Merging Incommensurable Possibilistic *DL-Lite* Assertional Bases

Salem Benferhat and **Zied Bouraoui** and **Sylvain Lagrue**
Univ Lille Nord de France, F-59000 Lille, France
UArtois, CRIL - CNRS UMR 8188, F-62300 Lens, France
{benferhat,bouraoui,lagrue}@cril.fr

Julien Rossit
Univ Paris Descartes, LIPADE-France
julien.rossit@parisdescartes.fr

Abstract

This short paper studies the problem of merging of different independent data sources linked to a lightweight ontology under the incommensurability assumption. In general, data are often provided by several and potentially conflicting sources of information having different levels of priority. To encode prioritized assertional bases, we use possibilistic *DL-Lite* logic. We investigate an egalitarist merging strategy that minimize dissatisfaction between the source involved in the merging process. We provide a safe way to merge incommensurable possibilistic *DL-Lite* assertional bases using the notion of compatible scales.

Introduction

Description Logics (DLs) are a well-known family of logic-based formalisms used to represent knowledge of a particular domain and make it available for reasoning. DLs are recognized as powerful frameworks that support ontologies. A DL knowledge base is formed by a terminological base, called TBox, and an assertional base, called ABox. The TBox contains intentional (or generic) knowledge of the application domain whereas the ABox stores data (individuals or constants) that instantiate terminological knowledge.

In the last years, there has been an increasingly interest in Ontology-based Data Access (OBDA) that studies how to query a set of data linked to a unified TBox (ontology). A lot of attention was given to *DL-Lite*, a family of lightweight DLs specifically dedicated for applications that use huge volumes of data, in which query answering is the most important reasoning task. *DL-Lite* offers a very low computational complexity for the reasoning process.

In many applications, data are often provided by several and potentially conflicting sources having different reliability levels. Moreover, a given source may provide different sets of uncertain data with different confidence levels. In such situation, there are two main attitudes that may be followed: the first attitude consists first in gathering sets of assertions provided by each sources which gives generally an inconsistent (prioritized or flat) assertional base and then coping with inconsistencies when performing inference using different inconsistency-tolerant inference strategies (*e.g.* (Lembo et al. 2010; Bienvenu 2012; Bienvenu and Rosati 2013)). The second one consists in *merging* the assertional bases using some aggregation strategies.

Knowledge bases merging or belief merging (*e.g.* (Bloch et al. 2001; Konieczny and Pérez 2002)), is a problem largely studied within the propositional logic setting. It focuses on aggregating pieces of information issued from distinct, and may be conflicting or inconsistent, sources in order to obtain a unified point of view by taking advantage of pieces of information provided by each source. Several merging approaches have been proposed which depend on the nature and the representation of knowledge such as merging propositional knowledge bases (*e.g.* (Konieczny and Pérez 2002)), prioritized knowledge bases (*e.g.* (Delgrande, Dubois, and Lang 2006)) or weighted logical knowledge bases (*e.g.* (Benferhat, Dubois, and Prade 1997)). Recently, some works (*e.g.* (Noy and Musen 2000; Kotis, Vouros, and Stergiou 2006; Moguillansky and Falappa 2007; Cóbe, Resina, and Wassermann 2013)), have proposed to merge ontologies.

In (Benferhat, Bouraoui, and Loukil 2013), the authors study the counterpart of the min-based merging (Benferhat, Dubois, and Prade 1997) when uncertain pieces of information are represented by a possibilistic *DL-Lite* knowledge base. The min-based merging operator, well-known as idempotent conjunctive operator, is suitable when sources are assumed to be dependent. In (Benferhat et al. 2014), a min-based merging operator based on conflict resolution is proposed to merge uncertain *DL-Lite* assertional bases linked to the same terminological base (*i.e.* a TBox) seen as merging integrity constraints.

This paper goes one step further by extending the min-based possibilistic merging operator in the case where uncertainty scales used by different sources are incommensurable. We will follow the idea of egalitarist merging operator proposed in (Benferhat, Lagrue, and Rossit 2007) based on the concept of comparable scales. In this paper, we assume that the TBox is coherent and fully certain and only assertional facts (ABoxes) issued from distinct sources may be somewhat certain.

A compatible scale is a re-assignment of certainty degrees to assertional facts such that the initial plausibility ordering inside each ABox (source) is preserved. We show, in particular, that merging a set of ABoxes under incommensurable assumption comes down to apply min-based possibilistic merging of ABox with respect to each compatible scale.

The rest of the paper is organized as follows: Section 2 gives brief preliminaries on *DL-Lite*. Section 3 recalls $DL\text{-}Lite^{\pi}$ an extension of *DL-Lite* within a possibility theory setting. Section 4 investigates min-based merging of multiple and uncertain *DL-Lite* ABoxs under the incommensurability assumption. Section 5 concludes the paper.

A brief refresh on *DL-Lite*

For the sake of simplicity, we only present $DL\text{-}Lite_{core}$ the core fragment of all the *DL-Lite* family (Calvanese et al. 2007) and we will simply use *DL-Lite* instead of $DL\text{-}Lite_{core}$. However, results of this paper are valid for $DL\text{-}Lite_{R}$ and $DL\text{-}Lite_{F}$, two important members of the *DL-Lite* family. The *DL-Lite* language is defined as follows:

$$R \longrightarrow P|P^{-} \quad B \longrightarrow A|\exists R \quad C \longrightarrow B|\neg B$$

where A is an atomic concept, P is an atomic role and P^{-} is the inverse of P. B (*resp.* C) is called basic (*resp.* complex) concept and role R is called basic role. A *DL-Lite* knowledge base (knowledge base) is a pair $\mathcal{K}$=$\langle\mathcal{T},\mathcal{A}\rangle$ where $\mathcal{T}$ is the TBox and $\mathcal{A}$ is the ABox. The TBox $\mathcal{T}$ includes a finite set of inclusion assertions of the form $B \sqsubseteq C$ where B and C are concepts. The ABox $\mathcal{A}$ contains a finite set of assertions on atomic concepts and roles of the form $A(a)$ and $P(a, b)$ where a and b are two individuals.

The semantics of a *DL-Lite* knowledge base is given in term of interpretations. An interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, .^{\mathcal{I}})$ consists of a non-empty domain $\Delta^{\mathcal{I}}$ and an interpretation function $.^{\mathcal{I}}$ that maps each individual a to $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$, each A to $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ and each role P to $P^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. Furthermore, the interpretation function $.^{\mathcal{I}}$ is extended in a straightforward way for complex concepts and roles: $(\neg B)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus B^{\mathcal{I}}$, $(P^{-})^{\mathcal{I}} = \{(y, x)|(x, y) \in P^{\mathcal{I}}\}$ and $(\exists R)^{\mathcal{I}} = \{x|\exists y$ s.t. $(x, y) \in R^{\mathcal{I}}\}$. An interpretation $\mathcal{I}$ is said to be a model of a concept inclusion axiom, denoted by $\mathcal{I} \models B \sqsubseteq C$, iff $B^{\mathcal{I}} \subseteq C^{\mathcal{I}}$. Similarly, we say that $\mathcal{I}$ satisfies a concept (*resp.* role) assertion, denoted by $\mathcal{I} \models A(a)$ (*resp.* $\mathcal{I} \models P(a, b)$), iff $a^{\mathcal{I}} \in A^{\mathcal{I}}$ (*resp.* $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in P^{\mathcal{I}}$).

An interpretation $\mathcal{I}$ is said to be a model of $\mathcal{K}$=$\langle\mathcal{T},\mathcal{A}\rangle$, denoted by $\mathcal{I} \models \mathcal{K}$, iff $\mathcal{I} \models \mathcal{T}$ and $\mathcal{I} \models \mathcal{A}$ where $\mathcal{I} \models \mathcal{T}$ (*resp.* $\mathcal{I} \models \mathcal{A}$) means that $\mathcal{I}$ is a model of all axioms in $\mathcal{T}$ (*resp.* $\mathcal{A}$). A knowledge base $\mathcal{K}$ is said to be consistent if it admits at least one model, otherwise $\mathcal{K}$ is said to be inconsistent. A *DL-Lite* TBox $\mathcal{T}$ is said to be incoherent if there exists at least a concept C such that for each interpretation I which is a model of $\mathcal{T}$, we have $C^{\mathcal{I}}$=$\emptyset$. Note that within a *DL-Lite* setting, the inconsistency problem is always defined with respect to some ABox since a TBox may be incoherent but never inconsistent.

Possibilistic DL-Lite

In this section, we recall the main notions of possibilistic *DL-Lite* framework (Benferhat and Bouraoui 2013), denoted by $DL\text{-}Lite^{\pi}$, as an adaptation of *DL-Lite* within a possibility theory setting (Dubois and Prade 1988). $DL\text{-}Lite^{\pi}$ provides an excellent mechanism to deal with uncertainty and to ensure reasoning under inconsistency while keeping a computational complexity identical to the one used in standard *DL-Lite*.

Possibility Distribution over *DL-Lite* Interpretation

Let Ω be a universe of discourse composed by a set of *DL-Lite* interpretations ($\mathcal{I}$=$(\Delta, .^{I}) \in \Omega$). The semantic counterpart of a $DL\text{-}Lite^{\pi}$ is given by a possibility distribution, denoted by π, which is a mapping from Ω to the unit interval $[0, 1]$ that assigns to each interpretation $\mathcal{I} \in \Omega$ a possibility degree $\pi(\mathcal{I}) \in [0, 1]$ that represents its compatibility or consistency with respect to the set of available knowledge. When $\pi(\mathcal{I})$=0, we say that $\mathcal{I}$ is impossible and it is fully inconsistent with the set of available knowledge, whereas when $\pi(\mathcal{I})$=1, we say that $\mathcal{I}$ is totally possible and it is fully consistent with the set of available knowledge. For two interpretations $\mathcal{I}$ and $\mathcal{I}'$, when $\pi(\mathcal{I}) > \pi(\mathcal{I}')$ we say that $\mathcal{I}$ is more consistent or more preferred than $\mathcal{I}'$ *w.r.t* available knowledge. Lastly, π is said to be normalized if there exists at least one totally possible interpretation, namely $\exists\mathcal{I} \in \Omega, \pi(\mathcal{I})$=1, otherwise, we say that π is sub-normalized. The concept of sub-normalization reflects the presence of conflicts in the set of available information.

Given a possibility distribution π defined on a set of interpretations Ω, one can define two measures on a *DL-Lite* axiom φ: A possibility measure $\Pi(\varphi)$=$\max_{\mathcal{I}\in\Omega}\{\pi(\mathcal{I}) : \mathcal{I} \models \varphi\}$ that evaluates to what extent an axiom φ is compatible with the available knowledge encoded by π and a necessity measure $N(\varphi)$=$1 - \max_{\mathcal{I}\in\Omega}\{\pi(\mathcal{I}) : \mathcal{I} \not\models \varphi\}$ that evaluates to what extent φ is certainty entailed from available knowledge encoded by π.

$DL\text{-}Lite^{\pi}$ Knowledge Base

Let $\mathcal{L}$ be a *DL-Lite* description language, a $DL\text{-}Lite^{\pi}$ knowledge base is a set of possibilistic axioms of the form $(\varphi, W(\varphi))$ where φ is an axiom expressed in $\mathcal{L}$ and $W(\varphi) \in\,]0, 1]$ is the degree of certainty/priority of φ. Namely, a $DL\text{-}Lite^{\pi}$ knowledge base $\mathcal{K}$ is such that $\mathcal{K}$=$\{(\varphi_i, W(\varphi_i)) : i = 1, ..., n\}$. Only somewhat certain information are explicitly represented in a $DL\text{-}Lite^{\pi}$ knowledge base. Namely, axioms with a null weight ($W(\varphi_i) = 0$) are not explicitly represented in the knowledge base. The weighted axiom $(\varphi, W(\varphi))$ means that the certainty degree of φ is at least equal to $W(\varphi_i)$. A $DL\text{-}Lite^{\pi}$ knowledge base $\mathcal{K}$ will also be represented by a couple $\mathcal{K}$=$\langle\mathcal{T},\mathcal{A}\rangle$ where both elements in $\mathcal{T}$ and $\mathcal{A}$ may be uncertain. It is important to note that, if we consider all $W(\varphi_i) = 1$ then we found a classical *DL-Lite* knowledge base: $\mathcal{K}^*$=$\{\varphi_i : (\varphi_i, W(\varphi_i)) \in \mathcal{K}\}$.

Given $\mathcal{K}$=$\langle\mathcal{T},\mathcal{A}\rangle$ a $DL\text{-}Lite^{\pi}$ knowledge base, we define the α-cut of $\mathcal{K}$ (*resp.* $\mathcal{T}$ and $\mathcal{A}$), denoted by $\mathcal{K}_{\geq\alpha}$ (*resp.* $\mathcal{T}_{\geq\alpha}$ and $\mathcal{A}_{\geq\alpha}$), the subbase of $\mathcal{K}$ (*resp.* $\mathcal{T}$ and $\mathcal{A}$) composed of axioms having weights at least greater than α.

We say that $\mathcal{K}$ is consistent if the standard knowledge base obtained from $\mathcal{K}$ by ignoring the weights associated with axioms is consistent. In case of inconsistency, we attach to $\mathcal{K}$ an inconsistency degree. The inconsistency degree of a $DL\text{-}Lite^{\pi}$ knowledge base $\mathcal{K}$, denoted by $Inc(\mathcal{K})$, is syntactically defined as follow: $Inc(\mathcal{K})$=$max\{W(\varphi_i)$:$\mathcal{K}_{\geq W(\varphi_i)}$ is inconsistent}.

Given a $DL\text{-}Lite^{\pi}$ knowledge base $\mathcal{K}$, one can associate to $\mathcal{K}$ a joint possibility distribution, denoted by $\pi_{\mathcal{K}}$, defined over the set of all interpretations $\mathcal{I}$=$(\Delta, .^{\mathcal{I}})$ by associating to each interpretation its level of consistency with the set of available knowledge, that is, with $\mathcal{K}$. Namely:

Definition 1. The possibility distribution induced from a $DL\text{-}Lite^{\pi}$ is defined as follows: $\forall \mathcal{I} \in \Omega$:

$$\pi_{\mathcal{K}}(\mathcal{I})=\begin{cases}1 \;\; if\,\forall\,(\varphi_i, W(\varphi_i)) \in \mathcal{K}, \mathcal{I} \models \varphi_i \\ 1\text{-}max\{W(\varphi_i):(\varphi_i,W(\varphi_i)) \in \mathcal{K}, \mathcal{I} \not\models \varphi_i\} \text{ otherwise}\end{cases}$$

A $DL\text{-}Lite^{\pi}$ knowledge base $\mathcal{K}$ is said to be consistent if its joint possibility distribution $\pi_{\mathcal{K}}$ is normalized. If not, $\mathcal{K}$ is said to be inconsistent and its inconsistency degree is defined semantically as follow: $Inc(\mathcal{K})$=$1 - \underset{\mathcal{I}\in\Omega}{max}\{\pi_{\mathcal{K}}(\mathcal{I})\}$.

It was shown in (Benferhat and Bouraoui 2013) that computing the inconsistency degree of a $DL\text{-}Lite^{\pi}$ knowledge base comes from the extension of the algorithm presented in (Calvanese et al. 2007) by modifying it to query for individuals with a given certainty degree.

Example 1. Let $\mathcal{K}$=$\langle\mathcal{T},\mathcal{A}\rangle$ be a $DL\text{-}Lite^{\pi}$ knowledge base where $\mathcal{T}$=$\{(A \sqsubseteq B, 1),\ (B \sqsubseteq \neg C, .9)\}$ and $\mathcal{A}$=$\{(A(a), .6), (C(b), .5)\}$. The possibility distribution $\pi_{\mathcal{K}}$ associated to $\mathcal{K}$ is computed using Definition 1 as follows where Δ=$\{a, b\}$:

$\mathcal{I}$	$.^{\mathcal{I}}$	$\pi_{\mathcal{K}}$
$\mathcal{I}_1$	A={a},B={},C={b}	0
$\mathcal{I}_2$	A={a},B={a},C={b}	1
$\mathcal{I}_3$	A={},B={},C={a,b}	.4
$\mathcal{I}_4$	A={a,b},B={a,b},C={}	.5

Table 1: Example of a possibility distribution induced from a $DL\text{-}Lite^{\pi}$ knowledge base

One can observe that $\pi_{\mathcal{K}}(I_2)$=1 meaning that $\pi_{\mathcal{K}}$ is normalized, and thus, $\mathcal{K}$ is consistent. □

Fusion-based on compatible scalings

This section studies min-based possibilistic merging operator in the case where uncertainty scales used by the different sources are incommensurable. Throughout this section, we assume that the TBox is coherent and fully certain and only assertional facts (ABoxes) may be somewhat certain. We first present merging using min-based operator of *DL-Lite* assertional bases under commensurability assumption.

Merging using the min-based operator

Let $\mathcal{A} = \{\mathcal{A}_1, ..., \mathcal{A}_n\}$ be a set of n uncertain ABoxes issued from n distinct sources, and let $\mathcal{T}$ be a *DL-Lite* TBox representing the integrity constraints to be satisfied. Let us assume that $\pi_1, ..., \pi_n$ are possibility distributions provided by n sources of information that share the same domain of interpretations (namely $\Delta_1^{\mathcal{I}} = ... = \Delta_n^{\mathcal{I}}$), and that all possibility distributions use the same scale to represent uncertainty. We suppose that each ABox is consistent with $\mathcal{T}$, namely each possibility distribution π_i that encodes $\mathcal{K}_i = \langle\mathcal{T}, \mathcal{A}_i\rangle$ is normalized. For the sake of simplicity, we use $\pi_{\mathcal{A}_i}$ instead of $\pi_{\mathcal{K}_i}$ to denote the possibility distribution associated to each $\mathcal{K}_i = \langle\mathcal{T}, \mathcal{A}_i\rangle$

Given n commensurable ABoxes, merging aims to compute $\Delta_{\mathcal{T}}(\mathcal{A})$, an ABox representing the result of the fusion of these ABoxes. In the literature, different methods for merging have been proposed. In this section, we perform merging of $\mathcal{A}_1$,...,$\mathcal{A}_n$ a set of ABoxes with respect to a TBox $\mathcal{T}$ using min-based merging operator proposed to aggregate $DL\text{-}Lite^{\pi}$ knowledge bases. This operator is a direct extension of the well-known idempotent conjunctive operator (*e.g.* (Benferhat, Dubois, and Prade 1997)) within possibilistic *DL-Lite* setting. It is recommended when distinct sources that provide information are assumed to be dependent.

We first introduce the notion of *profile* associated with an interpretation $\mathcal{I}$, denoted by $\nu_{\mathcal{A}}(\mathcal{I})$, and defined by

$$\nu_{\mathcal{A}}(\mathcal{I}) =< \pi_{\mathcal{A}_1}(\mathcal{I}), ..., \pi_{\mathcal{A}_n}(\mathcal{I}) > .$$

Namely, $\nu_{\mathcal{A}}(\mathcal{I})$ represents the possibility values of an interpretation $\mathcal{I}$ with respect to each source.

From a semantics point of view, the result of merging is a possibility distribution $\Delta_{\mathcal{T}}(\mathcal{A})$ obtained using two steps: i) the possibility degrees $\pi_{\mathcal{A}_i}(\mathcal{I})$'s are first combined with a merging operator (here we use the minimum operator), and the interpretations with height degrees are kept. This leads to define an order relation, denoted by $\triangleleft_{Min}$, between interpretations as follows: an interpretation $\mathcal{I}$ is preferred to another interpretation $\mathcal{I}'$ if the minimum element of the profile of $\mathcal{I}$ is higher than the minimum element of the profile of $\mathcal{I}'$. More formally:

Definition 2 (Definition of $\triangleleft_{Min}$)**.** Let $\mathcal{A} = \{\mathcal{A}_1, ..., \mathcal{A}_n\}$ be a set of ABoxes linked to a TBox $\mathcal{T}$. Let $\mathcal{I}$ and $\mathcal{I}'$ be two interpretations and $\nu_{\mathcal{A}}(\mathcal{I})$, $\nu_{\mathcal{A}}(\mathcal{I}')$ be their associated profiles. Then:

$$\mathcal{I} \triangleleft_{min}^{\mathcal{A}} \mathcal{I}' \iff Min(\nu_{\mathcal{A}}(\mathcal{I})) > Min(\nu_{\mathcal{A}}(\mathcal{I}'))$$

where

$$Min(\nu_{\mathcal{A}}(\mathcal{I})) = Min\{\pi_{\mathcal{A}_i}(\mathcal{I}) : i \in \{1, ..., n\}\}.$$

The result of the merging $\Delta_{\mathcal{T}}^{min}(\mathcal{A})$ is a $DL\text{-}Lite^{\pi}$ knowledge base whose models are interpretations which are models of a constraint $\mathcal{T}$ and which are maximal with respect to $\triangleleft_{Min}$. More formally:

Definition 3 (Min-based merging operator)**.** Let $\mathcal{A} = \{\mathcal{A}_1, ..., \mathcal{A}_n\}$ be a set of ABoxes and $\mathcal{T}$ be an integrity constraint. Let $\{\pi_{\mathcal{A}_1}, ..., \pi_{\mathcal{A}_n}\}$ possibility distributions associated with $(\langle\mathcal{T}, \mathcal{A}_1\rangle, ..., \langle\mathcal{T}, \mathcal{A}_n\rangle)$. The result of merging is a $DL\text{-}Lite^{\pi}$ knowledge base, denoted by $\Delta_{\mathcal{T}}^{min}(\mathcal{A})$ where its model are defined by:

$$Mod(\Delta_{\mathcal{T}}^{min}(\mathcal{A}))=\{\mathcal{I} \in Mod(\mathcal{T}) : \nexists \mathcal{I}' \in Mod(\mathcal{T}), \mathcal{I}' \triangleleft_{Min}^{\mathcal{A}} \mathcal{I}\}$$

In general, merging two $DL\text{-}Lite^{\pi}$ normalized possibility distributions may lead to a sub-normalized possibility distribution. The normalization process comes down to set the degrees of interpretations in $Mod(\Delta_{\mathcal{T}}^{min}(\mathcal{A}))$ to 1.

From a syntactic point of view, the min-based merging operator, denoted by $\Delta_{\mathcal{T}}^{min}(\mathcal{A})$ is the union of all ABox. Namely:

$\Delta_{\mathcal{T}}^{min}(\mathcal{A})$=$\langle\mathcal{T}, \mathcal{A}_1 \cup \mathcal{A}_2 \cup \ldots \cup \mathcal{A}_n\rangle$.

The aggregation of ABoxs is not guaranteed to be consistent. Namely, the resulting knowledge base $\langle\mathcal{T}, \Delta_{\mathcal{T}}^{min}(\mathcal{A})\rangle$ may be inconsistent. To restore the consistency of the resulting knowledge base a normalization step is required. The following definition gives the formal logical representation of the normalized knowledge base.

Definition 4. Let $\mathcal{T}$ be a TBox and $\Delta_{\mathcal{T}}^{min}(\mathcal{A})$ be the aggregation of $\mathcal{A}_1, ...\mathcal{A}_n$, n ABox using classical min-based operator. Let x=$\Delta_{\mathcal{T}}^{min}(\mathcal{A})$. Then, the normalized knowledge base, denoted $\Delta_{\mathcal{T}}^{min}(\mathcal{K})$ is such that:

$$\Delta_{\mathcal{T}}^{min}(\mathcal{K})=\{(f_{ij}, W(f_{ij})):(f, W(f_{ij}) \in \Delta_{\mathcal{T}}^{min}(\mathcal{A}) \text{ and } W(f_{ij}) > x\}\rangle$$

Example 2 (continued)**.** Let us continue with the TBox $\mathcal{T}$=$\{A \sqsubseteq B,\ B \sqsubseteq \neg C\}$ presented in Example 1 while assuming that the certainty degree of each axioms is set to 1. Let us consider the following set of ABoxes to be linked to $\mathcal{T}$: $\mathcal{A}_1$=$\{(A(a), .6), (C(b), .5)\}$, $\mathcal{A}_2$=$\{(C(a), .4), (B(b), .8), (A(b), .7)\}$. We have:

$\mathcal{I}$	$\cdot^{\mathcal{I}}$	$\pi_{\mathcal{A}_1}$	$\pi_{\mathcal{A}_2}$	$\Delta_{\mathcal{T}}^{min}(\mathcal{A})$
$\mathcal{I}_1$	A={a},B={a},C={b}	1	.2	.2
$\mathcal{I}_2$	A={},B={},C={a,b}	.4	.2	.4
$\mathcal{I}_3$	A={a,b},B={a,b},C={}	.5	.6	**.5**
$\mathcal{I}_4$	A={b},B={b},C={a}	.4	1	.4

Table 2: Example of merging of possibility distributions using min-based operator

One can check that the resulting possibility distribution ($\Delta_{\mathcal{T}}^{min}(\mathcal{A})$) is sub-normalized. To normalize $\Delta_{\mathcal{T}}^{min}(\mathcal{A})$, it is enough to set $\mathcal{I}_3$ = .5 to 1. At syntactic level, we have $\Delta_{\mathcal{T}}^{min}(\mathcal{A})$=$\langle\mathcal{T}, \{(A(a), .6), (C(b), .5), (C(a), .4), (B(b), .8), (A(b), .7)\}\rangle$. We have $Inc(\Delta_{\mathcal{T}}^{min}(\mathcal{A}))$=.5 and $\Delta_{\mathcal{T}}^{min}(\mathcal{K})$=$\mathcal{T}, \{(A(a), .6), (B(b), .8), (A(b), .7)\}$. □

In the next section, we investigate min-based merging under incommensurability assumption.

Using compatible scales

The min-based merging operator presented in the previous section is defined over the assumption that all the sources providing the ABoxs use the same scale to encode uncertainties between facts. In Example 2, when dealing with assertions, we assumed that the weight attached to $f \in \mathcal{A}_i$ can be compared to the weight associated with $g \in \mathcal{A}_j$ with $j \neq i$. In this section, we drop this assumption and we suppose that sources are incommensurable.

We investigate a min-based fusion operator to merge incommensurable *DL-Lite* assertional bases. To make sources using different scale commensurable, we use the notion of "compatible scale" on existing scales used by each source.

A ranking scale is said to be compatible with all sources if it preserves original order relations between assertions of each ABox. The new ranking, denoted by $\mathcal{R}$, defines a new ranking relations for each ABox to be merged. More formally,

Definition 5 (Compatible ranking scale)**.** Let $\mathcal{A} = \{\mathcal{A}_1, ..., \mathcal{A}_n\}$ where $\mathcal{A}_i = \{(f_{ij}, W_{\mathcal{A}_i}(f_{ij}))\}$. Then a ranking $\mathcal{R}$ is defined by:

$$\begin{array}{llll} \mathcal{R}: & \mathcal{A}_1 \cup ... \cup \mathcal{A}_n & \rightarrow &]0,1] \\ & (f_{ij}, W_{B_i}(f_{ij})) & \mapsto & \mathcal{R}(f_{ij}) \end{array}$$

A ranking $\mathcal{R}$ is said to be compatible with $W_{\mathcal{A}_1}, ..., W_{\mathcal{A}_n}$ if and only if:

$$\forall \mathcal{A}_i \in \mathcal{A},\ \forall f, W_{\mathcal{A}_i}(f)),\ (f', W_{\mathcal{A}_i}(f')) \in \mathcal{A}_i,$$
$$W_{\mathcal{A}_i}(f) \leq W_{\mathcal{A}_i}(f') \iff \mathcal{R}(f) \leq \mathcal{R}(f').$$

Definition 5 is basically the adaptation of the one given in (Benferhat, Lagrue, and Rossit 2007) for the context of *DL-Lite*.

Example 3 (continued)**.** Let us consider again the following set of ABoxes to be linked to $\mathcal{T}$ given in Example 2: $\mathcal{A}_1$=$\{(A(a), .6)$,$(C(b), .5)\}$, $\mathcal{A}_2$=$\{(C(a), .4)$, $(B(b), .8)$, $(A(b), .7)\}$. The following table gives examples of ranking scales.

	f_{ij}	$W_{\mathcal{A}_i}(f_{ij})$	$\mathcal{R}_1(f_{ij})$	$\mathcal{R}_2(f_{ij})$	$\mathcal{R}_3(f_{ij})$
A_1	$A(a)$	.6	.5	.4	.6
	$C(b)$	.5	.2	.7	.5
A_2	$C(a)$	.4	.3	.3	.4
	$B(b)$	.8	.7	.6	.8
	$A(b)$	.7	.4	.2	.7

Table 3: Examples of ranking scales

The scaling $\mathcal{R}_1$ is a compatible one, because it preserves the order inside each ABox. However, the scaling $\mathcal{R}_2$ is not a compatible one since it inverses priorities inside $\mathcal{A}_1$ and $\mathcal{A}_2$. □

According to Example 3, it is obvious that a compatible ranking scale is not unique. Let us denote by $\mathcal{R}(\mathcal{A})$ the set of compatible scaling associated with $\mathcal{A} = \{\mathcal{A}_1, ..., \mathcal{A}_n\}$. The set $\mathcal{R}(\mathcal{A})$ is non-empty and an immediate way to obtain a ranking relation over $\mathcal{A}$ is to consider $\mathcal{R}(f_{ij}) = W_{\mathcal{A}_i}(f_{ij})$ (For instance, the scale $\mathcal{R}_3(f_{ij})$ given in Example 3). Note that this ranking is compatible in the sense that it permits to preserve the relative ordering between assertions of each $\mathcal{A}_i$.

Given a compatible scales $\mathcal{R}$, we denote by $\mathcal{A}_i^{\mathcal{R}}$ the assertional base obtained from $\mathcal{A}_i$ by replacing each assertion $(f_{ij}, W_{\mathcal{A}_i}(f_{ij}))$ by $(f_{ij}, \mathcal{R}(f_{ij}))$. Similarly, we denote by $\mathcal{A}^{\mathcal{R}}$ the set obtained from $\mathcal{A}$ by replacing each $\mathcal{A}_i$ in $\mathcal{A}$ by $\mathcal{A}_i^{\mathcal{R}}$.

Now, given the set of all compatible scales $\mathcal{R}(\mathcal{A})$, different possibilities may exist in order to merge the ABoxs. For instance, one can only select one scale to perform merging or one can consider all the compatible ranking in $\mathcal{R}(\mathcal{A})$, etc. To avoid an arbitrary choice, we consider all compatible rankings to perform merging.

Semantics merging

We first introduce the notion of preference between interpretation according to the notion of compatible scales. An interpretation $\mathcal{I}$ is then said to be preferred to $\mathcal{I}'$, if for each compatible scale $\mathcal{R}$, $\mathcal{I}$ is preferred to $\mathcal{I}'$ using Definition 2 (namely, $\mathcal{I} \lhd_{Min}^{\mathcal{A}^{\mathcal{R}}} \mathcal{I}'$). More precisely,

Definition 6 (Ordering between interpretations)**.** Let $\mathcal{A} = \{\mathcal{A}_1, ..., \mathcal{A}_n\}$ be a set of $DL\text{-}Lite^{\pi}$ ABoxs and $\mathcal{R}(\mathcal{A})$ be the set of all compatible scalings associated with $\mathcal{A}$. Let $\mathcal{I}$, $\mathcal{I}'$ be two interpretations. Then:

$$\mathcal{I} <^{\mathcal{A}}_{\forall} \mathcal{I}' \iff \forall \mathcal{R} \in \mathcal{R}(\mathcal{A}),\ \mathcal{I} \lhd^{\mathcal{A}^{\mathcal{R}}}_{Min} \mathcal{I}'$$

where $\lhd^{\mathcal{A}^{\mathcal{R}}}_{Min}$ is the result of applying Definition 2 on $\mathcal{A}^{\mathcal{R}}$.

According to Definition 6, we have models of $\Delta^{\forall}_{\mathcal{T}}(\mathcal{A})$ are those which are models of $\mathcal{T}$ and minimal for $<^{\mathcal{A}}_{\forall}$, namely:

$$Mod(\Delta^{\forall}_{\mathcal{T}}(\mathcal{A}))=\{\mathcal{I}{\in}Mod(\mathcal{T}){:}\ \nexists\mathcal{I}'{\in}Mod(\mathcal{T}), \mathcal{I}'{<^{\mathcal{A}}_{\forall}}\mathcal{I}\}.$$

Note that $<^{\mathcal{A}}_{\forall}$ is only a partial order. The following proposition shows that an interpretation $\mathcal{I}$ is a model of $\Delta^{\forall}_{\mathcal{T}}(\mathcal{A})$ if and only if there exists a compatible scaling where this interpretation belongs to the result fusion, namely it is a model of $\Delta^{\mathcal{A}^{\mathcal{R}}}_{Min}(\mathcal{A})$. More formally:

Proposition 1. *Let $\mathcal{A}$ be a set of ABoxes linked to the same TBox $\mathcal{T}$. Then $\mathcal{I} \in Mod(\Delta^{\forall}_{\mathcal{T}}(\mathcal{A}))$, if and only if there exists a compatible scaling $\mathcal{R}$ such that $\mathcal{I} \in Mod(\Delta^{min}_{\mathcal{T}}(\mathcal{A}^{\mathcal{R}}))$.*

The following example illustrates the fusion based on all compatible scalings.

Example 4 (continued)**.** Let us consider again the following set of ABoxes given in Example 2: $\mathcal{A}_1$=$\{(A(a), .6),$ $(C(b), .5)\}$, $\mathcal{A}_2$=$\{(C(a), .4),$ $(B(b), .8),$ $(A(b), .7)\}$. Let us consider again $\mathcal{R}_1$ where $\mathcal{A}^{\mathcal{R}_1}_1 = \{(A(a), .8), (C(b), .4))\}$ and $\mathcal{A}^{\mathcal{R}_1}_2 = \{(C(a), .2), (B(b), .9), (A(b), .6)\}$ and a scaling $\mathcal{R}_2$ where $\mathcal{A}^{\mathcal{R}_2}_1 = \{(A(a), .4), (C(b), .2))\}$ and $\mathcal{A}^{\mathcal{R}_2}_2 = \{(C(a), .3), (B(b), .6), (A(b), .5)\}$. Both of them are compatible. Table 4 presents the profile of each interpretation for each scaling.

$\mathcal{I}$	$\nu_{\mathcal{A}^{\mathcal{R}_1}}(\mathcal{I})$	Min	$\nu_{\mathcal{A}^{\mathcal{R}_2}}(\mathcal{I})$	Min
$\mathcal{I}_1$	$< 1, .1 >$	.1	$< 1, .4 >$	.4
$\mathcal{I}_2$	$< .2, .1 >$	.1	$< .6, .4 >$	.4
$\mathcal{I}_3$	$< .6, .8 >$	**.6**	$< .8, .7 >$	**.7**
$\mathcal{I}_4$	$< .2, 1 >$	.2	$< .6, 1 >$	.6

Table 4: Two equivalent compatible scalings

Note that in both compatible scalings $\mathcal{R}_1$ and $\mathcal{R}_2$, $\mathcal{I}_3$ is the preferred one. □

Once preferred models are computed, query answering from a set of uncertain ABox under incommensurability assumption, is defined as follows:

Definition 7. Let $\mathcal{A} = \mathcal{A}_1, ..., \mathcal{A}_n$ be a set of ABoxes linked to the same TBox $\mathcal{T}$. A query q is said to be consequence of $\mathcal{A}$ under incommensurability assumption if $\forall \mathcal{I}, \mathcal{I} \in Mod(\Delta^{min}_{\mathcal{T}}(\mathcal{A}^{\mathcal{R}})), \mathcal{I} \models q$.

Example 5 (continued)**.** From Example 4, we have $Mod(\Delta^{min}_{\mathcal{T}}(\mathcal{A}^{\mathcal{R}}))$=$\{\mathcal{I}_3\}$ where $A^{\mathcal{I}_3} = \{a, b\}$, $B^{\mathcal{I}_3} = \{a, b\}$ and $C^{\mathcal{I}_3} = \{\}$. Let $q_1(x) \leftarrow A(x) \wedge B(x)$ be a conjunctive query. One can easily check that $< b >$ is an answer of $q_1(x)$ using $\Delta^{min}_{\mathcal{T}}(\mathcal{A}^{\mathcal{R}})$. Similarly, let $B(a)$ be an instance query, one can check that $B(a)$ follows from $\Delta^{min}_{\mathcal{T}}(\mathcal{A}^{\mathcal{R}})$. □

Using the set of all compatible scales may lead to a very cautious merging operation. One way to get rid of incommensurability assumption is to use some normalization function in the spirit of the ones used in clustering methods for gathering attributes having incommensurable domains. Let $\mathcal{A}_i$ be an ABox and $\alpha_{\mathcal{A}_i}$ be the set of different certainty degrees used in $\mathcal{A}_i$. Let $min_{\mathcal{A}_i}$ and $max_{\mathcal{A}_i}$ be respectively the minimum and maximum certainty degrees associated with assertional facts in $\alpha_{\mathcal{A}_i}$. Then an example of a normalization function is

$$N(\alpha_i) = \frac{\alpha_i - (min_{\mathcal{A}_i} - \epsilon)}{max_{\mathcal{A}_i} - (min_{\mathcal{A}_i} - \epsilon)} \quad (1)$$

Where α_i is a certainty degree belonging to $\alpha_{\mathcal{A}_i}$ and ϵ is a very small number (lower than $min_{\mathcal{A}_i}$).

The main advantage of only having one normalization function is that one can have an immediate syntactic counterpart. More precisely, it is enough to replace for each fact $(f_{ij}, W_{\mathcal{A}_i}(f_{ij}))$ by $(f_{ij}, N(W_{\mathcal{A}_i}(f_{ij})))$ where N is the normalization function given by Equation 1.

Example 6 (continued)**.** From Example 2, we have $\mathcal{A}_1$=$\{(A(a), .6),$ $(C(b), .5)\}$, $\mathcal{A}_2$=$\{(C(a), .4),$ $(B(b), .8),$ $(A(b), .7)\}$. We have $min_{\mathcal{A}_1} = .5$, $min_{\mathcal{A}_2} = .4$, $max_{\mathcal{A}_1} = .6$ and $max_{\mathcal{A}_2} = .8$. Let $\epsilon = .01$, then applying Equation 1 on $\mathcal{A}_1$ and $\mathcal{A}_2$, gives: $\mathcal{A}_1$=$\{(A(a), 1),$ $(C(b), .09)\}$, and $\mathcal{A}_2$=$\{(C(a), 0, 02),$ $(B(b), 1),$ $(A(b), .75)\}$.

Once the syntactic computation of normalized assertional bases is done, it is enough the reuse merging of commensurable possibilistic knowledge bases for query answering.

Example 7 (continued)**.** From Example 6, we have $\Delta^{min}_{\mathcal{T}}(\mathcal{A})$=$\langle\mathcal{T}, \{(A(a), 1), (C(b), .09), (C(a), .02),$ $(B(b), 1), (A(b), .75)\}\rangle$. We have $Inc(\Delta^{min}_{\mathcal{T}}(\mathcal{A}))$=.09 and $\Delta^{min}_{\mathcal{T}}(\mathcal{K})$=$\mathcal{T}, \{(A(a), 1), (B(b), 1), (A(b), .75)\}$.

Consider now $q_1(x) \leftarrow A(x) \wedge B(x)$ and $q_2 \leftarrow B(a)$, queries given in Example 5. One can check that $< b >$ is an answer of $q_1(x)$ from the and $B(a)$ holds from the resulting knowledge bases. □

Conclusions

This paper proposed a min-based possibilistic merging operation of uncertain assertional facts under incommensurability assumption. The idea is to reuse standard min-based merging, over a set of compatible scales. Future work includes developing a syntactic counterpart of incommensurable merging operation. A natural question is whether one can extend a polynomial time complexity algorithm, defined for query answering from a standard uncertain ABox, to the case where uncertainty scales are incommensurable.

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