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DRAFT 1

Parametrization of the Muon Response in the Tile Calorimeter

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Abstract

The TILECAL response to muons has been described by the convolution of Landau and Gaussian distribution. This function describes the muon signal distribution better than the Moyal function, in particular it correctly determines the most probable signal.

The software providing the mentioned fitting procedure has been worked out and a short description of its usage is given as well.

1 Landau Distribution

Let us consider a minimum ionizing particle (mip) passing through a thin block of matter. Its energy loss distribution obeys the Landau probability density function (see [1])

$$\mathcal{L}(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} exp(s\ln s + xs)\,\mathrm{d}s, \qquad c > 0 \tag{1}$$

The integral (1) is c-independent and describes the special Landau distribution with the maximum at $x_0 = -0.22278$ and a certain width [1]. Therefore, a general Landau distribution L depends on 3 parameters

$$L(x|p_1, p_2, p_3) = p_1 \times \mathcal{L}\Big(\frac{x - p_2}{0.5860 \times p_3}\Big) \tag{2}$$

where p_1 is a normalization factor. The parameters p_2 and p_3 are connected with the most probable value[1] (MOP) and the full width at half maximum (FWHM) of the distribution by the relations

$$\mathrm{MOP}(L) = p_2 - 0.13054 \times p_3 \tag{3a}$$

$$\mathrm{FWHM}(L) = 2\sqrt{2\ln 2} \times p_3 = 2.355 \times p_3 \tag{3b}$$

[1]The most probable value of a distribution represents the peak position of the respective probability density function.

The multiplication factor 0.5860 has been chosen in the formula (2) in order to the Landau FWHM dependence (3b) fulfil the same relation as a Gaussian.

The formula (1) cannot be rewritten into an analytic form, but it can be piecewise approximated by exponential and rational functions. The comparison of the probability density function (1) and the analytic approximation is shown in Fig. 1.

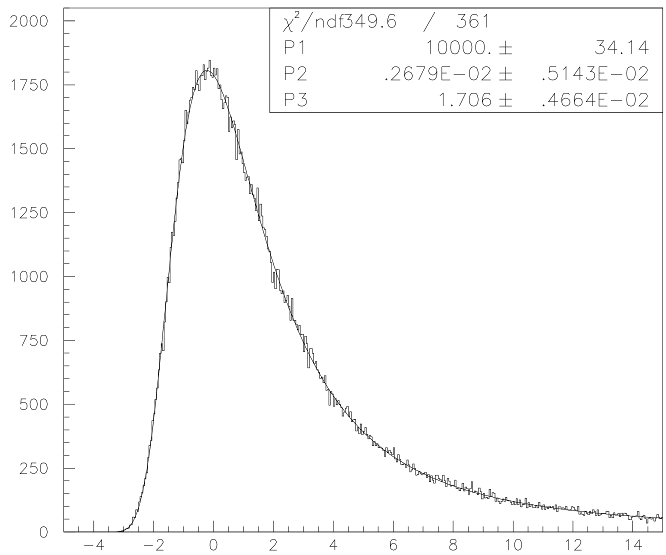

Figure 1: The Landau distribution generated according to the function (1) using the standard CERNLIB subroutines [2]. The analytic approximation of the general Landau distribution (2) *xandis.for* fits the distribution.

2 Moyal Function

The Moyal function represents a simple analytic approximation of the detector response to a mip passage. The respective probability density function reads (see [3])

$$\mathcal{M}(\lambda) = \frac{1}{\sqrt{2\pi}} exp\Big(-\frac{\lambda + exp(-\lambda)}{2}\Big) \quad (4a)$$

$$\lambda = R\,(x - x_p) \quad (4b)$$

where x_p means the most probable value and R is a constant depending on the absorber [4]. We used this distribution in the general 3-parametric form

$$M(\lambda) = p_1 \times exp\Big(-\frac{\lambda + exp(-\lambda)}{2}\Big) \quad (5a)$$

$$\lambda = 2.22 \times \frac{x - p_2}{p_3} \quad (5b)$$

The last parameter p_3 is connected with the respective FWHM by the relation

$$\text{FWHM}(M) = 1.6175 \times p_3 \tag{6}$$

3 The Detector Response

In the scintillator (the Tile calorimeter active medium), the deposited energy is converted to the light signal. The light originated in the scintillator is absorbed in the wavelength shifting fibre, re-emitted at a larger wavelength and conducted to the photomultiplier. Finally, the photons reaching the photocathode interact with its material and liberate the electrons. Moreover, the final signal is influenced by the pedestal spread. All these effects are of statistical nature, therefore the original energy loss distribution will be smeared by a Gaussian due to the above fluctuations. These considerations result in the convolution formula

$$S_i = L_i \otimes G_i \tag{7}$$

where S_i represents the signal measured by the i-th scintillator after a mip passage and G_i stands for a Gaussian. The resulting function (7) depends on four parameters :

$$S_i(x|p_1, p_2, p_3, p_4) = p_1 \times \int_{-\infty}^{\infty} \mathcal{L}\Big(\frac{y - p_2}{0.5860 \times p_3}\Big)\, exp\Big(-\frac{(x-y)^2}{2p_4^2}\Big)\mathrm{d}y \tag{8}$$

The first three parameters correspond to the Landau distribution and the last one describes the Gaussian smearing.

For the muons passing through a material, the ionization losses dominate over a large range of incident energies. Therefore, a high energy muon deposits the main part of its energy in the detector as it were a mip.

The total Tile calorimeter response to a muon is given by the sum of signals coming from the scintillators along the particle's track and is proportional to the total track length in the active medium. Therefore, the total signal will be also smeared due to the sampling fraction fluctuations, which are caused by the particle entrance angle and impact point fluctuations. Taking into account the formula (7) for each scintillator, the total response S obeys the formula (8) as well, but with different parameter values. A more detailed description of the function properties is given in Appendix A.

3.1 Comparison of the Moyal and Convolution Fits

The comparison of the Moyal (5a) and convolution (8) fits is shown in the Fig. 2. While the Moyal function fails to determine the correct MOP, the convolution function results in a good MOP prediction. The respective χ^2/ndf improves as well. The results of the mentioned fits applied on the experimental data for several pseudorapidities are listed in Tab. 1.

If one applies the same procedure to the signal distribution of separate module 0 depths, the mentioned behaviour of both functions is still the same.

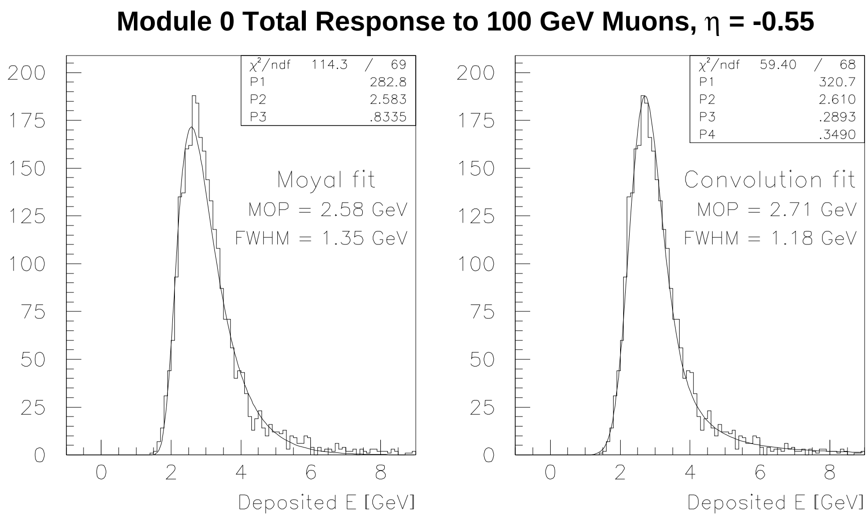

Figure 2: The comparison of the Moyal (5a) and convolution (8) fits to the experimental data – module 0 response to 100 GeV muons entering the calorimeter at the pseudorapidity $\eta = -0.55$.

Table 1: The results of the Moyal and convolution fits applied on the total TILECAL module 0 response to 100 GeV muons entering the calorimeter at different pseudorapidities η. Both the MOP and FWHM are given in GeV, the listed errors are those obtained from the fit (Moyal), resp. calculated according to the formula (12). The columns χ^2_n denote the respective χ^2/ndf.

$-\eta$	Moyal			Convolution		
	MOP	FWHM	χ^2_n	MOP	FWHM	χ^2_n
0.15	2.13 ± 0.02	1.43 ± 0.05	1.1	2.29 ± 0.04	1.32 ± 0.05	0.8
0.25	2.40 ± 0.02	1.42 ± 0.05	1.1	2.57 ± 0.05	1.33 ± 0.06	1.1
0.35	2.35 ± 0.02	1.35 ± 0.05	1.0	2.49 ± 0.04	1.23 ± 0.06	0.6
0.45	2.35 ± 0.01	1.22 ± 0.03	1.7	2.48 ± 0.03	1.15 ± 0.04	1.2
0.55	2.58 ± 0.01	1.35 ± 0.03	1.7	2.71 ± 0.02	1.18 ± 0.02	0.9
0.65	2.66 ± 0.03	1.73 ± 0.06	1.4	2.88 ± 0.04	1.64 ± 0.06	0.8
$0.75^\dagger$	1.96 ± 0.02	1.16 ± 0.04	1.2	2.11 ± 0.03	1.09 ± 0.04	0.7

† The MOP fall-off at $\eta = -0.75$ is due to geometry effect. The muons escape the barrel calorimeter module on its side after passing the first two longitudinal samples.

3.2 The Fit Parameters Properties

For the total TILECAL response, the Landau width parameter p_3 is approximately constant over all pseudorapidities. This is illustrated on Fig. 3 – when keeping the p_3 constant, the fit gives the same results as in the case of freely varying p_3.

The parameter p_4 (Gaussian sigma) slightly decreases with the increasing muon track length in the calorimeter.[2] This feature reflects the fact that for larger muon track lengths the signal fluctuations become smaller.

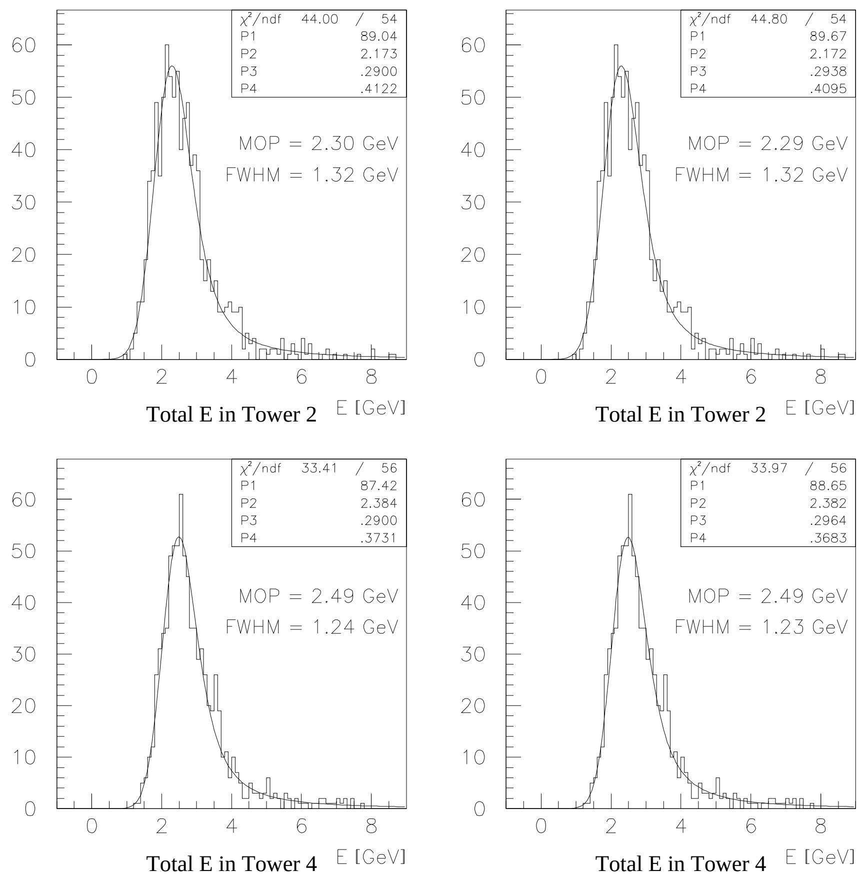

Figure 3: The convolution function fits with the fixed parameter $p_3 = 0.29$ GeV (left column) and with p_3 freely varying. The mentioned parameter is approximately constant over the whole pseudorapidity range for the TILECAL module 0 total response to muons.

[2] Sigma decreases with increasing value $|\eta|$ in the range $|\eta| \lesssim 0.7$. For higher values $|\eta|$, the muon track length in the TILECAL barrel part decreases (geometry effect).

4 Conclusion

The muon signal in TILECAL is rather poorly approximated with the Moyal function. It predicts the MOP value to be systematically lower than the convolution fit does.

The convolution function fits the experimental muon data better than the Moyal function over the whole range of studied pseudorapidities η (-0.15, -0.25, -0.35, -0.45, -0.55, -0.65, -0.75).

A short description on how to fit a histogram with the convolution function is given in Appendix B.

5 Acknowledgements

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References

[1] K. S. Koelbig, B. Schorr : *A Program Package for the Landau Distribution*, CERN DD-83-18, 1983

[2] *CERNLIB manual*, G110, CERN Program Library, 1993

[3] J. E. Moyal : *Theory of Ionization Fluctuations*, Phil.Mag. 46 (1955) 263

[4] R. K. Bock et al.: *Formulae and Methods in Experimental Data Evaluation with Emphasis on High Energy Physics*, Vol. 1, European Physical Society, 1984

A The Convolution Function Attributs

Although the general Landau distribution (2) is approximated by the analytic function *xandis.for*, the integral (8) has not analytic solution and therefore it is calculated numerically.

After the fit procedure, the goal is to predict the most probable value (MOP) of the muon response and its FWHM with the respective errors. Both the MOP and FWHM are p_1-independent. While the FWHM does not depend on the Landau peak parameter p_2, the MOP obeys the relation

$$\mathrm{MOP}(p_1, p_2, p_3, p_4) = \mathrm{MOP}(p_1, 0, p_3, p_4) + p_2 \tag{9}$$

The remaining dependences on the parameters p_3 and p_4 cannot be expressed analytically. As a consequence, the MOP has to be found numerically as the maximum of the fitted convolution function (8). The FWHM is then calculated as the difference

$$\mathrm{FWHM} = x_2 - x_1 \tag{10}$$

where x_1 , x_2 obey the relations (S is the convolution function (8) and x_0 stands for its MOP)

$$S(x_i|p_1, p_2, p_3, p_4) \quad = \quad \frac{1}{2} \times S(x_0|p_1, p_2, p_3, p_4) \ , \quad i = 1, 2 \tag{11a}$$

$$x_1 < \quad x_0 \quad < x_2 \tag{11b}$$

The equation (11a) is solved numerically. The parameters $\{p_i\}_{i=1}^4$ are those obtained from the fit.

In order to calculate the errors of MOP and FWHM properly, one has to take into account not only the parameter errors $\{\sigma_{p_i}\}_{i=1}^4$, but also the correlations between the mentioned parameters. In general, the MOP error reads

$$\sigma^2_{\mathrm{MOP}} = \sum_{i=1}^{4} \Big(\frac{\partial\,(\mathrm{MOP})}{\partial p_i} \, \sigma_{p_i} \Big)^2 + 2 \sum_{i<j}^{4} \frac{\partial\,(\mathrm{MOP})}{\partial p_i} \, \frac{\partial\,(\mathrm{MOP})}{\partial p_j} \, Cov(p_i, p_j) \tag{12}$$

where $Cov(p_i, p_j)$ represents the respective element of the covariance matrix. This general formula can be simplified by taking into account the relation (9) and the MOP independence on the parameter p_1. The error of the FWHM value is obtained in the same way.

B How to Perform the Convolution Function Fit

Several file have been prepared in order to provide the software for the convolution fit performance. They are located in the AFS directory

/afs/cern.ch/user/d/davidek/public/muonfit

One needs to copy these files :

- *glfit.kumac* – the main macro file

- *gslan1.f* – the convolution function (8)
- *glerror.f* – the Fortran source file with subroutines calculating the MOP and FWHM with the respective errors
- *gslan1.sl*, *glerror.sl* – the shared library files related to the mentioned Fortran sources. These files have been compiled on the HP-UX operating system. See the WWW document

  http://wwwcn.cern.ch/asdcgi/listpawnews.pl/paw.news20520#comis

  for instruction on how to compile *gslan1.f* and *glerror.f* on other systems.

To perform the fit, enter the following command in the PAW session

$$\text{exec glfit } n$$

where n represents the respective histogram ID. By default, the real vector $par(4)$ is created with appropriate input values. However, one can specify his own vector as the next parameter in the above command, which enables the fit procedure to start with other input parameter values. Although the fit procedure is stable, it might take some time (typically about 10 sec). Starting the fit with the parameter values very close to the resulting ones speeds up the procedure.

After that the fit is performed, the most probable value of the muon signal and the FWHM with their errors are calculated and displayed on the terminal.