Dialogue in Hierarchical Learning of a Concept Using Prototypes and Counterexamples

Extended Abstract

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While dealing with vague concepts often it puts us in fix to determine whether to a particular situation/case/state a particular concept applies or not. A human perceiver can determine some cases as the positive instances of the concept, and some as the negative instances of the same; but there always remain cases, which might have some similarities with some positive cases, and also have some similarities with some negative cases of the concept. So we propose to learn about the applicability of a concept to a particular situation using a notion of similarity of the situation with the available prototypes (positive instances) and counterexamples (negative instances) of the concept. Perceiving a vague concept, due to the inherent nature of vagueness, is subjective, and thus never can be exhausted by listing down all the positive and negative instances of the concept. Rather we may come to realize about the applicability, or non-applicability, or applicability to some extent, of a concept to a situation in a step-by-step hierarchical manner by initiating dialogue between a perceiver and the situation descriptor. Hence, the main key ingredients of this proposal are (i) prototypes and counterexamples of a concept, (ii) similarity based arguments in favour and against of applicability of a concept at a particular situation, and (iii) hierarchical learning of the concept through dialogues. Similarity based reasoning [3], hierarchical learning of concepts [1], dialogue in the context of approximation space [2] all are separately important directions of research. For our purpose, in this presentation we would concentrate on combining these aspects from a different angle.

In [4], a preliminary version of logic of prototypes and counterexamples has been set. To make this paper self-contained, we recapitulate the necessary definitions below.

We start with a set S of finitely many situations, say $\{s_1, s_2, \ldots, s_n\}$, and $\mathcal{A}$ of finitely many attributes $\{a_1, a_2, \ldots, a_m\}$. Each s_i, i = 1, 2, $\ldots n$, is considered to be a function $s_i : \mathcal{A} \mapsto [0, 1]$. Let the consolidated data of each situation is stored in the form of a set $\{\langle s_i(a_1), s_i(a_2), \ldots, s_i(a_m)\rangle : s_i \in S\}$, which is a subset of $[0, 1]^m$. Let $W \subseteq [0, 1]^m$ and $\{\langle s_i(a_1), s_i(a_2), \ldots, s_i(a_m)\rangle : s_i \in S\} \subseteq W$. Each member of W may be called a world. We now consider a fuzzy approximation space $\langle W, Sim\rangle$, where Sim is a fuzzy similarity relation between worlds of W. That is, $Sim : W \times W \mapsto [0, 1]$, and we assume Sim to satisfy the following properties.
(i) $Sim(\omega, \omega)$ = 1 (reflexivity)
(ii) $Sim(\omega, \omega')$ = $Sim(\omega', \omega)$ (symmetry)
(iii) $Sim(\omega, \omega') * Sim(\omega', \omega'') \leq Sim(\omega, \omega'')$ (transitivity).

Following [3], the fuzzy approximation space $\langle W, Sim\rangle$ is based on the unit interval $[0,1]$ endowed with a t-norm $*$ and a S-implication operation $\rightarrow$. We now propose to represent any (vague) concept α by a pair (α^+, α^-) consisting of the positive instances (prototypes) and negative instances (counterexamples) of α respectively, where $\alpha^+, \alpha^- \subseteq W$ and $\alpha^+ \cap \alpha^- = \phi$.

Definition 1 [4]. Given the fuzzy approximation space $\langle W, Sim\rangle$, and a concept α represented by (α^+, α^-), the degree to which α applies to a world $\omega \in W$, denoted by $gr(\omega \models \alpha)$, is given by: $gr(\omega \models \alpha) = 1$ if $\omega \in \alpha^+$,
= 0, if $\omega \in \alpha^-$, and
= $\overline{Sim}\alpha^+(\omega) * \neg\overline{Sim}\alpha^-(\omega)$, otherwise.

The fuzzy upper approximations $\overline{Sim}\alpha^+$ and $\overline{Sim}\alpha^-$ are defined following the Definition proposed in [3], i.e., $\overline{Sim}\alpha^+(\omega) = \sup_{u\in W} Sim(\omega, u) * \alpha^+(u) = \sup_{u\in\alpha^+} Sim(\omega, u)$. Similar is the case for $\overline{Sim}\alpha^-$. $\neg$ is considered to be the standard complementation operation defined as $\neg a$ = 1 - a.

Let us call $\overline{Sim}\alpha^+(\omega) = D_{af}(\omega, \alpha)$, the *degree of arguments in favour of ω qualifies α* and $\overline{Sim}\alpha^-(\omega) = D_{ag}(\omega, \alpha)$, the *degree of arguments against ω qualifies α*. So, given a concept α and world $\omega \notin \alpha^+, \alpha^-$, $gr(\omega \models \alpha) = D_{af}(\omega, \alpha) * \neg D_{ag}(\omega, \alpha)$.

Let us now pose the issue of the research in disguise of a practical need. Let we have a clinical record of n number of patients' details with respect to some m number of parameters/attributes. These parameters might be some objective values of some clinical tests, called signs, or some subjective features experienced by the patients, called symptoms. With respect to the state of each patient, the values corresponding to all these parameters are converted, by some mean, to the values over a common scale, say $[0,1]$. That is, if an m-tuple $\langle x_1, x_2, \ldots, x_m\rangle$ from $[0,1]^m$ represents the rates of the m parameters corresponding to a patient, then we say $\langle x_1, x_2, \ldots, x_m\rangle$ describes the state of a patient. Based on the rates assigned to all the parameters by each patient, i.e. a m-tuple of values $\langle x_1, x_2, \ldots, x_m\rangle$ from $[0,1]^m$, which cases representing the states of the patients are how much similar or dissimilar may be anticipated. Now, one task is to make a tentative diagnosis about a patient whose measurement concerning the m-tuple of parameters appears to be new with respect to the database of the n patients. Now with the above set-up, developed in [4], we may compute $gr(\omega \models d)$, the degree of applicability of a disease d for the newly appeared situation, say world ω. The value viz., $gr(\omega \models d)$, for different diseases, may help to make a hypothetical assumption regarding the plausible disease. For being more certain about the diagnosis, it is quite natural to enquire about some more factors/attributes. So the dialogue would have a role to play here. In order to incorporate dialogue in the previous set-up, below we would present the above mentioned theory in a broader framework.

Let us first fix the domain of the (vague) concepts of our concern. Let $\mathcal{A}$ be the set of all attributes (finitely many) required to understand all the concepts over the fixed domain. At time t_0, with respect to a set of attributes $\mathcal{A}_{t_0} \subseteq \mathcal{A}$ we have a set of finitely many situations, say S_{t_0}, at hand such that which situation is characterized by which concept is known to us. That is, given a situation s from S_{t_0}, s is characterized as a positive instance or negative instance of some of the concepts c over the domain of concern. So, we say S_{t_0}, a set of situations, is charaterized by the set of attributes

$\mathcal{A}_{t_0}$ at time t_0. Let $S_{t_0} = \{s_{1t_0}, s_{2t_0}, \ldots s_{nt_0}\}$ and $\mathcal{A}_{t_0} = \{a_1, a_2, \ldots a_m\}$. We say a database at time t_0 with respect to the set of situations S_{t_0}, denoted as $D_{S_{t_0}}$, is the set of all tuples of values for the attributes of $\mathcal{A}_{t_0}$ for each situations of S_{t_0}. That is, for each $a_i(s_{jt_0}) \in [0,1]$, $i \in \{1, 2, \ldots m\}$ and $j \in \{1, 2, \ldots, n\}$, $D_{S_{t_0}} = \{\langle a_1(s_{jt_0}), \ldots a_m(s_{jt_0})\rangle : s_{jt_0} \in S_{t_0}\} \subseteq [0,1]^m$. According to rough set literature $D_{S_{t_0}}$ is basically an information system. We assume that for each database $D_{S_{t_0}}$ there is a database manager, may be called decision maker, $dm_{(S_{t_0})}$.

Definition 2: A dialogue base at some time point t_k is a tuple $(G_1, G_2, \ldots G_r, R_i, R_e)$ such that each $G_j = \langle Ag_j, \mathcal{A}_j, R_i\rangle$ constitutes of a set of agents Ag_j, a set of attributes $\mathcal{A}_j$, and an accessibility relation R_i among the agents. R_i stands for internal accessibility relation among the agents of each group G_j. Each $Ag_j \supseteq S^j_{t_k} \cup \{dm_j\}$ for some time point t_k, where $S^j_{t_k}$ is the a set of situations characterized by $\mathcal{A}_j$. That is each Ag_j contains a set of situations $S^j_{t_k}$ and a database manager dm_j corresponding to $S^j_{t_k}$. For each $s \in S^j_{t_k}$, $R_i(s, dm_j)$ holds, and R_i is symmetric. The relation R_e is a reflexive, symmetric relation and it stands for external accessibility relation between different database managers. That is, for some j, l, $R_e(dm_j, dm_l)$ holds.

Intuitively each G_j of the dialogue base contains a set of nodes and relation R_i among the nodes. These nodes constitute the set Ag_j. Some of the nodes represent those situations which are characterized by $\mathcal{A}_j$, the set of attributes of G_j. dm_j is a node designated as database manager. $S^j_{t_k}$, the set of situations characterized by $\mathcal{A}_j$, generates a database $D_{S^j_{t_k}}$ of tuple of values for each attribute of $\mathcal{A}_j$ corresponding to each situation of $S^j_{t_k}$. $D_{S^j_{t_k}} \subseteq W_j$, and hence is embedded in the approximation space $(W_j, Sim_{\mathcal{A}_j})$. In each G_j the dm_j has access to the other nodes. Through dialogue it is expected that dm_j would enquire a particular situation (i.e. node) for information, and the particular situation would provide the information corresponding to the query. So, R_i has to be symmetric as both database manager and the situation descriptor should have access to make the communication. The external accessibility relation R_i allows accessing two database managers. A database manager can access her own information, and if dm_j can access dm_l, then the reverse also holds. So, R_e is reflexive and symmetric.

Summarizing the whole, we can say that each Ag_j of G_j is a set of nodes some of which are specific cases, already characterized by the set of attributes $\mathcal{A}_j$ of G_j, at some time point. dm_j can be considered as a dummy node which can access any other node. The rest of nodes can be any new case/situation appearing at some further point of time. That is why $Ag_j \supseteq S^j_{t_k} \cup \{dm_j\}$. On the other hand, the detailed information about the set of situations $S^j_{t_k}$, which are characterized by $\mathcal{A}_j$, are available in the corresponding database (or information system) $D_{S^j_{t_k}}$. The information system or database $D_{S^j_{t_k}}$ is also open to handle new information corresponding to new cases of Ag_j. That is why, $D_{S^j_{t_k}} \subseteq W_j$, where $(W_j, Sim_{\mathcal{A}_j})$ is a fuzzy approximation space based on the attributes $\mathcal{A}_j$. The following picture is a model of what we are trying to formalize through the notion of dialogue base and approximation spaces containing different databases characterized by different sets of attributes.

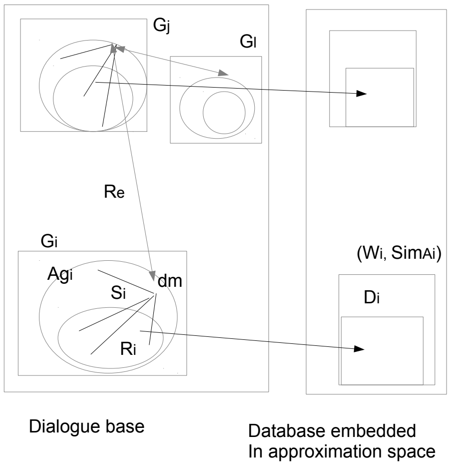

Fig. 1. Internal and external dialogues among the granules of a dialogue base and corresponding outcomes generated in the respective approximation spaces

Now given the prelude of the practical need, what do we expect from a dialogue? A dialogue at the first time point t_0, denoted as $diag_{t_0}$, would consists of two rounds r_1 and r_2. At round r_1 the database manager may ask the situation s to provide the values for a set of attributes $\langle a_1, \ldots a_m \rangle$. At the round r_2 the situation descriptor answers the query with the tuple $\langle a_1 = v_1, \ldots, a_m = v_m \rangle$, or in other words simply a tuple of values from $[0,1]^m$. So, as an output of a complete dialogue at time t_0 between a situation s and the corresponding database manager $dm^i_{t_0}$ we expect to receive a tuple of values from $[0,1]^m$. So, combining the both round we may write that output of a dialogue at time t_0 is given by, $diag_{t_0}(s, dm^i_{t_0}) = \omega \in W \subseteq [0,1]^m$. Though we are going to combine the two rounds in a single complete dialogue, there is a difference in the nature of the two rounds. The dialogue in r_1 throws a question, and the dialogue in r_2 provides an answer. So, the dialogue somehow moves the communication from the first agent's approximation space to the second agent's approximation space. So, the definition of dialogue is proposed as follows.

Definition 3: Given a dialogue base $(G_1, G_2, \ldots G_r, R_i, R_e)$ at time t_0, a dialogue between two agents ag_1, ag_2, denoted as $diag_{t_0}(ag_1, ag_2)$, is defined as follows. (i) $diag_{t_0}(ag_1, ag_2) = \omega \in W_i$ of the approximation space $(W_i, Sim_{\mathcal{A}t_i})$, if $R_i(ag_1, ag_2)$ holds for $ag_1, ag_2 \in Ag_i$ of the group $(Ag_i, \mathcal{A}t_i, R_i)$, and $W_i \supseteq D_{S^i_{t_0}}$ for

$S^i_{t_0} \subseteq Ag_i$.
(ii) $diag_{t_0}(ag_1, ag_2) = \omega \in W'$ of $(W', Sim_{At'})$, if $R_e(ag_1, ag_2)$ holds for $ag_1 \in Ag_i$ and $ag_2 \in Ag_j$, where $At' = \mathcal{A}t_i \cap \mathcal{A}t_j$, and $W' = W_j|_{At'}$ where $W_j \supseteq D_{S^j_{t_0}}$ for $S^j_{t_0} \subseteq Ag_j$.
$W_j|_{At'}$ denotes the restriction of W_j to the attribute set At'.

Now we cast our problem of hierarchical learning of a concept in the framework of dialogue. The idea is to start at time point t_0 with a set of situations S_{t_0} characterized by a set of attributes $\mathcal{A}_{t_0}$. The information corresponding to S_{t_0} would be available in the database $D_{S_{t_0}}$. The situations S_{t_0} corresponding to the attributes $\mathcal{A}_{t_0}$ is a part of a granule G_{t_0} of a dialogue base. Now given a new situation s first, the corresponding database manager dm_{t_0} of G_{t_0} would initiate a dialogue with the situation s. As an outcome of $diag_{t_0}(s, dm_{t_0})$ there will be a tuple of values for each attributes of $\mathcal{A}_{t_0}$. This tuple of values represents a world, say ω in W_{t_0}, the universe of the approximation space $(W_{t_0}, Sim_{\mathcal{A}_{t_0}})$, in which $D_{S_{t_0}}$ is already embedded. Now with respect to $(W_{t_0}, Sim_{\mathcal{A}_{t_0}})$, one can compute $gr(\omega \models_e c)$, the degree of applicability of a concept c to the world ω. In order to be more certain in the decision, the database manager at the next point of time $t_1 (> t_0)$ may initiate another dialogue with s asking for values for some additional attributes. In that situation the dialogue would proceed from the old approximation space to a new approximation space with respect to a bigger set of attributes.

Process of hierarchical learning of a concept

Step 1 We fix a set of attributes $\mathcal{A}$ (finitely many) for a fixed domain $\mathcal{C}$ of finitely many concepts, and consider all possible subsets of $\mathcal{A}$. We assume that for each possible $\mathcal{A}_1 \subseteq \mathcal{A}$ there is a set of situations S_1 characterized by $\mathcal{A}_1$ in the sense that each $s \in S_1$ is characterized as either a positive or a negative instance of some of the concepts of $\mathcal{C}$.

Step 2 For each set of situations S_i characterized by $\mathcal{A}_i$ there is a database D_{S_i} consisting of tuple of values for each attribute of $\mathcal{A}_i$ corresponding to each situation of S_i. We consider $W_i \supseteq D_{S_i}$ where $W_i \subseteq [0,1]^{|\mathcal{A}_i|}$. For each database D_{S_i} we assume the presence of a database manager dm_i.

Step 3 Now we start with a dialogue base $(G_{1t_0}, G_{2t_0}, \ldots G_{rt_0}, R_i, R_e)$ at time t_0. For each i = 1, 2, $\ldots r$, $G_{it_0} = \langle Ag_{it_0}, \mathcal{A}_{it_0}, R_i \rangle$ and $Ag_{it_0} \supseteq S_{it_0} \cup \{dm_{it_0}\}$, where S_{it_0} is the set of situations characterized by $\mathcal{A}_{it_0}$. Each S_{it_0} is embedded in an approximation space $(W_{it_0}, Sim_{\mathcal{A}_{it_0}})$ through its database $D_{S_{it_0}}$. To mark the time point t_0 corresponding to each component of a dialogue base we have used suffixes like it_0. But every S_{it_0} must coincide with some S_j of situations, about which we have discussed at Step 2, as either of the groups (G_{it_0}, $1 \leq i \leq r$) of the dialogue base constitutes of a set of agents and a set of attributes taken from $\mathcal{A}$.

Step 4 At time t_0, given a situation $s \in Ag_{it_0}$ - $\{dm_{it_0}\}$ a dialogue is initiated as $diag_{t_0}(s, dm_{it_0})$. The output of the dialogue would provide a tuple of values from W_{it_0}. Let us assume that $diag_{t_0}(s, dm_{it_0}) = \omega \in W_{it_0}$.

Step 5 Now in the fuzzy approximation space $(W_{it_0}, Sim_{\mathcal{A}_{it_0}})$, based on the development made in [4], we can compute $gr(\omega \models_e c)$ (see Definition 1) for a concept belonging to $\mathcal{C}$. Based on some (significantly high) value $gr(\omega \models_e c)$ for some c, we can make a hypothesis for the 'applicability of the concept c at the world ω (= $s(t_0)$)', the world assigned by situation s at time t_0.

Step 6 To be more certain regarding the decision, at time $t_1(> t_0)$ the second dialogue may be initiated as $diag_{t_1}(s, dm_{it_1})$, where dm_{it_1} is the same database manager dm_{it_0} at the next time point t_1, as the old dialogue base changes to $(G_{1t_1}, G_{2t_1}, \ldots G_{rt_1}, R_i, R_e)$ considering the new time point. The new G_{it_1} contains all the agents of G_{it_0} and preserves the same relation R_i among the agents of G_{it_0}. It differs in the set of attributes $\mathcal{A}_{it_1}$ where $\mathcal{A}_{it_1} \supseteq \mathcal{A}_{it_0}$. As, $\mathcal{A}_{t_0} \subseteq \mathcal{A}_{t_1}$, $S_{it_0} \subseteq S_{it_1}$. We take $G_{it_1} = G_{it_0} \cup S_{it_1}$, where S_{it_1} is the set of situations characterized by $\mathcal{A}_{t_1}$. So, $diag_{t_1}(s, dm_{it_1})$ would now provide a new world ω' from the approximation space $(W_{it_1}, Sim_{\mathcal{A}_{it_1}})$. W_{it_0} is embedded in W_{it_1} in the sense that $W_{it_1} \subseteq W_{it_0} \times [0,1]^{|\mathcal{A}_{t_1}|-|\mathcal{A}_{t_0}|}$, and for each $u = \langle u_1, \ldots u_{|\mathcal{A}_{t_0}|}\rangle \in W_{it_0}$, there is $u' \in W_{it_1}$ such that $u' = \langle u_1, \ldots u_{|\mathcal{A}_{t_0}|,0,0\ldots,0}\rangle$ having entry '0' for the rest of the $|\mathcal{A}_{t_1}| - |\mathcal{A}_{t_0}|$ components. When a dialogue at time t_i moves to a new approximation space from its previous approximation space at time t_{i-1} we call the dialogue proceeds.

Step 7 As all the subsets of the whole attribute set $\mathcal{A}$ is considered, the set of attributes $\mathcal{A}_{t_1} \subseteq \mathcal{A}$ must merge with some $\mathcal{A}_j$ considered in the begining. So, there is already a set of situations and corresponding database embedded in the approximation space $(W_{it_1}, Sim_{\mathcal{A}_{it_1}}) = (W_j, Sim_{\mathcal{A}_j})$, and the dialogue at time t_1 moves into the new approximation space with respect to $\mathcal{A}_j$. So, with respect to the approximation space $(W_j, \mathcal{A}_j)$ we can compute $gr(\omega' \models_e c)$.

Step 8 Now, if $gr(\omega \models_e c) < gr(\omega' \models c)$ where $s(t_0) = \omega$ and $s(t_1) = \omega'$, then we may consider the situation s to be ascribed as an instance of c.

Step 8 describes just a simple case for including a situation in the prototypes of a concept. In practice, the constraints for including a situation as a positive instance of a concept c, or as a negative instance of a concept c may have several layers of dialogues. In this presentation we would explore that idea more specifically. It is also to be noted that for each respective case of W, where $W \subseteq [0,1]^l$ for any finite natural number l, we assume the presence of a binary fuzzy similarity relation.

Below we present a simple application of the present proposal considering the same example taken in [4].

Example: Let we have a clinical database of a set of situations, $S = \{s_1, s_2, s_3, \ldots, s_9\}$ with respect to a set $\mathcal{A}$ of attributes, consisting of *temperature, blood-pressure, blood-tests, ecg, headache, sneezing, convulsion, vomiting, skin-rash, dizziness, stomach-upset, stomach pain*. Sequentially let us call these attributes as $a_1, a_2, a_3, \ldots, a_{12}$. As $a_1, \ldots, a_4$ are determined by some objective values of some tests these are called signs; the rest are symptoms, determined by some subjective values as experienced by particular patients. Let $C_B = \{Fev, Allergy, Stomach_{inf}, HBP, LBP, Vertigo, Unconsciousness\}$, and C is the union of C_B and $\{Fev_c, Fev_v, Stroke, Food\text{-}poisoning, Viral_{inf}, Peptic\text{-}ulcer\}$. The relations among the dependent and the independent concepts of C are as follows.

$Fev \subseteq_k Fev_c$; Fev, $Allergy \subseteq_k Fev_v$; Fev, HBP, $Vertigo$, $Unconsciousness \subseteq_k Stroke$; Fev, $Stomach_{inf} \subseteq_k$ *Food-poisoning*; Fev, $Stomach_{inf}$, $Allergy \subseteq_k Viral_{inf}$; and $Stomach_{inf} \subseteq_k$ *Peptic-ulcer*. Fev_c and Fev_v respectively stand for *fever due to cold* and *viral fever*. HBP, LBP, $Stomach_{inf}$, and $Viral_{inf}$ respectively stand for *high blood pressure*, *low blood pressure*, *stomach infection* and *viral infection*. Each s_i is identified with its state given by $\langle s_i(a_1), s_i(a_2), s_i(a_3), \ldots, s_i(a_{12})\rangle \in [0,1]^{12}$, and $W(\subseteq [0,1]^{12})$ contains $\langle s_i(a_1), s_i(a_2), s_i(a_3), \ldots, s_i(a_{12})\rangle$ for each s_i. The tuple of values corresponding to each s_i, and the positive and negative cases of each disease from C_B are given in the following table. Let us use d_1 for Fev, d_2 for $Allergy$, d_3 for $Stomach_{inf}$, d_4 for HBP, d_5 for LBP, d_6 for $Vertigo$, and d_7 for $Unconsciousness$. For each s_i if d_j receives $+$, then s_i is the positive case for d_j, and if it receives $-$, then s_i is the negative case of d_j. So, following the definitions proposed in [4] we can easily calculate $gr(s_i \models_e c)$ for any $s_i \in S$ and $c \in C$.

Table 1. Patients data table

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	d_1	d_2	d_3	d_4	d_5	d_6	d_7
s_1	.8	.3	.5	0	.7	.7	0	0	0	0	0	0	+		-	-		-	-
s_2	.5	.8	.7	.7	.7	0	.7	0	.3	.9	0	0			-	+	-	+	+
s_3	.9	.5	.7	0	.7	.8	0	0	0	0	0	0	+		-	-		-	-
s_4	.4	.3	.6	0	0	0	0	.5	0	0	.7	.6	-	-	+	-		-	-
s_5	.7	.1	.7	0	.3	0	0	.7	0	0	.9	.7		-	+	-		-	-
s_6	.7	.2	.8	0	.3	0	0	.5	.7	0	.7	.7		+	+	-		-	-
s_7	.9	.5	.8	0	.8	.8	0	0	.3	.2	0	0	+		-	-		-	-
s_8	.3	.1	.3	.8	.7	0	.7	0	0	.8	0	0	-	-	-	-	+	+	+
s_9	.6	.7	.7	.5	.5	0	0	.9	0	.2	.9	1		-	+		-	-	

Let a new situation s_{10}, with the tuple $\langle .5, .5, .5, .5, .7, 0, .8, .5, .7, .8, .5, 0\rangle$ of values corresponding to the respective attributes, appear. The task is to make a diagnosis for s_{10}. Now we pose the above problem in the framework of the present proposal in the following way.

Let us start at time t_0 with the dialogue base (G_{t_0}, R_e, R_i). For simplicity we have considered only one group of agents having internal relation R_i and external relation R_e. Let $S_{t_0} = S$, and at time t_0 the set of situations S_{t_0} is charaterized by $\mathcal{A} = \mathcal{A}_{t_0}$. The database at time t_0, denoted as $D_{S_{t_0}}$, is basically the set $\{\langle s_i(a_1), s_i(a_2), \ldots, s_i(a_{12})\rangle : s_i \in S_{t_0}\}$. Now $G_{t_0} \supseteq S_{t_0} \cup dm_{t_0}$, where dm_{t_0} is a dummy agent representing the database manager for $D_{S_{t_0}}$, and $D_{S_{t_0}} \subseteq W_{t_0} \subseteq [0,1]^{12}$. For each $s_i \in S_{t_0}$, $R_i(dm_{t_0}, s_i)$ holds, and R_i is symmetric. Now in appearance of a new situation s_{10}, the outcome of the dialogue between dm_{t_0} and s_{10} at the round

r_1 is $diag^{r_1}_{t_0}(dm_{t_0}, s_{10}) = \langle a_1, a_2, \ldots a_{12}\rangle$, and that of at the second round of the dialogue is $diag^{r_2}_{t_0}(s_{10}, dm_{t_0}) = \langle .5, .5, .5, .5, .7, 0, .8, .5, .7, .8, .5, 0\rangle$. Combining both the rounds we write $diag_{t_0}(s_{10}, dm_{t_0}) = \langle .5, .5, .5, .5, .7, 0, .8, .5, .7, .8, .5, 0\rangle = w$ (say). Now based on the proposal presented in [4], with respect to the fuzzy approximation space $(W_{t_0}, Sim_{A_{t_0}})$ one can calculate $gr(w \models_e c)$ for some $c \in C$. Let us denote the degree to which s_{10} qualifies c at time t_0 as $gr(w \models^{t_0}_e c)$. Let for $c = Stroke$, $gr(w \models^{t_0}_e c) = \frac{1}{2}$. In order to be more certain the decision maker may need to ask the patient for some more tests. Let the new test, i.e., the attribute a_{13} is $MRI\text{-}scan$ (Magnetic Resonance Imaging). So, the dialogue base at the next time point t_1 moves to (G_{t_1}, R_e, R_i) where $G_{t_1} = G_{t_0} \cup S_{t_1} \supseteq S_{t_1} \cup \{dm_{t_1}\}$, dm_{t_1} is the same as dm_{t_0} at the next point of time, and S_{t_1} is the set of situations characterized by $\mathcal{A}_{t_1} = \mathcal{A}_{t_0} \cup \{a_{13}\}$. According to the definition of a dialogue base $R_i(dm_{t_1}, s_i)$ holds for each $s_i \in S_{t_1}$. Now the new dialogue at time t_1 would be $diag^{r_1}_{t_1}(dm_{t_1}, s_{10}) = \langle a_1, a_2, \ldots a_{12}, a_{13}\rangle$, and let $diag^{r_2}_{t_1}(dm_{t_1}, s_{10}) = \langle s_{10}(a_1), s_{10}(a_2), \ldots s_{10}(a_{12}), s_{10}(a_{13})\rangle = w' \in W_{t_1}$, where $W_{t_1} \subseteq W_{t_0} \times [0, 1]$ such that for each $w = \langle x_1, x_2, \ldots x_{12}\rangle \in W_{t_0}$ there is $\langle x_1, x_2, \ldots x_{12}, 0\rangle \in W_{t_1}$. So, now at time t_1 with respect to the fuzzy approximation space $(W_{t_1}, Sim_{\mathcal{A}_{t_1}})$ one can calculate $gr(w' \models^{t_1}_e c)$, and based on $gr(w' \models^{t_1}_e c) \geq gr(w \models^{t_0}_e c)$ or $gr(w' \models^{t_1}_e c) \leq gr(w \models^{t_0}_e c)$ a decision regarding considering s_{10} as a positive or negative case of $Stroke$ may be taken through a hierarchical manner of learning.

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