1 Plenary talks

A PROOF OF THE ERDŐS-FABER-LOVÁSZ CONJECTURE

Deryk Osthus

University of Birmingham

(This talk is based on joint work with Dong-Yeap Kang, Tom Kelly, Daniela Kühn and Abhishek Methuku.)

MSC2000: 05D40,05C65

In 1972, Erdős, Faber, and Lovász conjectured the following equivalent statements. Let $n \in \mathbb{N}$.

(i) If $A_1, \ldots, A_n$ are sets of size n such that every pair of them shares at most one element, then the elements of $\bigcup_{i=1}^{n} A_i$ can be coloured by n colours so that all colours appear in each A_i.

(ii) If G is a graph that is the union of n cliques, each having at most n vertices, such that every pair of cliques shares at most one vertex, then the chromatic number of G is at most n.

(iii) If $\mathcal{H}$ is a linear hypergraph with n vertices, then the chromatic index of $\mathcal{H}$ is at most n.

Here the *chromatic index* $\chi'(\mathcal{H})$ of a hypergraph $\mathcal{H}$ is the smallest number of colours needed to colour the edges of $\mathcal{H}$ so that any two edges that share a vertex have different colours and a hypergraph is *linear* if two hyperedges share at most one vertex. Erdős considered this to be 'one of his three most favorite combinatorial problems'. The simplicity and elegance of its formulation initially led the authors to believe it to be easily solved. However, as the difficulty became apparent Erdős offered successively increasing rewards for a proof of the conjecture, which eventually reached $500.

We prove the Erdős-Faber-Lovász conjecture for every large n:

Theorem 1. *[1] For every sufficiently large n, every linear hypergraph $\mathcal{H}$ on n vertices has chromatic index at most n.*

In my talk, I will survey some background, related results and open problems. I will also discuss some of the ideas involved in the proof.

[1] Dong-Yeap Kang, Tom Kelly, Daniela Kühn, Abhishek Methuku and Deryk Osthus, A proof of the Erdős-Faber-Lovász conjecture, arxiv:2101.04698.

Monday 13:30, Zoom 1

Decomposing the edges of a graph into simpler structures

Marthe Bonamy

Université de Bordeaux

MSC2000: 05C15

We will review various ways to decompose the edges of a graph into few simple substructures. We will mainly focus on variants of edge colouring, and discuss specifically the discharging method and re-colouring techniques.

Codes and designs in Johnson graphs

Cheryl E. Praeger

The University of Western Australia

(This talk is based on joint work with R. A. Liebler, M. Neunhoeffer, and more recently J. Bamberg, A. C. Devillers and M. Ioppolo.)

MSC2000: 05C25, 20B25, 94B60

The Johnson graph $J(v,k)$ has, as vertices, all k-subsets of a v-set $\mathcal{V}$, with two k-subsets adjacent if and only if they share $k-1$ common elements of $\mathcal{V}$. Subsets of vertices of $J(v,k)$ can be interpreted as the block-set of an incidence structure, or as the set of codewords of a code, and automorphisms of $J(v,k)$ leaving the subset invariant are then automorphisms of the corresponding incidence structure or code.

This approach leads to interesting new designs and codes. For example, numerous actions of the Mathieu sporadic simple groups give rise to examples of Delandtsheer designs (which are both flag-transitive and anti-flag transitive), and codes with large minimum distance (and hence strong error-correcting properties). In my talk I will explore links between designs and codes in Johnson graphs which have a high degree of symmetry, and I will mention several open questions.

The partition complex: an invitation to combinatorial commutative algebra

Karim Adiprasito

Hebrew University of Jerusalem

(This talk is based on joint work with Geva Yashfe.)

MSC2000: 05E40

We provide a new foundation for combinatorial commutative algebra and Stanley-Reisner theory using the partition complex introduced in [1]. One of the main advantages is that it is entirely self-contained, using only a minimal knowledge of algebra and topology. On the other hand, we also develop new techniques and results using this approach. In particular, we provide

1. A novel, self-contained method of establishing Reisner's theorem and Schenzel's formula for Buchsbaum complexes.
2. A simple new way to establish Poincaré duality for face rings of manifolds, in much greater generality and precision than previous treatments.
3. A "master-theorem" to generalize several previous results concerning the Lefschetz theorem on subdivisions.
4. Proof for a conjecture of Kühnel concerning triangulated manifolds with boundary.

[1] Karim Adiprasito, *Combinatorial Lefschetz theorems beyond positivity*, 2018, preprint, arXiv:1812.10454.

Wednesday 9:00, Zoom 1

Base sizes and relational complexity of finite permutation groups

Colva Roney-Dougal

University of St Andrews

MSC2000: 20D05

This talk will start by briefly surveying what is known about the maximal subgroups of the finite simple groups. We will then see how this knowledge has been applied to bound some combinatorial invariants of finite permutation groups.

A *base* for a subgroup G of the symmetric group $\mathrm{S}(\Omega)$ is a subset Δ of Ω whose pointwise stabiliser in G is trivial. The first invariant we will look at is the size $b(G)$ of a *minimal* base for G. We will see that $b(G)$ gives a coarse estimate of the size of G, and survey results both old and new which bound $b(G)$. Next, we'll see how large an *irredundant* base for G can be: this is an ordered base $\Delta = (\delta_1, \ldots, \delta_k)$ such that the stabiliser $G_{\alpha_1,\ldots,\alpha_{i-1}} \neq G_{\alpha_1,\ldots,\alpha_{i-1},\alpha_i}$, for all i.

We'll end by linking these ideas to model theory, via the idea of relational complexity.

Hasse-Weil type theorems and relevant classes of polynomial functions

Daniele Bartoli

Università degli Studi di Perugia

MSC2000: 14-02

Several types of functions over finite fields have relevant applications in applied areas of mathematics, such as cryptography and coding theory. Among them, planar functions, APN permutations, permutation polynomials, and scattered polynomials have been widely studied in the last few years.

In order to provide both non-existence results and explicit constructions of infinite families, sometimes algebraic varieties over finite fields turn out to be a useful tool. In a typical argument involving algebraic varieties, the key step is estimating the number of their rational points over some finite field. For this reason, Hasse-Weil type theorems (such as Lang-Weil's and Serre's) play a fundamental role.

Borel combinatorics

Oleg Pikhurko

University of Warwick

MSC2000: 05C63, 03E05, 28A05

We give an introduction, aimed at non-experts, to Borel combinatorics (that studies definable graphs on topological spaces and looks for constructive assignments satisfying some given local combinatorial constraints). This is an emerging field on the borderline between combinatorics and descriptive set theory with deep connections to many other areas. The aim of this talk is to advertise this field to a wider research community.

Generating graphs randomly

Catherine Greenhill

UNSW Sydney

MSC2000: 05C85, 60J10, 68R05, 68W20, 68W40

Graphs are used in many disciplines to model the relationships that exist between objects in a complex discrete system. Researchers often wish to compare their particular graph to a “typical” graph from a family (or ensemble) of graphs which are similar to theirs in some way. Such a family might be the set of all graphs with a given number of vertices and edges, or the set of all graphs with a particular degree sequence.

One way to do this is to take several random samples from the family, to gather information about what is “typical”. This motivates the search for an algorithm which can generate graphs uniformly (or approximately uniformly) at random from the given family. Since many random samples may be required, the algorithm should also be efficient. Rigorous analysis of such algorithms is often challenging, involving both combinatorial and probabilistic arguments. I will discuss some algorithms for sampling graphs, and the methods used to analyse them.

Recent Advances on the Graph Isomorphism Problem

Martin Grohe

RWTH Aachen University

MSC2000: 05C60, 68R10, 20B25

The question of whether there is a polynomial time algorithm deciding if two graphs are isomorphic has been a one of the best known open problems in theoretical computer science for almost 50 years. Indeed, the graph isomorphism problem is one of the very few natural problems in the complexity class NP that is neither known to be solvable in polynomial time nor known to be NP-complete. Five years ago, Babai gave a quasipolynomial time isomorphism algorithm. Despite of this breakthrough result, the question for a polynomial algorithm remains wide open.

My talk will be a survey of recent progress on the isomorphism problem. I will focus on two generic algorithmic strategies that have proved to be useful and interesting in various contexts. The first is the combinatorial Weisfeiler-Leman algorithm with a wide range of applications from practical graph isomorphism testing to machine learning. The second is the group theoretic divide-and-conquer strategy, going back to Luks (1983), that is the foundation of Babai's quasi-polynomial time isomorphism test. In subsequent developments, it led to the design of isomorphism algorithms with a quasi-polynomial parameterised running time of the form $n^{\mathrm{polylog}\, k}$, where k is a graph parameter such as the maximum degree.

2 Minisymposia talks

Tuesday 10:30, Zoom 1

Diagonal semilattices and their graphs

Peter J. Cameron

University of St Andrews

(This talk is based on joint work with R. A. Bailey, Cheryl E. Praeger, Csaba Schneider and others.)

MSC2000: 05B15,05C25,20B05

Last year, with Rosemary Bailey, Cheryl Praeger and Csaba Schneider, I gave a descriptive and axiomatic approach to a class of geometries (that we called *diagonal semilattices*) associated with diagonal groups [3]. These groups arose as one of the classes of finite primitive permutation groups in the O'Nan–Scott Theorem, but in fact can be defined for any group, finite or infinite. Our theorem asserts that, in two dimensions, these structures are equivalent to Latin squares, but for higher dimensions, they are coordinatised by a group T (finite or infinite), and their full automorphism group is the diagonal group. (The Latin square which is coordinatised by a group is the *Cayley table* of that group.)

Associated with a diagonal semilattice is a graph, the *diagonal graph*. In the 2-dimensional case, this graph is the strongly regular *Latin square graph* associated with the Latin square. If the group T is C_2, then the graph is the distance-transitive *folded cube*. In view of these special cases, we think the graphs warrant further investigation. In particular, their chromatic number is determined in some cases by the use of graph homomorphisms and the *Hall–Paige conjecture* (now a theorem, proved by Wilcox, Evans and Bray). The question arose in connection with the theory of synchronization of finite automata, and shows that primitive diagonal groups are synchronizing. However, in other cases, the chromatic number of the graphs is unknown.

There are further interesting connections which time will not permit me to discuss; see the references below.

[1] R. A. Bailey and Peter J. Cameron, The diagonal graph, arXiv:2101.02451

[2] R. A. Bailey, Peter J. Cameron, Michael Kinyon and Cheryl E. Praeger, Diagonal groups and arcs over groups, arXiv:2010.16338

[3] R. A. Bailey, Peter J. Cameron, Cheryl E. Praeger and Csaba Schneider, The geometry of diagonal groups, arXiv:2007.10726

[4] John N. Bray, Qi Cai, Peter J. Cameron, Pablo Spiga and Hua Zhang, The Hall–Paige conjecture, and synchronization for affine and diagonal groups, *J. Algebra* **545** (2020), 27–42.

Mutually Orthogonal Frequency Squares

Nicholas Cavenagh

University of Waikato

(This talk is based on joint work with Ian Wanless, Adam Mammoliti, Thomas Britz and Fahim Rahim.)

MSC2000: 05B15, 05B20, 15B34, 62K15

A *frequency square* of type $(n;\lambda)$ is a $n \times n$ array such that each symbol from a set of size n/λ occurs λ times in each row and λ times each in column. Thus a frequency square of type $(n;1)$ is a Latin square of order n. Two frequency squares of type $(n;\lambda)$ are said to be *orthogonal* if each possible ordered pair occurs λ^2 times when the squares are overlapped. Sets of mutually orthogonal frequency squares (MOFS) occur more easily than sets of mutually orthogonal Latin squares (MOLS); for example there is a set of seventeen MOFS of type $(6;3)$; but, as Euler knew, there are no pairs of MOLS of order 6. We give new results on the existence and non-existence of MOFS and maximal sets of MOFS. These results feature Hadamard matrices and integral polytopes. We also mention applications to statistical factorial designs with double blocking.

Tuesday 11:40, Zoom 1

A lower bound on HMOLS

Peter Dukes

University of Victoria

(This talk is based on joint work with Michael Bailey and Coen del Valle.)

MSC2000: 05B10, 05B15

It is known that $N(n)$, the maximum number of mutually orthogonal latin squares of order n, satisfies the lower bound $N(n) \geq n^{1/14.8}$ for large n.

We consider *HMOLS*, or mutually orthogonal latin squares having a common equipartition into n 'holes' of a fixed size h. In a little more detail, each array is $nh \times nh$, and the diagonal $h \times h$ blocks are empty. Each row or column meeting the jth diagonal block contains, exactly once, every element not in the jth run of elements, say $(j-1)h+1, \ldots, jh$. Similar to the definition for orthogonal latin squares, two holey latin squares are orthogonal if, when superimposed, any ordered pair with not both elements from the same run appears exactly once. An example with $h = 2$ and $n = 4$, due to Dinitz and Stinson, is shown below.

		8	6	3	7	4	5
		5	7	8	4	6	3
7	6			1	8	5	2
5	8			7	2	1	6
4	7	2	8			3	1
8	3	7	1			2	4
3	5	6	2	4	1		
6	4	1	5	2	3		

		5	7	8	4	6	3
		8	6	3	7	4	5
5	8			7	2	1	6
7	6			1	8	5	2
8	3	7	1			2	4
4	7	2	8			3	1
6	4	1	5	2	3		
3	5	6	2	4	1		

For an alternate viewpoint, a set of $k-2$ HMOLS of type h^n is equivalent to a K_k-decomposition of the blow-up of the complete k-partite graph $K_{n,n,\ldots,n}$ by independent sets of size h.

Let $N(h^n)$ denote the maximum number of HMOLS of type h^n. We generalise a difference matrix method that had been used previously for explicit constructions of HMOLS. An estimate of R.M. Wilson on higher cyclotomic numbers guarantees our construction succeeds in suitably large finite fields. Feeding this into a generalized product construction, we obtain a modest lower bound $N(h^n) \geq (\log n)^{1/\delta}$ for any $\delta > 2$ and all $n > n_0(h, \delta)$.

Tuesday 12:15, Zoom 1

Ryser's conjecture and more

Liana Yepremyan

London School of Economics

(This talk is based on joint work with Peter Keevash, Alexey Pokrovskiy, Benny Sudakov.)

MSC2000: 05D15, 05D40, 05C35, 05C15, 05C35, 05C38, 05C48, 05B15

A Latin square of order n is an $n \times n$ array filled with n symbols such that each symbol appears only once in every row or column and a transversal is a collection of cells which do not share the same row, column or symbol. The study of Latin squares goes back more than 200 years to the work of Euler. One of the most famous open problems in this area is a conjecture of Ryser, Brualdi and Stein from 60s which says that every Latin square of order $n \times n$ contains a transversal of order $n-1$. A closely related problem is 40 year old conjecture of Brouwer that every Steiner triple system of order n contains a matching of size $(n-4)/3$. The third problem we'd like to mention asks how many distinct symbols in Latin arrays suffice to guarantee a full transversal? In this talk we discuss a relatively new approach to attack these problems.

Tuesday 10:30, Zoom 2

Counting solutions in the random k-SAT model

Andreas Galanis

University of Oxford

(This talk is based on joint work with Leslie Ann Goldberg, Heng Guo, and Kuan Yang.)

MSC2000: 68Q87

Random constraint satisfaction problems, such as the k-SAT model, have long posed various algorithmic and probabilistic challenges. In this talk, we focus on the *number* of solutions, the so-called partition function, which is a key quantity that captures the underlying phase transitions and typically requires a very fine understanding of the solution space.

We present a new algorithmic approach for counting the number of solutions in the random k-SAT model, when the density of the formula scales exponentially with k. This improves significantly upon the best previous counting algorithm by Montanari and Shah, which is based on belief propagation and works up to densities $(1 + o_k(1))2\frac{\log k}{k}$, the so-called Gibbs uniqueness threshold for the model. Instead, our algorithm harnesses a recent coupling technique by Moitra, based on the Lovász Local Lemma (LLL), to work for random formulas. The main challenge in our setting is to account for the presence of high-degree variables whose marginal distributions are hard to control and which cause significant correlations within the formula. The key ingredient in our approach is to control the correlation phenomena caused by high-degree variables based on a certain bootstrap percolation process.

Tuesday 11:05, Zoom 2

Local Colouring

Alexander E. Holroyd

University of Bristol

MSC2000: 60G10, 05C15, 60C05

Do local constraints demand global coordination? I'll address a particularly simple formulation of this question: can the vertices of a graph be assigned random colours in a stationary way, so that neighbouring colours always differ, but without long-range dependence? The quest to answer this has led to the discovery of a beautiful yet mysterious new stochastic process that seemingly has no right to exist, while overturning the conventional thinking on a fundamental 50-year old question.

Based on joint works with Tom Liggett, Tom Hutchcroft and Avi Levy.

Nonconvergence in the first order logic of permutations

Tobias Müller

Groningen University

(This talk is based on joint work with Fiona Skerman.)

MSC2000: 05A05, 05C80

Recently Albert, Bouvel and Féray introduced the *theory of two total orders* (TOTO) which allows one to express properties of permutations in first order logic. We are allowed to use the quantifiers $\forall, \exists$, variables $x, y, z, \dots$, the logical connectives $\wedge, \vee, \neg$, etc., brackets and the relation symbols $=, <_1, <_2$. Thinking of a permutation π as a map from $[n] := \{1, \dots, n\}$ two itself, if x, y represent two elements of $[n]$ then $x <_1 y$ just means that $x < y$ while $x <_2 y$ means that $\pi(x) < \pi(y)$. The occurrence of the pattern 231 can for instance be expressed as

$$\exists x, y, z : (x <_1 y) \wedge (y <_1 z) \wedge (z <_2 x) \wedge (x <_2 y).$$

We consider the probability that a given property expressible in TOTO holds for a random permutation. That is, a permutation π_n chosen uniformly at random from all $n!$ permutations of $[n]$. Answering a question of Albert, Bouvel and Féray in the negative, we show that there exists a property φ expressible in TOTO, such that

$$\lim_{n \to \infty} \mathbb{P}(\pi_n \text{ satisfies } \varphi) \text{ does not exist.}$$

That is, we construct a $\varphi \in$ TOTO such that probability that π_n satisfies it oscillates between zero and one. The construction builds on the seminal work of Shelah and Spencer on first order properties for the Erdős-Rényi random graph.

(Based on joint work with Fiona Skerman)

Oriented percolation with modified boundaries

Leonardo T. Rolla

University of Warwick

(This talk is based on joint work with Enrique Andjel.)

MSC2000: 60K35

Monotonicity questions in oriented percolation and contact process are more tricky than we may think, and nothing similar to enhanced percolation arguments seem to work. For example, it is immediate that the critical parameter of contact process is monotone in the dimension, but there is so far no proof that it is strictly monotone. In this talk, we consider two-dimensional directed percolation (or one-dimensional contact process) with two parameters: infection from the leftmost and rightmost occupied sites towards the exterior occurs with a different rate than in the interior. We show that the critical curve on the parameter space is strictly decreasing. In particular, any subcritical choice for the external parameter causes the critical internal parameter to increase strictly. In this regime, the process is still attractive but no longer additive, and most of the classical arguments break down. We also show that any supercritical choice for the external parameter makes the process percolate even if the internal parameter is critical (in this regime, the process is not attractive).

Clustered colouring of planar graphs

Louis Esperet

CNRS, Univ. Grenoble Alpes

(This talk is based on joint work with M. Bonamy, N. Bousquet, C. Groenland, C.-H. Liu, F. Pirot, and A. Scott, and with V. Dujmović, P. Morin, B. Walczak, and D.R. Wood.)

MSC2000: 05C10,05C12,05C15

A (not necessarily proper) vertex colouring of a graph has *clustering* at most C if every monochromatic component has at most C vertices. Note that a proper colouring is the same as a colouring with clustering at most 1. It is well-known that every planar graph has a proper 4-colouring. We show that there is a constant C such that every planar graph has a 3-colouring in which each monochromatic component has weak diameter at most C (which means that any two vertices in the component are at distance at most C in the graph). This directly implies the (known) result that planar graphs with maximum degree Δ are 3-colourable with clustering polynomial in Δ. We also prove that (without additional properties on the diameter of the monochromatic components) the clustering can be decreased to $O(\Delta^2)$, improving the previous bound of $O(\Delta^{37})$.

Dichotomizing k-vertex-critical H-free graphs for H of order four

Chính T. Hoàng

Wilfrid Laurier University

(This talk is based on joint work with Ben Cameron and Joe Sawada.)

MSC2000: 05C15

For every $k \geq 1$ and $\ell \geq 1$, we prove that there is a finite number of k-vertex-critical $(P_2+\ell P_1)$-free graphs. This result establishes the existence of new polynomial-time certifying algorithms for deciding the k-colorability of $(P_2+\ell P_1)$-free graphs. Together with previous research, our result also implies the following characterization: There is a finite number of k-vertex-critical H-free graphs for H of order and for fixed $k \geq 5$ if and only if H is one of $\overline{K_4}, P_4, P_2 + 2P_1$, or $P_3 + P_1$. We also improve the recent known result that there is a finite number of k-vertex-critical $(P_3 + P_1)$-free graphs for all k by showing that such graphs have at most $2k - 1$ vertices. We use this stronger result to exhaustively generate all k-vertex-critical $(P_3 + P_1)$-free graphs for $k \leq 7$.

Excluding a tree and a biclique

Sophie Spirkl

University of Waterloo

(This talk is based on joint work with Alex Scott and Paul Seymour.)

MSC2000: 05C15, 05C75

The Gyárfás-Sumner conjecture states that for every tree T, there is a function f such that graphs G with no induced T have chromatic number bounded by f of their clique number, that is, $\chi(G) \leq f(\omega(G))$. Hajnal and Rödl proved that if we replace "clique number" by "biclique number", that is, the largest t such that the graph G contains $K_{t,t}$ as a (not necessarily induced) subgraph, then the conjecture holds.

Bonamy, Bousquet, Pilipczuk, Rzazewski, Thomassé and Walczak recently showed further that in this biclique setting, if T is a path, then f can be chosen as a polynomial function. I will talk about a recent result, which extends this from paths to all trees.

Joint work with Alex Scott and Paul Seymour.

Burling graphs revisited

Nicolas Trotignon

CNRS, LIP, ENS de Lyon

(This talk is based on joint work with Pegah Pournajafi.)

MSC2000: 05C15

The Burling sequence is a sequence of triangle-free graphs of increasing chromatic number. Any graph which is an induced subgraph of a graph in this sequence is called a Burling graph. These graphs have attracted some attention because they have geometric representations and because they provide counter-examples to several conjectures about bounding the chromatic number in classes of graphs.

The goal of this talk is to provide new definitions of Burling graphs. Three of them are geometrical : they characterize Burling graphs as intersection graphs of various geometrical objects (line segments of the place, frame of the plane, axis-aligned boxes of $\mathbb{R}^3$). All these representations of Burling graphs were known. Our contribution is to add restrictions to the configurations of the geometrical objects so that there is an equivalence between the intersection graphs and the Burling graphs.

Among our new equivalent definitions of Burling graphs, one is of a more combinatorial flavour. It says how any Burling graph can be derived from a tree with some specific rules. This definition is convenient decide whether some given graph is Burling or not. We use it to give several generic examples of Burling graphs or rules to find edges whose subdivision preserves being a Burling graph. We also use it to find examples of graphs that are not Burling. Among several consequences of all this, one is that graphs that do not contain any subdivision of K_5 as an induced subgraph have unbounded chromatic number.

Wednesday 14:30, Zoom 2

Sidorenko systems of equations

Anita Liebenau

UNSW Sydney

(This talk is based on joint work with Nina Kamčev and Natasha Morrison.)

MSC2000: 05D99

A system of linear forms L over $\mathbb{F}_q$ is *Sidorenko* if the number of solutions to $L = 0$ in any subset A of $\mathbb{F}_q^n$ is asymptotically (as $n \to \infty$) at least the expected number of solutions in a random subset of $\mathbb{F}_q^n$ of density $|A|/q^n$. The systematic study of Sidorenko systems of linear equations was recently initiated by Saad and Wolf and follows an extensive research on Sidorenko's conjecture for graphs. Building on a result by Saad and Wolf, Fox, Pham and Zhao found a characterisation for one-equation systems that are Sidorenko.

In this talk, we report on recent progress towards characterising Sidorenko systems of two or more equations. In particular, we provide a simple necessary condition for a system to be Sidorenko by proving that the length of a shortest equation induced by the system must be even. We also find a large class of systems that are Sidorenko by combining Sidorenko equations in a certain way.

Spanning subgraphs in randomly perturbed graphs

Olaf Parczyk

London School of Economics and Political Science

(This talk is based on joint work with Julia Böttcher, Amedeo Sgueglia, and Jozef Skokan.)

MSC2000: 05C35, 05C80

We study the model of randomly perturbed dense graphs, which is the union of any n-vertex graph G_α with minimum degree αn and the binomial random graph $G(n,p)$. For the range $0 \leq \alpha < 1$ we are interested in the evolution of the threshold probability $\hat{p}(\alpha)$ that determines when $G_\alpha \cup G(n,p)$ asymptotically almost surely satisfies a given property. In this talk, we discuss questions on the containment of specific spanning structures, such as clique factors and powers of Hamilton cycles, and whole families of graphs, such as those with bounded maximum degree.

Progress towards Nash-Williams' conjecture on triangle decompositions

Michelle Delcourt

Ryerson University

(This talk is based on joint work with Luke Postle.)

MSC2000: 05C51

Partitioning the edges of a graph into edge disjoint triangles forms a triangle decomposition of the graph. A famous conjecture by Nash-Williams from 1970 asserts that any sufficiently large, triangle divisible graph on n vertices with minimum degree at least $0.75n$ admits a triangle decomposition. In the light of recent results, the fractional version of this problem is of central importance. A fractional triangle decomposition is an assignment of non-negative weights to each triangle in a graph such that the sum of the weights along each edge is precisely one.

We show that for any graph on n vertices with minimum degree at least $0.827327n$ admits a fractional triangle decomposition. Combined with results of Barber, Kühn, Lo, and Osthus, this implies that for every sufficiently large triangle divisible graph on n vertices with minimum degree at least $0.82733n$ admits a triangle decomposition. This is a significant improvement over the previous asymptotic result of Dross showing the existence of fractional triangle decompositions of sufficiently large graphs with minimum degree more than $0.9n$.

Uncommon systems of equations

Natasha Morrison

University of Victoria

(This talk is based on joint work with Nina Kamčev and Anita Liebenau.)

MSC2000: 05D99

A system of linear equations L over $\mathbb{F}_q$ is *common* if the number of monochromatic solutions to L in any two-colouring of $\mathbb{F}_q^n$ is asymptotically at least the expected number of monochromatic solutions in a random two-colouring of $\mathbb{F}_q^n$. Motivated by existing results for specific systems (such as Schur triples and arithmetic progressions), as well as extensive research on common and Sidorenko graphs, the systematic study of common systems of linear equations was recently initiated by Saad and Wolf. Building on earlier work of of Cameron, Cilleruelo and Serra, as well as Saad and Wolf, common linear equations have been fully characterised by Fox, Pham and Zhao.

In this talk I will discuss some recent progress towards a characterisation of common systems of two or more equations. In particular we prove that any system containing an arithmetic progression of length four is uncommon, confirming a conjecture of Saad and Wolf. This follows from a more general result which allows us to deduce the uncommonness of a general system from certain properties of one- or two-equation subsystems.

On optimal cryptographic Boolean functions

Lilya Budaghyan

University of Bergen

MSC2000: 06E30, 94A60, 94D10

Almost perfect nonlinear (APN) and almost bent (AB) functions are vectorial Boolean functions which are optimal against two powerful attacks on block ciphers - linear and differential cryptanalyses. Interestingly, these functions exhibit optimality properties also with respect to combinatorics, coding theory, sequence design, algebra and finite geometry. We will talk about recent developments in study of these functions.

On divisible linear codes

Michael Kiermaier

Universität Bayreuth

(This talk is based on joint work with Sascha Kurz.)

MSC2000: 94B05

Divisible codes have been introduced by Harold Ward in 1981. A linear code C is called Δ-*divisible* if the weight of all codewords is divisible by Δ. Divisible codes are interesting for various reasons, amongst others:

- Many good codes are divisible.
- (Hermitean) self-orthogonality of binary, ternary and quaternary codes can sometimes be generalized to divisibility.
- There are results and conjectures about the divisibility of Griesmer-optimal linear codes.
- There are interconnections to other research areas like Galois geometries and the theory of subspace codes.

In this talk, we will examine old and new results about divisible linear codes. Moreover, applications in Galois geometries and subspace codes will be discussed.

Thursday 16:10, Zoom 1

Distributed storage systems and finite geometry

Siaw-Lynn Ng

Information Security Group, Royal Holloway, University of London

(This talk is based on joint work with M. B. Paterson.)

MSC2000: 51E20,94B99

While existing geometric and combinatorial objects often provide constructions for various applications in communication and information security, these applications can also inspire new and interesting combinatorial structures. Here I will talk about two topics in distributed storage systems (specifically, functional repair codes) that illustrate this relationship between these applications and finite geometry.

A database can be coded and stored in multiple nodes in such a way that if a number of nodes fail, the data can still be recovered from the functioning nodes, and if a node should fail, it can be repaired using information in some of the functioning nodes so that the the recovery property of the system still holds. A functional repair code can be modelled as subspaces in a finite projective space [3]. The nodes are represented by subspaces, and to "repair" a subspace, we attempt to find smaller subspaces contained in some existing nodes, and use these to construct the target subspace.

Firstly we will see how this approach allows a simplified proof of the important upper bound [1] on the number of information symbols one can store in a system that is subject to storage and bandwidth constraints. Secondly we will see an interesting geometric structure that arises from a special class of functional repair codes, called *strictly* functional repair codes: there are nodes that cannot be replicated exactly when failed, even though they can be repaired to the extent that the recovery property of the system still holds. We will see some examples of this geometrical structure.

[1] A. G. Dimakis, P. B. Godfrey, Y. Wu, M. J. Wainwright and K. Ramchandran. Network coding for distributed storage systems. *IEEE Transactions on Information Theory*, 56(9):4539–4551, Sept. 2010.

[2] S. L. Ng and M. B. Paterson. Functional repair codes: a view from projective geometry. *Designs, Codes and Cryptography*, 87(11), 2701-2722

[3] H. D. L. Hollmann and W. Poh. Characterizations and construction methods for linear functional repair storage codes. IEEE International Symposium on Information Theory (ISIT), Istanbul, Turkey, July 2013, pp. 336–340.

On Security Properties of All-or-nothing Transforms

Douglas Stinson

University of Waterloo

(This talk is based on joint work with Navid Nasr Esfahani.)

MSC2000: 94A60

All-or-nothing transforms have been defined as bijective mappings on all s-tuples over a specified finite alphabet. These mappings are required to satisfy certain security conditions that upper-bound information about a subset of the inputs that can be inferred from the knowledge of some of the outputs. Alternatively, a purely combinatorial definition of AONTs has also been given. This definition involves certain kinds of "unbiased arrays," which are related to, but weaker than, orthogonal arrays.

In this talk, we examine the security provided by AONTs that satisfy the combinatorial definition. The security of the AONT can depend on the underlying probability distribution of the input s-tuples. We show that "perfect security" is obtained from an AONT if and only if the input s-tuples are equiprobable. However, in the case where the input s-tuples are not equiprobable, we still achieve a weaker security guarantee. We also consider the use of randomized AONTs to provide perfect security for a smaller number of inputs, even when those inputs are not equiprobable.

Interval-membership-width: a purely temporal parameter

Kitty Meeks

University of Glasgow

(This talk is based on joint work with Benjamin Bumpus.)

MSC2000: 05C85, 68R10, 05C40

Many algorithmic problems become harder on temporal graphs, in which each edge only appears at a specified set of times. In particular, the temporal versions of several problems remain intractable on temporal graphs even when the underlying graph has very restricted structure that would allow for efficient algorithms to solve the corresponding static problem – for example, when the underlying graph is a tree or even a path. This rules out the existence of efficient parameterised (FPT) algorithms for such problems with respect to most well-known graph parameters, and therefore poses a major challenge for the design of FPT algorithms in this setting.

In this talk, I will introduce a new parameter for temporal graphs, called *interval-membership-width*. This parameter does not depend on the structure of the underlying graph, but on the relationships between the sets of times at which each edge appears in the graph. I will illustrate how we can obtain efficient dynamic programming algorithms to solve a variety of temporal graph problems when this parameter is small, focussing in particular on temporal analogues of the problems of determining whether a graph is Hamiltonian or Eulerian respectively.

Towards Classifying the Polynomial-Time Solvability of Temporal Betweenness Centrality

Hendrik Molter

Ben-Gurion University of the Negev

(This talk is based on joint work with André Nichterlein, Rolf Niedermeier, and Maciej Rymar.)

MSC2000: 68Q25

In static graphs, the betweenness centrality of a graph vertex measures how many times this vertex is part of a shortest path between any two graph vertices. Betweenness centrality is efficiently computable and it is a fundamental tool in network science. Continuing and extending previous work, we study the efficient computability of betweenness centrality in temporal graphs (graphs with fixed vertex set but time-varying arc sets). Unlike in the static case, there are numerous natural notions of being a "shortest" temporal path (walk). Depending on which notion is used, it was already observed that the problem is #P-hard in some cases while polynomial-time solvable in others. In this conceptual work, we contribute towards classifying what a "shortest path (walk) concept" has to fulfill in order to gain polynomial-time computability of temporal betweenness centrality.

A Temporal Chase Is Harder Than You Think

Nils Morawietz

Philipps-Universität Marburg, Fachbereich Mathematik und Informatik, Marburg, Germany

(This talk is based on joint work with Carolin Rehs (Heinrich Heine Universität, Düsseldorf), Mathias Weller (Université Gustave Eiffel, Marne-la-Vallée), and Petra Wolf (Universität Trier).)

MSC2000: 68Q25

We consider the (parameterized) complexity of a cop and robber game on edge-periodic temporal graphs and a problem on periodic binary sequences to which these games relate intimately. We show that it is NP-hard to decide (a) whether a single cop can catch a single robber on an edge-periodic temporal graph, and (b) whether there is some common index at which all given periodic, binary sequences are 1. We further present results for various parameterizations of both problems and show that hardness not only applies in general, but also for highly limited instances. As one main result we show that even if the underlying graph is a directed or undirected cycle, the cop and robber game on periodic, temporal graphs is NP-hard and W[1]-hard when parameterized by the size of the underlying graph. Further, we present matching lower bounds for the relation between the length of the underlying cycle and the least common multiple of the lengths of binary strings describing edge-periodicies over which the graph is robber-winning. Finally, we improve the previously known EXPTIME upper bound on general edge-periodic graphs to PSPACE-membership. This closes several open problems stated in the introductory work by Erlebach and Spooner [SOFSEM 2020].

Distributed Algorithms over Temporal Networks?

Amitabh Trehan

Durham University

MSC2000: 68R10,05C85,05C90

Temporal graph theory and algorithms are often concerned with changing edges (temporal) and centralised algorithms with the graph given as input. In Distributed algorithms, on the other hand, the graph is usually the network itself. Distributed algorithms deal with both the situations where the graph is static or changing (dynamic). It will be interesting to explore the connections between distributed algorithms and temporal graph theory. We pose some some possibly interesting questions and/or extensions.

The common temporal/dynamic setting is for the edges of a graph to change over the same fixed set of vertices. A different setting is that of 'self-healing' - i.e. fault-tolerance by quick local repair of a system under adversarial attack to another good but possibly degraded state [1]. One version that can be imagined is playing a round-based game on a graph where one player (adversary) can remove or add one node per round with the other player (healer) adding or removing edges in the locality of the attack with the aim of preserving certain invariants throughout the history of the attacks and repairs. Can this inspire extensions of temporal graph theory in presence of churn (node additions/deletions)? How does one even state results when node additions are permitted?

Amnesiac Flooding [2] is among the simplest algorithms for information dissemination on a network. The basic algorithm is to forward a message to every node except the ones you just received the message from. This algorithm is 'stateless' and uses no additional memory. However, with such loss of history, it is possible that the messages could be regenerated ad-infinitum and circulated forever on the network. In practice, 'memory' flags are used explicitly to make sure a message is not circulated again. However, surprisingly and despite its simplicity, it turns out that amnesiac flooding terminates on every graph i.e. message circulation stops in a finite number of rounds. How would this behave for temporal graphs i.e. graphs whose edges may change over time?

[1] Amitabh Trehan. *Algorithms for self-healing networks.* Dissertation, University of New Mexico, 2010. *https://arxiv.org/abs/1305.4675.*

[2] Walter Hussak and Amitabh Trehan. *On the Termination of Flooding.* 37th International Symposium on Theoretical Aspects of Computer Science STACS 2020, March 10-13, 2020, Montpellier, France. Schloss Dagstuhl - Leibniz-Zentrum für Informatik. 2020.

3 Contributed talks

Iterated Integrals of Clique Polynomials

Hossein Teimoori Faal

Department of Mathematics and Computer Science, Allameh Tabataba'i University, Tehran, Iran

MSC2000: 05C31, 05C69, 30C15

Despite the significance of *graph polynomials* in algebraic graph theory, to the best of the authors' knowledge, the *graph-theoretical* interpretations of *iterated integrals* of graph polynomials have not been considered in the literature. A clique polynomial of a simple and undirected graph G is an *ordinary generating function* of the number of complete subgraphs on i vertices (i-cliques). The set of all i*-cliques* of G will be denoted by $\Delta_i(G)$. We also denote the number of i-cliques of G by $c_i(G)$. By convention $c_0(G) = 0$, for any graph G. We also note that by convention $C(G,x) = 1$, whenever G is the *order-zero* graph; that is, a graph without any vertices and edges. We give a *recursive* definition of the *generalized (open) neighborhood* $N_G(Q_l)$ of any higher order clique Q_l with l vertices by

$$N_G(Q_l) = \bigcap_{Q_l \in \Delta_{l-1}(G)} N_G(Q_{l-1}) \quad (l \geq 2).$$

where $N_G(Q_1)$ (or $N_G(v) = \{u \in V(G) \mid \{u,v\} \in E(G)\}$) is exactly the (open) *neighborhood* of G. Next we need one more interesting definition which orginally comes from *discrete Morse theory*. For the generalized neighborhood $N_G(Q_l)$ of any clique Q_l and a *locally injective* function $f_G : \ V(G) \cup E(G) \mapsto \mathbb{R}$, we define its corresponding *negative (open) neighborhood* denoted by $N_G^{<0}(Q_l)$, as follows.

$$N_G^{<0}(Q_l) = \Big\{ v \in N_G(Q_l) \mid f_G(v) - f_G(q) < 0, \ \forall q \in V(Q_l) \cup E(Q_l) \Big\}.$$

Moreover, for a given graph polynomial $f(G,x)$, we define its *iterated integral* of order k as $I_{k,f}(G,x) = \int_0^x \int_0^{t_{k-1}} \cdots \int_0^{t_1} f(G,s) ds dt_1 \cdots dt_{k-1}$.

Theorem 1 (Main Result). *Let $C(G,x)$ be the clique polynomial of a graph $G = (V,E)$ and $f_G : \ V(G) \cup E(G) \mapsto \mathbb{R}$ be a locally injective function defined on G. Then, we have the following combinatorial interpretations*

$$C(G,x) \;=\; \sum_{i=0}^{k-1} c_j(G) x^j + k! \sum_{Q_k \in \Delta_k(G)} I_{k,f_G}(G[N_G(Q)]), x), \qquad (k \geq 1), \tag{1}$$

$$C(G,x) \;=\; \sum_{i=0}^{k-1} c_j(G) x^j + x^k \sum_{Q_k \in \Delta_k(G)} C(G[N_G^{<0}(Q)], x), \qquad (k \geq 1). \tag{2}$$

The *proofs* are mainly based on counting arguments (the *generalized clique handshaking* lemma) and the *mathematical induction*. Finally, we will conclude our talk with several interesting open problems and conjectures.

New insight into introducing a $(2-\epsilon)$-approximation ratio for minimum vertex cover problem

Majid Zohrehbandian

Department of Mathematics, Karaj Branch, Islamic Azad University, Karaj, Iran.

MSC2000: 90C35 , 90C60

Vertex cover problem is a famous combinatorial problem, which its complexity has been heavily studied. It is known that it is hard to approximate to within any constant factor better than 2, while a 2-approximation for it can be trivially obtained. In this paper, new properties (e.g. existence of a feasible solution with cardinality smaller than $0.999|V|$, or existence of a connected subgraph of vertices with values $0.5+\epsilon$ and cardinality greater than $0.001|V|$, or existence of vertices with degrees greater than $0.001|V|$) and new techniques (contraction of the paired vertices to construct triangles) are introduced which lead to approximation ratios smaller than 2 on special graphs; e.g. Graphs for which their maximum cut optimal values are less than $0.85|E|$. In fact, we show that we can produce a $(2-\epsilon)$-approximation ratio on special graphs, where they don't satisfy some of the proposed assumptions. Then, by introducing a modified graph and corresponding ILP model along with satisfying all the proposed assumptions, we introduce new insight into solving this open problem and we introduce an approximation algorithm with performance ratio $(2-\epsilon)$ on arbitrary graphs.

Containment graphs and posets of paths in a tree

Martin Charles Golumbic

University of Haifa, Israel

(This talk is based on joint work with Vincent Limouzy.)

MSC2000: 05C75,06A07

We consider questions regarding the containment graphs of paths in a tree (CPT graphs), a subclass of comparability graphs. The posets defined by their transitive orientations are called CPT orders. In 1984, Corneil and Golumbic observed that a graph G may be CPT, yet not every transitive orientation of G necessarily has a CPT representation, illustrating this on the even wheels $W_{2k}(k \geq 3)$. Motivated by this example, we characterize the partial wheels that are the containment graphs of paths in a tree, and give a number of examples and obstructions for this class. Our characterization gives the surprising result that all partial wheels that admit a transitive orientation are CPT graphs. We then characterize the CPT orders whose comparability graph is a partial wheel. A survey of other results and open questions on CPT graphs and posets will conclude the talk.

A Linear Algorithm for Computing Independence Polynomials of Trees

Ohr Kadrawi

Ariel University

(This talk is based on joint work with Vadim E. Levit, Ron Yosef and Matan Mizrachi.)

MSC2020: 05C31, 05C05, 05C69

An independent set in a graph is a set of pairwise non-adjacent vertices. Let $\alpha(G)$ denote the cardinality of a maximum independent set in the graph $G = (V, E)$. Gutman and Harary defined the independence polynomial of G

$$I(G; x) = \sum_{k=0}^{\alpha(G)} s_k x^k = s_0 + s_1 x + s_2 x^2 + ... + s_{\alpha(G)} x^{\alpha(G)},$$

where s_k denotes the number of independent sets of cardinality k in the graph G [1]. A comprehensive survey on the subject is due to Levit and Mandrescu [2], where some recursive formulas are allowing to calculate the independence polynomial. A direct implementation of these recursions does not bring about an efficient algorithm. Yosef, Mizrachi, and Kadrawi developed an efficient way for computing the independence polynomials of trees with n vertices, such that a database containing all of the independence polynomials of all the trees with up to $n - 1$ vertices is required [3]. This approach is not suitable for big trees, as an extensive database is needed. On the other hand, using dynamic programming, it is possible to develop an efficient algorithm that prevents repeated calculations. In summary, our dynamic programming algorithm runs over a tree in linear time and does not depend on a database.

[1] Gutman I. , Harary F., "Generalizations of the matching polynomial," *Utilitas Mathematica*, Vol.24, 97-106, 1983.

[2] Levit V. E. and Mandrescu E., "The independence polynomial of a graph - a survey," *Proceedings of the 1st International Conference on Algebraic Informatics*, 231-252, 2005.

[3] Yosef R., Mizrachi M., Kadrawi O. "On unimodality of independence polynomials of trees", *https://arxiv.org/pdf/2101.06744v3.pdf*, 2021.

Monday 10:30, Zoom 2

On the Construction of Optimal Linear Codes from Hyperbolic Quadrics

Atsuya Kato

Osaka Prefecture University

(This talk is based on joint work with Keita Nomura and Tatsuya Maruta.)

MSC2000: 94B05, 94B27

An $[n, k, d]_q$ code is a linear code of length n, dimension k and minimum weight d over $\mathbb{F}_q$, the field of order q. A fundamental problem in coding theory is to find $n_q(k, d)$, the minimum length n for which an $[n, k, d]_q$ code exists for given q, k, d [1]. A $(q+1)^2$-set in $\mathrm{PG}(3, q)$ projectively equivalent to the set $V(x_0x_1 + x_2x_3)$ is called a hyperbolic quadric [2]. Divisible codes and the projective dual method are often used to construct optimal linear codes [3]. From two hyperbolic quadrics in $\mathrm{PG}(3, q)$ through the same four lines, we construct a q-divisible $[2q^2 - 3, 4, 2q^2 - 3q]_q$ code $\mathcal{C}$ for $q \geq 3$. As a projective dual of $\mathcal{C}$, we get a q-divisible $[q^3 - 2q^2 + q + 3, 4, q^3 - 3q^2 + 3q]_q$ code, which is optimal up to length for $3 \leq q \leq 13$.

[1] R. Hill, Optimal linear codes, in *Cryptography and Coding II*, C. Mitchell, Ed., Oxford Univ. Press, Oxford, 1992, pp. 75–104.

[2] J.W.P. Hirschfeld, Projective Geometries over Finite Fields 2nd ed., Clarendon Press, Oxford, 1998.

[3] Y. Inoue, T. Maruta, Geometric extending of divisible codes and construction of new linear codes, Finite Fields Appl. **71** (2021), 101773.

On the Nonexistence of Ternary Griesmer Codes

Daiki Kawabata

Osaka Prefecture University

(This talk is based on joint work with Tatsuya Maruta.)

MSC2000: 94B05, 94B27

An $[n, k, d]_q$ code $\mathcal{C}$ is a linear code of length n, dimension k and minimum weight d over $\mathbb{F}_q$, the field of order q. $\mathcal{C}$ is called *Griesmer* if it attains the Griesmer bound:

$$n \geq g_q(k, d) := \sum_{i=0}^{k-1} \left\lceil \frac{d}{q^i} \right\rceil,$$

where $\lceil z \rceil$ stands for the minimum integer $\geq z$. It is known for $k = 1, 2$ that the Griesmer bound is achieved for all d. So, we assume $k \geq 3$. For fixed q and k, it is also known that Griesmer codes with minimum weight d exist for all sufficiently large d [1]. One of the central problems in coding theory is to find $n_q(k, d)$, the minimum length n for which an $[n, k, d]_q$ code exists. A natural question is the following.

Problem. For fixed q and k, find the integer $D_{q,k}$ satisfying that $n_q(k, d) = g_q(k, d)$ for all $d > D_{q,k}$ and that $n_q(k, d) > g_q(k, d)$ for $d = D_{q,k}$. Then, determine $n_q(k, D_{q,k})$.

It is known that $D_{q,k} = (k-2)q^{k-1} - (k-1)q^{k-2}$ and that $n_q(k, D_{q,k}) = g_q(k, D_{q,k}) + 1$ for $q \geq k$ with $k = 3, 4, 5$ and for $q \geq 2k - 3$ with $k \geq 6$ [2, 4]. We posed a conjecture on $D_{3,k}$, which is valid for $4 \leq k \leq 7$ [3]. We give recent results about this conjecture.

[1] R. Hill, Optimal linear codes, in *Cryptography and Coding II*, C. Mitchell, Ed., Oxford Univ. Press, Oxford, 1992, pp. 75–104.

[2] Y. Kageyama, T. Maruta, On the geometric constructions of optimal linear codes, *Des. Codes Cryptogr.* **81** (2016) 469–480.

[3] D. Kawabata, T. Maruta, A conjecture on optimal ternary linear codes, Proceedings of 17th International Workshop on Algebraic and Combinatorial Coding Theory, 2020 Algebraic and Combinatorial Coding Theory (ACCT), IEEE Xplore (2021) 90–94.

[4] T. Maruta, On the achievement of the Griesmer bound, *Des. Codes Cryptogr.* **12** (1997) 83–87.

Scattered polynomials and their exceptionality

Ferdinando Zullo

Università degli Studi della Campania "Luigi Vanvitelli", Caserta, Italy.

(This talk is based on joint work with Daniele Bartoli and Giovanni Zini.)

MSC2000: 11T06, 51E20

Scattered polynomials over a finite field $\mathbb{F}_{q^n}$ have been introduced by Sheekey in 2016 [6] in connection with maximum scattered linear sets in $\mathrm{PG}(1, q^n)$ and maximum rank distance codes. A central open problem regards the classification of those that are *exceptional*. So far, only two families of exceptional scattered polynomials are known and some classification results have been provided, see [1, 2, 4]. More recently, Longobardi and Zanella in [5] weakened the property of being scattered by introducing the notion of *L-q^t-partially scattered* and *R-q^t-partially scattered* polynomials, for t a divisor of n. Indeed, a polynomial is scattered if and only if it is both L-q^t-partially scattered and R-q^t-partially scattered. In this talk, we will first survey on the known classification results about exceptional scattered polynomials and then we will investigate the exceptionality of both the properties of being L-q^t-partially scattered and R-q^t-partially scattered. Moreover, we will present a large family $\mathcal{F}$ of R-q^t-partially scattered polynomials, containing examples of exceptional R-q^t-partially scattered polynomials, which are connected with linear sets of so-called *pseudoregulus type*. We will introduce two different notions of equivalence preserving the property of being R-q^t-partially scattered and then we will analyze the inequivalent examples in $\mathcal{F}$. The results are based on the paper [3].

[1] D. Bartoli and M. Montanucci: On the classification of exceptional scattered polynomials, *J. Combin. Theory Ser. A* **179** (2021).

[2] D. Bartoli and Y. Zhou: Exceptional scattered polynomials, *J. Algebra* **509** (2018), 507–534.

[3] D. Bartoli, G. Zini and F. Zullo: Investigating the exceptionality of scattered polynomials, arXiv:2103.04591.

[4] A. Ferraguti and G. Micheli: Exceptional Scatteredness in prime degree, *J. Algebra*, **565** (2021), 691–701.

[5] G. Longobardi and C. Zanella: Partially scattered linearized polynomials and rank metric codes, arXiv:2009.11537.

[6] J. Sheekey: A new family of linear maximum rank distance codes, *Adv. Math. Commun.* **10**(3) (2016), 475–488.

Moore polynomial sets over finite fields

Giovanni Zini

University of Campania

(This talk is based on joint work with D. Bartoli and F. Zullo.)

MSC2000: 15A15, 94B27

Let q be a prime power, n and k be positive integers with $k \leq n$, and $A = (\alpha_1, \ldots, \alpha_k)$ be a k-tuple of elements of the finite field $\mathbb{F}_{q^n}$. For any k-set $I = \{i_0, \ldots, i_{k-1}\}$ of non-negative integers, define

$$M_{A,I} = \begin{pmatrix} \alpha_1^{q^{i_0}} & \cdots & \alpha_1^{q^{i_{k-1}}} \\ \vdots & \cdots & \vdots \\ \alpha_k^{q^{i_0}} & \cdots & \alpha_k^{q^{i_{k-1}}} \end{pmatrix}.$$

If $I = \{0, 1, \ldots, k-1\}$, $M_{A,I}$ is known as *Moore matrix* and satisfies the following property: $\det(M_{A,I}) = 0$ if and only if $\alpha_1, \ldots, \alpha_k$ are $\mathbb{F}_q$-linearly dependent. In [1], the set I was defined to be a *Moore exponent set* for q and n when the same property holds. A classification of Moore polynomial sets I for q and n was obtained in [1] under the assumption that $q > 5$ and the elements of I are small enough with respect to n. In this talk a generalization is provided, as follows. Let $\underline{f} = (f_1(x), \ldots, f_k(x))$ be q-linearized polynomials of normalized degree, i.e. polynomials of the form $\sum_{i=0}^{n-1} a_i x^{q^i} \in \mathbb{F}_{q^n}[x]$. Define

$$M_{A,\underline{f}} = \begin{pmatrix} f_1(\alpha_1) & \cdots & f_k(\alpha_1) \\ \vdots & \cdots & \vdots \\ f_1(\alpha_k) & \cdots & f_k(\alpha_k) \end{pmatrix}.$$

We call $\underline{f}$ a *Moore polynomial set* for q and n when $\det(M_{A,\underline{f}}) = 0$ if and only if $\alpha_1, \ldots, \alpha_k$ are $\mathbb{F}_q$-linearly dependent. Moore polynomial sets h are related to other combinatorial objects; for instance, they correspond to $\mathbb{F}_{q^n}$-linear maximum rank-distance (MRD) codes in $\mathbb{F}_q^{n\times n}$. To any Moore polynomial set $\underline{f}$ we attach an algebraic variety $\mathcal{V}$ defined over $\mathbb{F}_{q^n}$ not containing certain $\mathbb{F}_{q^n}$-rational points. We then apply algebraic-geometric tools to obtain some partial classification results for Moore polynomial sets. For example, if $q > 5$, $f_1(x) = x$, and the degrees of $f_1(x), \ldots, f_k(x)$ are small enough with respect to n (plus some non-restrictive assumptions), then $\underline{f} = (x, x^{q^s}, \ldots, x^{q^{(k-1)s}})$ for some s.

[1] D. Bartoli, Y. Zhou: Asymptotics of Moore exponent sets, *J. Combin. Theory Ser. A* 175 (2020), 105281.

[2] D. Bartoli, G. Zini, F. Zullo: Linear maximum rank distance codes of exceptional type. In preparation.

On deficiency problems for graphs

Andrea Freschi

University of Birmingham

(This talk is based on joint work with Joseph Hyde and Andrew Treglown.)

MSC2000: 05C35, 05C70

Motivated by analogous questions in the setting of Steiner triple systems and Latin squares, Nenadov, Sudakov and Wagner recently introduced the notion of *graph deficiency*. Given a global spanning property $\mathcal{P}$ and a graph G, the deficiency $\mathrm{def}(G)$ of the graph G with respect to the property $\mathcal{P}$ is the smallest non-negative integer t such that the join $G * K_t$ has property $\mathcal{P}$, where $G * K_t$ is the graph obtained by adding t new vertices to G and adding all edges incident to at least one of the new vertices. In particular, Nenadov, Sudakov and Wagner raised the question of determining how many edges an n-vertex graph G needs to ensure $G * K_t$ contains a K_r-factor (for any fixed $r \geq 3$). In this talk we present a solution to this problem. We also briefly discuss an analogous result which forces $G * K_t$ to contain any fixed bipartite $(n+t)$-vertex graph of bounded degree and small bandwidth.

arXiv Preprint: `https://arxiv.org/abs/2102.04389`

Extending perfect matchings to Hamiltonian cycles in line graphs

Domenico Labbate

Universitá degli studi della Basilicata - Potenza (Italy)

(This talk is based on joint work with M. Abreu, John Baptist Gauci, Giuseppe Mazzuoccolo and Jean Paul Zerafa.)

MSC2000: 05C70 (05C45 05C76)

A graph admitting a perfect matching has the Perfect-Matching-Hamiltonian property (for short the PMH-property) if each of its perfect matchings can be extended to a Hamiltonian cycle. In this talk we will present some sufficient conditions for a graph G which guarantee that its line graph $L(G)$ has the PMH-property. In particular, we prove that this happens when G is (i) a Hamiltonian graph with maximum degree at most 3, (ii) a complete graph, or (iii) an arbitrarily traceable graph. Further related questions and open problems will be stated.

A note on the covering dimension of a graph

E.C.M. Maritz

University of the Free State

MSC2000: 05C12,05C70

Let $W = \{w_1, w_2, \ldots, w_k\}$ be a vertex cover of a connected graph G. Each vertex v of G can be described by its distance towards the vertices in W using an ordered k-tuple $r(v|W) = (d(v, w_1), d(v, w_2), \ldots, d(v, w_k))$. The set W is a *resolving vertex cover of the graph* G if no two distinct vertices u and v have the same representation, i.e. $r(u|W) = r(v|W)$ implies $u = v$. The smallest k for which W is a resolving vertex cover of G is denoted by $\alpha_{dim}(G)$, the *covering dimension of* G. For certain classes of graphs, $\alpha_{dim}(G)$ is simply equal to the vertex covering number, $\alpha(G)$, or the metric dimension, $dim(G)$ of G. Boundaries for $\alpha_{dim}(G)$ are presented, as well as some results on graphs for which the covering dimension is not equal to $\alpha(G)$ or $dim(G)$.

Choosability with Separation of Cycles and Outerplanar Graphs

Olivier Togni

Université de Bourgogne

(This talk is based on joint work with Jean-Christophe Godin.)

MSC2000: 05C15, 05C38, 05C10

Let a, b, c and k be integers and let G be a graph. A k-list assignment L of G is a function which associates to each vertex a set of k integers. The list assignment L is *c-separating* if for any $uv \in E(G)$, $|L(u) \cap L(v)| \leq c$. The graph G is (a, b, c)-*choosable* if for any c-separating a-list assignment L, there exists an (L, b)-coloring of G, i.e. a coloring function φ on the vertices of G that assigns to each vertex v a subset of b elements from $L(v)$ in such a way that $\varphi(u) \cap \varphi(v) = \emptyset$ for any $uv \in E(G)$.

This type of restricted list coloring problem, called choosability with separation, has been introduced by Kratochvíl, Tuza and Voigt [1]. Most of the existing works concentrate on the case where $b = 1$ and sometimes $c = 1$. For planar graphs, the two following questions are still open (see [1, 2]): Does any planar graph is $(4, 1, 2)$-choosable? $(3, 1, 1)$-choosable?

In this paper we study this problem on cycles and outerplanar graphs. Our aim is to determine, for given a, b, $a \geq b$, the largest c such that G is (a, b, c)-choosable. We define the *separation number* $\mathsf{sep}(G, a, b)$ of G as $\mathsf{sep}(G, a, b) = \max\{c : G \text{ is } (a, b, c)\text{-choosable}\}$. We define analogously the free-separation number $\mathsf{fsep}(G, a, b)$ when an arbitrary vertex is precolored. Clearly, we have $0 \leq \mathsf{fsep}(G, a, b) \leq \mathsf{sep}(G, a, b) \leq a$ for any graph G.

We completely determine the separation and free-separation numbers of the cycle and derive tight bounds for these parameters on cactuses and (slightly less tight) bounds on arbitrary outerplanar graphs. In particular, we prove that for a cactus G of finite girth $g \geq 4$, we have $\mathsf{fsep}(G, a, b) = \mathsf{fsep}(C_g, a, b)$ and for an outerplanar graph G with finite girth $g \geq 5$, we have $\mathsf{fsep}(C_{g-1}, a, b) \leq \mathsf{fsep}(G, a, b) \leq \mathsf{fsep}(C_g, a, b)$, where C_n is the cycle of order n. Further details can be found in [3].

[1] J. Kratochvíl, Z. Tuza, M. Voigt *Brooks-type theorems for choosability with separation*, J. Graph Theory 27 (1998), 43–49.

[2] R. Škrekovski, *A note on choosability with separation for planar graphs*, Ars Comb. 58 (2001), 169–174.

[3] J.-C. Godin, O. Togni, *Choosability with Separation of Cycles and Outerplanar Graphs*, Discuss. Math. - Graph Theory (2021), in press.

Permutations of zerosum sets

Giovanni Falcone

Università di Palermo

(This talk is based on joint work with Marco Pavone.)

MSC2000: 05A18, 05B05, 11B75, 94B05

Given a n-dimensional vector space $\mathcal{P}$ over the Galois field GF(p), $p \geq 2$ a prime, and the family $\mathcal{B}_k$ of all the k-sets of elements of $\mathcal{P}$ summing up to zero, we characterise the permutations of $\mathcal{P}$ inducing permutations of $\mathcal{B}_k$ as the invertible linear mappings of the vector space $\mathcal{P}$ if p does not divide k, and as the invertible affinities of the affine space $\mathcal{P}$ if p divides k. The same question is answered also in the case where the elements of the k-sets are required to be all nonzero, and, in fact, the two cases prove to be intrinsically inseparable [1].

If the prime p is odd and divides k, then the pair $\mathcal{D} = (\mathcal{P}, \mathcal{B}_k)$ is a 2-(p^n, k, λ) design, whose parameter λ can be computed by a result in [3]. For $p = 2$, and k even, $\mathcal{D}$ is a 3-$(2^n, k, \lambda_3)$ design, and in [2] we compute the parameter λ_3 explicitly. Also, for any integer k, with $3 \leq k \leq 2^n - 4$, the k-sets of non-zero elements define a 2-$(2^n - 1, k, \lambda)$ design, and, again, we compute λ explicitly. In both cases, the blocks can be seen as codewords of weight k in the $(2^n - 1, 2^n - n - 1, 3)$ Hamming code (resp., in the extended binary Hamming code of length 2^n), thus they have also been widely studied in the context of Coding theory.

Our result describes the automorphism groups of the above design $\mathcal{D}$.

Moreover, it allows one to relax the definitions of the permutation automorphism groups of both the Hamming codes as the groups of permutations preserving just the codewords of a given Hamming weight (the case of the $(2^n - 1, 2^n - n - 1, 3)$ Hamming code being somehow known, although never explicitly stated).

[1] Falcone G., Pavone M.: Permutations of zero-sumsets in a finite vector space. Forum Math. 33(2), 349–359 (2021).

[2] Falcone, G., Pavone, M. Binary Hamming codes and Boolean designs. Des. Codes Cryptogr. (2021).

[3] J. Li, D. Wan, Counting subset sums of finite abelian groups, J. Combin. Theory, Ser. A 119 (1), pp. 170-182 (2012).

The cap set problem: standard diagrams and new proofs

Henry (Maya) Robert Thackeray

University of Pretoria

MSC2020: 51E20, 05B40, 05D99, 05B25, 51E15

An *n-dimensional cap* C of *size* s, or an *s-cap n-flat* C, is a pair (S_C, F_C), where F_C is an n-dimensional affine space over $\mathbb{Z}/3\mathbb{Z}$ and the subset S_C of F_C consists of s different points, of which no three are in a common line. The points in S_C are the *cap points* of C. For a given positive $n \in \mathbb{Z}$, the cap set problem is: what is the maximum possible s for an s-cap n-flat? It is known that the maximum sizes of 1-, 2-, 3-, 4-, 5-, and 6-dimensional caps are 2, 4, 9, 20, 45, and 112 respectively (Davis & Maclagan 2003, Edel et al. 2002, Potechin 2008). The problem is open for $n \geq 7$.

In this talk, we define *standard diagrams* – these are pictures that give an intuitive view of an established technique to solve the cap set problem – and we use them to find all 18-cap 4-flats up to isomorphism. That information is used to obtain new proofs that 45-cap 5-flats, 44-cap 5-flats, and 112-cap 6-flats are unique up to isomorphism, and to prove that every 43-cap 5-flat is a 45-cap 5-flat with two cap points removed, and every 111-cap 6-flat is a 112-cap 6-flat with one cap point removed. This talk is based on two upcoming papers by the author (Thackeray 2021a-b).

Davis, B. L. and Maclagan, D. 2003. The card game Set. *The Mathematical Intelligencer* 25 (3) 33–40. doi:https://doi.org/10.1007/BF02984846.

Edel, Y., et al. 2002. The classification of the largest caps in AG(5,3). *Journal of Combinatorial Theory Series A* 99 (1) 95–110. doi:https://doi.org/10.1006/jcta.2002.3261.

Potechin, A. 2008. Maximal caps in AG(6,3). *Designs, Codes and Cryptography* 46 (3) 243–259. doi:https://doi.org/10.1007/s10623-007-9132-z.

Thackeray, H. (M.) R. 2021a. The cap set problem and standard diagrams. Submitted to *Discrete Mathematics*.

Thackeray, H. (M.) R. 2021b. The cap set problem: 43-cap 5-flats and 111-cap 6-flats. To be submitted for publication.

Monday 11:20, Zoom 4

Additive and strongly additive block designs

Marco Pavone

Università di Palermo

MSC2000: 05B05, 05B25, 51E05

A block design is *additive* if it can be embedded in a commutative group in such a way that the sum of the points in any block is zero [1]. More precisely, a 2-(v, k, λ) design $\mathcal{D} = (\mathcal{P}, \mathcal{B})$ is additive if there exist a commutative group $(G, +)$ and an injective map $\psi : \mathcal{P} \longrightarrow G$ such that

$$\psi(X) + \psi(Y) + \psi(Z) = 0$$

for any block $\{X, Y, Z\} \in \mathcal{B}$.

Some geometric designs, such as the point-flat designs of an affine geometry $\mathrm{AG}(d, q)$ over the Galois field $\mathrm{GF}(q)$, and the point-flat designs of a projective geometry $\mathrm{PG}(d, 2)$ over $\mathrm{GF}(2)$, are basic examples of additive 2-designs, and are actually the main motivation behind the previous definition. In [1] and [2] it is shown that all symmetric 2-designs and all affine resolvable 2-designs are additive, whereas a Steiner triple system is additive if and only if it is a geometric STS.

A 2-(v, k, λ) design $\mathcal{D} = (\mathcal{P}, \mathcal{B})$ is said to be *strongly additive* if there exists a distinguished embedding of $\mathcal{P}$ in a commutative group $(G, +)$, in such a way that a k-subset of $\mathcal{P}$ is a block if and only if the sum of its elements is zero in G. An open problem is posed in [2] as to whether any additive design is also strongly additive.

We recently showed that there exists a 2-$(16, 4, 2)$ quasidouble of the affine plane of order 4 that is additive but not strongly additive [3].

[1] A. Caggegi, G. Falcone, M. Pavone, On the additivity of block designs, *J. Algebr. Comb.* **45**, 271–294 (2017).

[2] A. Caggegi, G. Falcone, M. Pavone, Additivity of affine designs, *J. Algebr. Comb.* **53**, 755–770 (2021).

[3] M. Pavone, A quasidouble of the affine plane of order 4 and the solution of a problem on additive designs, in preparation.

Group divisible designs with block size 4 and group sizes divisible by 3

Yudhistira A. Bunjamin

UNSW Sydney

(This talk is based on joint work with R. Julian R. Abel and Diana Combe.)

MSC2000: 05B05

A k-GDD, or *group divisible design* with block size k, is a triple $(X, \mathcal{G}, \mathcal{B})$ where X is a set of *points*, $\mathcal{G}$ is a partition of X into subsets (called *groups*) and $\mathcal{B}$ is a collection of k-element subsets of X (called *blocks*) such that any two points from distinct groups appear together in exactly one block and no two distinct points from any group appear together in any block. The *group type* (or *type*) of a k-GDD is the multiset $\{|G| : G \in \mathcal{G}\}$ which denotes the group sizes.

There are a number of known necessary conditions for the existence of a GDD with a particular group type which come from simple counting arguments. However, these conditions are not sufficient. We say that a multiset of positive integers is a *feasible* group type for a k-GDD if it satisfies the currently known necessary conditions.

This talk will focus on 4-GDDs. We will introduce the two most common techniques for constructing GDDs and show how they can be used to construct a 4-GDD for all but a finite number of feasible types 3^t6^s and 3^t9^s as well as other small 4-GDDs with groups sizes divisible by 3.

Path decompositions of random directed graphs

Alberto Espuny Díaz

Technische Universität Ilmenau

(This talk is based on joint work with Viresh Patel and Fabian Stroh.)

MSC2000: 05C70,05C80,05C20

We consider the problem of decomposing the edges of a directed graph into as few paths as possible. There is a natural lower bound for the number paths needed in an edge decomposition of a directed graph in terms of its degree sequence: for each vertex v whose outdegree $d^+(v)$ is larger than its indegree $d^-(v)$, at least $d^+(v) - d^-(v)$ paths must start at v, and conversely, if the indegree is larger than the outdegree, then at least $d^-(v) - d^+(v)$ paths must end at v. Adding this condition over all vertices, we conclude that any edge decomposition of a directed graph D into paths must consist of at least $\frac{1}{2}\sum_{v\in V(D)} |d^+(v) - d^-(v)|$ paths.

A conjecture due to Alspach, Mason and Pullman from 1976 concerning edge decompositions of tournaments into as few paths as possible states that this bound is correct for tournaments of even order. The conjecture was recently resolved for large tournaments in works by Lo, Patel, Skokan and Talbot, and by Girão, Granet, Kühn, Lo and Osthus. In our new work, we investigate to what extent the conjecture holds for directed graphs in general. In particular, we prove that the conjecture holds with high probability for the random directed graph $D_{n,p}$ for a large range of p (thus proving that it holds for most directed graphs). To be more precise, we define a deterministic class of directed graphs for which we can show the conjecture holds, and later show that the random digraph belongs to this class with high probability. Our techniques involve absorption and flows.

Trees in tournaments

Alistair Benford

University of Birmingham

(This talk is based on joint work with Richard Montgomery.)

MSC2000: 05C20

Given an n-vertex oriented tree T, what is the smallest size a tournament G must be, in order to guarantee G contains a copy of T? A strengthening of Sumner's conjecture poses that it is enough for G to have $(n+k-1)$ vertices, where k is the number of leaves of T. Recently, Dross and Havet used a method of median orders to prove that this is true for arborescences – i.e. trees with edges oriented outwards from a specified root vertex. We show that median orders can make further progress towards $(n+k-1)$, by proving that there exists a constant C such that $|G| = (n+Ck)$ is enough, as well as confirming a separate conjecture that $|G| = (n+k-2)$ is enough, provided we allow n to grow large with k fixed. In this talk we shall discuss these results and further progress that could be made.

Spanning trees in dense directed graphs

Amarja Kathapurkar

University of Birmingham

(This talk is based on joint work with Richard Montgomery.)

MSC2000: 05C20, 05C05, 05C35

In 2001, Komlós, Sárközy and Szemerédi proved that for sufficiently large n, every n-vertex graph with minimum degree at least $n/2 + o(n)$ contains a copy of every n-vertex tree with maximum degree at most $O(n/\log n)$. We prove the corresponding result for directed graphs. That is, we show that for sufficiently large n, every n-vertex directed graph with minimum semidegree at least $n/2 + o(n)$ contains a copy of every n-vertex oriented tree whose underlying maximum degree is at most $O(n/\log n)$.

This improves a recent result of Mycroft and Naia, which requires the oriented trees to have underlying maximum degree at most Δ, for any constant Δ and sufficiently large n. In contrast to the previous work on spanning trees in dense directed or undirected graphs, our approach does not use Szemerédi's regularity lemma.

Monday 11:45, Zoom 5

Path decompositions of tournaments

Bertille Granet

University of Birmingham

(This talk is based on joint work with António Girão, Daniela Kühn, Allan Lo, and Deryk Osthus.)

MSC2000: 05C20, 05C35, 05C38, 05C70, 05D40

In 1976, Alspach, Mason, and Pullman conjectured that any tournament T of even order can be decomposed into exactly $\mathrm{ex}(T)$ paths, where $\mathrm{ex}(T) \coloneqq \frac{1}{2}\sum_{v\in V(T)} |d_T^+(v) - d_T^-(v)|$ ($d_T^+(v)$ and $d_T^-(v)$ denote the out and indegree of v in T, respectively). We prove this conjecture for all sufficiently large tournaments. We also prove an asymptotically optimal result for tournaments of odd order.

Improved bounds on the cop number of a graph drawn on a surface

Florian Lehner

Graz University of Technology

(This talk is based on joint work with Joshua Erde.)

MSC2000: 05C57; 91A46; 91A24

The cops-and-robber game is a game on a graph played between two players controlling a set of cops and a single robber, respectively. The rules of the game are as follows: In the first round both the cops and the robber choose starting vertices. After that, in even rounds each cop can move to a neighbouring vertex, and in odd rounds the robber can move to a neighbouring vertex. The cops win, if after some finite number of rounds one of them occupies the same vertex as the robber.

A graph G is called k-cop win, if there is a winning strategy for a set of k cops, the cop number $c(G)$ is the least number k such that G is k-cop win. It is known that if a connected graph G can be embedded in a surface of genus g, then $c(G)$ can be bounded by a linear function in g. Schröder showed that $c(G) \leq \frac{3}{2}g + 3$ and conjectured that this can be improved to $c(G) \leq g(G)+3$. Recently, by relating the cops-and-robber to a topological game, Bowler, Erde, Pitz, and I gave the current best known bound $c(G) \leq \frac{4g(G)}{3} + \frac{10}{3}$.

In this talk we show, how this topological game can be combined with techniques introduced by Schröder to improve this bound and show that $c(G) \leq (1 + o(1))(3 - \sqrt{3})g$.

The Toucher-Isolator game

Mirjana Mikalački

Department of Mathematics and Informatics, Faculty of Sciences, University of Novi Sad, Novi Sad, Serbia

(This talk is based on joint work with Chris Dowden, Mihyun Kang and Miloš Stojaković.)

MSC2000: 05C57, 91A46

In this talk we introduce a new positional game called 'Toucher-Isolator', which is a quantitative version of a Maker-Breaker type game. The playing board is the edge set of a given graph G. The two players, Toucher and Isolator, claim edges alternately. The aim of Toucher is to 'touch' as many vertices as possible (i.e. to maximise the number of vertices that are incident to at least one of her chosen edges), and the aim of Isolator is to minimise the number of vertices that are so touched.

We analyse the number of untouched vertices $u(G)$ at the end of the game when both Toucher and Isolator play optimally, obtaining results both for general graphs and for particularly interesting classes of graphs, such as cycles, paths, trees, and k-regular graphs.

Burning giant sequoias

Paul Bastide

ENS Rennes

(This talk is based on joint work with Marthe Bonamy, Pierre Charbit, Théo Pierron, Mikaël Rabie.)

MSC2000: 05C05

How fast can a rumor propagate in a graph? One measure of that, introduced by Bonato, Janssen and Roshanbin [2], is the *burning number* $b(G)$ of a graph G. At step 1, we set a vertex on fire. At every step $i \geq 2$, all neighbours of a vertex on fire catch fire themselves, and we set a new vertex on fire. If at the end of step k the whole graph is on fire, then the graph is k-burnable. The burning number of G is defined to be the least k such that G is k-burnable.

A graph with n isolated vertices is trivially not $(n-1)$-burnable. We therefore focus on connected graphs. Paths are an interesting special case. For a path P_n on n vertices, it is not hard to check that $b(P_n) = \lceil\sqrt{n}\rceil$. When introducing the notion, Bonato et al. [2] conjectured that paths are, essentially, the worst case for the burning number of a graph.

Conjecture 1 (Bonato et al. [2]). *Every connected graph G satisfies $b(G) \leq \lceil\sqrt{|V(G)|}\rceil$.*

Conjecture 1 is only known to hold with a constant factor. We focus on trees that are taller than they are wide. More formally, we use the following definition.

Definition 2. *The* growth *of a tree T is the smallest k such that all vertices in T are within distance k of some path P in T.*

Note that a caterpillar has growth at most 1. The conjecture has only been confirmed for trees with growth at most 2. We refer the reader to a nice recent survey on the topic for further details [1]. In this talk we present the following theorem.

Theorem 3. *For any tree T on n vertices, if n is large enough compared to the growth of T, then $b(T) \leq \lceil\sqrt{n}\rceil + 1$.*

We also obtain that for any fixed k, in order to prove Conjecture 1 for trees of growth at most k, it suffices to verify it for a finite number of them.

[1] Anthony Bonato. A survey of graph burning. *arXiv preprint arXiv:2009.10642*, 2020.

[2] Anthony Bonato, Jeannette Janssen, and Elham Roshanbin. Burning a graph is hard, 2015.

Combinatorial aspects of Abelian and stochastic sandpile models on complete graphs

Thomas Selig

Xi'an Jiaotong-Liverpool University

MSC2000: 05A15, 05A19, 60J10

The sandpile model is a dynamic process on a graph G. At each unit of time, a grain of sand is added to a randomly selected vertex of G. When this causes the number of grains at that vertex to exceed a certain threshold (usually its degree), that vertex is said to be unstable, and topples, sending grains to (some of) its neighbours in G. In the standard Abelian sandpile model (ASM), topplings are deterministic: one grain is sent to each neighbour of an unstable vertex. In the stochastic sandpile model (SSM) an unstable vertex flips a coin for each neighbour to decide whether it should send a grain or not [1]. Of central interest in sandpile model research are the recurrent states, those that appear infinitely often in the long-time running of the model.

In this talk, we focus on the Abelian and stochastic sandpile models on complete graphs. We first recall the Cori-Rossin bijection between the set of recurrent states on the complete graph K_{n+1} and the set of n-parking functions [2]. We then study the SSM on complete graphs. We show that the set of recurrent states of the SSM is given by the integer lattice points in the parking function polytope. This allows us to recover a well-known result: that the number of integer lattice points in the n-dimensional permutation polytope is the number of labeled spanning forests on n vertices.

[1] Y. Chan, J.-F. Marckert, and T. Selig. A natural stochastic extension of the sandpile model on a grpah. *Journal of Combinatorial Theory - Series A*, 120(7): 1913-1928, 2013.

[2] R. Cori and D. Rossin. On the sandpile group of a graph. *European Journal of Combinatorics*, 21:447–459, 2000.

[3] T. Selig. On the stochastic sandpile model on complete graphs. In preparation.

Monday 15:00, Zoom 1

Clique immersions and independence number

Daniel Quiroz

Instituto de Ingeniería Matemática, Universidad de Valparaíso, Chile

(This talk is based on joint work with Sebastián Bustamante, Maya Stein, José Zamora.)

MSC2000: 05C15, 05C83

We give lower bounds for the order of the largest clique immersion in a graph with fixed independence number. This problem is motivated by an immersion-analogue of Hadwiger's conjecture, which, if true, would imply that every n-vertex graph with independence number α has a clique immersion of order at least n/α. Our bounds improve, for all $\alpha \geq 3$, previous results of Gauthier, Le and Wollan.

New Results on α-critical Graphs

Craig Larson

Virginia Commonwealth University

(This talk is based on joint work with Jack Edmonds & Mark Kayll.)

MSC2000: 05C69,05C75

A graph G is α-critical if, for every edge xy, the independence number of $G - xy$ is greater than the independence number of G. The study of critical graphs goes back to Dirac in the 1950s, and of α-critical graphs to Erdős and Gallai in the 1960s. They are deeply connected to the study of the independence structure of graphs and to the study of the stable set polytope. We present some new results—developed in connection with an investigation of two graph decompositions—as well as new proofs of classical results of Wessel, Andrásfai and Lovász.

Monday 15:00, Zoom 2

On the 486-vertex distance-regular graphs of Koolen–Riebeek and Soicher

Robert F. Bailey

Grenfell Campus, Memorial University of Newfoundland

(This talk is based on joint work with Daniel R. Hawtin.)

MSC2000: 05C25, 05E30, 20B25, 94B25

In this talk, we consider three imprimitive distance-regular graphs with 486 vertices and diameter 4: the Koolen–Riebeek graph (which is bipartite), the Soicher graph (which is antipodal), and the incidence graph of a symmetric transversal design obtained from the affine geometry $\mathrm{AG}(5,3)$ (which is both). We will show that each of these is preserved by the same rank-9 action of the group $3^5 : (2 \times \mathrm{M}_{10})$, and the connection is explained using the ternary Golay code.

[1] R. F. Bailey and D. R. Hawtin, On the 486-vertex distance-regular graphs of Koolen–Riebeek and Soicher, *Electronic J. Combin.* **27** (2020), P3.13 (12pp).

[2] A. E. Brouwer, J. H. Koolen and R. J. Riebeek, A new distance-regular graph associated to the Mathieu group M_{10}, *J. Algebraic Combin.* **8** (1998), 153–156.

[3] L. H. Soicher, Three new distance-regular graphs, *European J. Combin.* **14** (1993), 501–505.

On some recent results on 2-(v,k,λ) symmetric designs with small λ

Sanja Rukavina

University of Rijeka

(This talk is based on joint work with Dean Crnković .)

MSC2000: 05B05, 94B05

Fundamental problems of design theory are those of existence and classification of designs with certain parameter set. In this talk we are interested in biplanes and triplanes, i.e., in 2-$(v,k,2)$ and 2-$(v,k,3)$ symmetric designs.

The existence of a biplane with parameters $(121,16,2)$ is an open problem. Recently, in [1], it has been proved by Alavi, Daneshkhah and Praeger that the order of an automorphism group of a possible biplane $\mathcal{D}$ of order 14 divides $2^7 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 13$. We show that such a biplane do not have an automorphism of order 11 or 13, and thereby establish that $|Aut(\mathcal{D})|$ divides $2^7 \cdot 3^2 \cdot 5 \cdot 7$. Further, we exclude a possible action of some small groups of order divisible by five or seven, on a biplane with parameters $(121,16,2)$.

Triplanes of order 12, i.e. symmetric $(71,15,3)$ designs, have the greatest number of points among all known triplanes and it is not known if a triplane $(v,k,3)$ exists for $v > 71$. All 146 previously known $(71,15,3)$ designs admit an action of an automorphism of order 3. We give the first example of a triplane of order 12 that does not admit an automorphism of order 3, obtained by using binary linear codes.

[1] S. H. Alavi, A. Daneshkhah, C. E. Praeger, Symmetries of biplanes, Des. Codes Cryptogr. 88 (2020), 2337–2359.

[2] D. Crnković, D. Dumičić Danilović, S. Rukavina, On automorphism groups of a biplane (121,16,2), preprint, arXiv:2010.12944

[3] D. Crnković, S. Rukavina, L. Simčić, On triplanes of order twelve admitting an automorphism of order six and their binary and ternary codes, Util. Math. 103 (2017), 23–40.

[4] S. Rukavina, Some new triplanes of order twelve, Glas. Mat. Ser. III 36(56) (2001), 105–125.

On intersecting families of independent sets in trees

Glenn Hurlbert

Virginia Commonwealth University, USA

(This talk is based on joint work with Vikram Kamat, Villanova University, USA.)

MSC2000: 05D05 (05C05)

A family of sets is *intersecting* if every pair of its sets intersect. A star is a family with some element (a *center*) in each of its sets. The classical 1961 result of Erdős, Ko, and Rado states that every intersecting family of r-sets with $r \leq n/2$ has size at most that of a star. There are many clever and beautiful proofs of this result; we recently discovered a new injective proof [2].

Let G be a graph, $\alpha(G)$ be its independence number, and $\mu(G)$ be the size of the smallest maximal independent set in G. We say that G is r-EKR if, among all intersecting families of independent r-sets of G, the largest is attained by a star. Holroyd and Talbot conjectured that every graph G is r-EKR for all $1 \leq r \leq \mu(G)/2$. We verified the conjecture in [1] for all chordal graphs containing an isolated vertex.

For graphs without isolated vertices it is difficult to determine the center of the largest star, which is often necessary to prove that they are EKR. A tree has the *r-leaf property* if its largest r-star occurs on one of its leaves. We proved in [1] that every tree T has the r-leaf property when $r \leq 4$, but counterexamples exist when $r \geq 5$. Thus we are led to ask which trees T have the *leaf property*: the r-leaf property for all $r \leq \alpha(T)$.

A *split* vertex in a tree is a vertex of degree at least 3. A *spider* is a tree with exactly one split vertex. In [3] we prove that all spiders have the leaf property, and we characterize which of its leaves are maximum star centers. A *pendant* tree is one for which each of its split vertices is adjacent to a leaf. Estrugo and Pastine recently showed that all pendant trees have the leaf property. Here we also consider pendant trees with exactly two split vertices and provide partial results on the locations of their maximum star centers.

[1] G. Hurlbert and V. Kamat, *Erdős–Ko–Rado theorems for chordal graphs and trees*, J. Combin. Theory Ser. A **118** (2011), no. 3, 829–841.

[2] G. Hurlbert and V. Kamat, *New injective proofs of the Erdős-Ko-Rado and Hilton-Milner theorems*, Discrete Math. **341** (2018), no. 6, 1749–1754.

[3] G. Hurlbert and V. Kamat, *On intersecting families of independent sets in trees*, arXiv:1620.08153 (2021).

The number of locally p-stable functions on Q_n

Asier Calbet

Queen Mary University of London

MSC2000: 05A16

A Boolean function $f : V \to \{-1, 1\}$ on the vertex set of a graph $G = (V, E)$ is *locally p-stable* if for every vertex v the proportion of neighbours w of v with $f(v) = f(w)$ is exactly p. This notion was introduced by Gross and Grupel in [1] while studying a problem on scenery reconstruction. They give an exponential type lower bound for the number of isomorphism classes of locally p-stable functions when $G = Q_n$ is the n-dimensional Boolean hypercube and ask for more precise estimates. We provide such estimates by improving the lower bound to a double exponential type lower bound and finding a matching upper bound. We also show that for a fixed k and increasing n, the number of isomorphism classes of locally $(1 - k/n)$-stable functions on Q_n is eventually constant. The proofs use the Fourier decomposition of functions on the Boolean hypercube.

[1] Renan Gross and Uri Grupel. *Indistinguishable sceneries on the Boolean hypercube.* Combinatorics, Probability and Computing, Volume 28, Issue 1, January 2019 , pp. 46 - 60.

Monday 15:00, Zoom 4

Del Pezzo Surfaces of Rank Two over Finite Fields

Anton Betten

Colorado State University

(This talk is based on joint work with Fatma Karaoglu.)

MSC2000: 05E14

A del Pezzo surface of rank two is a double cover of a plane quartic curve. Smooth quartics have 28 bitangents and are related to cubic surfaces with 27 lines using projection from a point not on any line. The classification problem is the problem of determining the equivalence classes of the objects under the action of the various projective groups. Despite a long history of research, the classification problem for cubic surfaces, del Pezzo surfaces and quartic curves is still open. Over finite fields, the problem can be attacked using computer.

We will describe recent progress on the classification of del Pezzo surfaces over small fields. This is based on earlier work of the speakers on the classification of cubic surfaces with 27 lines over small finite fields. Our goal is to identify interesting infinite families of objects, and to determine their properties, including their automorphism groups. On the geometric side, we are interested in Kowalevski points, which are points where four bisecants intersect. These are similar to Eckardt points on cubic surfaces, where three lines meet. Kowalevski in 1884 observed that such points are related to involutorial automorphisms of the curve.

Anton Betten and Fatma Karaoglu. *Cubic surfaces over small finite fields.* Des. Codes Cryptogr., 87(4):931–953, 2019.

I. Dolgachev, *Endomorphisms of complex abelian varieties, Milan, February 2014* http://www.math.lsa.umich.edu/ idolga/MilanLect.pdf

F.E. Eckardt. *Ueber diejenigen Flächen dritten Grades, auf denen sich drei gerade Linien in einem Punkte schneiden*, Math. Ann. 10 (1876), 227-272.

J. W. P. Hirschfeld. *Finite projective spaces of three dimensions.* Oxford Mathematical Monographs. The Clarendon Press, Oxford University Press, New York, 1985. Oxford Science Publications.

J. W. P. Hirschfeld. *del Pezzo surfaces over finite fields.* Tensor (N.S.), 37(1):79–84, 1982.

S. Kowalevski, *Über Reduction einer bestimmten Klasse Abelscher Integrale 3ten Ranges auf elliptische Integrale*, Acta Mathematica **4** (1884), 393–416.

Generalized pentagonal geometries

Tony Forbes

(ORCID: 0000-0003-3805-7056)

(This talk is based on joint work with Carrie Rutherford (LSBU).)

MSC2000: 05B25

In 2013, S. Ball, J. Bamberg, A. Devillers and K. Stokes introduced the concept of a *pentagonal geometry* as a generalization of the pentagon. A pentagonal geometry PENT(k, r) is a partial linear space where every line is incident with k points, every point is incident with r lines, and for each point x there is a line incident with precisely those points that are not collinear with x. The pentagon is a PENT(2,2).

In this talk we go a step further to define a *generalized pentagonal geometry*, PENT(k, r, w): a partial linear space where every line is incident with k points, every point is incident with r lines, and for each point x the points not collinear with x form the point set of a Steiner system $S(2,k,w)$ whose blocks are lines of the geometry. This is a reasonable extension. An $S(2,k,w)$ has the same relevant property as a single line: any two points are collinear. Observe that a line in a PENT(k, r) is an $S(2,k,k)$ and that the pentagon is a PENT(2,2,2).

We explore basic properties and the existence spectrum of these structures.

Explicit asymptotic formulae for multiplicative combinatorial sructures.

B.L. Granovsky

MSC2000: 05A16,05A17

We obtain explicit formulae for the solution of the Khintchine type equation for multiplicative combinatorial models. The latter setting encompasses a wide class of weighted partitions with generating functions of multiplicative form. Our main result states that the solution is given by an infinite power series with coefficients which are polynomials specified explicitly by their roots. As in the case of the original Khintchine equation(1954), the solution obtained is the point of minimum of the entropy(=logarithm of the generating function) of the associated scheme of statistical mechanics.

Our presentation is based on the preprint [1] and it continues the author's paper [2] and two joint papers [3],[4] with Dudley Stark.

[1] Granovsky, B. Explicit asymptotic formulae for multiplicative combinatorial sructures, (2021), preprint

[2] Granovsky, B. Asymptotic enumeration by Khintchine- Meinardus probabilistic method: Necessary and sufficient conditions for exponential growth, Ramanujan J.(2018),45:739-765.

[3] Granovsky, B. and Stark, D. (2012), A Meinardus theorem with multiple singularities. *Comm. Math. Phys.* **314** 329–350.

[4] Granovsky, B, Stark, D (2015), Developments in the Khintchine-Meinardus probabilistic method for asymptotic enumeration, *EJC*, **22,4**.

THE $m = 2$ AMPLITUHEDRON AND THE HYPERSIMPLEX: SIGNS, CLUSTERS, TRIANGULATIONS, EULERIAN NUMBERS

Matteo Parisi

University of Oxford, *Mathematical Institute*
Princeton University, *Visiting Sachs Scholar*

(This talk is based on joint work with L. K. Williams and M. Sherman-Bennett [1].)

MSC2000: 05E14, 13F60

The *hypersimplex* $\Delta_{k+1,n}$ is the image of the Grassmannian $\mathrm{Gr}_{k+1,n}$ and the positive Grassmannian $\mathrm{Gr}_{k+1,n}^{\geq 0}$ under the well-knwon moment map [2]. It is a polytope of codimension 1 inside $\mathbb{R}^n$. Meanwhile, the *amplituhedron* $\mathcal{A}_{n,k,2}$ is the projection of the positive Grassmannian $\mathrm{Gr}_{k,n}^{\geq 0}$ into the Grassmannian $\mathrm{Gr}_{k,k+2}$ under the amplituhedron map $\tilde{Z}$ [3]. Introduced in the context of the physics of *scattering amplitudes*, it is not a polytope and has full dimension $2k$ inside $\mathrm{Gr}_{k,k+2}$. Nevertheless, as was first discovered in [4], these two objects appear to be closely related via *T-duality*. In this work we draw new striking connections between $\Delta_{k+1,n}$ and $\mathcal{A}_{n,k,2}$. We show that inequalities cutting out *positroid polytopes* – images of positroid cells of $\mathrm{Gr}_{k+1,n}^{\geq 0}$ under the moment map – translate into sign conditions characterizing the T-dual *Grasstopes* – images of positroid cells of $\mathrm{Gr}_{k,n}^{\geq 0}$ under the amplituhedron map. Moreover, we subdivide the amplituhedron into *chambers* enumerated by the Eulerian numbers, just as the hypersimplex can be subdivided into simplices enumerated by the Eulerian numbers. We use this to prove one direction of the conjecture of [4]: whenever a collection of positroid polytopes triangulates the hypersimplex, the collection of T-dual Grasstopes triangulates the amplituhedron. Along the way, we prove several more conjectures: Arkani-Hamed–Thomas–Trnka's conjecture that $\mathcal{A}_{n,k,2}$ can be characterized using sign conditions [5], Lukowski–Parisi–Spradlin–Volovich's conjectures about *generalized triangles* (Grasstopes in a triangulation of $\mathcal{A}_{n,k,2}$), and $m = 2$ *cluster adjacency* [6]. Finally, we find novel cluster structures in the amplituhedron.

[1] MP, M. Sherman-Bennett, and L. K. Williams. *The $m = 2$ amplituhedron and the hypersimplex: signs, clusters, triangulations, Eulerian numbers.* Preprint, `arXiv:2104.08254`.

[2] I. M. Gelfand, R. M. Goresky, R. D. MacPherson, and V. V. Serganova. *Combinatorial geometries, convex polyhedra, and Schubert cells.* Advances in Mathematics, 63(3):301–316, 1987.

[3] N. Arkani-Hamed and J. Trnka. *The Amplituhedron.* J. of High Energy Physics 2014, 30.

[4] T. Lukowski, MP, L. K. Williams. *The positive tropical Grassmannian, the hypersimplex, and the $m = 2$ amplituhedron.* Preprint, `arXiv:2002.06164`.

[5] N. Arkani-Hamed, H. Thomas, and J. Trnka. *Unwinding the Amplituhedron in Binary.* J. High Energy Physics 2018, 16.

[6] T. Łukowski, MP, M. Spradlin A. Volovich. *Cluster Adjacency for $m = 2$ Yangian Invariants.* J. High Energy Physics 2019, 10.

The language of self-avoiding walks, part I

Wolfgang Woess

Institute of Discrete Mathematics, Graz University of Technology, Austria

(This talk is based on joint work with with Christian Lindorfer [1].)

MSC2000: 05C25, 20F10, 68Q45

Let X be the Cayley graph of a finitely generated (typically infinite) group with respect to a finite, symmetric set Σ of generators. We consider the edges to be oriented, so that each edge is labelled by an element of Σ. Consider the language of all words over Σ which can be read along a self-avoiding walk starting at the group identity. We characterise under which conditions on the graph structure this language is regular or context-free. This is the case if and only if the graph has more than one end, and the size of all ends is 1, or at most 2, respectively. More generally, this holds for any deterministically labelled, quasi-transitive graph.

[1] Ch. Lindorfer and W. Woess: *The language of self-avoiding walks,* Combinatorica **40** (2020) 691-720.

The language of self-avoiding walks, part II

Christian Lindorfer

Institute of Discrete Mathematics, Graz University of Technology, Austria

(This talk is based on joint work with Florian Lehner [1].)

MSC2000: 05C25, 20F10, 68Q45

Let X be the Cayley graph of a finitely generated group with respect to some finite, symmetric generating set, where each directed edge is labelled with its generator. The language of self-avoiding walks consists of all words which can be read along self-avoiding walks on X.
In this talk we discuss a recent characterisation of the language of self-avoiding walks on virtually free groups. This language is k-multiple context-free if and only if the size of all ends of X is at most $2k$. More generally, this result also holds for deterministically labelled quasi-transitive graphs. Moreover, our approach shows that the connective constant of any thin-ended quasi-transitive graph is an algebraic number.

[1] F. Lehner and C. Lindorfer: *Self-avoiding walks and multiple context-free languages,* preprint, arXiv:1205.2525 [math.CO] (2020).

Large Arcs in Small Planes

Awss Alogaidi

Middle Technical University

(This talk is based on joint work with Anton Betten from Colorado State University.)

MSC2000: 14Nxx

An arc of degree d in a projective plane is a set of n points with no more than d of them collinear. It is denoted as $(n, d)-$arc. Examples arise from algebraic curves of degree d. An important task is to determine for each value of d and q the largest value of n for which an $(n, d)-$arc exists. We are interested in studying large arcs of degree d in $\mathrm{PG}(2, q)$ for small q. A related problem is that of classifying arcs up to projective equivalence. The talk will survey some of the techniques which are used to classify arcs: Complete searches with classification using poset classification; liftings of smaller arcs using techniques of Cook, Ball and others; isomorph classification using canonical forms; parallel computing. Iterestingly, largest arcs do not always arise from curves of degree d, so it is of interest to build models for the known examples. Such models may lead to new constructions of arcs and perhaps to infinite families. We will consider specific problems from the plane $\mathrm{PG}(2, 11)$, with a particular emphasis on arcs of degree 5.

Small complete caps in $\mathrm{PG}(4n+1, q)$

Francesco Pavese

Polytechnic University of Bari

(This talk is based on joint work with A. Cossidente, B. Csajbók and G. Marino.)

MSC2000: 51E22, 51E20

Let $\mathrm{PG}(r, q)$ denote the r-dimensional projective space over the finite field with q elements $\mathbb{F}_q$. A *k-cap* of $\mathrm{PG}(r, q)$ is a set of k points no three of which are collinear. A k-cap of $\mathrm{PG}(r, q)$ is said to be *complete* if it is not contained in a $(k+1)$-cap of $\mathrm{PG}(r, q)$. The study of caps is not only of geometrical interest, but arises from coding theory. Indeed, by identifying the representatives of the points of a complete k-cap of $\mathrm{PG}(r, q)$ with columns of a parity check matrix of a q–ary linear code, it follows that (apart from three sporadic exceptions) complete k-caps of $\mathrm{PG}(r, q)$ with $k > r+1$ and non-extendable linear $[k, k-r-1, 4]_q$ 2–codes are equivalent objects.

One of the main issue is to determine the spectrum of the sizes of complete caps in a given projective space and in particular their maximal and minimal possible values. For the size $t_2(r, q)$ of the smallest complete cap in $\mathrm{PG}(r, q)$, the trivial lower bound is $t_2(r, q) > \sqrt{2} q^{\frac{r-1}{2}}$. Apart from the cases q even and r odd, all known infinite families of complete caps explicitly constructed in $\mathrm{PG}(r, q)$ have size far from the trivial bound.

In this talk I will describe the construction of a complete cap of $\mathrm{PG}(4n+1, q)$ of size $2(q^{2n} + \ldots + 1)$ that is obtained by projecting two disjoint Veronese varieties of $\mathrm{PG}(n(2n+3), q)$ from a suitable $(2n^2-n-2)$-dimensional projective space. This establishes that the trivial lower bound on $t_2(4n+1, q)$ is essentially sharp.

Power sum polynomials and the ghosts behind them

Silvia M.C. Pagani

Università Cattolica del Sacro Cuore, Brescia, Italy

(This talk is based on joint work with Silvia Pianta.)

MSC2000: 11T06, 51E15, 52C05

The Rédei factor of a point $P \in \mathrm{PG}(n, q)$ is the linear polynomial in $n + 1$ variables, whose coefficients are the coordinates of P. Given a subset S of $\mathrm{PG}(n, q)$, its power sum polynomial is the sum of the $(q-1)$-th powers of the Rédei factors associated to the points of S [3]. Differently from the well-known Rédei polynomial, a same power sum polynomial may be shared by several subsets. Such a lack of uniqueness offers a straightforward connection to other inverse problems, in particular to discrete tomography, where the aim is to reconstruct the internal of an object, seen as a density function, from the knowledge of its projections. A central role is played by ghosts, which are functions with null projections and therefore can be added to a solution of a tomographic problem to obtain another solution [1].

In this talk, partly published as [2], we will deal with the two-dimensional case and define the counterpart of tomographic ghosts in the finite geometry context as those subsets with associated null power sum polynomial, which are called ghosts as well. These are also called generalized Vandermonde sets in [3]. The binary operation which makes subsets and ghosts interact is the multiset sum (modulo the field characteristic).

We will prove some general results on ghosts in $\mathrm{PG}(2, q)$, present some classes of examples and explicitly compute their number in case q is a prime.

[1] L. Hajdu, R. Tijdeman. Algebraic aspects of discrete tomography. *J. Reine Angew. Math.* 534, pp. 119–128 (2001).

[2] S.M.C. Pagani, S. Pianta. Power sum polynomials in a discrete tomography perspective. *Lecture Notes in Comput. Sci.* 12708, pp. 325-337 (2021).

[3] P. Sziklai. *Polynomials in Finite Geometry.* Manuscript. Available online at http://web.cs.elte.hu/ sziklai/polynom/poly08feb.pdf

On the Multiplicity of a Nonintersecting Chord Diagram Generated by Chord Expansions

Tomoki Nakamigawa

Shonan Institute of Technology

MSC2000: 05A15

For a set of vertices V on a circumference, let $C(V)$ denote the set of chords having their endvertices in V. A chord diagram $E \subset C(V)$ is a set of chords such that they share no endvertex with each other. The expansion of a chord diagram E with respect to a pair of crossing chords $S = \{ac, bd\} \subset E$ is an operation replacing E with two chord diagrams $E_1 = (E \setminus S) \cup \{ad, bc\}$ and $E_2 = (E \setminus S) \cup \{ab, cd\}$. Beginning from a chord diagram E, by a finite sequence of expansions, we have a multiset of nonintersecting chord diagrams $\mathcal{NCD}(E)$, which is uniquely determined not depending on the choice of expansions. For a chord diagram E, and for a nonintersecting chord diagram F, let $m(E, F)$ denote the multiplicity of F in $\mathcal{NCD}(E)$.

The main purpose of the paper is to study $m(E, F)$ in a general case. For a triangulation T on V and for a chord diagram $E \subset C(V)$, let $w_T(E)$ denote a Laurent polynomial of E with respect to T, which is naturally defined in the setting of cluster algebra. A variable of $w_T(E)$ is corresponding to a chord of T, and if all chords of F are chords of T, then $w_T(F)$ is a monomial. In this paper, it is shown that for $F \subset T$, $m(E, F)$ equals the coefficient of a monomial $w_T(F)$ in a Laurent polynomial $w_T(E)$.

Acknowledgements. This work was supported by JSPS KAKENHI Grant Number 19K03607.

Graphs Reconstructible from One Card and a One–Dimensional Eigenspace

Irene Sciriha

Department of Mathematics, Faculty of Science, University of Malta, Msida, Malta

MSC2000: 05C60, 05C07 05C50 15A18 05B20

The deck $\mathcal{D}$ of a graph G is its multiset of one-vertex deleted subgraphs. We prove that a graph G with a given generator of the eigenspace of any simple eigenvalue μ of the 0–1–adjacency matrix is reconstructed uniquely from one μ–card of $\mathcal{D}$, that is, a one-vertex deleted subgraph that does not have μ as an eigenvalue. If the generator of the μ–eigenspace is known to be full, that is if it has no zero entries, the graph is said to be a μ–nut graph. For a μ–nut graph, the reconstruction holds from any card. No two non-isomorphic μ–nut graphs having a common μ–card, have the same associated one-dimensional eigenspace. Moreover two non-isomorphic μ–nut graphs with the same simple eigenvalue and associated eigenspace have no card in common.

DIAMETER OF A GENERALIZATION OF GENERALIZED PETERSEN GRAPHS

Laila Loudiki

LMC Laboratory, Department of Mathematics and Computer Science, Polydisciplinary Faculty of Safi, Cadi Ayyad University, Safi, Morocco

(This talk is based on joint work with Mustapha Kchikech and EL Hassan Es-Saky.)

MSC2000: 05C12, 05C75

The edge contraction of the generalized Petersen graph $GPG(n,s)$ gives the well known circulant graph $C_n(1,s)$. By a reverse procedure, $GPG(n,s)$ can be obtained from $C_n(1,s)$. This natural relation between these graphs led Beenker and Van Lint [1] to prove that if $C_n(1,s)$ has diameter d, then $GPG(n,s)$ has diameter at least $d+1$, and at most $d+2$.

In [2], we are providing necessary and sufficient conditions so that the diameter of $GPG(n,s)$ is equal to $d+1$, and sufficient conditions so that the diameter of $GPG(n,s)$ is equal to $d+2$. Afterwards, we are also showing that there exists an algorithm computing the diameter of generalized Petersen graphs with running time $O(\log n)$. And, we are giving exact values for the diameter of $GPG(n,s)$ for almost all cases of n and s.

In this work, we present a generalization of generalized Petersen graphs, which we call $GGPG$-graphs, that differs from generalized Petersen graphs in allowing the number of chords to be greater than 1. The purpose is to determine the diameter of $GGPG$-graphs. In particular, we prove that if the circulant graph has diameter d, then the $GGPG$-graph has diameter at least $d+1$ and at most $d+2$. Then, we provide necessary and sufficient conditions so that the diameter of $GGPG$-graphs is equal to $d+1$, and sufficient conditions so that the diameter of $GGPG$-graphs is equal to $d+2$.

[1] G. F. M. Beenker and J. H. Van Lint, Optimal generalized Petersen graphs, *Philips. J. Res.* **43**(2) (1988) 129–136.

[2] L. Loudiki, M. Kchikech and E. H. Essaky, Diameter of generalized Petersen graphs, *arXiv preprint arXiv:2102.10397* (2021).

Algorithm for Computing the Total Vertex Irregularity Strength of the Generalized Petersen Graphs

Rikayanti

Institut Teknologi Bandung, Indonesia

(This talk is based on joint work with Suhadi Wido Saputro and Edy Tri Baskoro.)

MSC2000: 05C78, 05C85

The total vertex irregularity strength of a graph was introduced by Bača, Jendrol', Miller, and Ryan (2002). Let $G = (V, E)$ be a graph. Any surjective mapping $\alpha : V \cup E \to \{1, 2, \cdots, t\}$ is called a t-labeling on G. The *weight* $wt(u)$ of a vertex u in G, under α, is defined as $wt(u) = \alpha(u) + \sum_{uw \in E} \alpha(uw)$. The labeling α of G is called a *total vertex irregular labeling* if all weights of the vertices are distinct, namely $wt(u) \neq wt(w)$ for any distinct vertices u and w. The *total vertex irregularity strength* of the graph G, denoted by $tvs(G)$, is the smallest integer k such that G admits a total vertex irregular k-labeling. Nurdin, et al. (2010) has derived a lower bound of the total vertex irregularity strength of any connected graph G with minimum degree δ and maximum degree Δ, namely:

$$tvs(G) \geq \max\left\{ \left\lceil \tfrac{\delta+n_\delta}{\delta+1} \right\rceil, \left\lceil \tfrac{\delta+n_\delta+n_{\delta+1}}{\delta+2} \right\rceil, \cdots, \left\lceil \tfrac{\delta+\sum_{j=\delta}^{\Delta} n_j}{\Delta+1} \right\rceil \right\},$$

where n_i is the number of vertices of degree i in G. For $n \geq 3$ and $1 \leq k \leq \lfloor n/2 \rfloor$, the generalized Petersen graph $P(n, k)$ is defined as the graph with vertex set $V(P(n, k)) = \{v_i, u_i : 0 \leq i \leq n-1\}$ and edge set $E(P(n, k)) = \{v_i v_{i+1}, v_i u_i, u_i u_{i+k} : 0 \leq i \leq n-1$, all subscripts are in mod $n\}$. In this talk, we present an algorithm for computing $tvs(P(n, k))$ for any $n \geq 3$ and $1 \leq k \leq \lfloor n/2 \rfloor$.

Keywords: total vertex irregularity strength, generalized Petersen graphs, algorithm.

Covering Symmetric Subsets of the Boolean Boolean cube with Affine Hyperplanes

S. Venkitesh

Indian Institute of Technology Bombay

MSC2000: 05D40,68R05

Alon and Füredi (1993) proved that any family of hyperplanes that covers every point of the Boolean cube $\{0,1\}^n$, except one, must contain at least n hyperplanes. We obtain two extensions of this result to hyperplane covers of symmetric subsets of the Boolean cube (subsets that are closed under permutations of coordinates), over the reals.

As a main tool for proving our results, we introduce *finite-degree hyperplane closures*, a family of closure operators defined using hyperplane covers, for subsets of the Boolean cube. We obtain a combinatorial characterization of the hyperplane closures of symmetric subsets of the Boolean cube, over the reals, which enables us to compute these hyperplane closures efficiently. This characterization may also be of independent interest.

On transversals, near transversals, and diagonals in iterated groups and quasigroups

Anna Taranenko

Sobolev Institute of Mathematics, Novosibirsk, Russia

MSC2000: 05B15; 05D15; 05A16; 05E15; 20N05

A d-dimensional latin hypercube of order n is a d-dimensional array of the same order filled by n symbols so that in each line all symbols are different. Latin hypercubes can be considered as the Cayley tables of d-ary quasigroups of order n, latin hypercubes of dimension 2 are known as latin squares. A transversal in a latin hypercube of order n is a collection of n entries hitting each hyperplane exactly once and containing all n different symbols of the hypercube.

In this talk we focus on transversals in latin hypercubes corresponding to d-iterated quasigroups whose studies were initiated in [2]. Given a binary quasigroup G with the operation $*$, define the d-iterated quasigroup $G[d]$ to be the $(d+1)$-ary quasigroup such that

$$G[d](x_1, \ldots, x_{d+1}) = x_0 \Leftrightarrow (\ldots((x_1 * x_2) * x_3) * \ldots * x_d) * x_{d+1} = x_0.$$

It is known that the Hall–Paige conjecture gives a condition when the Cayley table of a group has a transversal. Here we show that if a group G satisfies the Hall–Paige condition, then every d-iterated group $G[d]$ has a transversal, otherwise $G[d]$ has transversals only if d is even. Moreover, if the number of transversals in $G[d]$ is nonzero, then it is equal to $\frac{n!}{|G'|n^{n-1}} \cdot n!^d(1 + o(1))$ as $d \to \infty$, where G' is the commutator subgroup of G. The obtained asymptotic of the number of transversals is similar to one from [1] for Cayley tables of groups of large order.

We present a method that allows us to prove analogous asymptotics for transversals in any iterated quasigroup. Moreover, we can count not only transversals but other types of diagonals and structures. For instance, we show that all iterated quasigroups of large enough dimensions have many near transversals.

[1] S. Eberhard, F. Manners, R. Mrazović. An asymptotic for the Hall–Paige conjecture. ArXiv:2003.01798.

[2] A. A. Taranenko. Transversals, plexes, and multiplexes in iterated quasigroups. *Electron. J. Combin.* 25(4), 2018, P. 4.30, 1–17.

Alternating sign hypermatrices and Latin-like squares

Cian O'Brien

National University of Ireland, Galway

(This talk is based on joint work with Rachel Quinlan.)

MSC2000: 05B15, 05B20

To any $n \times n$ Latin square L, we may associate a unique sequence of n mutually orthogonal $n \times n$ permutation matrices $P = P_1, P_2, ..., P_n$ such that

$$L = L(P) = \sum kP_k.$$

Brualdi and Dahl [1] described a generalisation of a Latin square, called an *alternating sign hypermatrix Latin-like square (ASHL)*, by replacing P with an *alternating sign hypermatrix (ASHM).*

An ASHM is an $n \times n \times n$ (0,1,-1)-hypermatrix in which the non-zero elements in each row, column, and vertical line alternate in sign, beginning and ending with 1. Since every sequence of n mutually orthogonal $n \times n$ permutation matrices forms the planes of a unique $n \times n \times n$ ASHM, this generalisation of Latin squares follows very naturally, with an ASHM A having corresponding ASHL

$$L = L(A) = \sum kA_k,$$

where A_k is the k^{th} plane of A.

This talk presents problems posed in [1], some of which I have addressed in [2], and some of which are current work.

[1] R. Brualdi, G. Dahl. *Alternating Sign Matrices and Hypermatrices, and a Generalization of Latin Squares.* Advances in Applied Mathematics, **95**(10): 1016, 2018.

[2] C. O'Brien. *Alternating Sign Hypermatrix Decompositions of Latin-like Squares.* Advances in Applied Mathematics, **121**, 2020.

Omniversal Latin squares

Ian Wanless

Monash University

(This talk is based on joint work with Tony Evans, Adam Mammoliti.)

MSC2000: 05B15

A partial transversal of a Latin square is a set of entries in which no row, column or symbol is repeated. It is maximal if it is not contained in a larger partial transversal. A Latin square of order n is omniversal if it possesses a maximal partial transversal of every size from $\lceil \frac{n}{2} \rceil$ to n. We show that omniversal Latin squares exist iff $n \not\equiv 2 \bmod 4$ and $n \notin \{3, 4\}$. We also show that group tables are very far from omniversal (as are random Latin squares). In the process we encounter an interesting problem in combinatorial group theory.

A new Latin square isomorphism invariant arisen from fractal image patterns

Raúl M. Falcón

Universidad de Sevilla

MSC2000: 05B15, 20N05, 68T10

The analysis and recognition of fractal image patterns derived from Cayley tables have recently turned out to play a relevant role for distributing distinct types of algebraic and combinatorial structures into isomorphism classes. This talk delves into this topic by focusing on the study of standard sets of image patterns associated to a given Latin square [2] (see also [1, 3]). Based on the differential box-counting method, the mean fractal dimension of these standard sets enables one to describe a new Latin square isomorphism invariant, which enables one to characterize isomorphism classes of non-idempotent Latin squares in an efficient computational way.

References

[1] V. Dimitrova, S. Markovski, *Classification of quasigroups by image patterns*. In: Proceedings of the Fifth International Conference for Informatics and Information Technology, Bitola, Macedonia, 2007; 152–160.

[2] R. M. Falcón, *Recognition and analysis of image patterns based on Latin squares by means of Computational Algebraic Geometry*, Mathematics **9** (2021), paper 666, 26 pp.

[3] R. M. Falcón, V. Álvarez, F. Gudiel, *A Computational Algebraic Geometry approach to analyze pseudo-random sequences based on Latin squares*, Adv. Comput. Math. **45** (2019), 1769–1792.

Distance-regular graphs obtained from the Mathieu groups and new block designs

Andrea Švob

University of Rijeka

(This talk is based on joint work with Dean Crnković and Nina Mostarac.)

MSC2000: 05E18, 05E30, 94B05, 05B05.

In this talk we will present distance-regular graphs admitting a transitive action of the Mathieu groups. We also study codes spanned by the adjacency matrices of the constructed DRGs. From the code spanned by the adjacency matrix of the strongly regular graph with parameters (176,70,18,34) we obtain new block designs having the full automorphism groups isomorphic to the Higman-Sims finite simple group.

Grid Major of Graph Drawings

Claire Hilaire

LaBRI, University of Bordeaux

(This talk is based on joint work with Cyril Gavoille.)

MSC2000: 05C83,05C62

Motivated by the Grid-Minor Theorem of Robertson and Seymour, a.k.a. the Excluded Grid Theorem, we study the following problem on the relation between graph drawings and grid minors:

Question 1. *For any given planar graph H with a polyline drawing on a $p \times q$ grid, what is the smallest area $A = A(p, q)$ of a grid having H as minor?*

Since a grid of area A is minor of a square grid of side $O(\sqrt{A})$, the Excluded Grid Theorem implies that any planar graph G that excludes such a square grid, and thus excluding H, has a treewidth at most $O(\sqrt{A})$. Since H is a planar graph with at most pq vertices, a classical result in Graph Minor Theory implies that H is minor of a square grid of side $2pq-4$, yielding the upper bound $A(p, q) = O((pq)^2)$. More recently, Dieng and Gavoille [1] showed that $A(p, q) = O(p^2 q)$, leaving open the question whether $A(p, q) = O(pq)$ or not. This upper bound would be optimal since clearly $A(p, q) \geq pq$ if H is a $p \times q$ grid.

In this study, we proved that finding the smallest area of a grid having H as minor is NP-hard, and also that $A(p, q) = O(pq)$ holds for several large classes of n-vertex planar graphs with dense drawing, i.e., with drawing area $O(n)$.

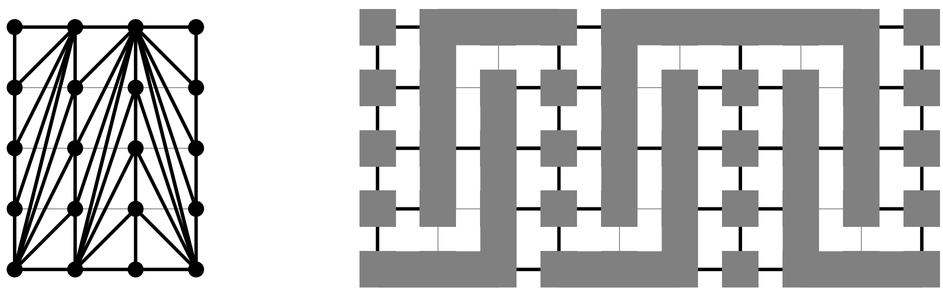


Figure 1: A dense drawing on a $p \times q$ grid for a planar graph with $p = 5$ and $q = 4$, and minor drawing on a $p \times (3q-2)$ grid of the same graph.

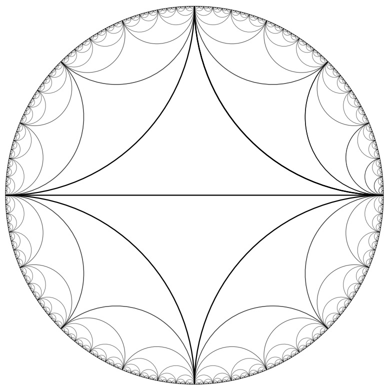
[1] Y. Dieng and C. Gavoille, *On the treewidth of planar minor free graphs*, in 4th EAI Int'nl Conf. on Innov. and Interdisciplinary Sol. for Underserved Areas (InterSol), vol. 321 of LNICST series, Springer, Cham, Mar. 2020, pp. 238–250.

The Farey graph

Jan Kurkofka

Universität Hamburg

MSC2000: 05C63, 05C55, 05C40, 05C83, 05C10



The Farey graph, shown above and surveyed in [1, 2], plays a role in a number of mathematical fields ranging from group theory and number theory to geometry and dynamics [1]. Curiously, graph theory is not among these. We will see that the Farey graph plays a central role in graph theory too: it is one of two infinitely edge-connected graphs that must occur as a minor in every infinitely edge-connected graph. Previously it was not known that there was any set of graphs determining infinite edge-connectivity by forming a minor-minimal list in this way, let alone a finite set.

[1] M. Clay and D. Margalit. *Office Hours with a Geometric Group Theorist*, Princeton University Press, 2017.

[2] A. Hatcher. *Topology of numbers*, Book in preparation, 2017. Available online.

[3] Jan Kurkofka. *Every infinitely edge-connected graph contains the Farey graph or $T_{\aleph_0}*t$ as a minor*, 2020. Available at arXiv:2004.06710.

[4] Jan Kurkofka. *The Farey graph is uniquely determined by its connectivity*, 2020. Positively evaluated by Journal of Combinatorial Theory, Series B. Available at arXiv:2006.12472.

[5] Jan Kurkofka. *Ubiquity and the Farey graph*, European Journal of Combinatorics **95** (2021) 103326. Available at arXiv:1912.02147.

Covers of complete graphs and related association schemes

Ludmila Tsiovkina

Krasovsky Institute of Mathematics and Mechanics

MSC2000: 05E18, 05E30

A distance-regular antipodal cover of the complete graph K_n is equivalently defined as a connected graph, whose vertex set admits a partition into n cells (called antipodal classes) of the same size $r \geq 2$ such that each cell induces an r-coclique, the union of any two distinct cells induces a perfect matching, and any two non-adjacent vertices that lie in distinct cells have exactly $\mu \geq 1$ common neighbours.

Distance-regular antipodal covers of complete graphs form a vast class of graphs that is closely related to many important algebraic or combinatorial objects such as generalised Hadamard matrices, projective planes, generalised quadrangles, divisible designs, and codes. Despite the fact that the complete their classification seems unreachable, one may hope to obtain new their constructions and a partial classification under additional constraints of group nature, such as vertex-transitivity. Since every vertex-transitive cover can be constructed as a union of some graphs of basis relations of a schurian association scheme, the following problem naturally arises: *describe schurian association schemes, for which a union of some graphs of basis relations is a distance-regular antipodal cover of a complete graph.*

In this talk, we investigate schurian association schemes $\mathrm{Inv}(G)$ of a permutation group G, for which the (underlying) graph Γ of a basis relation is a distance-regular antipodal cover of a complete graph. The group G can be regarded as an edge-transitive automorphism group of Γ and induces a 2-homogeneous permutation group G^{Σ} on the set Σ of its antipodal classes, which is either almost simple, or affine. We will present some recent results on classification of such schemes and covers in the almost simple case for G^{Σ}.

Some Ramsey $(C_4, K_{1,n})$-minimal graphs

Maya Nabila

Institut Teknologi Bandung, Indonesia

(This talk is based on joint work with Hilda Assiyatun and Edy Tri Baskoro.)

MSC2000: 05C55, 05D10

Let F, G and H be simple graphs. The notation $F \to (G, H)$ means that for any red-blue coloring on the edges of graph F, there exists either a red copy of G or a blue copy of H. A graph F is called a Ramsey (G, H)-minimal graph if it satisfies two condiditions: (i) $F \to (G, H)$ and (ii) $F - e \nrightarrow (G, H)$ for any edge e of F. The class of all Ramsey (G, H)-minimal graphs is denoted by $\mathcal{R}(G, H)$. The pair (G, H) is called Ramsey-finite if $\mathcal{R}(G, H)$ is finite, otherwise it is called Ramsey-infinite. The study of Ramsey minimal graphs was introduced by Burr et al. [1]. In general, finding the Ramsey (G, H)-minimal graphs is an interesting problem but also a difficult one. Burr et al. [2] proved that $(G, K_{1,n})$ is Ramsey-infinite if G is any 2-connected graph. Borowiecki et al. [3] characterized all graphs in $\mathcal{R}(K_3, K_{1,2})$. Vetrík et al. [4] gave an infinite family of Ramsey $(C_4, K_{1,2})$-minimal graphs of any diameter $k \geq 4$. In this talk, we present an infinite family of Ramsey (G, H)-minimal graphs where $G \cong C_4$ and $H \cong K_{1,n}$ for any $n \geq 3$.

Keywords: Ramsey minimal graph, cycle, star.

[1] S.A. Burr, P. Erdős, and L. Lovász, On graphs of Ramsey type, *Ars Combin.* **1** (1976), 167–190.

[2] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau, and R.H. Schelp, Ramsey minimal graphs for the pair star-connected graph, *Studia Sci. Math. Hungar.*, **15** (1980), 265–273.

[3] M. Borowiecki, I. Schiermeyer, and E. Sidorowicz, Ramsey $(K_{1,2}, K_3)$-Minimal Graphs, *Electron. J. Combin.* **12** (2005), #R20.

[4] T. Vetrík, L. Yulianti, and E.T. Baskoro, On Ramsey $(K_{1,2}, C_4)$-minimal graphs. *Discuss. Math. Graph Theory*, **30** (4) (2010), 637–649.

On the size-Ramsey number of grid graphs

Meysam Miralaei

Institute for Research in Fundamental Sciences (IPM)

(This talk is based on joint work with Dennis Clemens, Damian Reding, Mathias Schacht and Anusch Taraz..)

MSC2000: 05C55, 05D10

Abstract

For two graphs F and G, we say that G is *Ramsey for* F and write $G \longrightarrow F$, if every 2-coloring of the edges of G yields a monochromatic copy of F. Erdős, Faudree, Rousseau, and Schelp defined the *size-Ramsey number* $\hat{r}(F)$ of F to be the smallest integer m such that there exists a graph G with m edges that is Ramsey for F, i.e.,

$$\hat{r}(F) = \min\{e(G)\colon G \longrightarrow F\}\,.$$

Size-Ramsey numbers of graphs have been studied for almost 50 years with particular focus on the case of trees and bounded degree graphs (sparse graphs).

Addressing a question posed by Conlon and Nenadov we focus on 2-dimensional grids. The $s \times t$ *grid graph* $G_{s,t}$ is defined on the vertex set $[s] \times [t]$ with edges uv present, whenever u and v differ in exactly one coordinate by exactly one. We prove that the size-Ramsey number of the grid graph on $\sqrt{n} \times \sqrt{n}$ vertices is bounded from above by $n^{3/2+o(1)}$.

The Multipartite-Size Ramsey Number of Complete Bipartite Graphs

I Wayan Palton Anuwiksa

Institut Teknologi Bandung, Indonesia

(This talk is based on joint work with Rinovia Simanjuntak and Edy Tri Baskoro.)

MSC2000: 05C55, 05D10

The multipartite-size Ramsey number and the multipartite-set Ramsey number are variations of the classical Ramsey number. These two types of Ramsey numbers for complete multipartite graphs were introduced by Burger et al. (2004a and 2004b) and generalized by Syafrizal et al. (2005).

For $j, s \geq 1$, let $K_{j\times s}$ denote a complete multipartite graph having j classes with s vertices in each class. For simple graphs $G_1, ..., G_k$, the *multipartite-set Ramsey number* $M_s(G_1, ..., G_k)$ is defined as the smallest positive integer j such that any k-coloring of the edges of $K_{j\times s}$ contains a monochromatic copy of G_i for some $i, 1 \leq i \leq k$. Similarly, the *multipartite-size Ramsey number* $m_j(G_1, ..., G_k)$ is defined as the smallest positive integer t such that any k-coloring of the edges of $K_{j\times t}$ contains a monochromatic copy of G_i for some $i, 1 \leq i \leq k$. In 2019, Perondi and Carmelo proved that the multipartite-set Ramsey number $M_m(K_{2,m(n-1)+1}, K_{2,m(n-1)+1}) = 4n - 1$ if there are a strongly regular graph with parameter $(4n-2, 2n-2, n-2, n-1)$ and a symmetric Hadamard matrix of order m with $m \geq 4n$.

In this talk, utilising the results of Perondi and Carmelo, we present the exact value of the multipartite-size Ramsey number $m_{4n-3}(K_{2,j(n-1)+1}, K_{2,j(n-1)+1})$.

Keywords: multipartite-size Ramsey number, multipartite-set Ramsey number, bipartite graph

Minimum degree of asymmetric Ramsey-minimal graphs

Thomas Lesgourgues

UNSW Sydney

(This talk is based on joint work with Anurag Bishnoi, Simona Boyadzhiyska, Dennis Clemens, Pranshu Gupta, and Anita Liebenau.)

MSC2000: 05C55

A graph G is q-Ramsey for a q-tuple of graphs $(H_1, \ldots, H_q)$, denoted by $G \rightarrow_q (H_1, \ldots, H_q)$, if every q-colouring $c : E(G) \rightarrow [q]$ contains a monochromatic copy of H_i in colour i, for some $i \in [q]$. The graph G is called q-Ramsey-minimal for $(H_1, \ldots, H_q)$ if it is q-Ramsey for $(H_1, \ldots, H_q)$ but no proper subgraph of G possesses this property. Let $s_q(H_1, \ldots, H_q)$ denote the smallest minimum degree of G over all graphs G that are q-Ramsey-minimal for $(H_1, \ldots, H_q)$.

The study of the parameter s_2 was initiated by Burr, Erdős and Lovász [1] in 1976 when they showed that for cliques, $s_2(K_k, K_t) = (k-1)(t-1)$. In the past two decades the parameter s_q has been studied extensively, focusing on its symmetric version with $H_i = H$ for all i (H being a clique, a cycle, certain bipartite graph or from some sporadic classes of graphs). We present three new results in the asymmetric setting, two exact results with 2 colours for the parameters $s_2(K_k, C_\ell)$ and $s_2(C_k, C_\ell)$ (where C_ℓ is a cycle of length ℓ), and find various upper bounds on $s_q(K_k, \ldots, K_k, C_\ell, \ldots, C_\ell)$, depending on the range of parameters.

[1] S. A. Burr, P. Erdős, and L. Lovász. On graphs of ramsey type. *Ars Combin.*, 1:167-190, 1976.

Shattering with Permutations

Belinda Wickes

Queen Mary University of London

(This talk is based on joint work with J. Robert Johnson.)

MSC2000: 05D99

Many problems in extremal set theory have analogues in terms of permutations, we consider such a problem regarding shattering. We view a permutation as an ordering of the elements in $[n]$. A family of permutations $\mathcal{F} \subseteq S_n$ is said to shatter a triple $\{x, y, z\} \subseteq [n]$ if there exist six permutations in $\mathcal{F}$ where the elements x, y, z can be seen in a different order in each of the chosen permutations. We consider families that shatter all possible triples from $[n]$ and ask how small these families can be. For instance when $n = 4$ we have the following family of permutations shattering every triple

$$\{1234,\ 2413,\ 3412,\ 1432,\ 4231,\ 3214\}.$$

This definition of shattering can be extended from triples to k-tuples, where all $k!$ orderings of the k-tuple must appear in permutations from $\mathcal{F}$.

The question we explore is to find the size of the smallest family of permutations of $[n]$ that shatters every triple and the k-tuple extension of this. Clearly the above example is as small as possible when $n = 4$, there are 6 different orderings of any given triple and each permutation contains exactly one ordering, meaning we need at least 6 permutations to shatter any triple. We show that, when n is large, families that shatter every k-tuple have size $O(\log n)$ for all $k \geq 3$ and give constructions of shattering families with this size. We also explore a partial variant of shattering in which every k-tuple must have at least t out of the total $k!$ orders appearing in the permutations of $\mathcal{F}$.

An exact characterization of saturation for permutation matrices

Benjamin Aram Berendsohn

Freie Universität Berlin

MSC2000: 05D

A 0-1 matrix M *contains* a 0-1 matrix *pattern* P if we can obtain P from M by deleting rows and/or columns and turning arbitrary 1-entries into 0s. The saturation function $\mathrm{sat}(P,n)$ for a 0-1 matrix pattern P indicates the minimum number of 1s in an $n \times n$ 0-1 matrix that does not contain P, but changing any 0-entry into a 1-entry creates an occurrence of P.

Saturation for 0-1 matrices was introduced by Brualdi and Cao [1]. Fulek and Keszegh [2] started a systematic study, and showed that each pattern has a saturation function either in $\mathcal{O}(1)$ or in $\Theta(n)$. They exhibited large classes of patterns with linear saturation function, in particular the *decomposable* patterns, i.e., 0-1 matrices of the form $\left(\begin{smallmatrix} A & \mathbf{0} \\ \mathbf{0} & B \end{smallmatrix}\right)$ or $\left(\begin{smallmatrix} \mathbf{0} & A \\ B & \mathbf{0} \end{smallmatrix}\right)$.

Subsequently, Geneson [3] showed that almost all *permutation matrices* have bounded saturation functions. We complete the classification of permutation matrices by showing that each indecomposable permutation matrix has bounded saturation function. This involves constructions based on *oscillations* in indecomposable permutations.

[1] Richard A. Brualdi and Lei Cao. Pattern-avoiding (0,1)-matrices. *arXiv e-prints*, 2020.

[2] Radoslav Fulek and Balázs Keszegh. Saturation problems about forbidden 0-1 submatrices. *arXiv e-prints*, 2020.

[3] Jesse Geneson. Almost all permutation matrices have bounded saturation functions. *The Electronic Journal of Combinatorics*, 28(2):P2.16, 2021.

Between Monotone and Geometric Griddability of Permutation Classes and Beyond

Bogdan Alecu

University of Warwick

(This talk is based on joint work with Robert Ferguson, Mamadou Kanté, Vadim Lozin, Vince Vatter and Viktor Zamaraev.)

MSC2000: 05C75, 05C62, 05A05, 05C99

Generalising previous work, the notion of *monotone griddability* of a permutation class was introduced in [3] as an enumerative tool. Later, a stronger notion of *geometric griddability* was developed [1]. While more restrictive, this stronger condition guarantees very desirable enumerative properties of the class, as well as well-quasi-orderability under the subpattern relation. The requirements for monotone griddability are very well understood, with a concise and transparent characterisation of monotone griddable classes in terms of minimal obstructions [3]. The boundary between monotone and geometric griddability, however, is a different story. In [2], we uncovered a conceptual similarity between geometric griddability of a permutation class and boundedness of a certain graph parameter in the corresponding class of permutation graphs. This parameter, called *lettericity*, was introduced in [4].

In this talk, we start by sketching a proof of a conjecture from [2]: a class $\mathcal{X}$ is geometrically griddable if and only if the corresponding class $\mathcal{G}_{\mathcal{X}}$ of permutation graphs has bounded lettericity. As a consequence, understanding the boundary between monotone and geometric griddability is the same as understanding the behaviour of lettericity in the class of permutation graphs. We then briefly discuss our current progress in this direction. Finally, we remark that the study of lettericity in the universe of all graphs reveals an intriguing hierarchy of structure – we propose some open problems on this topic.

[1] M.H. Albert, M.D. Atkinson, M. Bouvel, N. Ruškuc, V. Vatter, Geometric grid classes of permutations, *Trans. Amer. Math. Soc.* 365 (2013), 5859–5881.

[2] B. Alecu, V. Lozin, D. de Werra, V. Zamaraev, Letter graphs and geometric grid classes of permutations: characterization and recognition, *Discrete Applied Mathematics* 283 (2020), 482–494.

[3] S. Huczynska, V. Vatter, Grid classes and the Fibonacci dichotomy for restricted permutations. *Electron. J. Combin.* 13 (2006), R54, 14 pp.

[4] M. Petkovšek, Letter graphs and well-quasi-order by induced subgraphs, *Discrete Mathematics*, 244 (2002) 375–388.

Distribution of mesh patterns

Sergey Kitaev

University of Strathclyde

MSC2000: 05A15

The notion of a mesh pattern, generalizing several classes of permutation patterns, was introduced in 2011 by Brändén and Claesson to provide explicit expansions for certain permutation statistics as, possibly infinite, linear combinations of (classical) permutation patterns. There is a long line of research papers dedicated to the study of mesh patterns and their generalizations.

In this talk, I will discuss a systematic study of avoidance and distribution of mesh patterns of short length, and some general results on distributions of several infinite families of mesh patterns.

Synchronizing Times for k-sets in Automata

Natalie C. Behague

Ryerson University, Toronto

(This talk is based on joint work with Robert Johnson.)

MSC2000: 68Q45, 05D99

An automaton consists of a finite set of states and a collection of functions from the set of states to itself. An automaton is *synchronizing* if there is a word (that is, a sequence of functions) that maps all states onto the same state. Černý's conjecture on the length of the shortest such word is one of the most famous open problem in automata theory. We consider the closely related question of determining the minimum length of a word that maps some k states onto a single state.

For synchronizing automata, we have improved the upper bound on the minimum length of a word that sends some triple to a a single state from $0.5n^2$ to $\approx 0.19n^2$. I will discuss this result and some related results, including a generalisation of this approach leading to an improved bound on the length of a synchronizing word for 4 states and 5 states.

Cocyclic two-circulant core Hadamard matrices.

Santiago Barrera Acevedo

Monash University

(This talk is based on joint work with Padraig Ó Catháin and Heiko Dietrich.)

MSC2000: 05B20

The two-circulant core (TCC) construction for Hadamard matrices (HMs) uses two sequences with almost perfect autorrelation to construct a HM. A research problem of K. Horadam asks whether such matrices are cocyclic. Using ideas from permutation groups, we prove that the order of a cocyclic TCC HM coincides with the order of a HM of Paley type, Sylvester type or certain multiples of these orders. In addition, we show that there exit cocyclic TCC HMs at all allowable order less or equal to 1000 with at most one exception.

Towards characterising locally common graphs

Robert Hancock

Heidelberg University

(This talk is based on joint work with Daniel Král', Matjaž Krnc and Jan Volec.)

MSC2000: 05C35, 05C55

A graph H is *common* if the number of monochromatic copies of H in a 2-edge-colouring of the complete graph is asymptotically minimised by the random colouring. The classification of common graphs is one of the most intriguing problems in extremal graph theory. In this talk we will consider this notion in a local setting as considered by Csóka, Hubai and Lovász where the graph is required to be the minimiser with respect to perturbations of the random 2-edge-colouring, and give a complete characterisation of graphs H into three categories, in regard to a possible behaviour of the 12 initial terms in the Taylor series determining the number of monochromatic copies of H in such perturbations:

- graphs of Class I are locally common,
- graphs of Class II are not locally common, and
- graphs of Class III cannot be determined to be locally common or not based on the initial 12 terms.

As a corollary, we obtain new necessary conditions on a graph to be common and new sufficient conditions on a graph to be not common.

The Sunflower Problem

Suchakree Chueluecha

Lehigh University

(This talk is based on joint work with Tolson Bell and Lutz Warnke.)

MSC2000: 05D05, 05D40

A sunflower with p petals consists of p sets whose pairwise intersections are identical. The goal of the sunflower problem is to find the smallest $r = r(p, k)$ such that every family of at least r^k k-element sets must contain a sunflower with p petals. Major breakthroughs by Alweiss-Lovett-Wu-Zhang and others show that $r = O(p \log(pk))$ suffices. In this talk, we present our improvement to $r = O(p \log(k))$.

Joint work with Tolson Bell and Lutz Warnke, see `https://arxiv.org/abs/2009.09327`

ON WEAKLY HADAMARD DIAGONALIZABLE GRAPHS

Mahsa N. Shirazi

University of Regina

MSC2000: 05C50; 15A18

An interesting question in the spectral graph theory is about the structure of the eigenvectors of matrices associated with graphs. A graph is *weakly Hadamard diagonalizable (WHD)* if its Laplacian matrix L can be diagonalized with a weakly Hadamard matrix [1]. In other words, if $L = PDP^{-1}$, where D is a diagonal matrix and P has the property that all entries in P are from $\{0, -1, 1\}$ and that P^tP is a tridiagonal matrix. In this talk, I will present some necessary and sufficient conditions for a graph to be WHD. Also some families of graphs which are WHD, will be presented.

[1] M. Adm, K. Almuhtaseb, S. Fallat, K. Meagher, S. Nasserasr, M.N. Shirazi and A.S. Razafimahatratra. Weakly Hadamard diagonalizable graphs. *Linear Algebra and its Applications*, 610, 86-119, 2021.

Finite ϵ-unit distance graphs

Mike Krebs

California State University, Los Angeles

MSC2000: 05C15

In 2005, Exoo posed the following question. Fix ϵ with $0 \leq \epsilon < 1$. Let G_ϵ be the graph whose vertex set is the Euclidean plane, where two vertices are adjacent iff the Euclidean distance between them lies in the closed interval $[1-\epsilon, 1+\epsilon]$. What is the chromatic number $\chi(G_\epsilon)$ of this graph? The case $\epsilon = 0$ is precisely the classical "chromatic number of the plane" problem. In a 2018 preprint, de Grey shows that $\chi(G_0) \geq 5$; the proof relies heavily on machine computation. In 2016, Grytczuk et al. proved a weaker result with a human-comprehensible but nonconstructive proof: whenever $0 < \epsilon < 1$, we have that $\chi(G_\epsilon) \geq 5$. (This lower bound of 5 was later improved by Currie and Eggleton to 6.) The De Bruijn - Erdős theorem (which relies on the axiom of choice) then guarantees the existence, for each ϵ, of a finite subgraph H_ϵ of G_ϵ such that $\chi(H_\epsilon) \geq 5$. In this paper, we explicitly construct such finite graphs H_ϵ. We find that the number of vertices needed to create such a graph is no more than $2\pi(15 + 14\epsilon^{-1})^2$. Our proof can be done by hand without the aid of a computer.

Spined Categories: generalising tree-width beyond graphs

Benjamin Merlin Bumpus

University of Glasgow

(This talk is based on joint work with Zoltan Kocsis.)

MSC2000: 05C75, 18B99, 05C85

Problems that are NP-hard in general are often tractable on inputs that have a recursive structure. For instance consider classes defined in terms of 'graph decompositions' such as of bounded tree- or clique-width graphs. Given the algorithmic success of graph decompositions, it is natural to seek analogues of these notions in other settings. What should a 'tree-width-k' digraph or lattice or temporal graph even look like?

Since most notions of decomposition are defined in terms of the internal structure of the decomposed object, generalizing such decompositions to a larger class of objects tends to be an arduous task. In this talk I will show how this difficulty can be reduced significantly by finding a characteristic property formulated purely in terms of the *category* that the decomposed objects inhabit, which defines the decomposition independently of the internal structure.

I will introduce an abstract characterisation of tree-width by defining our new notions of *spined categories* and *S-functors*. Our results can be thought as a vast generalisation of Halin's definition of tree-width as the maximal graph parameter sharing certain properties with the Hadwiger number and chromatic number. Spined categories provide a roadmap to the discovery of new tree-width-like parameters: they can be seen as a black box taking as input some category satisfying two axioms and yielding an appropriate tree-width analogue as output.

Sharp Thresholds in Random Temporal Graphs

Viktor Zamaraev

Department Of Computer Science, University of Liverpool

(This talk is based on joint work with Arnaud Casteigts, Michael Raskin, and Malte Renken.)

MSC2000: 05C80

A graph whose edges only appear at certain points in time is called a temporal graph (among other names). Such a graph is temporally connected if each ordered pair of vertices is connected by a path which traverses edges in chronological order (i.e., a temporal path). In this paper, we consider a simple model of random temporal graph, obtained by permuting the edges of an Erdős–Rényi random graph $G_{n,p}$ uniformly at random.

We show that this model exhibits a surprisingly regular sequence of thresholds related to temporal reachability. In particular, we show that at $p = \log n/n$ any fixed pair of vertices can a.a.s. reach each other, at $2 \log n/n$ at least one vertex (and in fact, any fixed node) can a.a.s. reach all others, and at $3 \log n/n$ all the vertices can a.a.s. reach each other (i.e., the graph is temporally connected). All these thresholds are sharp. In addition, we show that at $p = 4 \log n/n$ the graph contains a spanning subgraph of minimum possible size that preserves temporal connectivity, i.e., it admits a temporal spanner with $2n - 4$ edges.

Maximal Matroids in Weak Order Posets

Bill Jackson

Queen Mary University of London

(This talk is based on joint work with Katie Clinch and Shin-Ichi Tanigawa.)

MSC2000: Primary 05B35; Secondary 05C35

Let $\mathcal{X}$ be a family of subsets of a finite set E. A matroid M on E is said to be an $\mathcal{X}$-matroid if each set in $\mathcal{X}$ is a circuit in M. We consider the problem of determining when there exists a unique maximal $\mathcal{X}$-matroid in the weak order poset of all $\mathcal{X}$-matroids on E (defined by putting $M_1 \preceq M_2$ if every independent set in M_1 is independent in M_2).

The critical group of orientable ribbon graphs

Criel Merino

UNAM

(This talk is based on joint work with Iain Moffatt and Steven D. Noble.)

MSC2000: 05C50,05C10

The critical group of a connected graph is now a well-stablished structure in Combinatorics. It is usually defined using the reduced Laplacian of the graph. However, the critical group is also isomorphic to the quotient $\mathbb{Z}^m/(\mathcal{C}\oplus\mathcal{C}^*)$, where m, $\mathcal{C}$ and $\mathcal{C}^*$ are the number of edges, cycle space and cocycle space, respectively, of the graph. We use that an orientable ribbon graph is a even delta-matroid representable by a principal unimodular matrix to associate an abelian group to any orientable ribbon graph. For planar graphs both groups are isomorphic. As a byproduct we obtain a formula for the number of quasi-trees of an orientable ribbon graph by computing a determinant.

Thursday 10:30, Zoom 1

An experimental approach to Gauss diagram realizability

Alexei Lisitsa

University of Liverpool

(This talk is based on joint work with Abdullah Khan and Alexei Vernitski, University of Essex.)

MSC2000: 05A99, 05C75, 57M25, 57M99

A chord diagram consists of a circle and some chords inside it. Chord diagrams are a well-established tool in the study of topology of knots and of planar and spherical curves. Not every chord diagram corresponds to a knot (or an immersed curve); if it does, it is called realizable. A classical question of computational topology asked by Gauss is which chords diagrams are realizable. Many variants of the criteria have been proposed since then and it has turned out that realizability of diagrams can be expressed in terms of circle graphs (chord interlacement graphs) associated with diagrams. The vertices of the chord interlacement graph correspond to the chords of the diagram, and there is an edge between vertices iff the corresponding chords intersect. In this talk we report on the experimental investigation [3] of various Gauss diagram realizability descriptions expressed in terms of their chord interlacement graphs. A novel efficient algorithm is used for the generation of chord diagrams.

We discuss the number of chord diagrams (of a given size) satisfying various realizability descriptions and corresponding chord interlacement graphs, and apply these numbers in two ways. Firstly, some of these sequences of numbers are new, including [4], and expand on known results. Secondly, we find counterexamples showing that recently published Gauss diagram realizability descriptions in [1, 2] are incorrect.

[1] O. Biryukov, Parity conditions for realizability of Gauss diagrams, *Journal of Knot Theory and Its Ramifications*, vol 28, No 01, pp 1950015, 2019

[2] A. Grinblat and V. Lopatkin, On Realizability Of Gauss Diagrams And Constructions Of Meanders, *Journal of Knot Theory and Its Ramifications*, vol 29, No 05, pp 2050031, 2020

[3] A. Khan and A. Lisitsa and A. Vernitski, Experimental Mathematics Approach to Gauss Diagrams Realizability, *arXiv,2103.02102*, 2021

[4] The On-Line Encyclopedia of Integer Sequences, A343358, http://oeis.org/A343358

Irreducibility and Tutte polynomials of graphs in surfaces

Iain Moffatt

Royal Holloway, University of London

(This talk is based on joint work with A. Goodall, S.D. Noble, and L. Vena.)

MSC2000: 05C31

The Tutte polynomial is a polynomial-valued invariant of graphs, and is arguably the most important and best studied graph polynomial. It's important not only because it encodes a large amount of combinatorial information about a graph, but also because of its applications to areas such as statistical physics (as the Ising and Potts models) and knot theory (as the Jones and homfly polynomials). Unsurprisingly, given its role in combinatorics, the Tutte polynomial has been extended to many different settings for many different purposes.

In this talk I'll discuss extensions of the Tutte polynomial to graphs embedded in surfaces. After giving a brief overview of the various "topological Tutte polynomials" in the literature, I'll focus on the "ribbon graph polynomial". This is a natural two-variable Tutte polynomial for cellularly embedded graphs that is closely related to the "Bollobás–Riordan polynomial". In particular, I'll characterise when the ribbon graph polynomial is irreducible in terms of the connectivity of the embedded graph, a result that is joint work with A. Goodall, S.D. Noble, and L. Vena.

Thursday 11:20, Zoom 1

A Tutte Polynomial for Embedded Graphs

Maya Thompson

Royal Holloway, University of London

(This talk is based on joint work with Iain Moffatt.)

MSC2000: 05C31

The Tutte polynomial is one of the most important graph invariant polynomials. Part of its relevancy comes from its ability to capture so much combinatorial information about a graph. In recent years, several extensions of the Tutte polynomial to graphs cellularly embedded in surfaces have appeared, one of the most recent of which being a polynomial by Goodall, Litjens, Regts and Vena that captures information such as the number of flows and tensions of a graph or the number of quasi-trees.

In this talk, I will show how extending to the family of decorated, (potentially) non-cellularly embedded graphs in pseudo-surfaces facilitates a recursive formula for the Goodall, Litjens, Regts and Vena polynomial. I hope to convey through my talk why this is the natural family of embedded graphs to consider when talking about edge deletion and contraction.

The corona product of two graphs on n vertices with the metric-location-domination number $n/2$

Zulfaneti

Institut Teknologi Bandung, Indonesia

(This talk is based on joint work with Hilda Assiyatun and Edy Tri Baskoro.)

MSC2000: 05C69

Let $G = (V, E)$ be a simple connected graph. A set $H \subseteq V$ is said to be a *dominating set* of graph G if for every $u \in V - H$ there exists a vertex $h \in H$ so that $d(u, h) = 1$. For an ordered subset $X = \{x_1, x_2, \cdots, x_p\}$ of V and $w \in V$, the *representation* of vertex w with respect to X is the p-vector $(d(w, x_1), d(w, x_2), \cdots, d(w, x_p))$, where $d(w, x)$ is the distance between vertices w and x in G. A set X is called a *resolving set* of G if all vertices of G have distinct representations with respect to X. If a dominating set H of a graph G is also a resolving set, then H is called a *metric-locating-dominating set* (MLD-set) of G. The *dominating number* $\gamma(G)$ (*the metric dimension* $\beta(G)$ and the *metric-location-domination number* $\gamma_M(G)$, resp.) of a graph G is defined as the size of a minimum dominating set (a resolving set and a MLD-set of G, resp.). The study on the MLD-number of any graph was introduced by Brigham et al. (2003). They showed that for any graph G, $max\{\gamma(G), \beta(G)\} \leq \gamma_M(G) \leq min\{\gamma(G) + \beta(G), n - 1\}$. The *corona product* of two graphs G and H, denoted by $G \odot H$, is defined as the graph formed by taking $|V(G)|$ copies of graph H and connecting the i-th vertex of G to all vertices of the i-th copy of H. In this talk, we consider the metric-location-domination number of the corona product $G \odot H$. We characterize all graphs H with $\gamma_M(G \odot H) = \frac{1}{2}|V(G \odot H)|$.

Keywords : domination number, metric dimension, MLD-number, Corona product.

Width parameters and graph classes: the case of mim-width

Andrea Munaro

School of Mathematics and Physics, Queen's University Belfast

(This talk is based on joint work with Flavia Bonomo-Braberman, Nick Brettell, Jake Horsfield, Giacomo Paesani, and Daniël Paulusma.)

MSC2000: 05C85, 68Q25, 68R10

A large number of NP-hard graph problems become polynomial-time solvable on graph classes where the mim-width is bounded and quickly computable. Hence, when solving such problems on special graph classes, it is helpful to know whether the graph class under consideration has bounded mim-width. We extend the toolkit for proving (un)boundedness of mim-width of graph classes and initiate a systematic study into bounding mim-width from the perspective of hereditary graph classes. We present summary theorems of the current state of the art for the boundedness of mim-width for (H_1, H_2)-free graphs and observe several interesting consequences. We also study the mim-width of generalized convex graphs. This allows us to re-prove and strengthen a large number of known results.

[1] Flavia Bonomo-Braberman, Nick Brettell, Andrea Munaro, and Daniël Paulusma. Solving problems on generalized convex graphs via mim-width. WADS 2021, accepted.

[2] Nick Brettell, Jake Horsfield, Andrea Munaro, Giacomo Paesani, and Daniël Paulusma. Bounding the mim-width of hereditary graph classes. *Proc. IPEC 2020, LIPIcs*, 180:6:1–6:18, 2020.

[3] Nick Brettell, Jake Horsfield, Andrea Munaro, and Daniël Paulusma. List k-colouring P_t-free graphs: a mim-width perspective. CoRR, abs/2008.01590, 2020.

Uncountably many minimal hereditary classes of graphs of unbounded clique-width

Dan Cocks

Department of Mathematics and Statistics, The Open University

(This talk is based on joint work with Robert Brignall.)

MSC2000: 05C85/05C75

Clique-width is a graph parameter which is important in algorithmic graph theory owing to its use in understanding algorithmic tractability. A range of decision problems defined on graphs that are in general NP-hard can be solved in polynomial time when the input is restricted to graphs with bounded clique-width. In seeking to better understand graph characteristics that result in bounded clique-width, much attention has recently been directed to identifying minimal hereditary classes of graphs of unbounded clique-width.

In this talk, I will show that given an infinite word α over the alphabet $\{0, 1, 2, 3\}$, we can define a class of bipartite hereditary graphs $\mathcal{G}^\alpha$, such that $\mathcal{G}^\alpha$ has unbounded clique-width unless α contains at most finitely many non-zero letters.

I will show that $\mathcal{G}^\alpha$ is minimal of unbounded clique-width if and only if α belongs to a precisely defined collection of words Γ. The set Γ includes all almost periodic words containing at least one non-zero letter, which both enables us to exhibit uncountably many pairwise distinct minimal classes of unbounded clique width, and also proves one direction of a conjecture due to Collins, Foniok, Korpelainen, Lozin and Zamaraev. It can then be demonstrated that the other direction of the conjecture is false, since Γ also contains words that are *not* almost periodic.

Feedback Vertex Set and Even Cycle Transversal for H-Free Graphs: Finding Large Block Graphs

Giacomo Paesani

Department of Computer Science, Durham University, UK

(This talk is based on joint work with Daniël Paulusma and Paweł Rzążewski.)

MSC2000: 05C85

We prove new complexity results for Even Cycle Transversal and Feedback Vertex Set on H-free graphs, that is, graphs that do not contain some fixed graph H as an induced subgraph. In particular, we prove that both problems are polynomial-time solvable for sP_3-free graphs for every integer $s \geq 1$. Our results show that both problems exhibit the same behaviour on H-free graphs (subject to some open cases). This is in part explained by a new general algorithm we design for finding in a graph G a largest induced subgraph whose blocks belong to some finite class $\mathcal{C}$ of graphs. We also compare our results with the state-of-the-art results for the Odd Cycle Transversal problem, which is known to behave differently on H-free graphs.

Thursday 11:45, Zoom 2

Tree-width dichotomy

Vadim Lozin

University of Warwick

(This talk is based on joint work with Igor Razgon.)

MSC2000: 05C75

We prove that the tree-width of graphs in a hereditary class defined by a finite set F of forbidden induced subgraphs is bounded if and only if F includes a complete graph, a complete bipartite graph, a tripod (a forest in which every connected component has at most 3 leaves) and the line graph of a tripod.

Coloring of Graphs Avoiding Bicolored Paths of a Fixed Length

Alaittin Kırtışoğlu

Hacettepe University

(This talk is based on joint work with Lale Özkahya.)

MSC2000: 05

The problem of finding the minimum number of colors to color a graph properly without containing any bicolored copy of a fixed family of subgraphs has been widely studied. Most well-known examples are star coloring and acyclic coloring of graphs (Grünbaum, 1973) where bicolored copies of P_4 and cycles are not allowed, respectively. We introduce a variation of these problems and study proper coloring of graphs not containing a bicolored path of a fixed length and provide general bounds for all graphs. A P_k-coloring of an undirected graph G is a proper vertex coloring of G such that there is no bicolored copy of P_k in G, and the minimum number of colors needed for a P_k-coloring of G is called the P_k-*chromatic number* of G, denoted by $s_k(G)$. We provide bounds on $s_k(G)$ for all graphs, in particular, proving that for any graph G with maximum degree $d \geq 2$, and $k \geq 4$, $s_k(G) = 6\sqrt{10}d^{\frac{k-1}{k-2}}$. Moreover, we find the exact values for the P_k-chromatic number of the products of some cycles and paths for $k = 5, 6$.

F-Perfect Graphs

James Alex

Curtin University

(This talk is based on joint work with Louis Caccetta.)

MSC2000: 05C15, 05C17, 05C69

Perfect graphs and their associated conjectures which were introduced by Berge during the early 1960s have significantly influenced the development of graph theory over the last fifty years. Inspired by Berge's perfect graphs, Ravindra introduced a new class of graphs in 2011 called F-perfect graphs and defined them as follows: Let $\mathscr{F}$ be a class of well-defined graph and $F \in \mathscr{F}$. Let $\theta_F(G)$ denote the minimum cardinality of a partition $\mathscr{C}$ of the vertex set of G such that each set $C_i \in \mathscr{C}$ induces an F in G. Let $\alpha_F(G)$ denote the maximum cardinality of a subset $S \subseteq V(G)$ such that no two distinct vertices in S lie in the same F. Obviously $\theta_F(G) \geq \alpha_F(G)$ and a graph G is F-perfect if $\theta_F(H) = \alpha_F(H)$ for every induced subgraph H of G. Similarly, we denote χ_F as the minimum number of colors required to color the graph G such no two vertices in the same F receive the same color and ω_F as the number of vertices of a maximum F contained in the graph G. Obviously, $\chi_F(G) \geq \omega_F(G)$ and graphs for which the coloring behaves in a controlled and structured way depending on the F are called *χ_F-perfect*. Simultaneously this class is quite large and includes many important classes. If F is a complete graph, then F-perfect graphs mean Berge's perfect graphs. Thus F-Perfect graphs enrich the purview of further research in perfect graph theory by generalizing it. If F is a star, then we have star-perfect graphs and with respect to these graphs Ravindra's conjecture that a graph G is θ_F-perfect if and only if G is C_3-free, C_{3n+1}-free and C_{3n+2}-free for $n \geq 1$. This conjecture was settled in affirmative by Alex and Caccetta. For a suitable F, studying the F-perfectness is worthy of investigation.

Some graphs of order n with dominating partition dimension $n-3$

Muhammad Ridwan

Institut Teknologi Bandung, Indonesia

(This talk is based on joint work with Hilda Assiyatun and Edy Tri Baskoro.)

MSC2000: 05C12

Let G be a connected graph of order n. Let $\Pi = \{S_1, \cdots, S_k\}$ be a k-partition of $V(G)$. The *representation* of a vertex $u \in V(G)$ with respect to Π, denoted by $r(u|\Pi)$, is the vector $(d(u, S_1), d(u, S_2), \cdots, d(u, S_k))$, where $d(u, S)$ represents the distance between vertex u and a set S in G. The partition Π is called a *resolving partition* of G if all vertices have distinct representations with respect to Π. Additionally, if Π also satisfies the property that for every vertex v of G, there exists $S_j \in \Pi$ for some j such that $d(v, S_j) = 1$, then Π is called a *resolving dominating partition* of G. The *dominating partition dimension* of G is the minimum cardinality of a resolving dominating partition of G. In general, characterizing all graphs of order n with certain dominating partition dimension k (where $k \leq n$) is a difficult problem. Only few cases are already solved, namely the characterization studies of such graphs for $k = n-2, n-1$, or n. In this talk, we shall focus on studying the graphs G of order $n \geq 4$ with the dominating partition dimension $n-3$. We shall characterize all graphs of order $n \geq 4$ with dominating partition dimension $n-3$ and diameter two.

Keywords: resolving dominating, dominating partition dimension, partition dimension

Directed Cordial Labeling of Some Graphs

Sarang Sadawarte

Department of Mathematics, Sharda University, Greater Noida

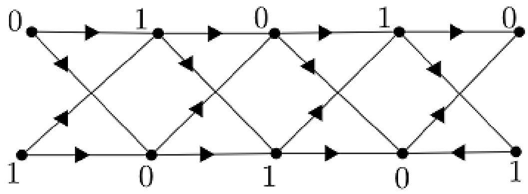
(This talk is based on joint work with Dr. Sweta Srivastav.)

MSC2000: 05C78

Abstract: A graph K is said to be directed, if each edge of K has an orientation. A binary vertex labeling of K is directed cordial if it satisfies certain properties. A graph which preserves directed cordiality is known as directed cordial graph. This study introduce directed cordial labeling of double graph of directed path P_r and double graph of directed cycle C_r. We also emphasized directed cordiality of these graphs, for $r \equiv 0, 1, 2, 3(mod4)$.

Theorem 1. *The double graph of directed path preserves directed cordial labeling.*

Example 1 : The double graph of directed path P_5 is directed cordial under certain condition $r \equiv 1(mod4)$ is elaborated by figure given below.



Theorem 2. *The double graph of directed cycle preserves directed cordial labeling.*

References

[1] Cahit, I., (1987). Cordial graphs : A Weaker version of graceful and harmonious graphs. *Ars combinatorica*, *23*, 201–207.

[2] Burton, D.M., (1990). *Elementary Number Theory*, Brown Publishers, Seventh Edition.

[3] Gallian, J.A., (2016). A Dynamic survey of graph labeling, *The Electronics Journal of Combinatorics*, DS6.

[4] Harary, F., (1972). *Graph Theory*, Addison-Wesley, Reading, Massachusetts.

[5] Al-Shamiri, M.M.A., Nada, S.I., Elrokh, A.I. and Elmshtay, Y. (2020). Some Results on Cordial Digraphs.*Open Journal of Discrete Mathematics*, *10*, 4–12.

On Stability Number and Chromatic Number of Markovian Random Graphs

Akshay Gupte

School of Mathematics, University of Edinburgh

(This talk is based on joint work with Yiran Zhu.)

MSC2000: O5C80, O5C15, 60J10, 60G42

The stability number $\alpha(G)$ of a graph $G = (V, E)$ is the maximum size of an independent set in G. Although hard to approximate in polynomial time within factor $|V|$, for the Erdös-Rényi random graph $\mathcal{G}_{n,p}$, there have been numerous studies on bounding $\alpha(\mathcal{G}_{n,p})$ asymptotically and more so about the chromatic number $\chi(\mathcal{G}_{n,p})$ and its concentration. We consider a random graph model in which edges are *not* i.i.d. Bernoulli r.v. like in $\mathcal{G}_{n,p}$, but are generated with respect to a Markov process. In particular, given $p \in (0, 1)$ and a decay parameter $\delta \in (0, 1]$, the probability p_{ij} of an edge (i, j), for $1 \le i < j \le n$, depends on the presence of the edge $(i-1, j)$; if the latter edge does not exist then $p_{ij} = p_{i-1,j}$, otherwise $p_{ij} = \delta\, p_{i-1,j}$. Taking $\delta = 1$ retrieves the Erdös-Rényi graph $\mathcal{G}_{n,p}$.

Theorem 1. *Let $\delta \in (0, 1)$. Denote $\sigma = \frac{1}{1-\delta}$ and let $\gamma = \frac{\delta}{\lambda(1-\delta)}$ for $\lambda > 1$. We have w.h.p.*[1] *that $\Omega(n^{\frac{1}{\sigma+1}}) = \alpha(\mathcal{G}^{\delta}_{n,p}) \le \left(1 + \frac{2}{3e} - e^{-\gamma}\right) \cdot n$.*

We prove this using nontrivial bounds on the total probabilities for each edge and on the probability that a subset of vertices forms an independent set. For the asymptotic lower bound, we analyse the size of the maximal independent set found by a greedy algorithm. The Motzkin-Straus theorem, Hoffman's lower bound on $\chi(G)$, and Theorem 1 imply some spectral properties of adjacency matrix of $\mathcal{G}^{\delta}_{n,p}$.

Our second theorem shows that the average vertex degree in $\mathcal{G}^{\delta}_{n,p}$, denoted by $d^{\delta}_{n,p}$, scaled by a constant factor of $\log n$ concentrates to 2.

Theorem 2. *For $\epsilon > 0$, we have w.h.p. that $\left| \frac{d^{\delta}_{n,p}}{\sigma \log n} - 2 \right| < \epsilon$.*

We use the second moment method (Chebyshev's inequality). Due to the absence of independence structure between r.v.'s, we cannot apply the Chernoff/Hoeffding inequalities, and use of martingale tail inequalities also does not help. For fixed p, this theorem shows $\mathcal{G}^{\delta}_{n,p}$ to be more sparse than $\mathcal{G}_{n,p}$. Our result is consistent with the intuition that a denser graph has smaller stability number because for dense $\mathcal{G}_{n,p}$ it is known that w.h.p. $\alpha(\mathcal{G}_{n,p}) \approx 2 \log_{\frac{1}{1-p}} n$ for fixed p.

Since $\alpha(G) \ge \lceil \frac{|V|}{\chi(G)} \rceil$ for any graph G, Theorem 1 implies a lower bound on $\chi(\mathcal{G}^{\delta}_{n,p})$. Vizing's theorem and the analysis in Theorem 2 imply bounds on the edge chromatic number $\chi'(\mathcal{G}_{n,p})$.

[1] Probability of the claim being true converges to 1 as $n \to \infty$ (also called asymptotically almost surely)

Best Response Dynamics on Random Graphs

Calina Durbac

(This talk is based on joint work with Jordan Chellig, Nikolaos Fountoulakis.)

MSC2000: 05C99, 91A22

‘We consider evolutionary games on a population whose underlying topology of interactions is determined by a binomial random graph $G(n, p)$. Our focus is on 2-player symmetric games with 2 strategies played between the incident members of such a population. Players update their strategies synchronously. At each round, each player selects the strategy that is the best response to the current set of strategies its neighbours play. We show that such a system reduces to generalised majority and minority dynamics. We show rapid convergence to unanimity for p in a range that depends on a certain characteristic of the payoff matrix. In the presence of a bias among the pure Nash equilibria of the game, we determine a sharp threshold on p above which the largest connected component reaches unanimity with high probability. For p below this critical value, where this does not happen, we identify those substructures inside the largest component that remain discordant throughout the evolution of the system.’

Component Counts of Random Injections

Dudley Stark

Queen Mary, University of London

MSC2000: 05C20, 05C30, 05C80

Similarly to the way that the digraph representing a permutation can be uniquely decomposed into cycles, the digraph representing an injection between two finite sets can be uniquely decomposed into cycles and paths. The component structure of a random injection undergoes a phase transition between cycles dominating and paths dominating as certain parameters change.

Thursday 11:45, Zoom 4

Large complete minors in random subgraphs

Joshua Erde

Graz University of Technology

(This talk is based on joint work with Mihyun Kang and Michael Krivelevich.)

MSC2000: 05C80,05C83

Let G be a graph of minimum degree at least k and let G_p be the random subgraph of G obtained by keeping each edge independently with probability p. We are interested in the size of the largest complete minor that G_p contains when $p = \frac{1+\varepsilon}{k}$ with $\varepsilon > 0$. We show that with high probability G_p contains a complete minor of order $\tilde{\Omega}(\sqrt{k})$, where the $\sim$ hides a polylogarithmic factor. Furthermore, in the case where the order of G is also bounded above by a constant multiple of k, we show that this polylogarithmic term can be removed, giving a tight bound.

Maximizing the distance spectral radius of graphs

Aysel Erey

Gebze Technical University

MSC2000: 05C15, 05C50

The distance spectral radius of a graph is the largest eigenvalue of its distance matrix. In this talk, I will discuss recent results and open questions on the problem of finding extremal graphs with largest distance spectral radius within the family of connected graphs with fixed chromatic number and order.

Matchings in k-partite k-graphs

Candida Bowtell

University of Oxford

(This talk is based on joint work with Richard Mycroft.)

MSC2000: 05C65, 05C70, 05D15

Let H be a k-partite k-graph with parts $V_1, \ldots, V_k$ each of size n, such that, for every $i \in [k]$, every $(k-1)$-set in $\prod_{j \in [k] \setminus \{i\}} V_j$ lies in at least a_i edges. Suppose further that $a_1 \geq \ldots \geq a_k$. Han, Zang and Zhao showed that for every $\epsilon > 0$ and sufficiently large n, with $a_1, a_2 \geq \epsilon n$, H contains a matching of size at least $\min\{n-1, \sum_{i \in [k]} a_i\}$, answering and generalising a question of Rödl and Ruciński. Their arguments use complex absorbing methods which fail when all of $a_2, \ldots, a_k$ are small. We consider the remaining cases and, in particular, show that when $\sum_{i=2}^{k} a_i \leq \sqrt{\frac{n}{k+1}}$, H in fact contains a matching of size at least $\min\{n, \sum_{i \in [k]} a_i\}$. Our proof uses a novel approach, making use of Aharoni and Haxell's 'Hall's Theorem for Hypergraphs' and rainbow matchings.

Thursday 11:20, Zoom 5

TURÁN PROBLEMS FOR k-GEODETIC DIGRAPHS

James Tuite

Open University

(This talk is based on joint work with Grahame Erskine, Ervin Győri, Nika Salia and Casey Tompkins.)

MSC2000: 05C35 05C20

A Turán-type problem asks for the largest possible size of a graph with given order n with some family $\mathcal{F}$ of forbidden subgraphs. Such questions have been extensively investigated for undirected graphs, but less so for directed graphs; some Turán problems for directed graphs have been considered in [1,2,4].

A digraph G is k-geodetic if for any pair of (not necessarily distinct) vertices u, v of G there is at most one walk in G from u to v of length $\leq k$; this parameter has been studied in a recent generalisation of the degree/girth problem to directed graphs [3]. Therefore it is of interest to ask for the largest size of a k-geodetic digraph with order n.

In fact, this problem turns out to be relatively trivial, with extremal digraphs given by orientations of complete bipartite graphs. The problem becomes much more interesting when we add the condition of strong connectivity. Therefore in this talk we discuss the following problem: what is the largest possible size of a strongly connected k-geodetic digraph with order n? We solve this problem for $k = 2$ and classify the extremal digraphs. For larger k we conjecture that the answer is of order $\frac{n^2}{k^2}$. We present constructions that achieve this bound and prove a strong upper bound. Finally we present some results on generalised Turán problems for k-geodetic digraphs.

[1] Heydemann, M.C., *On cycles and paths in digraphs*, Discrete Math., 31 (2) (1980), 217-219.

[2] Huang, Z., Lyu, Z. and Qiao, P., *A Turán problem on digraphs avoiding distinct walks of a given length with the same endpoints.* Discrete Math. 342 (6) (2019), 1703-1717.

[3] Miller, M., Miret, J.M. and Sillasen, A.A., *On digraphs of excess one.* Discrete Appl. Math. (238) (2018), 161-166.

[4] Ustimenko, V.A. and Kozicki, J., *On extremal directed graphs with large hooves.* Topics in Graph Theory, (2013), 26-35.

Thursday 11:45, Zoom 5

Pósa-type results for Berge Hypergraphs

Nika Salia

Alfréd Rényi Institute of Mathematics

(This talk is based on joint work with Ervin Győri.)

MSC2000: 05C45, 05C65, 05C38

A Berge-path of length k in a hypergraph $\mathcal{H}$ is a sequence $v_1, h_1, v_2, h_2, \ldots, v_k, h_k, v_{k+1}$ of distinct vertices and hyperedges with $v_{i+1} \in h_i, h_{i+1}$ for all $i \in [k]$. Füredi, Kostochka, and Luo recently gave Dirac-type minimum degree conditions that force non-uniform Hypergraphs to have Hamiltonian cycles. We give Pósa-type lower bounds for degree sequences for r-uniform and non-uniform Hypergraphs that force Hamiltonian cycles. We also show that those bounds can not be strengthened.

Permutation-Generated Maps on Dyck Paths

Yozef Tjandra

Calvin Institute of Technology, Jakarta, Indonesia

(This talk is based on joint work with Kevin Limanta and Hopein Christofen Tang.)

MSC2000: 05

In [1], Elizalde and Deutsch defined bijective maps between Dyck paths which are beneficial in enumerating Dyck paths with certain statistics as well as pattern-avoiding permutations. In this talk, we generalise the maps in such a way that they are generated by any permutation in S_{2n}. The generalised maps induce several ways to partition S_{2n} which lead to a new proof of an existing combinatorial identity involving double factorials and other interesting results. As the generalisation relaxes several features of the map, in certain condition, the maps are no longer bijective. We then provide a characterisation of permutations which generate bijections. Moreover, we introduce a statistic that is the number of unpaired steps among some consecutive circular steps of a Dyck path, whose distribution is identical to other well-known height statistics of Dyck paths.

[1] Elizalde S. and Deutsch E. "A simple and unusual bijection for Dyck paths and its consequences". In: *Annals of Combinatorics* 7.3 (2003), pp. 281–297.

On the RIP of Paley ETF and related combinatorial results

Shohei Satake

Kumamoto University, Japan

MSC2000: 94A08, 05C20, 05C69

Matrices with the *restricted isometry property* (*RIP*) play an important role in compressed sensing. In particular, constructing *deterministic* RIP matrices breaking the *square-root bottleneck* on the RIP is a challenging problem. In [1], Bandeira, Fickus, Mixon and Wong considered the RIP of a matrix, called *Paley ETF*, defined by quadratic residues of the p-element field where p is an odd prime, and they conjectured that Paley ETF could break the square-root bottleneck. Later Bandeira, Mixon and Moreira ([2]) proved that this conjecture is true when $p \equiv 1 \pmod 4$ and a predicted character sum estimation holds. Also it was proved in [2] that if Paley ETF breaks the square-root bottleneck, then a significantly improved upper bound on the clique number of *Paley graph* can be obtained.

In this talk, we consider the case of general odd primes p. We first prove that Paley ETF breaks the square-root bottleneck assuming that a widely-believed conjecture, namely, the *Paley graph conjecture*, holds. Moreover we show that if Paley ETF breaks the square-root bottleneck, then we have significantly improved upper bounds on the maximum size of transitive subtournaments in *Paley tournament* as well as on the clique number of Paley graph. Finally, we discuss an application of our results to Paley graph extractor as well.

[1] A. S. Bandeira, M. Fickus, D. G. Mixon, P. Wong, The road to deterministic matrices with the restricted isometry property, *J. Fourier Anal. Appl.* **19** (2013), 1123–1149.

[2] A. S. Bandeira, D. G. Mixon, J. Moreira, A conditional construction of restricted isometries, *Int. Math. Res. Not.* **2017** (2017), 372–381.

[3] S. Satake, On the restricted isometry property of the Paley matrix, *arXiv:2011.02907.*

The maximal number of 3-term arithmetic progressions in finite sets in different geometries

Shoni Gilboa

The Open University of Israel

(This talk is based on joint work with Itai Benjamini.)

MSC2000: 05D99, 51F99

Let M be a metric space. We say that $(a,b,c) \in M^3$ is a 3-*term arithmetic progression* in M if $d_M(a,b) = d_M(b,c) = \frac{1}{2}d_M(a,c)$, where d_M is the metric of M. For every positive integer n, let $\mu_n(M)$ be the maximal number of 3-term arithmetic progressions in n-element subsets of M.

In 2008, Green and Sisask showed that $\mu_n(\mathbb{Z}) = \lceil n^2/2 \rceil$ for every n; the same argument shows that $\mu_n(\mathbb{R}) = \lceil n^2/2 \rceil$ for every n; this yields, by a simple projection argument, that the same is true for Euclidean spaces of any dimension.

We show that this extends to a rather large class of metric spaces, including the hyperbolic spaces, and more generally, any *Cartan–Hadamard manifold*, i.e., complete simply connected Riemannian manifold that has everywhere nonpositive sectional curvature.

On the other hand, we show that the result of Green and Sisask does not extend to some other natural metric spaces. Starting with spherical geometry, we show that for every $n \neq 2$,

$$\mu_n(S^1) = \frac{1}{2}n^2 + \begin{cases} n & n \bmod 4 = 0, \\ \frac{1}{2}n & n \bmod 4 = 1, \\ 2 & n \bmod 4 = 2, \\ \frac{1}{2}n - 1 & n \bmod 4 = 3; \end{cases}$$

in particular, $\mu_n(S^1) > \lceil n^2/2 \rceil$ for every $n \geq 4$; we further show that $\mu_n(S^2) > \mu_n(S^1)$ for every $n \geq 5$. For the r-regular tree $\mathbb{T}_r$, with respect to the graph metric, we show that

$$\limsup_{n\to\infty} \frac{\mu_n(\mathbb{T}_r)}{n^2} \geq \frac{1}{2} + \frac{(r-2)^2}{2r^2},$$

and for the ℓ-dimensional lattice graph $\mathbb{Z}^\ell$ (where two vertices are adjacent if the Euclidean distance between them is 1), again with respect to the graph metric, we show that $\mu_n\left(\mathbb{Z}^\ell\right) = \Omega\left(n^{3-\frac{1}{\ell}}\right)$. Finally, we show that the maximum of μ_n over all metric spaces is $\frac{1}{4}n^3 - \frac{1}{2}n^2 + \Theta(n)$.

A preprint, containing all the proofs and additional details, is available at `arXiv:2011.04410`.

Prevalence of Braess' Paradox?

Vadim Zverovich

University of the West of England, Bristol

MSC2000: 05C90, 05D40

The well-known Braess' paradox illustrates situations when adding a new link to a transport network might not reduce congestion in the network but instead increase it. This is due to individual entities acting selfishly/separately when making their travel plan choices and hence forcing the system as a whole not to operate optimally. Deeper insight into this paradox from the viewpoint of the structure and characteristics of networks may help transport planners to avoid the occurrence of Braess-like situations in real-life networks.

A generally accepted belief is that Braess' paradox is widespread. This was confirmed by some researchers who claimed that the likelihood of the paradox is 50%, or even higher under some assumptions. In this talk, we will discuss our recent results devoted to the probability of Braess' paradox to occur in the classical network configuration introduced by Braess.

V. Zverovich, *Modern Applications of Graph Theory*, 2021, Oxford, UK: Oxford University Press, 416 pages.

Ramsey simplicity of random graphs

Dennis Clemens

Hamburg University of Technology

(This talk is based on joint work with Simona Boyadzhiyska, Shagnik Das and Pranshu Gupta.)

MSC2000: 05D10, 05C80

The classic Ramsey problem asks for the minimum number of vertices in a graph G that is q-Ramsey for H; that is, such that any q-edge-colouring of G leads to a monochromatic copy of H. This central line of research was then broadened in the seminal work of Burr, Erdős and Lovász to include the investigation of other extremal parameters of Ramsey graphs, including the minimum degree of minimal Ramsey graphs.

By a simple application of the pigeonhole principle it follows that if G is a minimal q-Ramsey graph for H, we must have $\delta(G) \geq q(\delta(H) - 1) + 1$, and we call a graph H q-Ramsey simple if this bound can be attained. Grinshpun showed that the random graph $G(n, p)$ is almost surely 2-Ramsey simple in the range $\frac{\log n}{n} \ll p \ll n^{-2/3}$. We shall explore this question further, asking for which pairs p and q we can expect $G(n, p)$ to be q-Ramsey simple, and we will in particular uncover some interesting behaviour in the range $n^{-2/3} \ll p \ll n^{-1/2}$.

Towards the 0-statement of the Kohayakawa-Kreuter Conjecture

Joseph Hyde

University of Birmingham

MSC2000: 05C55, 05C80

We study asymmetric Ramsey properties in $G_{n,p}$. Specifically, for fixed graphs $H_1, \ldots, H_r$, we study the asymptotic threshold function for the property $G_{n,p} \rightarrow (H_1, \ldots, H_r)$ which denotes that given any colouring of the edges of $G_{n,p}$ with colours from the set $[r] := \{1, \ldots, r\}$ there exists $i \in [r]$ and a copy of H_i in $G_{n,p}$ where every edge has been given colour i. Rödl and Ruciński determined the threshold function for the general symmetric case; that is, when $H_1 = \cdots = H_r$. Kohayakawa and Kreuter conjectured the threshold function for the asymmetric case. Recently, the 1-statement of this conjecture was confirmed by Mousset, Nenadov and Samotij.

Building on work of Marciniszyn, Skokan, Spöhel and Steger, we reduce the 0-statement of Kohayakawa and Kreuter's conjecture to a more approachable, deterministic conjecture. To demonstrate the potential of this approach, we show our conjecture holds for almost all pairs of regular graphs. This therefore resolves the 0-statement for all such pairs of graphs.

Friday 11:20, Zoom 1

MINIMAL RAMSEY GRAPHS WITH MANY VERTICES OF SMALL DEGREE

Pranshu Gupta

Hamburg University of Technology

(This talk is based on joint work with Simona Boyadzhiyska and Dennis Clemens.)

MSC2000: 05D10

Given any graph H, a graph G is said to be q-Ramsey for H if every colouring of the edges of G with q colours yields a monochromatic subgraph isomorphic to H. Further, such a graph G is said to be minimal q-Ramsey for H if additionally, no proper subgraph G' of G is q-Ramsey for H. In 1976, Burr, Erdős, and Lovász initiated the study of the parameter $s_q(H)$, defined as the smallest minimum degree among all minimal q-Ramsey graphs for H. In this talk, we consider the problem of determining how many vertices of degree $s_q(H)$ a minimal q-Ramsey graph for H can contain. Specifically, we seek to identify graphs for which a minimal q-Ramsey graph can contain arbitrarily many such vertices. We call a graph satisfying this property s_q-abundant. Among other results, we prove that every cycle of length at least 4 is s_q-abundant for any integer $q \geq 2$. We also discuss the cases when H is a clique or a clique with a pendant edge, extending previous results of Burr et al. and Fox et al., respectively. To prove our results and construct suitable minimal Ramsey graphs, we develop certain gadget graphs, called pattern gadgets, which generalize and extend earlier constructions that have been proven useful in the study of minimal Ramsey graphs.

On the Ramsey numbers for the tree graphs versus certain generalised wheel graphs

Zhi Yee Chng

UNSW Sydney

(This talk is based on joint work with Ta Sheng Tan and Kok Bin Wong.)

MSC2000: 05C55, 05D10

Given two simple graphs G and H, the Ramsey number $R(G, H)$ is the smallest integer n such that for any graph of order n, either it contains G or its complement contains H. Let T_n be a tree graph of order n and $W_{s,m}$ be the generalised wheel graph $K_s + C_m$. In this research, we show that for $n \geq 5, s \geq 2$, $R(T_n, W_{s,6}) = (s+1)(n-1)+1$ and for $n \geq 5, s \geq 1$, $R(T_n, W_{s,7}) = (s+2)(n-1)+1$.

[1] J. A. Bondy, Pancyclic graphs, *Journal of Combinatorics, Ser. B* **11** (1971), 80–84.

[2] Y. Chen, Y. Zhang and K. Zhang, The Ramsey numbers of stars versus wheels, *European Journal of Combinatorics* **25** (2004), 1067–1075.

[3] Y. Chen, Y. Zhang and K. Zhang, The Ramsey numbers of paths versus wheels, *Discrete Mathematics* **290** (2005), 85–87.

[4] Y. Chen, Y. Zhang and K. Zhang, The Ramsey numbers of trees versus W_6 or W_7, *European Journal of Combinatorics* **27** (2006), 558–564.

[5] Y. Chen, Y. Zhang and K. Zhang, The Ramsey numbers $R(T_n, W_6)$ for $\Delta(T_n) \geq n-3$, *Applied Mathematics Letters* **17** (2004), 281–285.

[6] Q. Lin, Y. Li and L. Dong, Ramsey goodness and generalized stars, *European Journal of Combinatorics* **31** (2010), 1228–1234.

[7] L. Wang and Y. Chen, The Ramsey numbers of trees versus generalized wheels, *Graphs and Combinatorics* **35** (2019), 189–193.

[8] Y. Zhang, On Ramsey numbers of short paths versus large wheels, *Ars Combinatoria* **89** (2008), 11–20.

Perfect Colorings of Generalized Petersen Graphs

Hamed Karami

Iran University of Science and Technology

MSC2000: 05C15

For a graph G and an integer m, a mapping $T : V(G) \to \{1, ..., m\}$ is called a perfect m-coloring with matrix $A = (a_{ij}), i, j \in \{1, ..., m\}$, if it is surjective, and for all i, j, for every vertex of color i, the number of its neighbors of color j is equal to a_{ij}. There is another term for this concept in literature as "equitable partition". In this talk, we present some important results about enumerating parameter matrices of all perfect 2-colorings and perfect 3-colorings of generalized Petersen graphs $GP(n, k)$ (see [1, 2, 3]).

[1] Alaeiyan Mehdi, Karami Hamed, Perfect 2-colorings of generalized Petersen graphs, Proc. Indian Acad. Sci. (Math. Sci.) Vol. 126, No. 3, August 2016, pp. 289–294.

[2] Alaeiyan Mehdi, Karami Hamed, Siasat Sajjad, Perfect 3-colorings of GP(5,2), GP(6,2), and GP(7,2), JOURNAL OF THE INDONESIAN MATHEMATICAL SOCIETY, Vol. 24, No. 2, October 2018, pp. 47-53.

[3] Karami Hamed, Perfect 2-colorings of generalized Petersen graph GP(n,3), **preprint** (arXive: 2009.07120 [math.CO]).

$L(2,1)$-number of the Mycielski of graphs

Kamal Dliou

National School of Applied Sciences (ENSA), Ibnu Zohr University, Agadir, Morocco

(This talk is based on joint work with Hicham El Boujaoui and Mustapha Kchikech.)

MSC2000: 05C78

An $L(2,1)$-Labeling of a graph $G = (V, E)$ is a function f from the vertex set V to the set of all nonnegative integers such that $|f(x) - f(y)| \geq 2$ if $d(x, y) = 1$ and $|f(x) - f(y)| \geq 1$ if $d(x, y) = 2$. The span of f is the difference between the largest and the smallest label used by f. The $L(2,1)$-number or λ-number of a graph G, denoted by $\lambda(G)$, is the minimum span over all $L(2,1)$-Labelings of G. The Mycielski's construction is a well-known construction which transform a k-chromatic graph G into $(k+1)$-chromatic graph $M(G)$, called the Mycielski graph of G, which has the same clique number as G.

In this paper we show that for any graph G, we have $max(n+1, 2(\triangle+1)) \leq \lambda(M(G)) \leq n + \lambda(G) + 1$, where n and $\triangle$ are respectively the order and the maximum degree of G. We show some graphs having $\lambda(M(G)) = n + \lambda(G) + 1$. Next we give a condition for a graph implying $\lambda(M(G)) = n + 1$. Then we determine the λ-number of the Mycielski graph generated from the graph path P_n and the graph cycle C_n, using that and the lower bound $max(n + 1, 2(\triangle + 1))$, we characterize connected graphs realising $\lambda(M(G))$ equal to 4, 6 and 7, which are the smallest values for $\lambda(M(G))$ for any non-trivial connected graph. Finally, we conjecture that $\lambda(M(G)) \leq n + \triangle^2 + 1$, for any graph G of order n and maximum degree $\triangle$.

Discrepancies of Spanning Trees

Peleg Michaeli

Tel Aviv University

(This talk is based on joint work with Lior Gishboliner and Michael Krivelevich.)

MSC2000: 05C35, 05D10, 11K38

Discrepancy theory is concerned with colouring elements of a ground set so that each set in a given set system is as balanced as possible. In the graph setting, the ground set is the edge set of a given graph, and the set system is a family of subgraphs. In this talk, I shall discuss the discrepancy of the set of spanning trees in general graphs, a notion that has been recently studied by Balogh, Csaba, Jing and Pluhár. More concretely, for every graph G and a number of colours r, we look for the maximum D such that in any r-colouring of the edges of G, one can find a spanning tree with at least $(n-1+D)/r$ edges of the same colour. As our main result, we show that under very mild conditions (for example, if G is 3-connected), D is equal, up to a constant factor, to the minimal integer s such that G can be separated into r equal parts by removing s vertices. This strong and perhaps surprising relation between the extremal quantity D and a geometric quantity allows us to estimate the spanning-tree discrepancy for many graphs of interest. In particular, we reprove and generalize results of Balogh et al., as well as obtain new ones.

Partial colouring of graphs: What to do if you don't have enough colours?

Xinyi Xu

London School of Economics and Political Science

(This talk is based on joint work with Jan van den Heuvel.)

MSC2000: 05C15

Suppose you are given a graph G for which the vertices can be properly coloured with k colours, but you only have $s < k$ colours available. Then it is an easy observation that you can properly colour at least a fraction $\dfrac{s}{k}$ of the vertices of G. (More formally: There exists an induced subgraph H of G such that H is s-colourable and $|V(H)| \geq \dfrac{s}{k}|V(G)|$.)

In this talk we look at this idea of partial colouring for some other colouring concepts, such as list colouring, fractional colouring and multicolouring. Our guiding question will always be: If we have only a fraction α of the required colour-set available, can we always colour at least a fraction α of the graph?

Friday 10:30, Zoom 3

Vertex stability and edge stability for the chromatic index of graphs

Saeid Alikhani

Department of Mathematics, Yazd University, Iran

(This talk is based on joint work with Mohammad R. Piri.)

MSC2000: 05C15, 05C25

Let $G = (V, E)$ be a simple graph. A function $c : E \to \{c_1, \ldots, c_k\}$ with $c(e_1) \neq c(e_2)$ for any two adjacent edges e_1 and e_2 is a proper k-edge coloring of G. The minimum k for which G admits a proper k-edge coloring is the chromatic index of G, and denoted by $\chi'(G)$. The vertex stability of the chromatic index is denoted by $vs_{\chi'}(G)$, is the minimum number of vertices of G whose removal results in graph $H \subseteq G$ with $\chi'(H) \neq \chi'(G)$. Also the edge stability of the chromatic index is denoted by, $es_{\chi'}(G)$, is the minimum number of edges of G such that their deletion results in a graph H with $\chi'(H) \neq \chi'(G)$. In this talk we present our new results on these two parameters. More precisely, we give some general bounds for $vs_{\chi'}(G)$ and $es_{\chi'}(G)$ and determine these parameters exactly for specific classes of graphs such as joins of graphs, and corona of graphs.

[1] S. Akbari, S. Klavžar, N. Movarraei, M. Nahvi, *Nordhaus-Gaddum and other bounds for the chromatic edge-stability number*, European J. Combin. 84 (2020), 103042, 8 pp.

[2] S. Alikhani and S. Soltani, *Stabilizing the distinguishing number of a graph*, Comm. Alg. 46(12) (2018) 5460-–5468.

[3] M.A. Henning and M. Krzywkowski, *Total domination stability in graphs*, Discrete Appl. Math. 236(19) (2018) 246–255.

[4] A. Kemnitz, M. Marangio, *On the ρ-edge stability number of graphs,* Disscus. Math. Graph Theory. Available at `https://doi.org/10.7151/dmgt.2255`.

[5] A. Kemnitz, M. Marangio, N. Movarraei, *On the chromatic edge stability number of graphs*, Graphs Combin. 34 (2018) 1539-–1551.

A better upper bound of the locating-chromatic number of trees

~~Edy Tri Baskoro~~

UPDATE: This talk will now be presented by Devi Imulia Dian Primaskun.

Institut Teknologi Bandung, Indonesia

(This talk is based on joint work with Devi Imulia Dian Primaskun.)

MSC2000: 05C12, 05C15

Let $G = (V, E)$ be a simple connected graph. The distance $d(u, S)$ from a vertex u to a set S in G is defined as $\min\{d(u, v)|v \in S\}$. The color code $a_c(u)$ of vertex u in G under a proper k-coloring c is defined as the k-tuple $(d(u, C_c^{(1)}), d(u, C_c^{(2)}), \cdots, d(u, C_c^{(k)}))$, where $C_c^{(i)}$ is the set of all vertices of color i. A proper k-coloring c is called a locating coloring of G if for any pair of distinct vertices u and v, we have $a_c(u) \neq a_c(v)$. The locating-chromatic number of a graph G is the smallest integer k such that G admits a locating k-coloring. The study on the locating-chromatic number of graphs was introduced by Chartrand et al. (2002). This notion is, in fact, a special case of the partition dimension of graphs. This study has received much attention. However, the results are still very limited. The locating-chromatic numbers of some trees have been discovered, such as amalgamations of stars by Asmiati et al. (2011), complete n-ary trees by Welyyanti et al. (2013), and all trees with locating-chromatic number 3 by Baskoro and Asmiati (2013). However for most classes of trees, their locating-chromatic numbers are still open. Recently, Furuya and Matsumoto (2019) proposed an algorithm to derive an upper bound of the locating chromatic number of any tree T. This upper bound depends on the number of leaves and the number local end-branches in T. In this talk, we propose an algorithm for deriving a better upper bound of the locating-chromatic number of any tree T. This algorithm is based on subtrees called palms of the tree T.

Keywords: locating-chromatic number, tree, algorithm, upper bound

Chromatic identities on maximal triangle-free graphs

Ez-Zobair Bidine

Hassan First University of Settat, Morocco

(This talk is based on joint work with M. Kchikech, O. Togni and T. Gadi.)

MSC2000: 05C15,05C70,05C12

A graph is maximal triangle-free if no edge may be added without producing a triangle. A triangle-free graph is maximal triangle-free if and only if its diameter is two. The neighborhood of every vertex in triangle-free graphs is an independent set. Then, in such graphs, it is evident that $\Delta(G) \leq \alpha(G)$, where $\Delta(G)$ and $\alpha(G)$ stand for the maximum degree and the independence number of a graph G, respectively.

In 1964, Vizing [1] showed that every graph G has edge-chromatic number $\chi'(G)$ either $\Delta(G)$ (known as Class I graphs) or $\Delta(G)+1$ (known as Class II graphs). Deciding the class of a given graph is $\mathcal{NP}$-complete problem [2], even when restricted to triangle-free graphs with $\Delta = 3$ [3].

A k-packing coloring of a graph G with vertex set V, for some integer k, is a mapping $f : V \to \{1, 2, \dots, k\}$ such that for any two distinct vertices u and v from V : if $f(u) = f(v) = i$, then $d_G(u, v) > i$, where $d_G(u, v)$ is the distance between u and v in G. The packing chromatic number $\chi_\rho(G)$ of a graph G is the smallest integer k such that the graph G has a k-packing coloring [4]. A well known upper bound of $\chi_\rho(G)$ for some graph G is $|G| - \alpha(G) + 1$ with equality if the diameter of G is two [4]. In this work, we prove the existence of class I maximal triangle-free graph where the parameters α, Δ and χ_ρ coincide, i.e maximal triangle-free graph G such that

$$\alpha(G) = \Delta(G) = \chi_\rho(G) = \chi'(G).$$

[1] Vadim G Vizing. On an estimate of the chromatic class of a p-graph. *Discret Analiz*, 3:25–30, 1964.

[2] I. Holyer. The np-completeness of edge-coloring. *SIAM Journal on computing*, 10(4):718–720, 1981.

[3] D. P. Koreas. The NP-completeness of chromatic index in triangle free graphs with maximum vertex of degree 3. *Applied mathematics and computation*, 83(1):13–17, 1997.

[4] W. Goddard, S.M. Hedetniemi, S.T. Hedetniemi, J M. Harris;, and D.F. Rall. Broadcast chromatic numbers of graphs. *Ars Combinatoria*, 86:33–50, 2008.

Friday 11:45, Zoom 3

Acyclic, Star and Injective Colouring for H-free graphs

Jan Bok

Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic

(This talk is based on joint work with Nikola Jedličková, Barnaby Martin, Pascal Ochem, Daniël Paulusma, and Siani Smith.)

MSC2000: 05C15, 05C85

A (proper) colouring is acyclic, star, or injective if any two colour classes induce a forest, star forest or disjoint union of vertices and edges, respectively. Hence, every injective colouring is a star colouring and every star colouring is an acyclic colouring. The corresponding decision problems are Acyclic Colouring, Star Colouring and Injective Colouring (the last problem is also known as $L(1,1)$-Labelling). A classical complexity result on Colouring is a well-known dichotomy for H-free graphs (a graph is H-free if it does not contain H as an *induced* subgraph). In contrast, there is no systematic study into the computational complexity of Acyclic Colouring, Star Colouring and Injective Colouring despite numerous algorithmic and structural results that have appeared over the years. We perform such a study and give almost complete complexity classifications for Acyclic Colouring, Star Colouring and Injective Colouring on H-free graphs (for each of the problems, we have one open case). Moreover, we give full complexity classifications if the number of colours k is fixed, that is, not part of the input. From our study it follows that for fixed k the three problems behave in the same way, but this is no longer true if k is part of the input. To obtain several of our results we prove stronger complexity results that in particular involve the girth of a graph and the class of line graphs of multigraphs.

Jan Bok, Nikola Jedličková, Barnaby Martin, Daniël Paulusma and Siani Smith. Acyclic Colouring, Star Colouring and Injective Colouring for H-Free Graphs. *Proc. ESA 2020, LIPIcs* 173, 22:1-22:22, 2020.

Jan Bok, Nikola Jedličková, Barnaby Martin, Daniël Paulusma and Siani Smith. Injective colouring for H-free graphs. *Proc. CSR 2021, LNCS*, to appear.

Jan Bok, Nikola Jedličková, Barnaby Martin, Pascal Ochem, Daniël Paulusma and Siani Smith. Acyclic Colouring, Star Colouring and Injective Colouring for H-Free Graphs. `https://arxiv.org/abs/2008.09415`, 2021.

Efficient generation of elimination trees and Hamilton paths on graph associahedra

Arturo Merino

TU Berlin + University of Warwick

(This talk is based on joint work with Jean Cardinal and Torsten Mütze.)

MSC2000: 05C45, 52B05

Graph associahedra are a large class of polytopes defined with respect to a finite graph G [1, 5], and they generalize many classical polytopes, such as standard associahedra, permutahedra, hypercubes, stellohedra, cyclohedra etc., which arise for particular choices of G. Their vertices are all elimination trees of G, i.e., all ways of removing vertices of G one by one, and their edges correspond to tree rotations. Manneville and Pilaud [4] proved that for any graph G with at least two edges, the graph associahedron has a *Hamilton cycle*, but their proof does not translate into an efficient algorithm.

In this work we present a simple and efficient algorithm for computing a Hamilton path on the graph associahedron for the case when G is a chordal graph, which includes many interesting subclasses, such as paths, stars, trees, k-trees, complete graphs, interval graphs, and split graphs. The algorithm runs in time $\mathcal{O}(m+n)$ per generated elimination tree of G, where m and n are the number of edges and vertices of G, respectively, which can be improved to $\mathcal{O}(1)$ for trees G. We made this algorithm available for experimentation on the Combinatorial Object Server [2]. Our algorithm generalizes several known Gray codes, and gives new Gray codes for interesting combinatorial objects. Moreover, it produces a Hamilton cycle on the graph associahedron of G, rather than just a Hamilton path, if G is chordal and 2-connected. The algorithm also characterizes chordality, i.e., it fails to compute a Hamilton path for non-chordal graphs G. These results are obtained by applying the permutation language framework proposed by Hartung, Hoang, Mütze, and Williams [3], and by encoding elimination trees by permutations.

[1] M. P. Carr and S. L. Devadoss. Coxeter complexes and graph-associahedra. *Topology Appl.*, 153(12):2155–2168, 2006.

[2] The Combinatorial Object Server: `http://www.combos.org/elim`.

[3] E. Hartung, H. P. Hoang, T. Mütze, and A. Williams. Combinatorial generation via permutation languages. I. Fundamentals. To appear in *Trans. Amer. Math. Soc.*, 2021.

[4] T. Manneville and V. Pilaud. Graph properties of graph associahedra. *Sém. Lothar. Combin.*, 73:Art. B73d, 31, 2015.

[5] A. Postnikov. Permutohedra, associahedra, and beyond. *Int. Math. Res. Not. IMRN*, (6):1026–1106, 2009.

Reconstructing trees from small cards

Jane Tan

University of Oxford

(This talk is based on joint work with Carla Groenland, Tom Johnston and Alex Scott.)

MSC2000: 05C60

The ℓ-deck of a graph G is the multiset of all induced subgraphs of G on ℓ vertices. Kelly and Ulam's classical reconstruction conjecture states that all graphs on $n \geq 3$ vertices are uniquely determined by (or *reconstructible* from) their $(n-1)$-decks. While well-studied, this conjecture is very much open. When it comes to smaller cards, a basic observation is that reconstruction becomes more difficult as ℓ decreases. Thus, given a class of graphs that is reconstructible from the $(n-1)$-deck, we can ask: what is the smallest ℓ for which graphs in that class are also reconstructible from the ℓ-deck? We shall discuss this question for trees. In particular, we will show that trees are reconstructible from their ℓ-decks when $\ell \geq (8/9 + o(1))n$, improving on a result of Giles from 1976 which required $\ell \geq n-2$. This is good news for a conjecture of Nýdl, although we have some bad news for that conjecture as well.

Graphs with two moplexes are more than perfect

Matjaž Krnc

University of Primorska,
Faculty of Mathematics, Natural Sciences and Information Technologies

(This talk is based on joint work with Clément Dallard,
Robert Ganian, Meike Hatzel, and Martin Milanič.)

MSC2000: 05C75, 05C45

A well-known result by Dirac (1961) states that every chordal graph contains a simplicial vertex. This theorem proved to be very useful for structural and algorithmic applications. Moplexes, in the setting of general graphs, are an analogue of simplicial vertices in chordal graphs, as Berry and Bordat (1998) proved that every non-complete graph contains at least two moplexes.

There are results on the structure of chordal graphs with a bounded number of simplicial modules, for example the chordal graphs having at most two simplicial modules are interval. This motivates the research of graphs with a bounded number of moplexes. As only complete graphs have exactly one moplex, we consider the smallest interesting case: the class of graphs with at most two moplexes. Berry and Bordat (2001) proved that this class of graphs contains all connected proper interval graphs and is contained in the class of AT-free graphs. We strengthen the latter inclusion in two ways. First, we generalise it by proving that the asteroidal number yields a lower bound on the number of moplexes. Second, as our main structural result, we show that graphs with at most two moplexes are cocomparability.

So, as the class of connected graphs with at most two moplexes is sandwiched between the connected proper interval graphs and cocomparability graphs, this leads to the natural question of whether the presence of at most two moplexes guarantees a sufficient amount of structure to efficiently solve problems that are known to be intractable on cocomparability graphs, but not on proper interval graphs. For two such problems, namely Graph Isomorphism and Max-Cut, we show that they stay hard on the graphs with two moplexes. On the other hand, we prove that every connected graph with two moplexes contains a Hamiltonian path.

Erdős-Hajnal conjecture for Galaxies with Spiders

Soukaina Zayat

Lebanese University

(This talk is based on joint work with Salman Ghazal.)

MSC2000: 05C20

Abstract

The celebrated Erdős-Hajnal conjecture states that for every undirected graph H there exists $\epsilon(H) > 0$ such that every undirected graph on n vertices that does not contain H as an induced subgraph contains a clique or a stable set of size at least $n^{\epsilon(H)}$. This conjecture has a directed equivalent version stating that for every tournament H there exists $\epsilon(H) > 0$ such that every $H-$free $n-$vertex tournament T contains a transitive subtournament of order at least $n^{\epsilon(H)}$. This conjecture is known to hold for a few infinite families of tournaments. In this talk I will discuss a joint work with Salman Ghazal in which we construct a new infinite family of tournaments - the family of so-called galaxies with spiders and prove the correctness of the conjecture for every galaxy with spiders.

On Hamilton Cycles in Kneser Graphs

Namrata

University of Warwick

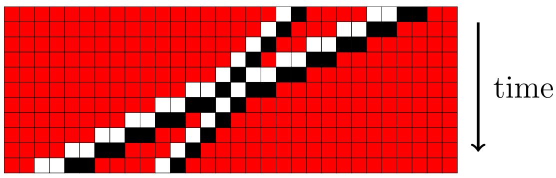
(This talk is based on joint work with Arturo Merino, Torsten Mütze and Pascal Su.)

MSC2000: 05C45

For integers $k \geq 1$ and $n \geq 2k+1$, the *Kneser graph* $K(n,k)$ has as vertices all k-element subsets of $\{1,\dots n\}$ and its edges connect pairs of subsets that are disjoint. These graphs were introduced by Lovász [4] in his celebrated proof of Kneser's conjecture, showing that the chromatic number of $K(n,k)$ equals $n-2k+2$. Also, the maximum size of an independent set in $K(n,k)$ equals $\binom{n-1}{k-1}$ by the famous Erdős-Ko-Rado theorem.

It has long been conjectured that all Kneser graphs $K(n,k)$ have a Hamilton cycle, with one notable exception, namely the Petersen graph $K(5,2)$. This is a special case of an even more general conjecture due to Lovász [3], which asserts that every connected vertex-transitive graph has a Hamilton cycle, apart from $K(5,2)$ and four additional exceptions. To date, the conjecture about Kneser graphs has been verified in the dense case when $n \geq 2.62k+1$ [1] and in the sparse case when $n = 2k+2^a$, $a \geq 0$ [5].

In this work we prove that Kneser graphs $K(n,k)$ have a Hamilton cycle for all $n \geq 7$ that are a prime number or twice a prime number. These results are obtained from a new construction of a cycle factor in $K(n,k)$ via the parenthesis matching approach of Greene and Kleitman [2]. It turns out that these cycles can be described and analyzed by a physical system of multiple moving 'gliders' that participate in collisions and overtakings, while preserving kinetic energy. The figure shows the collision of two gliders.



[1] Y. Chen. Triangle-free Hamiltonian Kneser graphs. *J. Combin. Theory Ser. B*, 89(1):1–16, 2003.

[2] C. Greene and D. J. Kleitman. Strong versions of Sperner's theorem. *J. Combin. Theory Ser. A*, 20(1):80–88, 1976.

[3] L. Lovász. Problem 11. In *Combinatorial Structures and Their Applications (Proc. Calgary Internat. Conf., Calgary, Alberta, 1969)*. Gordon and Breach NY, 1970.

[4] L. Lovász. Kneser's conjecture, chromatic number, and homotopy. *J. Combin. Theory Ser. A*, 25(3):319–324, 1978.

[5] T. Mütze, J. Nummenpalo, and B. Walczak. Sparse Kneser graphs are Hamiltonian. To appear in *J. London Math. Soc.*, 2021.

Friday 10:55, Zoom 5

On 12-regular nut graphs

Riste Škrekovski

University of Ljubljana & Faculty of Information Studies in Novo Mesto

(This talk is based on joint work with Nino Bašić & Martin Knor.)

MSC2000: 05C50, 15A18

A nut graph is a simple graph whose adjacency matrix is singular with 1-dimensional kernel and corresponding eigenvector with no zero elements. For each $d \in \{3, 4, \ldots, 11\}$ are known all values n for which there exists a d-regular nut graph of order n. In the talk, we consider all values n for which there exists a 12-regular nut graph of order n.

On Proximity and Remoteness in Graphs and Digraphs

Sonwabile Mafunda

University of Johannesburg, South Africa

(This talk is based on joint work with Peter Dankelmann, Betsie Jonck & another joint work with Jiangdong Ai, Stefanie Gerke, Gregory Gutin.)

MSC2000: 05C12

In a connected, finite graph or a strong, finite digraph G of order n, the distance $d_G(u,v)$ from a vertex u to a vertex v is the length of a shortest $u-v$ (di)path in G. The *average distance* $\bar{\sigma}(x)$ of a vertex x is the average of the distance from x to all other vertices in G, that is, $\bar{\sigma}(x) = (|V(G)|-1)^{-1} \sum\limits_{y \in V(G)} d_G(x,y)$. The *proximity* $\pi(G)$ and *remoteness* $\rho(G)$ of a (di)graph G are defined by $\pi(G) = \min\limits_G\{\bar{\sigma}(x)|\ x \in V\}$ and $\rho(G) = \max\limits_G\{\bar{\sigma}(x)|\ x \in V\}$ respectively. For a graph, the *minimum degree*, $\delta(G)$ is the smallest of the degrees of the vertices of G.
Bounds on proximity and remoteness in terms of order were given by Aouchiche and Hansen in 2011 for graphs. In 2015 Dankelmann strengthened these bounds by taking into account also the minimum degree.
In this talk we show that these bounds can be improved for triangle-free graphs and for graphs not containing a 4-cycle. We also present results on proximity and remoteness of directed graphs.

Optimal Adjacent Vertex-Distinguishing Edge-Colorings of Circulant Graphs

Souad Slimani

Laboratoire LaROMaD, sfr Maths à Modeler. U.S.T.H.B Université, Faculté des Mathématiques

(This talk is based on joint work with Sylvain Gravier and Hippolyte Signargout.)

MSC2000: 05C15-05C38

A k-proper edge-coloring of a graph G is called *adjacent vertex-distinguishing* if any two adjacent vertices are distinguished by the set of colors appearing in the edges incident to each vertex. The smallest value k for which G admits such coloring is denoted by $\chi'_a(G)$. This notion was introduced by Zhang et al.[5] and they conjectured that if G is a simple connected graph on at least 3 vertices and $G \neq C_5$ (a cycle of order 5) then $\Delta(G) \leq \chi'_a(G) \leq \Delta(G) + 2$. For $n \in \mathbb{N}^*$ and $S \subset \mathbb{Z}_n$, the *circulant graph* $C_n(S)$ is the non-directed graph whose n vertices are the elements of $\mathbb{Z}_n$ with an edge (i, j) if and only if $|i - j| \in S$. In this paper we will say two vertices i and j are at distance d and that an edge (i, j) is of length d if $|i - j| = d$. We write $[\![a, b]\!] = \{i \in \mathbb{N} | a \leq i \leq b\}$. We prove that $\chi'_a(G) = 2R + 1$ for most circulant graphs $C_n([\![1, R]\!])$.

[1] P.N. Balister, E. Györi, J. Lahel and R. H. Schelp. Adjacent Vertex Distinguishing Edge-Colorings. *SIAM Journal on Discrete Mathematics*, 21(1) :237, 2007.

[2] J. L. Baril, H. Kheddouci and O. Togni. Adjacent Vertex Distinguishing Edge-Colorings of Meshes and Hypercubes. *The Australasian Journal of Combinatorics*, 35 : 89–102, 2006

[3] H. Hatami. δ+300 is a Bound on the Adjacent Vertex-Distinguishing Edge Chromatic Number. *Journal of Combinatorial Theory, Series B*, 95(2): 246–256, 2005

[4] W. Wang and Y. Wang. Adjacent Vertex Distinguishing Edge-Colorings of Graphs With Smaller Maximum Average Degree. *Journal of Combinatorial Optimization*, 19(4) : 471–485, 2010

[5] Z. Zhang, L. Liu and J. Wang. Adjacent Strong Edge Colorings of Graphs. *Applied Mathematics Letters*, 15 (5) : 623–626, 2020

Colour-bias problems for dense graphs

Andrew Treglown

University of Birmingham

(This talk is based on joint work with József Balogh, Béla Csaba and András Pluhár; Andrea Freschi, Joseph Hyde and Joanna Lada.)

MSC2000: 05C35, 05C15

The study of colour-biased structures in graphs concerns the following problem. Given graphs H and G, what is the largest t such that in any r-colouring of the edges of G, there is always a copy of H in G that has at least t edges of the same colour? Note if H is a subgraph of G, one can trivially ensure a copy of H with at least $|E(H)|/r$ edges of the same colour; so one is interested in when one can achieve a colour-bias significantly above this. The 2-colour version of the problem is often stated in the language of *graph discrepancy.*

The topic was first raised by Erdős in the 1960s but has seen a resurgence of interest in the last couple of years. In this talk we survey recent progress in the area, including results on Hamilton cycles, trees and clique factors. We will also highlight several open problems.

The Erdős-Rothschild problem

Katherine Staden

University of Oxford

(This talk is based on joint work with Oleg Pikhurko.)

MSC2000: 05D99, 05C15, 05B20

Consider an n-vertex graph G whose edges are coloured with s colours so that there is no monochromatic clique of size k, and call such a colouring of G valid. The Erdős-Rothschild problem from 1974 is to determine the maximum number of valid colourings over all n-vertex graphs G. This problem is in general wide open and an exact (or even asymptotic) answer is only known for a few pairs (k, s). In this talk I will discuss a method for obtaining new exact results.

Orthogonal Colourings of Random Graphs

Kyle MacKeigan

Dalhousie University, Canada

(This talk is based on joint work with Jeannette Janssen.)

MSC2000: 05C15, 05C60, 05C80,

Two colourings of a graph are orthogonal if they have the property that when two vertices receive the same colour in one colouring, then those vertices must receive distinct colours in the other colouring. In this talk, orthogonal colourings of random geometric graphs are discussed. It is shown that sparse random geometric graphs have optimal orthogonal colourings with high probability. Then, an upper bound on the orthogonal chromatic number for dense random geometric graphs is obtained.

Conflict-free coloring game

Paola Tatiana P. Huaynoca

Fluminense Federal University

(This talk is based on joint work with Simone Dantas and Rodrigo Marinho.)

MSC2000: 05C15, 05C57

In Cellular Networks, communication between bases and mobile devices is established via radio frequencies. Interference occurs if one particular device communicates with two different bases that have the same frequency. Thus, every device must contact a base with a unique frequency and, since having a lot of different frequencies is expensive, it is important to minimize the number of frequencies without interference between them.

In order to study the aforementioned problem, in 2002, Even, Lotker, Ron and Smorodinsky [4] introduced the concept of Conflict Free coloring in a geometric scenario. The *Conflict-Free coloring problem* of a graph G consists of assigning different colors to the vertices of G such that, for every vertex v there exists a vertex v' in the neighborhood of v, such that the color of v' differs from the color of every other vertex in the neighborhood of v. This problem has attracted a lot of attention in the last decades. In 2009, for instance, Cheilaris considered these colorings not only on graphs, but also on hypergraphs [3].

Inspired by the Conflict-Free Coloring problem and by the well known coloring game (Gardner [5], Bodlaender [2], Beaulieu, Burke and Duchêne [1]), we study the *Conflict-Free k-coloring game*. The game starts with an uncoloured graph G, $k \geq 2$ different colors, and two players, Alice and Bob, who alternately take turns coloring the vertices of G. Both players respect the following rule: for every vertex v if the neighborhood N_v of v is fully colored then there exists a color which was used only once in N_v. Alice wins the game if the final coloring is a *Conflict-Free k-coloring*, otherwise Bob wins. We consider both players playing optimally and we allow both to start the game. In this work, we determine necessary and sufficient conditions for Alice to win, and we analyze the game played on graph classes.

[1] G. Beaulieu, K. Burke and E. Duchêne: Impartial Coloring games, Theoretical Computer Science, 485 (2013) 49–60.

[2] H. Bodlaender, On the complexity of some coloring games, Graph-Theoretic Concepts in Computer Science of Lecture Notes in Computer Science 484 (1991) 30–40.

[3] P. Cheilaris, Conflict-free coloring, City University of New York, New York, 2009.

[4] G. Even, Z. Lotker, D. Ron, S. Smorodinsky, Conflict–free colorings of simple geometric regions with applications to frequency assignment in cellular networks, SIAM Journal on Computing 33 (2003) 94–136.

[5] M. Gardner, Mathematical Games, Scientific American 23 (1981) 18–23.

Small sums of five roots of unity

Ben Barber

University of Manchester

MSC2000: 11B75

How small can can the sum of k complex nth roots of unity be? The best bounds [1, 2] on the minimum $f(k,n)$ have the shape

$$k^{-n} \leq f(k,n) \leq n^{-k/4+o(1)},$$

with the upper bound valid only when n and k are both even. The cases $k \leq 4$ can be treated exactly, leading Myerson [1] to suggest $k = 5$ as holding particular interest.

One of the difficulties with $k = 5$ is that it prevents naive pigeonhole arguments. I'll describe new upper bounds of $O(n^{-4/3})$, improving to $O(n^{-7/3})$ infinitely often, that can be viewed as a precision application of the pigeonhole principle in a tiny part of the configuration space.

The corresponding configurations were suggested by examining exact minimum values computed for $n \leq 221000$. These minima can be explained at least in part by selection of the best example from multiple families of competing configurations related to close rational approximations.

[1] Gerald Myerson. Unsolved Problems: How Small Can a Sum of Roots of Unity Be? *Amer. Math. Monthly*, 93(6):457–459, 1986.

[2] Terry Tao. How small can a sum of a few roots of unity be? MathOverflow, 2010. https://mathoverflow.net/q/46068 (version: 2010-11-14).

Friday 14:25, Zoom 2

Four Dimensional Association Schemes Have Cyclotomic Character Values

Roghayeh Maleki

University of Regina

(This talk is based on joint work with Allen Herman.)

MSC2000: 16Z05, 05E30

In 1980, Simon P. Norton posed the Cyclotomic Eigenvalue Question (CEQ) which asks whether the entries of the character table of a commutative association scheme always lie in a cyclotomic number field. The adjacency algebras of association schemes are a special type of standard integral table algebras with integral multiplicites (SITAwIMs). Character formulas for complete graphs, strongly regular graphs, and doubly regular tournaments imply the CEQ is true in dimensions 2 and 3.

In this talk we will show that the values of irreducible characters of SITAwIMs of dimension up to 4 lie in cyclotomic number fields. We also give an example of a SITAwIM with noncyclotomic character values of dimension 5. This is joint work with Allen Herman.

Counting substructures of highly symmetric structures

Samuel Braunfeld

University of Maryland, College Park

MSC2000: 03C15, 05A16, 20B27

In the 1970s, Peter Cameron began studying orbit-counting for a group acting on a countable space. This problem may be alternately viewed as starting with a countable structure M satisfying the strong symmetry condition of homogeneity, and considering the growth rate of the hereditary class of finite substructures of M, i.e. counting the number of (unlabelled) substructures of size n for each $n \in \mathbb{N}$.

We give a description of the spectrum of possible subexponential growth rates and of the homogeneous structures that realize them. In particular, we prove there is a jump from polynomial growth to the partition function as well as infinite families of further jumps, and that these jumps in growth rate reflect jumps in the structural complexity of M. This confirms some longstanding conjectures of Cameron and Macpherson.

The methods are primarily model-theoretic, but no prior knowledge will be assumed. One goal will be to show that model theory and combinatorics can be very closely aligned.

[1] Samuel Braunfeld, *Monadic stability and growth rates of ω-categorical structures*, ***arXiv preprint***, arXiv:1910.04380 (2019).

How many finiite rings are there?

Simon R. Blackburn

Royal Holloway University of London

(This talk is based on joint work with K. Robin McLean, University of Liverpool.)

MSC2000: 05A16

For a positive integer n, write $f(n)$ for the number of isomorphism classes of rings of order n. What can we say about $f(n)$?

Determining $f(n)$ exactly for all n looks unrealistic, but in 1970 Kruse and Price [2] stated an asymptotic result that gives the growth rate of $f(n)$ as n goes to infinity. Sadly, as pointed out by Knopfmacher [1], there is a problem with their proof. I will talk about the problem, how to fix it, and how to improve the error term of the Kruse–Price result. I will assume no knowledge of ring theory above a first undergraduate course.

[1] John Knopfmacher, 'Arithmetical properties of finite rings and algebras, and analytic number theory. III. Finite modules and algebras over Dedekind domains', *J. Reine Angew. Math.* **259** (1973), 157–170.

[2] R.L. Kruse and D.T. Price, Enumerating finite rings, *J. Lond. Math. Soc.* (2) **2** (1970) 149–159.

Friday 14:00, Zoom 3

From Convexity to Threshold Logic with Lattices on the Path

M. Reza Emamy-K

UPR at Rio Piedras, San Juan, PR, USA

(This talk is based on joint work with Gustavo M. Meléndez Ríos.)

52A99, 52C99, 06D99

A cut-complex over the geometric unit n-cube is a proper cubical complex whose vertices are strictly separable from the rest of the vertices of the n-cube by a hyperplane of R^n. These objects render geometric presentations for threshold Boolean functions, and their study leads to a convex geometric connection to threshold logic. We present an overview of some of the results on this connection and define a poset on the set of all cut-complexes that turns out to be a distributive lattice for dimensions $n \leq 4$.

Keywords: Convexity, Hyperplanes, Hypercube Cuts, Distributive lattices.

Voronoi Games on the Discrete Hypercube

Robert Johnson

Queen Mary, University of London

(This talk is based on joint work with A. Nicholas Day.)

MSC2000: 05C57, 91A46

Suppose that X is a metric space and S is a finite subset of X. The *Voronoi cell* of $i \in S$ is the set of all points of X which are closer to i than to any other element of S. In a *Voronoi game* we think of the elements of S as competing players with the payoff to i being the volume (under some measure) of its Voronoi cell (to complete the specification of the game we need to state how the players can locate themselves or move within X).

A classical result in this area is the *Median Voter Theorem* which considers the setting of two players with X being a real interval. This can be interpreted as a *spatial voting* model; we have two candidates competing for vote share in an election where voters' (and candidates') opinions are represented by points on a continuous 1-dimensional (left-wing/rightwing) spectrum. In higher dimensions the situation is much more complicated and there is no analogue of the Median Voter Theorem.

Discrete Voronoi games have been much less studied than continuous ones and lead to some appealing combinatorial problems. We will consider the underlying space X being the discrete hypercube (in spatial voting terms, this can be thought of as an opinion space described by d binary issues). The game is rather simple with two players on a hypercube with uniform measure. However, with more players, or a different measure it becomes much more interesting. We exhibit a variety of results and open questions, particularly focussing on the existence of equilibria.

Friday 14:50, Zoom 3

Hyperplane coverings with multiplicities

Simona Boyadzhiyska

Freie Universität Berlin

(This talk is based on joint work with Anurag Bishnoi, Shagnik Das, and Tamás Mészáros.)

MSC2000: 05B40

A well-known result of Alon and Füredi states that n hyperplanes are needed to cover all nonzero points of $\mathbb{F}_2^n$ while avoiding the origin. In this talk, we will generalize this result to the setting where the points of $\mathbb{F}_2^n \setminus \{\vec{0}\}$ must be covered at least k times, while the origin can be covered by at most $k-1$ hyperplanes. Exploiting a connection to coding theory, and using combinatorial and probabilistic arguments, we will provide tight bounds in certain ranges of the parameters n and k.

Location-Domination in Binary Hamming Spaces: An Improved Lower Bound

Tuomo Lehtilä

Université Claude Bernard Lyon 1, LIRIS, Lyon, France, and University of Turku, Finland.

(This talk is based on joint work with Ville Junnila and Tero Laihonen from University of Turku, Finland.)

MSC2000: 94B60; 94B65; 05C69; 05C76

A set of vertices C in a graph $G = (V, E)$ is a *locating-dominating code* if each vertex in $V \setminus C$ has a distinct neighbourhood in C and none of these neighbourhoods is empty. The cardinality of the smallest such code in graph G is denoted by $\gamma^{LD}(G)$. Locating-dominating codes have originally been introduced by Slater and Rall in 1980s and have been widely studied since then. We concentrate on locating-dominating codes in the *binary Hamming space* (or Hamming graph or hypercube) denoted by $\mathbb{F}^n$.

The earliest lower bound for locating-dominating codes in Hamming spaces

$$\gamma^{LD}(\mathbb{F}^n) \geq 2^{n+1}/(n+3)$$

is due to Slater in 2002. After that the lower bound has been improved by Honkala et al. in 2004 to

$$\gamma^{LD}(\mathbb{F}^n) \geq 2^{n+1}/(n+2+3/n-2/n^2).$$

The proof of Slater was based on a technique called *share* and the proof of Honkala et al. on a *father-son argument*. In [2], we have combined these two methods together with new schemes of shifting the share. We get a new improved lower bound

$$\gamma^{LD}(\mathbb{F}^n) \geq \begin{cases} \frac{2^{n+1}}{n+1+2(n-1)/(3(n-4))}, & \text{if } 10 \leq n \leq 12; \\ \frac{2^{n+1}}{n+2-4/(3n)+2/(n^2-5n)}, & \text{if } 13 \leq n. \end{cases}$$

The strength of our approach is illustrated by the case $n = 11$, where the lower bound is improved from 309 to 317 and the best known construction has cardinality 320 ([1]). This gap of size four is smaller than what is known in $\mathbb{F}^n$ for $7 \leq n \leq 10$, [2, Table 1]. Moreover, the small gap between the upper and lower bound also highlights how strong the 320 size code is and we conjecture it to be the best possible.

[1] V. Junnila and T. Laihonen and T. Lehtilä. On regular and new types of codes for location-domination. *Discrete Appl. Math.*, 247:225–241, 2018.

[2] V. Junnila and T. Laihonen and T. Lehtilä. Improved Lower Bound for Locating-Dominating Codes in Binary Hamming Spaces. arXiv:2102.05537, 2021.

The minimum degree of minimal Ramsey graphs for cliques

Anurag Bishnoi

TU Delft

(This talk is based on joint work with John Bamberg and Thomas Lesgourgues.)

MSC2000: 05D10, 05C55, 51E12

We will present a new upper bound of $s_r(K_k) = O(k^5 r^{5/2})$ on the Ramsey parameter $s_r(K_k)$ introduced by Burr, Erdős and Lovász in 1976, which is defined as the smallest minimum degree of a graph G such that any r-colouring of the edges of G contains a monochromatic K_k, whereas no proper subgraph of G has this property. This improves the previous upper bound of $s_r(K_k) = O(k^6 r^3)$ proved by Fox et al. The construction used in our proof relies on a group theoretic model of generalised quadrangles introduced by Kantor in 1980.

The Ramsey number for 4-uniform tight cycles

Vincent Pfenninger

University of Birmingham

(This talk is based on joint work with Allan Lo.)

MSC2000: 05C35, 05C65, 05D10

The *Ramsey number* for a k-graph (k-uniform hypergraph) H is the least integer N such that any 2-edge-colouring of the complete k-graph on N vertices contains a monochromatic copy of H. A *k-uniform tight cycle* is a k-graph with a cyclic ordering of its vertices such that its edges are precisely the sets of k consecutive vertices in that ordering. We prove that the Ramsey number for the 4-uniform tight cycle on $4n$ vertices is $(5+o(1))n$. This is asymptotically tight and confirms a special case of a conjecture of Haxell, Łuczak, Peng, Rödl, Ruciński and Skokan.

Bounds on Ramsey Games via Alterations

He Guo

Georgia Institute of Technology

(This talk is based on joint work with Lutz Warnke.)

MSC2000: 05C55, 05C80, 05D10, 05D40

In this talk we introduce a refined alteration approach for constructing H-free graphs: we show that removing all edges in H-copies of the binomial random graph does not significantly change the independence number (for suitable edge-probabilities); previous alteration approaches of Erdos and Krivelevich remove only a subset of these edges. We present two applications to online graph Ramsey games of recent interest, deriving new bounds for Ramsey, Paper, Scissors games and online Ramsey numbers.

H. Guo and L. Warnke. Bounds on Ramsey Games via Alterations. Preprint (2019). arXiv:1909.02691.

Orientation Ramsey thresholds

Tássio Naia

Instituto de Matemática e Estatística, USP

(This talk is based on joint work with G. Barros, B. P. Cavalar, G. O. Mota and Y. Kohayakawa.)

MSC2000: 05D10,05C80,05C20

If G is a graph and $\vec{H}$ is an oriented graph, we write $G \to \vec{H}$ to say that every orientation of G contains $\vec{H}$ as a subdigraph. Since every graph admits an acyclic orientation, we have that $G \not\to \vec{H}$ whenever $\vec{H}$ has a directed cycle $v_1 \to v_2 \to \cdots \to v_m \to v_1$. We consider the case in which G is the binomial random graph $G(n,p)$ and study the threshold $p_{\vec{H}} = p_{\vec{H}}(n)$ for the property $G(n,p) \to \vec{H}$, where $\vec{H}$ is an acyclically oriented graph. In this talk we shall present some recent results about $p_{\vec{H}}$.

For any graph or oriented graph G, the *maximum density* and (when $v(G) \geq 3$) the *maximum 2-density* of G are, respectively,

$$m(G) := \max_{\substack{J \subseteq G \\ v(J) \geq 1}} \frac{e(J)}{v(J)} \qquad \text{and} \qquad m_2(G) := \max_{\substack{J \subseteq G \\ v(J) \geq 3}} \frac{e(J)-1}{v(J)-2}.$$

It can be shown that $p_{\vec{H}}$ is always bounded above by $n^{-1/m_2(\vec{H})}$ (we ignore multiplicative constants). We show a matching lower bound for some classes of directed graphs. Interestingly, we also show that if $\vec{H}$ is a transitive triangle, or is formed by a rooted product of an oriented tree and a transitive triangle, then $p_{\vec{H}} \ll n^{-1/m_2(\vec{H})}$. This contrasts to the classical edge-colouring threshold for the presence of a monochromatic copy of a graph H in $G(n,p)$, where the threshold is $n^{-1/m_2(H)}$ whenever, for instance, H has a cycle.

For example, we show that if $\vec{H}$ is a transitive tournament or acyclically oriented cycle, then

$$p_{\vec{H}} = \begin{cases} n^{-1/m_2(\vec{H})} & \text{if } v(\vec{H}) \geq 4, \\ n^{-1/m(\vec{H})} & \text{if } v(\vec{H}) = 3. \end{cases}$$

Tangled Paths: A Random Graph Model from Mallows Permutations

John Sylvester

University of Glasgow

(This talk is based on joint work with Jessica Enright, Kitty Meeks, and William Pettersson.)

MSC2000: 05C80, 05A05, 68Q87, 05C78.

We introduce a new random graph model $\mathcal{P}(n,q)$ which results from taking the union of two paths of length $n \geq 1$, where the vertices of one path have been relabelled according to a permutation sampled from the Mallows distribution with real parameter $0 < q(n) \leq 1$.

The aforementioned Mallows distribution [1] samples an n element permutation σ with probability proportional to $q^{\mathsf{inv}(\sigma)}$, where $\mathsf{inv}(\sigma)$ counts the number of inverted pairs of elements in σ. Increasing q has the following effect on the resulting random graph $\mathcal{P}(n,q)$: if q is close to 0 the graph bears resemblance to a path and as q tends to 1 it becomes an expander.

In order to further understand the effect of the parameter q on the structure of $\mathcal{P}(n,q)$ we obtained bounds on the treewidth and cutwidth in terms of q, and show the diameter is constant for fixed $q < 1$. We also prove a sharp threshold for the property of having a separator of size one.

[1] C. L. Mallows. Non-null ranking models. I. *Biometrika*, 44:114–130, 1957.

Prague Dimension of Random Graphs

Kalen Patton

Georgia Institute of Technology

(This talk is based on joint work with He Guo, Lutz Warnke.)

MSC2000: 05C80, 05C15, 05C62

The Prague dimension of graphs was introduced by Nesetril, Pultr and Rodl in the 1970s. Proving a conjecture of Furedi and Kantor, we show that the Prague dimension of the binomial random graph is typically of order $n/\log n$ for constant edge-probabilities. The main new proof ingredient is a Pippenger-Spencer type edge-coloring result for random hypergraphs with large uniformities, i.e., edges of size $O(\log n)$.

Based on joint work with He Guo and Lutz Warnke, see `https://arxiv.org/abs/2011.09459`.

The jump of the clique chromatic number of random graphs

Lutz Warnke

Georgia Institute of Technology

(This talk is based on joint work with Lyuben Lichev and Dieter Mitsche.)

MSC2000: 05C80,05C15,60C05

The clique chromatic number of a graph is the smallest number of colors in a vertex coloring so that no inclusion-maximal clique is monochromatic (ignoring isolated vertices). Settling an open problem of McDiarmid, Mitsche and Prałat, in this talk we explain the surprising polynomial 'jump' of the clique chromatic number of the binomial random graph $G_{n,p}$ around edge-probability $p \approx n^{-1/2}$. Our proof resolves this unusually steep transition by a mix of approximation and concentration arguments, which enables us to go beyond Janson's inequality used in previous work. As a by-product, we also determine the clique chromatic number of $G_{n,p}$ up to logarithmic factors for all edge-probabilities p.

On a k-matching algorithm and finding k-factors in random graphs with minimum degree $k+1$ in linear time

Michael Anastos

Freie Universität Berlin

MSC2000: 05C80, 05C85

We prove that for $k+1 \geq 3$ and $c > (k+1)/2$ w.h.p. the random graph on n vertices, cn edges and minimum degree $k+1$ contains a (near) perfect k-matching. As an immediate consequence we get that w.h.p. the $(k+1)$-core of $G_{n,p}$, if non empty, spans a (near) spanning k-regular subgraph. This improves upon a result of Chan and Molloy [2] and completely resolves a conjecture of Bollobás, Kim and Verstraëte [1]. In addition, we show that w.h.p. such a subgraph can be found in linear time. A substantial element of the proof is the analysis of a randomized algorithm for finding k-matchings in random graphs with minimum degree $k+1$.

[1] B. Bollobás, J.H. Kim and J. Verstraëte, Regular subgraphs of random graphs, *Random Structures & Algorithms*.29(2006), 1-13.

[2] S.O. Chan and M. Molloy, (k+1)-cores have k-factors,*Combinatorics, Probability and Computing*, 21(6)(2012), 882-896.

Space vectors forming rational angles

Alexander Kolpakov

Université de Neuchâtel

(This talk is based on joint work with Kiran S. Kedlaya, Bjorn Poonen, and Michael Rubinstein.)

MSC2000: 52B10, 11R18, 14Q25, 51M04

We classify all sets of non-zero vectors in $\mathbb{R}^3$ such that the angle formed by each pair is a rational multiple of π. The special case of four-element subsets lets us classify all tetrahedra whose dihedral angles are multiples of π, solving a 1976 problem of Conway and Jones: there are 2 one-parameter families and 59 sporadic tetrahedra, all but three of which are related to either the icosidodecahedron or the B_3 root lattice. This becomes possible by applying a blend of algebraic geometry, group theory, combinatorics, and polyhedral geometry.

We start by determining all possible rational-angled 4-line configurations, of which the rational-angled tetrahedra are a subset. Geometry reduces the problem to solving a polynomial equation whose variables are constrained to lie in the set μ of all roots of unity. There are two known methods for solving equations in roots of unity: one is based on classifying vanishing sums of roots of unity, and the other is based on computing torsion closures of semi-abelian varieties. The complexity of each algorithm grows faster than exponentially: given that our polynomial equation has 105 monomials in 6 variables none of these approaches will work directly! The previous record was only 12 monomials.

The key idea, never before used to solve equations in roots of unity in characteristic 0, of building upon the recent result of Dvornicich and Zannier by working first in the quotient $\mathbb{Z}[\mu]/(2)$ of the subring $Z[\mu] \subset \mathbb{C}$. This makes the problem barely solvable in a number of steps.

First, reducing modulo 2 yields a polynomial equation in $\mathbb{Z}[\mu]/(2)$ with only 12 monomials. We adapt the classification of vanishing sums of unity to parametrise all solutions in μ to such equations in $\mathbb{Z}[\mu]/(2)$. This restricts the possible 6-tuples to lie in finitely many families, each parametrized by at most 3 variables. Second, substituting each parametrisation back into the original equation yields a number of simpler polynomial equations, no longer mod 2, in at most 3 variables. We solve each of these by computing torsion closures and making use of the extremely rich $W(D_6)$ symmetry of the original equation.

Finally, we proceed to finding all maximal rational-angled line configurations.

The manuscript is available as `https://arxiv.org/abs/2011.14232`. The code for the computations therein, written in `c++`, `Magma`, `SageMath`, and `Singular` is available at `https://github.com/kedlaya/tetrahedra/`

On the Erdős-Ko-Rado theorem for transitive groups

Sarobidy Razafimahatratra

University of Regina

(This talk is based on joint work with Karen Meagher and Pablo Spiga.)

MSC2000: 05C35, 05C69

A set of permutations $\mathcal{F}$ of a finite transitive group $G \leq \mathrm{Sym}(\Omega)$ is *intersecting* if any two permutations in $\mathcal{F}$ agree on an element of Ω. The *intersection density* of the intersecting set $\mathcal{F} \subset G$ is the rational number $\rho(\mathcal{F}) := \frac{|\mathcal{F}|}{|G_\omega|}$, where $\omega \in \Omega$. The intersection density of the group G is the number $\rho(G) := \max\{\rho(\mathcal{F}) : \mathcal{F} \subset G \text{ is intersecting}\}$. The group G is said to have the *Erdős-Ko-Rado (EKR) property* if $\rho(G) = 1$.

The standard tool used to study the EKR property of the transitive group G is its *derangement graph* Γ_G. This graph is the Cayley graph of G with connection set equal to the set of all derangements of G (i.e., the fixed-point-free elements).

I will talk about some recent progress on the construction of transitive groups that do not have the EKR property. I will focus on the transitive groups with complete multipartite derangement graphs. I will also present some open problems on the intersection density of transitive groups of certain degrees.

Series–Parallel Delta-Matroids

Steven Noble

Birkbeck, University of London

(This talk is based on joint work with Criel Merino and Iain Moffatt.)

MSC2000: 05B35, 05C10, 05C31, 05C75

Series–parallel networks form a familiar class of graphs. They are the graphs that can be built from a circuit with two edges by repeatedly adding edges in parallel or series with existing edges. It is possible to characterize them in several ways, for example, they are the 2-connected graphs:

1. for which the coefficient of x in the Tutte polynomial is equal to 1;
2. containing no K_4-minor;
3. constructed from circuits with 2 or 3 edges or the graph comprising 3 parallel edges by repeatedly using 2-sums.

We show that there is an analogous class of ribbon graphs (or equivalently 2-cell embedded graphs) which has characterizations corresponding to each of those described above.

Hamilton transversals in random Latin squares

Tom Kelly

University of Birmingham

(This talk is based on joint work with Stephen Gould.)

MSC2000: 05B15, 05C45, 05C80

Gyárfás and Sárközy [3] conjectured that every $n \times n$ Latin square has a "cycle-free" partial transversal of size $n-2$. We confirm this conjecture in a strong sense for almost all Latin squares, by showing that as $n \to \infty$, all but a vanishing proportion of $n \times n$ Latin squares have a *Hamilton* transversal, i.e. a full transversal for which any proper subset is cycle-free. In fact, we prove a counting result that is best possible up to lower order terms. This result strengthens a result of Kwan [4] (which in turn implies that almost all Latin squares also satisfy the famous Ryser-Brualdi-Stein conjecture [2, 5, 6]). As part of the proof, we also prove that almost all $n \times n$ Latin squares have no symbol appearing more than $\omega(\log n / \log \log n)$ times on the leading diagonal.

The Ryser-Brualdi-Stein conjecture and the Gyárfás-Sárközy conjecture can be equivalently stated as questions concerning the existence of nearly spanning rainbow subgraphs in proper arc-colorings of the complete directed graph and can thus be viewed as "directed analogues" of Andersen's conjecture [1]. We propose a common strengthening of all three of these conjectures, which holds for almost all Latin squares by our main result.

[1] L. D. Andersen. Hamilton circuits with many colours in properly edge-coloured complete graphs. *Math. Scand.*, 64:5–14, 1989.

[2] R. A. Brualdi and H. J. Ryser. *Combinatorial matrix theory.* Cambridge University Press, 1991.

[3] A. Gyárfás and G. N. Sárközy. Rainbow matchings and cycle-free partial transversals of latin squares. *Discrete Math.*, 327:96–102, 2014.

[4] M. Kwan. Almost all Steiner triple systems have perfect matchings. *Proc. Lond. Math. Soc.*, 121(6):1468–1495, 2020.

[5] H. J. Ryser. Neuere Probleme der Kombinatorik. In *Vorträge über Kombinatorik*, pages 69–91, 1967.

[6] S. K. Stein. Transversals of Latin squares and their generalizations. *Pacific J. Math.*, 59(2):567–575, 1975.

Index