*Supporting Online Material*

**GRACE Measurements of Mass Variability in the Earth System**

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# 1. Supporting Text

## 1.1 Degree correlations for GRACE and GLDAS annual cycles

Comparisons between maps can be an imprecise method of quantifying the correlation between two sets of spherical harmonic coefficients; however, meaningful confidence limits can be established by segregating the comparison by degrees (S1). The geopotential is commonly described by a set of gravitational spherical harmonic coefficients, $C_{lm}$ and $S_{lm}$, where $l$ and $m$ are the degree and order, respectively (S2). The scale of a spatial feature in the geoid is related to the spherical harmonic degree (half-wavelength ≈ 20,000/$l$); therefore, a measure of the power as a function of spatial scale can be described by the degree variance

$$\sigma_l^2 = \sum_{m=0}^{l}\left(C_{lm}^2 + S_{lm}^2\right), l \geq 1$$

The degree correlation between two sets of global harmonics denoted as set $A$ and set $B$ is given by

$$r_l = \frac{1}{\sigma_l^{(A)}\sigma_l^{(B)}}\sum_{m=0}^{l}\left(C_{lm}^{(A)}C_{lm}^{(B)} + S_{lm}^{(A)}S_{lm}^{(B)}\right) \ , \ l \geq 1$$

Comparisons between spacecraft observations and geophysical models for the annual geoid variations using degree correlations have been previously limited to half-wavelengths of ~5000 km and larger (S3). The two sets of coefficients from GRACE and GLDAS allowed for comparison of the annual cycle at much smaller spatial scales (Fig. S1). With increasing degree (decreasing spatial scales), GRACE and GLDAS are relatively well correlated at the lowest degrees (longest spatial scales), becoming completely uncorrelated somewhere between degree 30-40 (spatial features of ~500-650 km).

The discrepancy between GRACE and the GLDAS model was due to a number of factors. First, the GRACE errors grow with increasing degree (S4). For small enough spatial scales, the errors will be larger than the time-variable gravity signal. Furthermore, the GLDAS model is an incomplete representation of the expected gravity signal detectable by GRACE. The model does not contain deep groundwater variations, it does not account for lake and river storage, and variations in Greenland and Antarctic are omitted (S5). Finally, the GRACE signal will reflect the unmodeled or incorrectly modeled atmosphere and ocean mass variation.

A degree correlation analysis alone does not indicate that GRACE time-variable gravity results are accurate to a specific spatial scale. In fact, there are no current tests that can validate the accuracy of the time-variable gravity results from GRACE on a global level. However, the results presented here suggest that there is sensible signal to at least 500-650 km when compared to an independent data source. Other methods are required to more precisely determine what level of error is being admitted at specific spatial scales.

## 1.2 Error versus smoothing radius

The error in geoid height calculation at any geographic location depends on errors in all spherical harmonic coefficients of the geopotential, theoretically to infinite degrees, and orders. However, in practicality, the error estimates are limited to a selected maximum degree and order available in the geopotential models.

An areal average, centered at any geographic location, can be calculated from the harmonics of the geopotential. A Gaussian weighted average over a circular disk, centered at that geographic location, offers a means of calculating smoothed estimates of geoid height that does not depend on the high degree terms of the geopotential expansion. This is useful particularly for visualizing the gravity variability phenomena observed from space geodetic methods in a way that is not susceptible to the lack of knowledge of the local or regional static gravity features that are generally ill-known.

In analogy, the spectral representation of the weighting functions may be used to propagate the variances of the geopotential spherical harmonic coefficient errors to the variances of errors in calculation of the areal average geoid height. The error propagation can be used to obtain the global variance (or standard deviation) of geoid height errors for each weighting function (Fig. S2).

## 1.3 Geoid height error generation

The error covariance matrix **P** predicts the likely power of the error for each gravity coefficient as well as the correlation between the coefficient estimates, but the actual errors are unknown. To obtain an illustration of the possible geoid height error, a realization of the individual coefficient errors is required. The process involves factoring the covariance **P** into its ‘square-root’, a triangular matrix **S** that when multiplied by its transpose gives back the matrix **P**. A vector of randomly generated numbers with a Gaussian distribution, zero mean and a standard deviation of 1 are multiplied by the matrix **S**, providing a vector of individual coefficient errors. A geoid height error map can then be calculated (Fig. S3).

**2. Supporting figures**

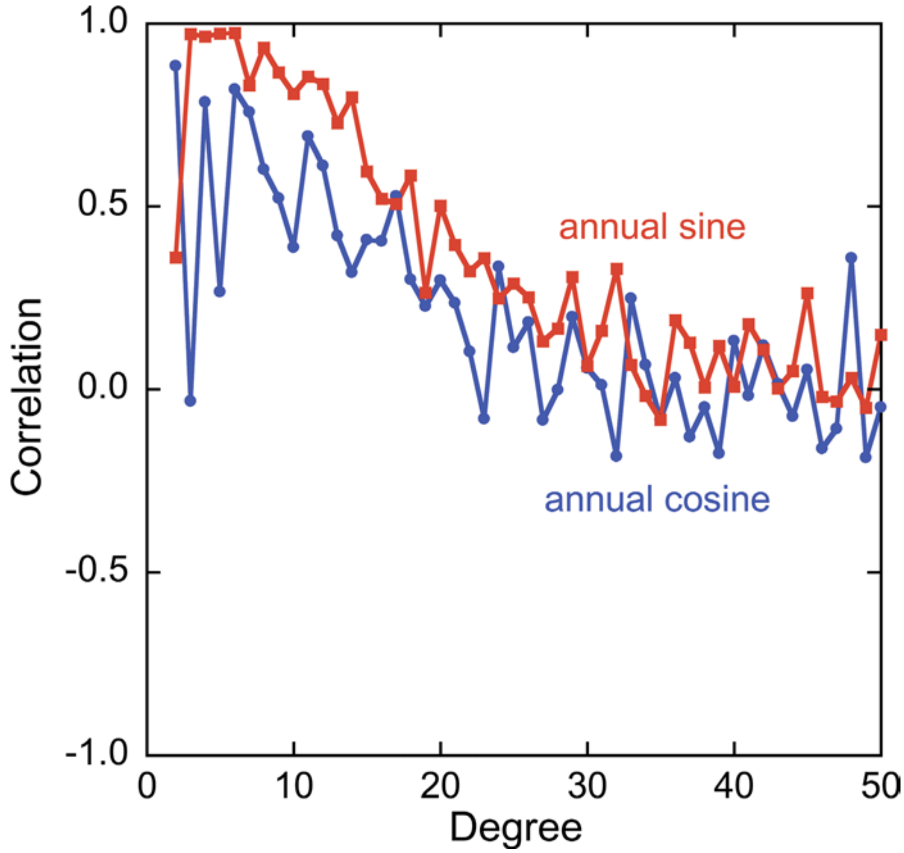


**Fig. S1.** A comparison of the degree correlations for the annual cycle from GRACE and GLDAS is illustrated. Since a description of the annual cycle consists of two parameters, the annual cosine and annual sine contributions were considered separately. A value of 1 indicates perfect correlation, -1 indicates anti-correlation, and 0 indicates uncorrelated. Beyond degree 50 the GRACE and GLDAS coefficients were clearly uncorrelated and are not included in the figure.

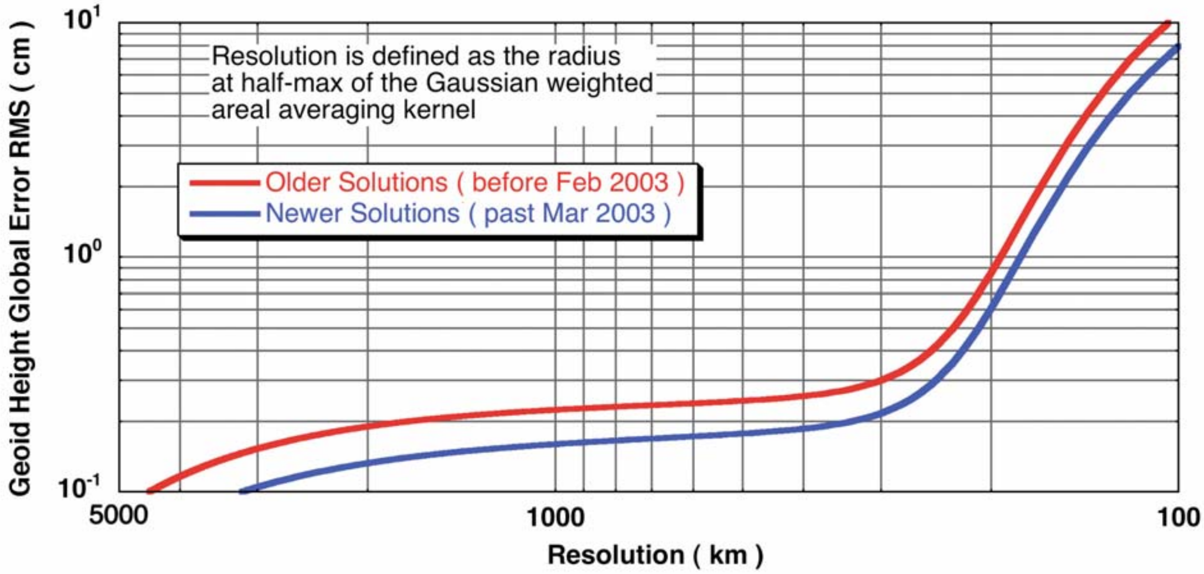


**Fig. S2.** The radius at which the Gaussian weighting function is half the peak at the origin is called the "resolution" associated with that areal average. For each such resolution on the X-axis, the global root-mean-square geoid height error is shown on the Y-axis. Two curves are shown, one from the propagation of the higher errors for GRACE fields in 2002 and another from the propagation of improved GRACE gravity fields starting in 2003.

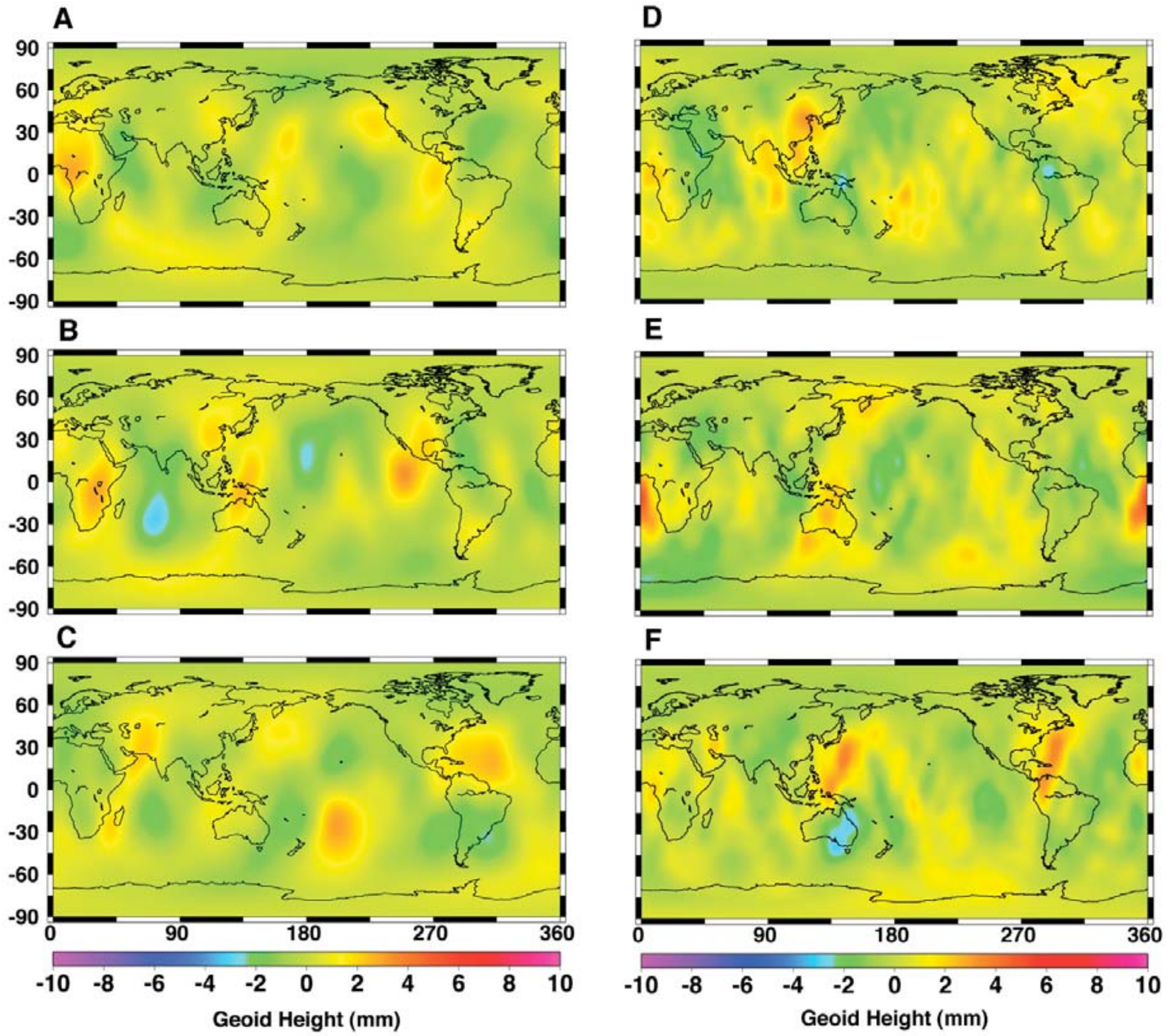


**Fig. S3.** The figure illustrates additional examples of random realizations of the predicted geoid height error for the 2002 gravity solutions using 1000 km smoothing (**A**, **B**, **C**) and for the 2003 solutions using 600 km smoothing (**D**, **E**, **F**). The exact distribution of the errors varies with each realization, but the nature of the errors (amplitude and structure) is similar from map to map. It is generally useful to generate several such realizations. For example, the realization of the errors illustrated in map (**A**) appears to be more benign than the realizations illustrated in the other maps.

## 3. Supporting References

*S1*. D. H. Eckhardt, *Mathematical Geology* **16**, 155 (1984).
*S2*. For example, see W.M. Kaula, *Theory of Satellite Geodesy* (Blaisdell, Waltham, MA, 1966).
*S3*. R. S. Nerem, R. J. Eanes, P. F. Thompson, J. L. Chen, *Geophys. Res. Lett.* **27**, 1783 (2000).
*S4*. J. Kim, B. Tapley, *AIAA J. of Guidance*, *Control and Dynamics* **25**, 1100 (2002).
*S5*. M. Rodell *et al.*, *Bull. Am. Meteorol. Soc.* **85**, 381 (2004).