by $\mathtt{Afra}(\cdot)$ the function mapping each structure $\mathfrak{A}$ to its corresponding AFRA-extension $\mathcal{E}_{\mathfrak{A}}$.

**Proposition 5.** *The following assertions hold:*

*i) if $\mathcal{E}$ is AFRA-complete (or just a closed AFRA-conflict-free extension), then $\mathtt{Afra}(\mathfrak{A}_{\mathcal{E}}) = \mathcal{E}$, and*

*ii) if $\mathfrak{A}$ is a complete structure, then $\mathfrak{A}_{\mathtt{Afra}(\mathfrak{A})} = \mathfrak{A}$.* □

**Theorem 3.** *The function $\mathtt{Afra}(\cdot)$ is a one-to-one correspondence between the sets of all complete (resp. preferred and stable) structures and the set of all AFRA-complete (resp. preferred and stable) extensions.* □

Note that given the one-to-one correspondence between preferred structures and AFRA-preferred extensions, there are AFRA-preferred extensions that do not correspond to arg-preferred ones and thus, they do not contain a maximal set of arguments. For instance, $\{a, b, \beta\}$ and $\{a, \alpha\}$ are both AFRA-preferred extensions in Example 4, but only the former contains a maximal set of arguments.

An interesting consequence of Theorem 3 and Proposition 12 in [4] is that complexity for RAFs' semantics does not increase w.r.t. Dung's frameworks. That is, credulous acceptance w.r.t. the complete, preferred and the stable semantics is NP-complete. Sceptical acceptance w.r.t. the preferred (resp. stable) semantics is $\Pi_2^{\mathrm{P}}$-complete (resp. coNP-complete) [12].

**Example 2 (cont'd)** *For the framework represented in Fig. 2, there is a unique AFRA-complete, AFRA-preferred and AFRA-stable extension: $\mathcal{E} = \{a, g, p, \alpha, \epsilon, \gamma\}$. Note that $\delta \notin \mathcal{E}$ whereas $\mathcal{E} = \mathcal{E}_{\mathfrak{A}_0}$. Indeed, no AFRA-admissible extension contains $\delta$. Analogously, we have $\mathcal{E}_{\mathfrak{A}_1} = \mathcal{E}_{\mathfrak{A}_2} = \mathcal{E}_{\mathfrak{A}_3} = \{a, \epsilon\}$. Moreover, among the AFRA-admissible extensions, we find $\{a, g, \epsilon, \gamma\}$ which is not closed. The associated structure $\mathfrak{A}_4 = \langle \{a, g\}, \{\epsilon, \gamma\} \rangle$ is not an admissible structure.*

## 5 Conservative generalisation

As mentioned in the introduction, our theory aims to be a conservative generalisation of Dung's theory (**P2**). Indeed, given the one-to-one correspondence between complete, preferred and stable structures and their corresponding AFRA-extensions and between the latter and Dung's extensions [4] in the case of non-recursive frameworks, it immediately follows that there exists a one-to-one correspondence between complete, preferred and stable structures and their corresponding Dung's extensions.

On the other hand, this is not the case when we consider only conflict-freeness or admissibility. As mentioned in the introduction, $\{a, b\}$ is an AFRA-conflict-free extension of the non-recursive argumentation framework of Example 3. From Proposition 3, this implies that the corresponding structure $\langle \{a, b\}, \varnothing \rangle$, is a conflict-free structure.

It is worth to note that, in Dung's argumentation frameworks, every attack is considered as "valid" in the sense that it may affect its target. The following definition strengthens the notion of structure by adding a kind of reinstatement principle on attacks, that forces every attack that cannot be defeated to be "valid".

**Definition 11** (D-structure)**.** *A* d-structure *$\mathfrak{A} = \langle S, \Gamma \rangle$ is a structure that satisfies $(Acc(\mathfrak{A}) \cap \mathbf{K}) \subseteq \Gamma$.* □

**Definition 12.** *A* conflict-free *(resp.* admissible, complete, preferred, stable*)* d-structure *is a conflict-free (resp. admissible, complete, preferred, stable) structure which is also a d-structure.* □

As a direct consequence of Definition 7, we have:

**Observation 2.** *Every complete structure is also a d-structure.* □

Observation 2 plus Theorem 3 immediately imply the existence of a one-to-one correspondence between complete (resp. preferred or stable) d-structures and their corresponding AFRA and Dung's extensions. In order to establish a correspondence between conflict-free (resp. admissible) d-structures and their corresponding Dung's extensions, we need to define what it means for a set of arguments to be an extension of some recursive framework.

**Definition 13** (Argument extensions)**.** *A set of arguments $S \subseteq \mathbf{A}$ is* conflict-free *(resp.* admissible, complete, preferred, stable*) iff there is some $\Gamma \subseteq \mathbf{K}$ such that $\mathfrak{A} = \langle S, \Gamma \rangle$ is a conflict-free (resp. admissible, complete, preferred, stable) d-structure.* □

Definition 13 allows us to talk about sets of arguments instead of structures. Before formalising the fact that Definition 13 characterizes a conservative generalisation of Dung's argumentation framework, we define the attack relation associated with some framework in a similar way to the attack relation associated with some structure: $\mathbf{R_{RAF}} \overset{\text{def}}{=} \{ (\mathbf{s}(\alpha), \mathbf{t}(\alpha)) \mid \alpha \in \mathbf{K} \}$. Note that, since every structure $\mathfrak{A} = \langle S, \Gamma \rangle$ satisfies $\Gamma \subseteq \mathbf{K}$, it clearly follows that $\mathbf{R}_{\mathfrak{A}} \subseteq \mathbf{R_{RAF}}$. We also precise what we mean by non-recursive framework:

**Definition 14** (Non-recursive framework)**.** *A framework $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ is said to be* non-recursive *iff $\mathbf{R_{RAF}} \subseteq \mathbf{A} \times \mathbf{A}$.* □

That is, non-recursive frameworks are those in which no attack targets another attack. Given a non-recursive framework $\mathbf{RAF}$, it is easy to observe that $\mathbf{AF} = \langle \mathbf{A}, \mathbf{R_{RAF}} \rangle$ is a D-framework (Definition 1). In this sense, by $\mathbf{RAF}^D \overset{\text{def}}{=} \langle \mathbf{A}, \mathbf{R_{RAF}} \rangle$, we denote the D-framework associated with some $\mathbf{RAF}$.

**Observation 3.** *Every d-structure $\mathfrak{A} = \langle S, \Gamma \rangle$ of any non-recursive framework satisfies $\Gamma = \mathbf{K}$.* □

**Theorem 4.** *A set of arguments $S \subseteq \mathbf{A}$ is conflict-free (resp. admissible, complete, preferred or stable) w.r.t. some non-recursive $\mathbf{RAF}$ (Definition 13) iff it is conflict-free (resp. admissible, complete, preferred or stable) w.r.t. $\mathbf{RAF}^D$ (Definition 3).* □

Due to Observation 2, it follows directly that:

**Corollary 1.** *A structure $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ is complete (resp. preferred, stable) w.r.t. a non-recursive* **RAF** *(Definition 7) iff $S$ is complete (resp. preferred or stable) w.r.t.* $\mathbf{RAF}^D$ *(Definition 3).* □

It is worth to note that the notion of d-structure provides alternative semantics for the principles of conflict-freeness and admissibility.

**Example 1 (cont'd)** *Among the conflict-free structures that are not admissible, only five are conflict-free d-structures:* $\langle \varnothing, \{\alpha, \beta\} \rangle$, $\langle \{a\}, \{\alpha, \beta\} \rangle$, $\langle \{b\}, \{\alpha, \beta\} \rangle$, $\langle \{a, b\}, \{\beta\} \rangle$, $\langle \{b\}, \{\beta\} \rangle$. *Similarly, among the admissible structures that are not complete, only five are admissible d-structures:* $\langle \{a, c\}, \{\beta\} \rangle$, $\langle \{b, c\}, \{\beta\} \rangle$, $\langle \{a\}, \{\beta\} \rangle$, $\langle \{c\}, \{\beta\} \rangle$ *and* $\langle \varnothing, \{\beta\} \rangle$. □

**Example 2 (cont'd)** *There are admissible structures w.r.t. the framework represented in Fig. 2 that are not d-structures: for instance $\mathfrak{A}_1$ and $\mathfrak{A}_2$. Indeed, each d-structure must contain the attacks thas are not targeted by any other attack, that is, $\{\epsilon, \alpha, \delta\}$. Moreover each d-structure containing $a$ must also contain $\gamma$.* □

## 6 Inhibited attacks

In this section, the intuition behind the concept of inhibited attacks is deepened and precisely defined. Indeed, we may expect that attacks that are inhibited do not have any effect on their targets, that is, we may remove them without modifying the condition of the structure.

**Example 6.** *Let* **RAF** *be the recursive argumentation framework of Fig. 6 and $\mathfrak{A} = \langle \{a, b, c\}, \{\beta\} \rangle$ its unique complete structure. It is easy to check that $\alpha$ is inhibited w.r.t. $\mathfrak{A}$ because $c$ and $\beta$ belong to the structure and $\alpha$ is the target of $\beta$. According to the above intuition, we may expect that this would imply that there is a "somehow" corresponding structure $\mathfrak{A}'$ which is complete w.r.t. some* $\mathbf{RAF}'$ *obtained by removing $\alpha$. Note that, in this case, removing $\alpha$ also implies removing $\beta$ because there cannot be attacks without target. In fact, the resulting* $\mathbf{RAF}'$ *is a recursive framework with arguments $\{a, b, c\}$ and no attack. It is easy to check that $\mathfrak{A}' = \langle \{a, b, c\}, \varnothing \rangle$ is complete (also preferred and stable) w.r.t* $\mathbf{RAF}'$ *and that it shares with $\mathfrak{A}$ the set of "acceptable" arguments.* □

Let us now formalise this intuition:

**Definition 15.** *Given some framework* **RAF** *and two different attacks $\beta, \alpha$, we define: $\beta \prec \alpha$ iff there is some chain of attacks $\delta_1, \delta_2, \ldots \delta_n$ such that $\delta_1 = \beta$, $\delta_n = \alpha$ and $\mathbf{t}(\delta_i) = \delta_{i+1}$ for $1 \leq i < n$.* □

For instance, in the argumentation framework of Fig. 6, we have that $\beta \prec \alpha$. On the other hand, neither $\alpha \prec \beta$, nor $\beta \prec \alpha$ hold for the argumentation framework of Fig. 10. Note that $\prec$ is the empty relation for any non-recursive framework. As usual, by $\preceq$ we denote the reflexive closure of $\prec$.

Given an attack $\alpha$, and a set of attacks $\Gamma$, by $\Gamma^{-\alpha} \stackrel{\text{def}}{=} \Gamma \backslash \{ \beta \in \mathbf{K} \mid \beta \preceq \alpha \}$ we denote the set of attacks obtained by removing the attack $\alpha$ from $\Gamma$. Furthermore, by $\mathbf{RAF}^{-\alpha} = \langle \mathbf{A}, \mathbf{K}^{-\alpha}, \mathbf{s}^{-\alpha}, \mathbf{t}^{-\alpha} \rangle$, with $\mathbf{s}^{-\alpha}$ and $\mathbf{t}^{-\alpha}$ the restrictions of $\mathbf{s}$ and $\mathbf{t}$ to $\mathbf{K}^{-\alpha}$, we denote the framework obtained by removing the attack $\alpha$ from $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$. Similarly, by $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$ we denote the structure obtained by removing the attack $\alpha$ from the structure $\mathfrak{A} = \langle S, \Gamma \rangle$.

**Example 1 (cont'd)** *Let* **RAF** *be the recursive argumentation framework of Fig. 6. Then $\mathbf{RAF}^{-\alpha} = \langle \mathbf{A}, \varnothing, \mathbf{s}^{-\alpha}, \mathbf{t}^{-\alpha} \rangle$ with $\mathbf{A} = \{a, b, c\}$ because $\beta \prec \alpha$ implies that $\beta \notin \mathbf{K}^{-\alpha}$. Furthermore, if $\mathfrak{A} = \langle \{a, b, c\}, \{\beta\} \rangle$, then $\mathfrak{A}^{-\alpha} = \langle \{a, b, c\}, \varnothing \rangle$ which is a stable structure of* $\mathbf{RAF}^{-\alpha}$. □

Proposition 6 below formalises the intuitions presented in the previous example.

**Proposition 6.** *Let* **RAF** *be some framework, $\mathfrak{A}$ be some conflict-free (resp. admissible, complete, preferred, stable) structure and $\alpha \in Inh(\mathfrak{A})$ be some inhibited attack w.r.t. $\mathfrak{A}$. Then, $\mathfrak{A}^{-\alpha}$ is a conflict-free (resp. admissible, complete, preferred, stable) structure of* $\mathbf{RAF}^{-\alpha}$. □

## 7 Conclusion and future works

In this work we have extended Dung's abstract argumentation framework with recursive attacks. One of the essential characteristics of this extension is its conservative nature with respect to Dung's approach (when d-structures are considered). The other one is that semantics are given with respect to the notion of "valid attacks" which play a role analogous to attacks in Dung's frameworks. The notions of "grounded attack" and "valid attack" have been introduced in [7]. However, these notions have been encoded through a two-step translation into a meta-argumentation framework. In the first step, a meta-argument is associated to an attack, and a support relation is added from the source of the attack to the meta-argument. In the second step, a support relation is encoded by the addition of a new meta-argument and new attacks. So [7] uses a method for flattening a recursive framework. As a consequence, extensions contain different kinds of argument. In contrast, we propose a theory where valid attacks remain explicit, and distinct from arguments, within the notion of structure. Despite these differences with respect to other generalisations, we proved a one-to-one correspondence with AFRA-extensions in the case of the complete, preferred and stable semantics, while retaining a one-to-one correspondence with Dung's frameworks in the case of conflict-free and admissible extensions.

For a better understanding of the RAF framework, future work should include the study of other semantics (stage, semi-stable, grounded and ideal), extending our approach by taking into account bipolar interactions [9; 16] (case when arguments and attacks may be attacked or supported), and enriching the translation proposed by [5; 6; 13; 15] from Dung's framework into propositional logic and ASP, in order to capture RAF.

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