Center-of-mass energy determination using dimuon events at ILC

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Key issue: systematic control for the absolute scale of center-of-mass energy (in collision...) and reconstructed mass at **all** center-of-mass energies.

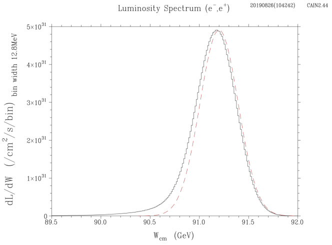
ILC

The ILC has been designed with an emphasis on an **initial-stage Higgs factory** starting at $\sqrt{s} = 250$ GeV and **expandable in energy** to run at higher energies for pair production of top quarks and Higgs bosons, and potentially to ≥ 1 TeV.

The **unique feature** of **longitudinally polarized electron and positron beams** and the **higher energies** open up many new measurement possibilities. These are very complementary to those feasible with $\mathrm{e^+e^-}$ circular colliders.

The ILC is designed primarily to explore the 200 – 1000 GeV energy frontier regime. This has been the focus in making the case for the project. It is also capable of running at the **Z** and **WW** threshold.

Need $\sqrt{s}$ method(s) that works across all energies. High precision required for ultimate Z physics runs. Momentum-scale calibration (needed for $\sqrt{s_p}$ method at all energies) also benefits from Z runs.



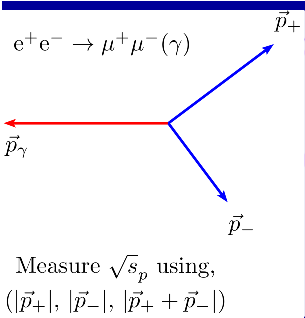
Z running – see Yokoya, Kubo, Okugi

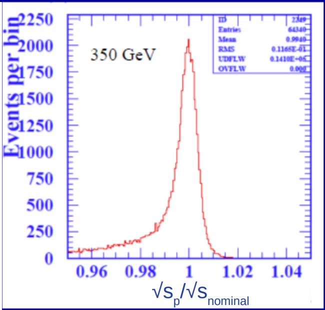
Center-of-Mass Energy Measurement

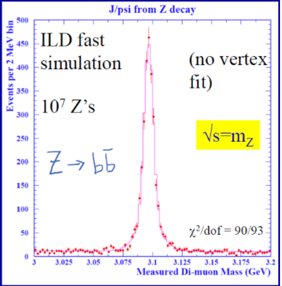
Critical input for M_{t}, M_{W}, M_{H}, M_{Z}, M_{X}, Γ_{Z} measurements

1. Standard precision of $\mathcal{O}(10^{-4})$ in $\sqrt{s}$ for M_{t} straightforward
2. Targeting precision of $\mathcal{O}(10^{-5})$ in $\sqrt{s}$ for M_{W} given likely systematics
3. For M_{Z} - helps to do even better. Now targeting of $\mathcal{O}(10^{-6})$.

Use dilepton **momenta** method, with $\sqrt{s_p} \equiv E_+ + E_- + |\vec{p}_{+-}|$ as $\sqrt{s}$ estimator.
Tie detector p-scale to particle mass scales (J/ψ known to 1.9 ppm).







Measure $< \sqrt{s} >$ and luminosity spectrum with same events. Expect statistical uncertainty of 1.0 ppm on p-scale per 1.2M $J/\psi \to \mu^+\mu^-$ (4×10^9 hadronic Z's).

Introduction to Center-of-Mass Energy Issues

- Proposed $\sqrt{s}_p$ method uses only the momenta of leptons in dilepton events.
- Critical issue for $\sqrt{s}_p$ method: calibrating the **tracker momentum scale**.
- Can use $\mathrm{K^0_S}$, Λ, $J/\psi \to \mu^+\mu^-$ (mass known to 1.9 ppm).

For more details see studies of $\sqrt{s}_p$ from ECFA LC2013, and of momentum-scale from AWLC 2014. Recent $\mathrm{K^0_S}$, Λ studies at LCWS 2021 – much higher precision feasible ... few **ppm** (not limited by parent mass knowledge or J/ψ statistics).

Today,

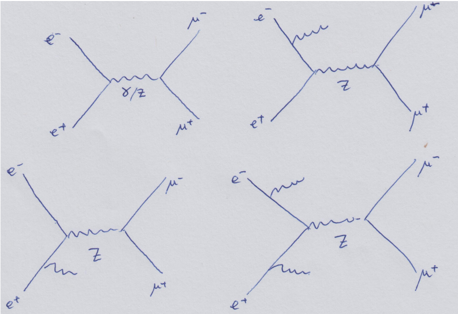
- Look more carefully at the $\sqrt{s}_p$ method prospects with $\mu^+\mu^-$
- Include crossing angle, full simulation and reconstruction with ILD, track error matrices, vertex fitting, and updated ILC $\sqrt{s} = 250$ GeV beam spectrum
- Treatment of detected ISR/FSR photons in progress
- Bonus. Physics: M_Z. Beam knowledge: **luminosity spectrum**, dL/d$\sqrt{s}$, and beam-energy/interaction-vertex correlations.

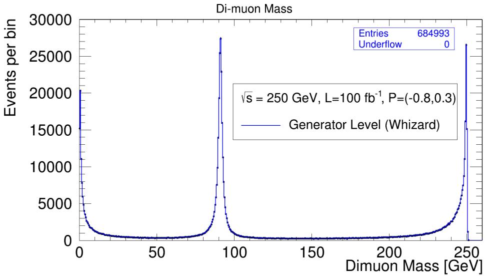
Dimuons

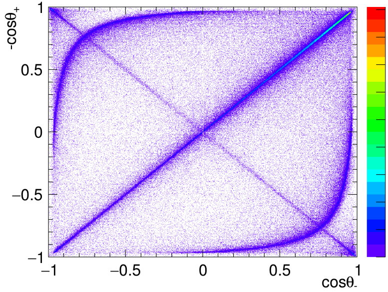
Three main kinematic regimes.

1. **Low** mass, $m_{\mu\mu} < 50$ GeV
2. **Medium** mass, $50 < m_{\mu\mu} < 150$ GeV
3. **High** mass, $m_{\mu\mu} > 150$ GeV

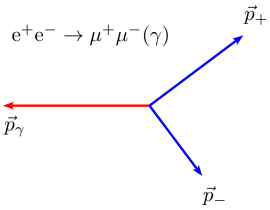
- Back-to-back events in the full energy peak.
- Significant radiative return (ISR) to the Z and to low mass.







$\sqrt{s}_p$ Method in a Nutshell



Measure $\sqrt{s}_p$ using,
$(|\vec{p}_+|, |\vec{p}_-|, |\vec{p}_+ + \vec{p}_-|)$

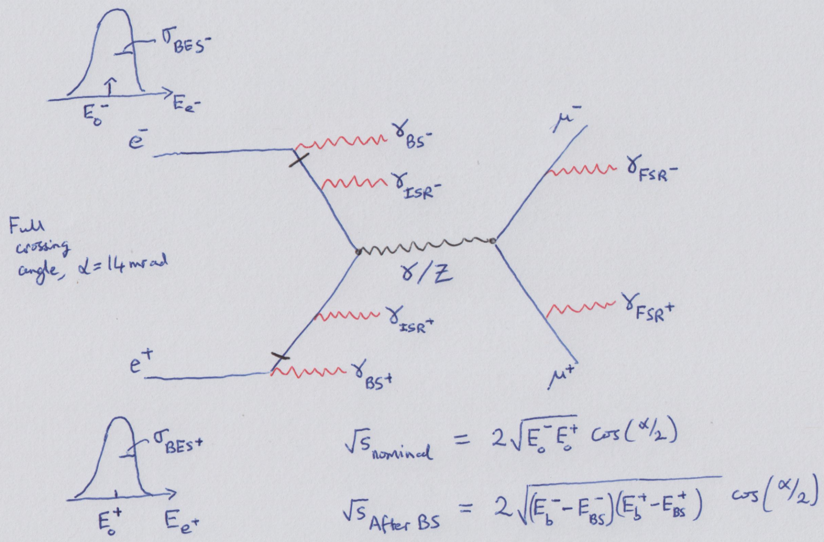
Assuming,

- Equal beam energies, E_{b}
- The lab is the CM frame, $(\sqrt{s} = 2\,E_{\mathrm{b}},\ \sum \vec{p}_i = 0)$
- The system recoiling against the dimuon is massless

$$\sqrt{s} = \sqrt{s}_p \equiv E_+ + E_- + |\vec{p}_+ + \vec{p}_-|$$

$$\sqrt{s}_p = \sqrt{p_+^2 + m_\mu^2} + \sqrt{p_-^2 + m_\mu^2} + |\vec{p}_+ + \vec{p}_-|$$

An estimate of $\sqrt{s}$ using only the (precisely measurable) muon momenta

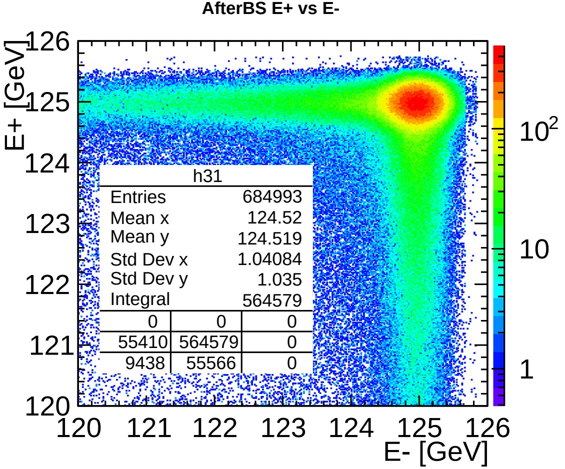


See backup for more detailed explanations

Ideally, the 2-d distribution of the **absolute beam energies** after beamstrahlung. From this we would know the distribution of both $\sqrt{s}$ and the initial state momentum vector (especially the z component).

Now let's look at the related 1-d distributions $(E_+, E_-, \sqrt{s}, \; p_z)$ with empirical fits.

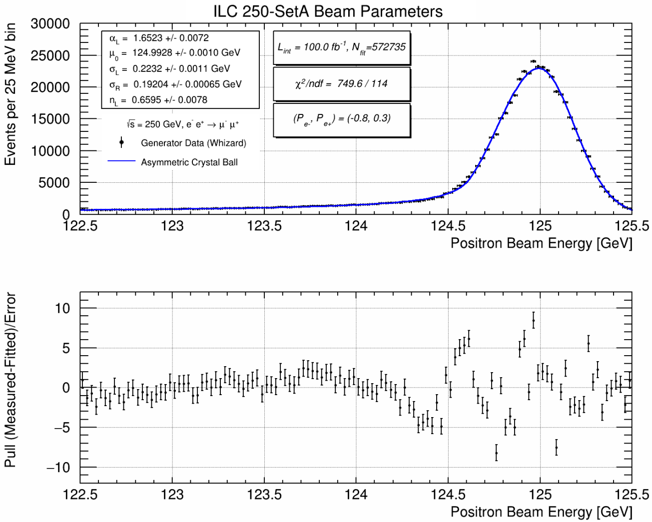
[dL/d$\sqrt{s}$: see work by Frary, Miller, Moenig, Sailer, Poss]



Whizard 250 GeV SetA $e^+e^- \to \mu^+\mu^-(\gamma)$ events

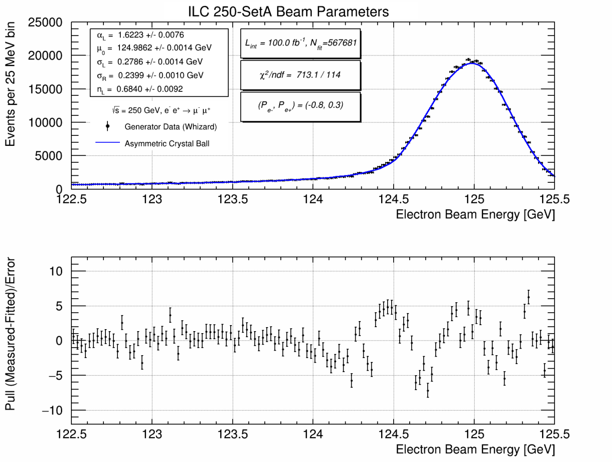
Positron Beam Energy (After Beamstrahlung)

Fits use asymmetric Crystal Ball with 5 parameters (details in backup)



$\sigma_{\mathrm{R}}/E = 0.1536 \pm 0.0005\%$ (cf 0.152% in TDR)

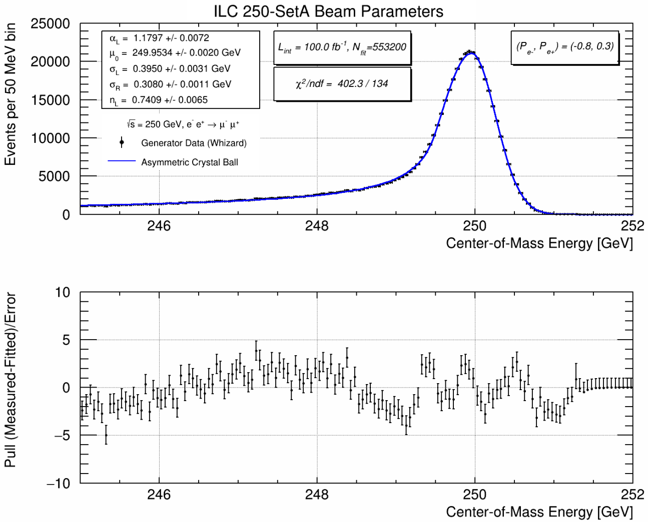
Electron Beam Energy (After Beamstrahlung)



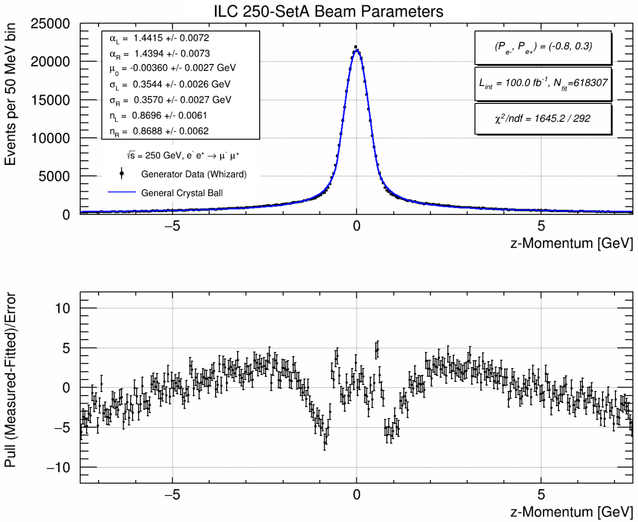
$$\sigma_{\rm R}/E = 0.1919 \pm 0.0008\% \text{ (cf 0.190\% in TDR)}$$

Note an undulator bypass could reduce this spread when one e^- cycle is used purely for e^+ production.

Center-of-Mass Energy (After Beamstrahlung)



$\sigma_{\rm R}/\sqrt{s} = 0.1232 \pm 0.0004\%$ (cf 0.122% in TDR ($0.190\% \oplus 0.152\%)/2$)



$\sigma/\sqrt{s} = 0.1416 \pm 0.0007\%$ (cf 0.122% from beam energy spread alone)

Initial State Kinematics with Crossing Angle

Define the two beam energies (after beamstrahlung) as E_{b}^{-} and E_{b}^{+} for the e^{-} and e^{+} beam respectively.

Initial-state energy-momentum 4-vector (neglecting m_{e})

$$E = E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}$$

$$p_x = (E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}) \sin(\alpha/2)$$

$$p_y = 0$$

$$p_z = (E_{\mathrm{b}}^{-} - E_{\mathrm{b}}^{+}) \cos(\alpha/2)$$

The corresponding center-of-mass energy is

$$\sqrt{s} = 2\sqrt{E_{\mathrm{b}}^{-} E_{\mathrm{b}}^{+}} \cos(\alpha/2)$$

Hence if α (crossing-angle) is known, evaluation of the center-of-mass energy of this collision amounts to measuring the two beam energies. Introducing,

$$E_{\mathrm{ave}} \equiv \frac{E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}}{2} \;, \overline{\Delta E_{\mathrm{b}}} \equiv \frac{E_{\mathrm{b}}^{-} - E_{\mathrm{b}}^{+}}{2}$$

then with this notation,

$$\sqrt{s} = 2\sqrt{E_{\mathrm{ave}}^{2} - (\overline{\Delta E_{\mathrm{b}}})^{2}} \cos(\alpha/2)$$

Final State Kinematics and Equating to Initial State

Let's look at the final state of the $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-(\gamma)$ process. Denote the μ^+ and μ^- as particles 1, 2, and the rest-of-the event (RoE) as system 3.
So the final-state system 4-vector is

$$(E_1 + E_2 + E_3,\ \vec{p_1} + \vec{p_2} + \vec{p_3})$$

Then applying $(E, \vec{p})$ conservation and assuming $m_3 = 0$ we obtain,

$$(E_1 + E_2 + E_3) = E_1 + E_2 + p_3 = 2\ E_{\mathrm{ave}} \tag{1}$$

$$\vec{p_1} + \vec{p_2} + \vec{p_3} = (2\ E_{\mathrm{ave}} \sin(\alpha/2), 0, 2\ \overline{\Delta E_{\mathrm{b}}} \cos(\alpha/2)) \equiv \vec{p}_{\mathrm{initial}} \tag{2}$$

In general the RoE may not be fully detected and needs to be inferred using $(E, \vec{p})$ conservation. We have 4 equations and 5 unknowns, namely the 3 components of the RoE momentum ($\vec{p_3}$) and both E_{ave} and $\overline{\Delta E_{\mathrm{b}}}$.
One approach is to solve for E_{ave} with assumptions on $\overline{\Delta E_{\mathrm{b}}}$. Specifically we then focus on using the simplifying assumption that $\overline{\Delta E_{\mathrm{b}}} = 0$. Note this is often a poor assumption event-by-event for the p_z conservation component.

The Averaged Beam Energy Quadratic

The outlined approach results in a quadratic equation in E_{ave}, ($AE_{\mathrm{ave}}^2 + BE_{\mathrm{ave}} + C = 0$), with coefficients of

$$A = \cos^2(\alpha/2)$$

$$B = -E_{12} + p_{12}^x \sin(\alpha/2)$$

$$C = (M_{12}^2)/4 + p_{12}^z \overline{\Delta E_{\mathrm{b}}} \cos(\alpha/2) - \overline{\Delta E_{\mathrm{b}}}^2 \cos^2(\alpha/2)$$

Based on this, there are three particular cases of interest to solve for E_{ave}.

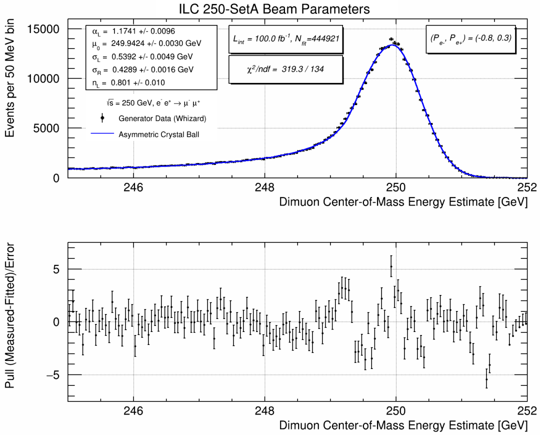
1. Zero crossing angle, $\alpha = 0$, and zero beam energy difference.
2. Crossing angle and zero beam energy difference.
3. Crossing angle and non-zero beam energy difference.

The original formula,

$$\sqrt{s} = E_1 + E_2 + |\vec{p}_{12}|$$

arises trivially in the first case. In the rest of this talk I will use the $\sqrt{s}$ estimate from the largest positive solution of the second case as what I now mean by $\sqrt{s}_p$. Obviously it is also a purely muon momentum dependent quantity.

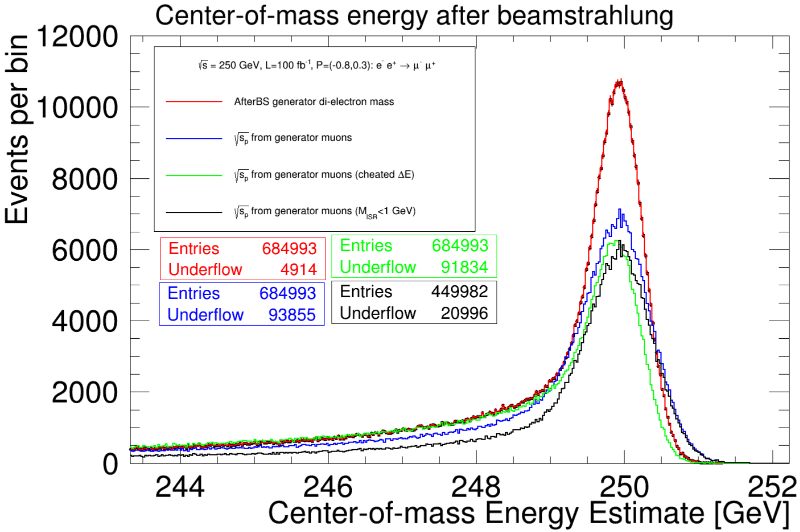
Dimuon Estimate of Center-of-Mass Energy (After BS)



$\sigma_{\rm R}/\sqrt{s} = 0.1716 \pm 0.0006\%$ (cf 0.1232% with true $\sqrt{s}$)

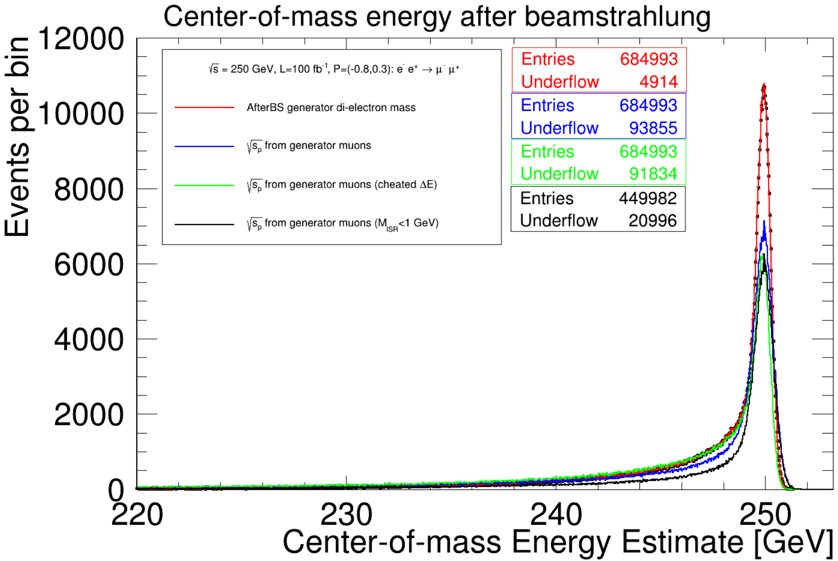
- This is the generator-level $\sqrt{s}_p$ calculated from the 2 muons
- Why so broad? Why fewer events?
- Likely because some events violate the assumptions that $\overline{\Delta E_{\rm b}} = 0$ and $m_3 = 0$
- The former is no surprise given the p_z distribution
- The latter can be associated with events with 2 or more non-collinear ISR/FSR photons

Comparisons (After BS)

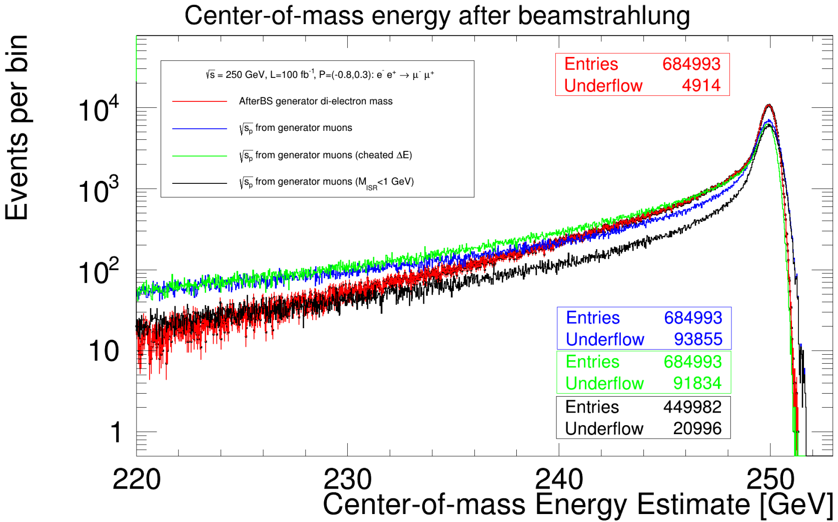


Note: Underflow statistics refer to < 220 GeV. Next 2 slides - same but wider scale

Comparisons I (After BS) Linear

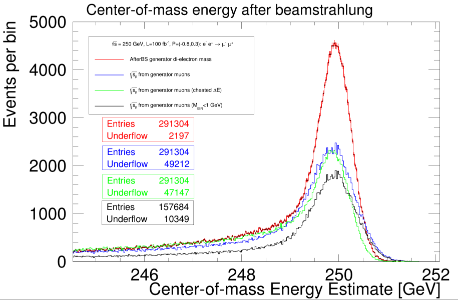


Comparisons II (After BS) Log

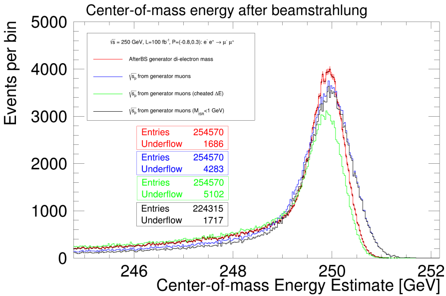


What's Going On?

$50 < m_{\mu\mu}^{\text{gen}} < 150$ GeV



$m_{\mu\mu}^{\text{gen}} > 150$ GeV



- For lower dimuon mass events, only about half are reconstructed close to $\sqrt{s}$
- Most higher dimuon mass events reconstructed close to the original $\sqrt{s}$

Conclusion

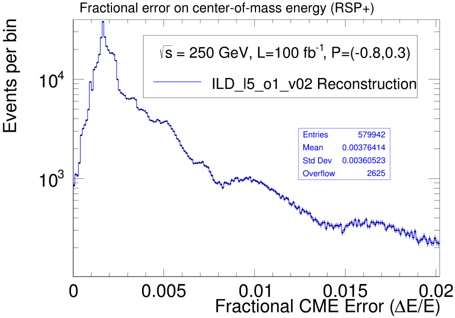
Lower dimuon mass events are more likely to violate the assumptions.

Event Selection Requirements

Currently rather simple.

Use latest full ILD simulation/reconstruction at 250 GeV.

- Require exactly two identified muons
- Opposite sign pair
- Require uncertainty on estimated $\sqrt{s}_p$ of the event of less than 0.8% based on propagating track-based error matrices
- Categorize reconstruction quality as **gold** (<0.15%), **silver** ([0.15, 0.30]%), **bronze** ([0.30, 0.80]%)
- Require the two muons pass a vertex fit with p-value > 1 %



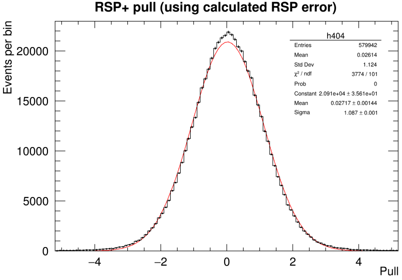
Selection efficiencies for (80%/30%) beam polarizations:

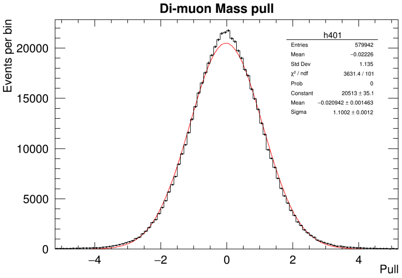
- $\varepsilon_{-+} = 69.77 \pm 0.06$ %
- $\varepsilon_{+-} = 67.35 \pm 0.06$ %
- $\varepsilon_{--} = 69.47 \pm 0.05$ %
- $\varepsilon_{++} = 67.72 \pm 0.06$ %

Backgrounds not yet studied in detail, ($\tau^+\tau^-$ is small:0.15%, of no import for the $\sqrt{s}$ peak region).

Dimuon Pull Distributions

- Pull $\equiv$ (meas - true)/error.
- Track-based estimates of the errors on both the $\sqrt{s}_p$ quantity (left) and the di-muon mass (right) agree well with the modeled uncertainties.





- In both cases the fitted rms over this range is about 10% larger than ideal. Central range well described. Suspect tails should be non-Gaussian given the non-Gaussian tails of multiple scattering.
- In practice this is rather encouraging.

Vertex Fit: Exploit ILC nanobeams

With well modeled track errors, and given that the 2 muons should originate from a common vertex consistent with the interaction point, we can perform:

- Vertex Fit: Constrain the two tracks to a common point in 3-d
- Beam-spot Constrained Vertex Fit

The ILC beam-spot size (no pinch) is $(\sigma_x, \sigma_y) = (515, 7.7)$ nm, $\sigma_z = 0.202$ mm

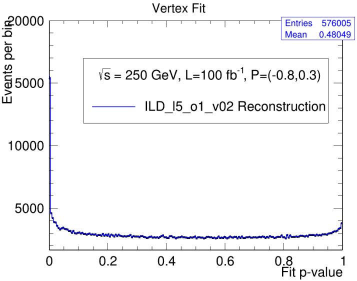
- Vertex fit (see AWLC2014 talk) implemented using the fully simulated and reconstructed data
- Also have explored beam-spot constraints

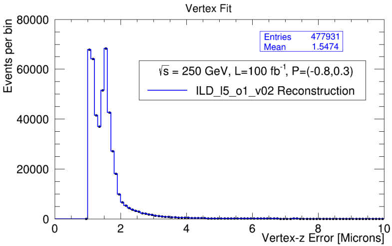
What good is this?

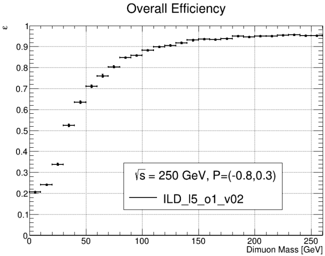
- Residual background rejection (eg. $\tau^+\tau^-$ reduced by factor of 20)
- Additional handle for rejecting or deweighting mis-measured events
- Some modest improvement in precision of di-muon kinematic quantities
- Also useful for $\mathrm{H} \to \mu^+\mu^-$ and for ZH recoil
- Interaction point measurement ($\mathcal{O}$ $(1\mu\mathrm{m})$ resolution per event) **can** be used to correlate with (E_-, E_+) for understanding beamstrahlung effects

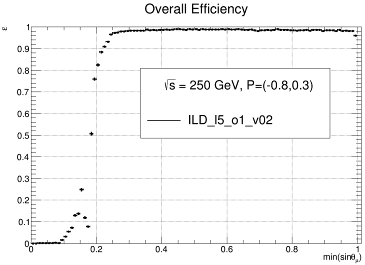
Note: simulated data does not currently simulate the transverse beam-spot ellipse

Event Selection Aspects: Vertex Fit and Overall Efficiency



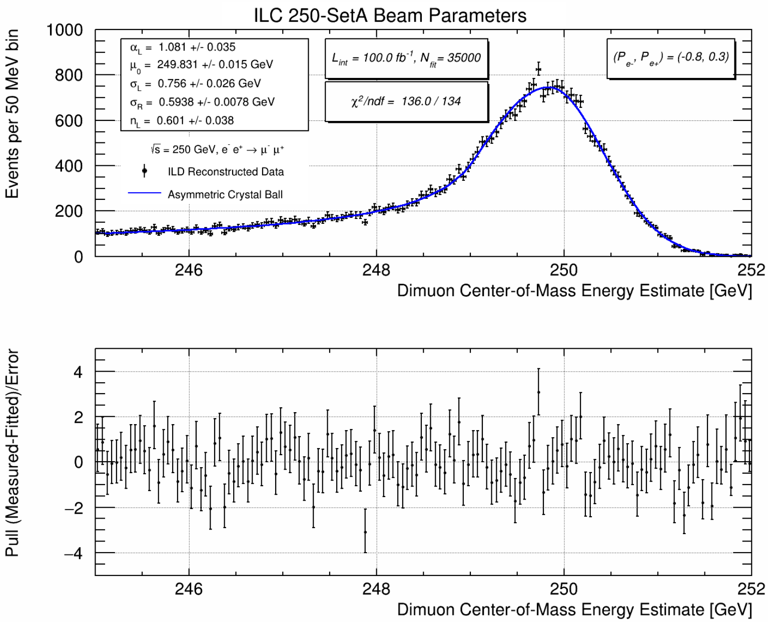




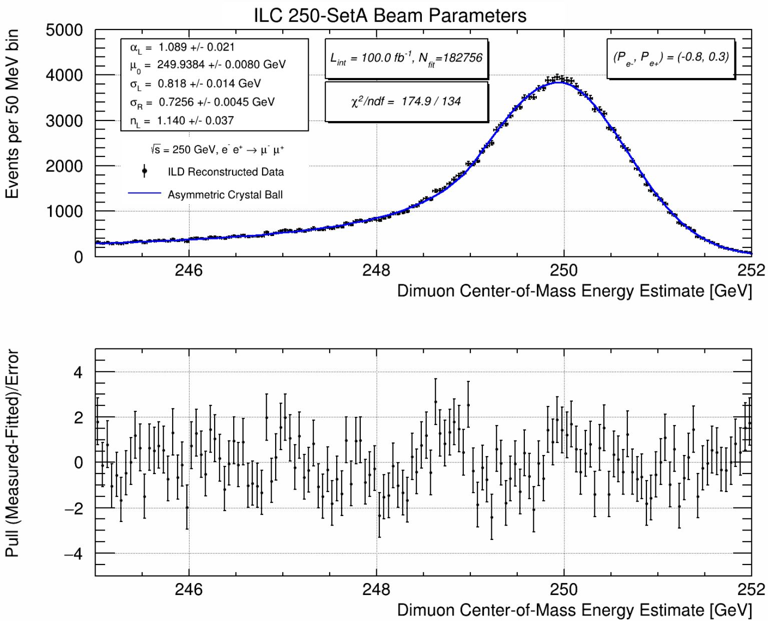


Efficiency rather mass dependent. Mostly due to geometrical acceptance.

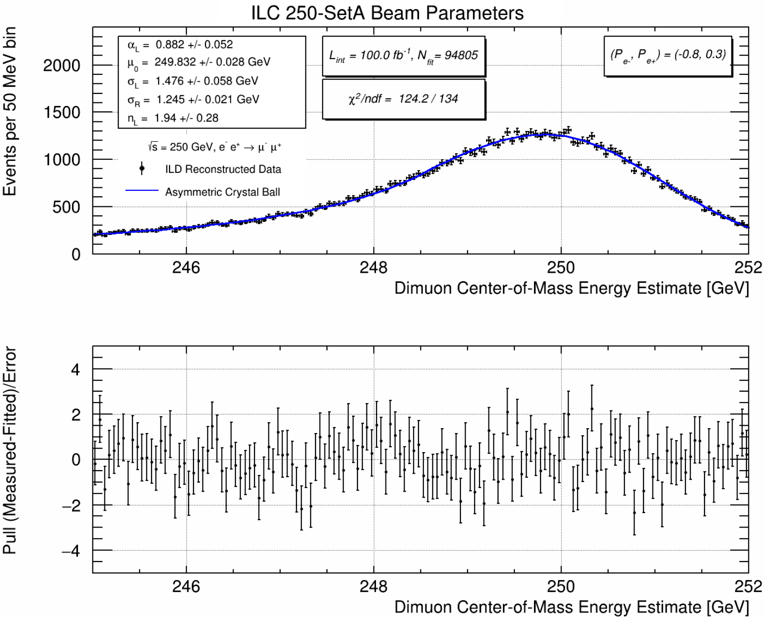
Gold Quality Dimuon PFOs (After BS)



Silver Quality Dimuon PFOs (After BS)



Bronze Quality Dimuon PFOs (After BS)



Strategy for Absolute $\sqrt{s}$ and Estimate of Precision

Prior Estimation Method

- Guesstimate how well the peak position of the Gaussian can be measured using the observed $\sqrt{s}_p$ distributions in bins of fractional error

Current Thinking

- The **luminosity spectrum** and **absolute center-of-mass energy** are the same problem or at least very related. How well one can determine the absolute scale depends on knowledge of the shape (input also from Bhabhas).
- **Beam energy spread** likely to be well constrained by spectrometer data
- Likely need either a convolution fit (CF) or a reweighting fit
- We are currently working on a CF by parametrizing the underlying (E_-, E_+) distribution, and modeling quantities related to $\sqrt{s}$ and p_z after convolving with detector resolution (and ISR, FSR and cross-section effects)

Current Estimation Method

- Follow a similar approach to before, but using estimates of the statistical error on μ_0 for 5-parameter Crystal Ball fits to fully simulated data with the 4 shape parameters fixed to their best fit values. Fits are done in the various resolution categories (example gold, silver, bronze fits in backup slides).
- Next slide has these estimates

$\sqrt{s}$ Sensitivity Estimate at $\sqrt{s} = 250$ GeV

Statistical uncertainties in ppm on $\sqrt{s}$ for $\mu^+\mu^-$ channel

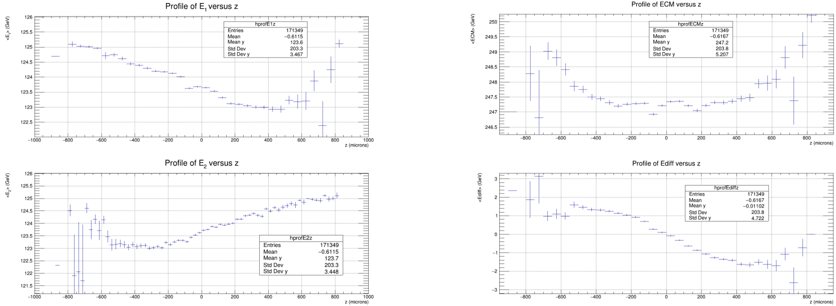
$L_{\rm int}$ [ab^{-1}]	Poln [%]	Gold	Silver	Bronze	G+S+B
0.9	$-80, +30$	6.5	3.1	8.5	2.7
0.9	$+80, -30$	7.7	3.4	9.6	3.0
0.1	$-80, -30$	26	12.1	33	10.4
0.1	$+80, +30$	29	13.0	41	11.4
2.0	–	4.8	2.2	6.2	**1.9**

Fractional errors on μ_0 parameter (mode of peak) when fitting with 5-parameter Crystal Ball function with all 4 shape parameters fixed to their best-fit values.

Also the e^+e^- channel should be used. The additional benefit of the much larger statistics from more forward Bhabhas is offset by the poorer track momentum resolution at forward angles.

Can the vertex info be used to decode beamstrahlung?

Pinch effect, disruption, and beamstrahlung. (x, y, z, x', y').
Dependence of the **means** of the e^- and e^+ beam energies (E_1, E_2), $\sqrt{s}$, ($E_1 - E_2$) on the **z of the interaction.** Used `guinea-pig++` incl. energy spread.

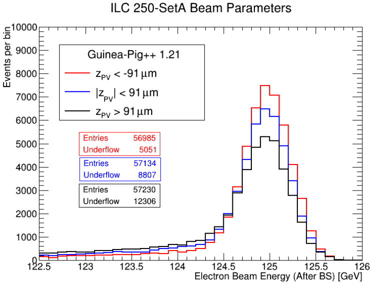


As we saw, z can be measured with a few μm resolution. Luminous region has σ_z=200 μm. Indeed the energy distributions of each beam depend on z (related to traversal of the opposing bunch). Statistically may also measure x vs z.

Obvious scope for more refined analysis of dL/d$\sqrt{s}$ and $\sqrt{s}$. Envisage reconstructing $f(x_1, x_2, z)$. Useful also for accelerator diagnostics?
NB: physics generators currently only simulate $f(x_1, x_2)$.

Divide interactions in 3 equi-probability parts according to z_{PV}. Preferentially

1. e^+e^- collisions occuring more on the initial e^- side ($z < 0$)
2. e^+e^- collisions mostly central
3. e^+e^- collisions preferentially on the initial e^+ side ($z > 0$)

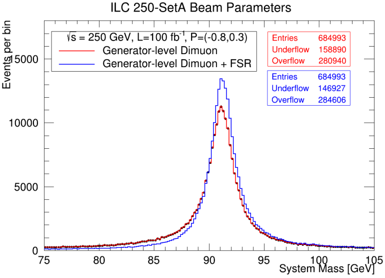


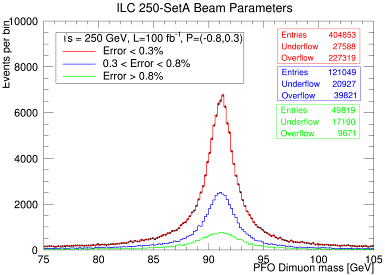
The beamstrahlung tail grows and the peak shrinks for e^- as z increases, and, for e^+ as z decreases. In both cases, the largest beamstrahlung tail occurs when the interacting e^- or e^+ has on average traversed more of the opposing bunch.

Thus both $\sqrt{s}$ and $p_z = E_- - E_+$ distributions depend on z. Likely needs to be taken into account for $\sqrt{s}$, $dL/d\sqrt{s}$, Higgs recoil, kinematic fits ...

Measuring M_Z using $m_{\mu^+\mu^-}$ with high energy running

Look at $\sqrt{s} = 250$ GeV running with latest beam parameters and full simulation





Adding in FSR photon(s) reduces the peak width to be consistent with Γ_Z. Improves statistical sensitivity on mode by 10–20%.

$m_{\mu^+\mu^-}$ resolution is much less than Γ_Z. Sensitivity estimates from prior study (slide n+2) with smeared MC will be reasonable.

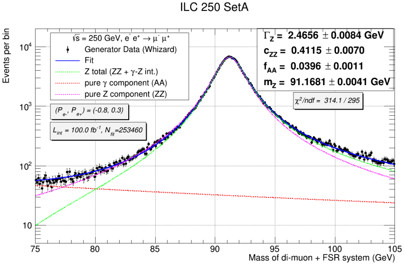
Main systematics:

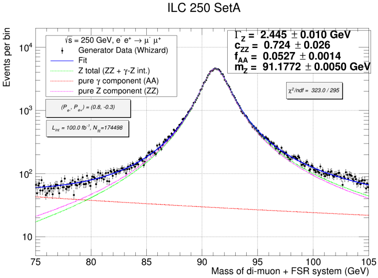
1. momentum-scale
2. FSR modeling/treatment
3. Electron p-scale in the e^+e^- channel

Also direct measurement of Γ_Z

Radiative return to the Z for M_Z and Γ_Z

Expected stat. precision on M_Z and Γ_Z is driven by the no. of events and Γ_Z.





Semi-empirical physics-based parametrization. Shape given by a relativistic Breit-Wigner with additional shape contributions from pure photon-exchange and $\gamma - Z$ interference using Born-level $\sigma(\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-)$ at ISR reduced $\sqrt{s'}$. Fits generator-level distribution (after BS and ISR) surprisingly well.

Using similar fits to gen.-level distributions (but for dimuon events passing event selection criteria): uncertainty of 1.0 MeV on M_Z and 2.2 MeV on Γ_Z for 2 ab^{-1} at $\sqrt{s} = 250$ GeV (just $\mu^+\mu^-$ channel)

Measuring M_Z from $m_{\mu^+\mu^-}$

Revisited old study of $\sqrt{s}_p$ at $\sqrt{s} = 250$, 350, 500, 1000 GeV. Used smeared MC. Fitted $m_{\mu^+\mu^-} \in [75, 105]$ GeV with sum of two Voigtians. Statistical uncertainties on the peak parameter, M_Z, scaled to full ILC program using simulations with TDR beam parameters

Statistical uncertainties for $\mu^+\mu^-$ channel

$\sqrt{s}$ [GeV]	L_{int} [ab^{-1}]	Poln [%]	Sharing [%]	ΔM_Z [MeV]
250	2.0	80/30	(45,45,5,5)	1.20
350	0.2	80/30	(67.5,22.5,5,5)	5.99
500	4.0	80/30	(40,40,10,10)	2.55
1000	8.0	80/20	(40,40,10,10)	5.75
All	14.2	–	–	1.05

- Current PDG uncertainty on M_Z is 2.1 MeV
- FSR makes effective Breit-Wigner width larger and shifts the peak
- Treatment of FSR and especially inclusion of e^+e^- channel should decrease stat. uncertainty to **0.7 MeV**. Similarly Γ_Z to 1.5 MeV.
- Sensitivity dominated by $\sqrt{s} = 250$ GeV running
- Main systematic - tracker p-scale. Target at most 2.5 ppm in this context.

Concluding Remarks

Progress

- New high precision method for momentum-scale using especially $\mathrm{K^0_S}$ and Λ. Promises 2.5 ppm uncertainty per 10M hadronic Zs.
- More detailed investigation of dimuons for $\sqrt{s}$ and $\mathrm{dL}/\mathrm{d}\sqrt{s}$ reconstruction
- Measurement of M_{Z} using dimuon mass for $\sqrt{s} \gg M_{\mathrm{Z}}$ to 1.0 MeV - dominated by $\sqrt{s} = 250$ GeV data
- Beamstrahlung energy/vertexing correlations look very promising

Conclusions

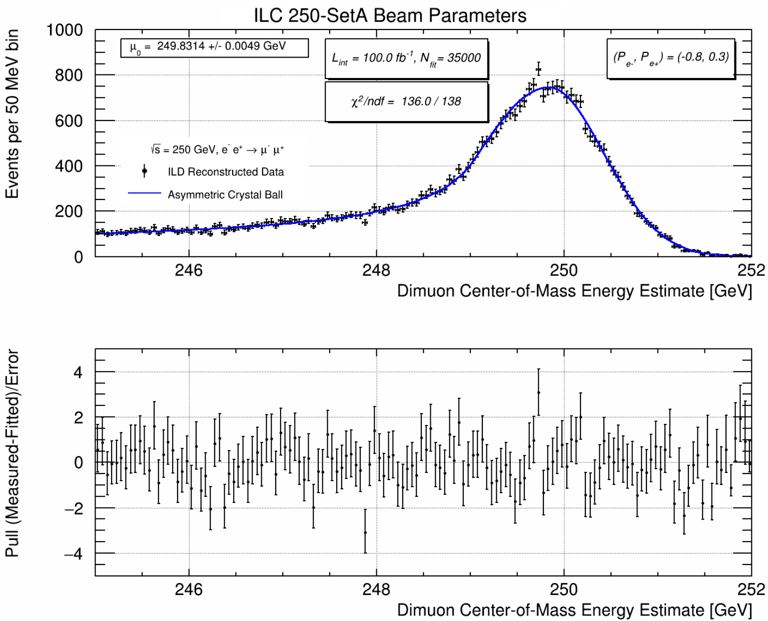
- ILC tracking detectors have the potential to measure beam energy related quantities with precision similar to the intrinsic energy spread using dimuon events (and also wide-angle Bhabha events)
- At $\sqrt{s} = 250$ GeV, dimuon estimate of 2 ppm stat. precision on $\sqrt{s}$. More than sufficient (10 ppm needed) to not limit measurements such as M_{W}.
- Potential to improve M_{Z} by a factor of three using 250 GeV di-lepton data
- Applying the same techniques to running at the Z-pole will enable a high precision electroweak measurement program for ILC that takes advantage of absolute center-of-mass energy scale knowledge

Acknowledgments

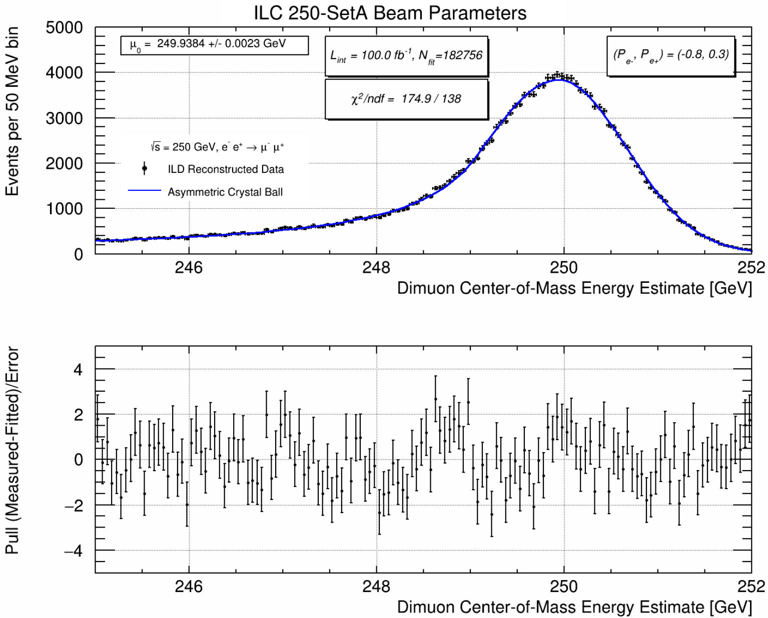
- KU graduate student, Brendon Madison, now working on aspects of the center-of-mass energy studies including luminosity spectrum.
- Input from Michael Peskin on Z lineshape helped with the semi-empirical fit parametrization.
- Mikael Berggren for help with Guinea-PIG setup consistent with ILD's Whizard event generation.
- Work done producing the central ILD simulated samples.

Backup Slides

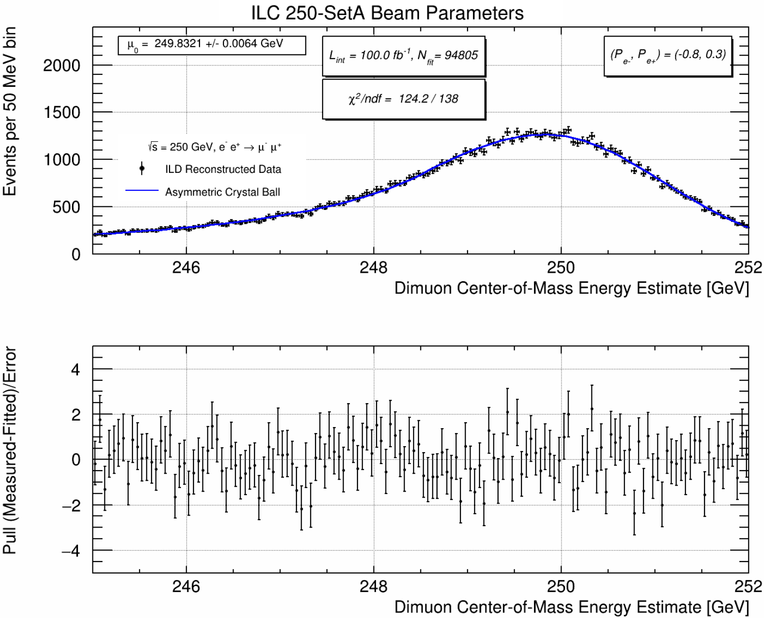
Gold Quality Dimuon PFOs (After BS)



Silver Quality Dimuon PFOs (After BS)



Bronze Quality Dimuon PFOs (After BS)



Beam Effects

The main idea is to use the kinematics of $\mathrm{e^+e^-} \to \mu^+\mu^-(\gamma)$ events and measurements of the final-state particles to measure the distribution of the center-of-mass energy of collisions.

We identify 3 effects needed to make a more realistic model of the collision:

0. Nominal. Each beam is a δ-function centered at a particular beam energy.
1. **Beam energy spread.** Each beam has a Gaussian distribution with rms width, σ_E, centered at a particular beam energy.
2. **Beamstrahlung.** The collective interaction of the two beams leads to radiation of collinear photons from the beams, resulting in the colliding e^+ and e^- having a *beamstrahlung-reduced center-of-mass energy*.
3. **Initial-state-radiation (ISR).** All $\mathrm{e^+e^-}$ physics processes may have ISR, where the invariant mass of the annihilating e^+ and e^- and the resulting particle system is further reduced cf 2 due to the emitted ISR photon(s).

We are primarily concerned with evaluating the **beamstrahlung-reduced center-of-mass energy**. This is *after* beam energy spread and beamstrahlung radiation, but *before* emission of any ISR photons. We should allow for differences in the energy of each beam and for a **beam crossing angle**, α, defined as the horizontal plane angle between the two beam lines. For ILC, α, is 14 mrad.

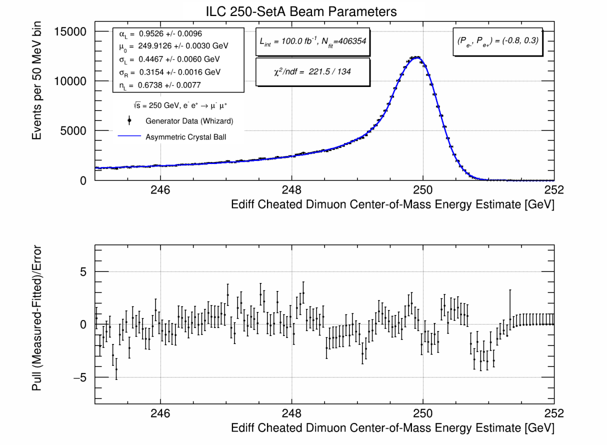
Aside on Crystal Ball Empirical Fit Functions

- The 1-d distributions generally feature a Gaussian **peak** associated with beam energy spread and a long **tail** with harder beamstrahlung
- These can be fit qualitatively well - although not well enough - with a Crystal Ball function. This piece-wise function has a Gaussian core and a power-law tail with a continuous first-derivative at the transition points.
- The generalized asymmetric double-sided Crystal Ball is

$$f(E; \mu_0, \sigma_L, \alpha_L, n_L, \sigma_R, \alpha_R, n_R)$$

  where μ_0 is the Gaussian peak **mode**, σ_i are the Gaussian **widths** (on L&R), α_i are the Gaussian/power-law **transition points** in units of σ_i (on L&R), and n_i are the power law **exponents** (on L&R)
- With the beam energy related distributions, only a 5-parameter version is applicable with parameters, $\mu_0, \sigma_L, \alpha_L, n_L, \sigma_R$ with the right-hand power-law tail disabled. The classic 1-sided Crystal Ball (4-parameters) $\mu_0, \sigma_L, \alpha_L, n_L$ fits are included for reference in the backup slides.
- See RooCrystalBall for implementation details

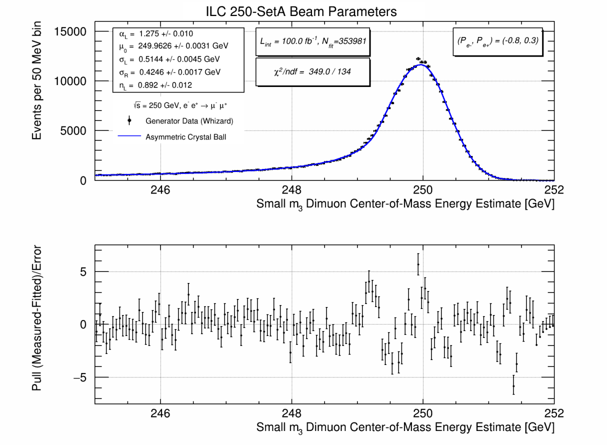
Cheated Dimuon Estimate of $\sqrt{s}$ (After BS)



$\sigma_{\mathrm{R}}/\sqrt{s} = 0.1259 \pm 0.0007\%$ (cf 0.1232% with true $\sqrt{s}$)

- This is the generator-level $\sqrt{s}_p$ calculated from the 2 muons
- But using the true $\overline{\Delta E_{\mathrm{b}}}$ in the equations
- Why so few events in range?

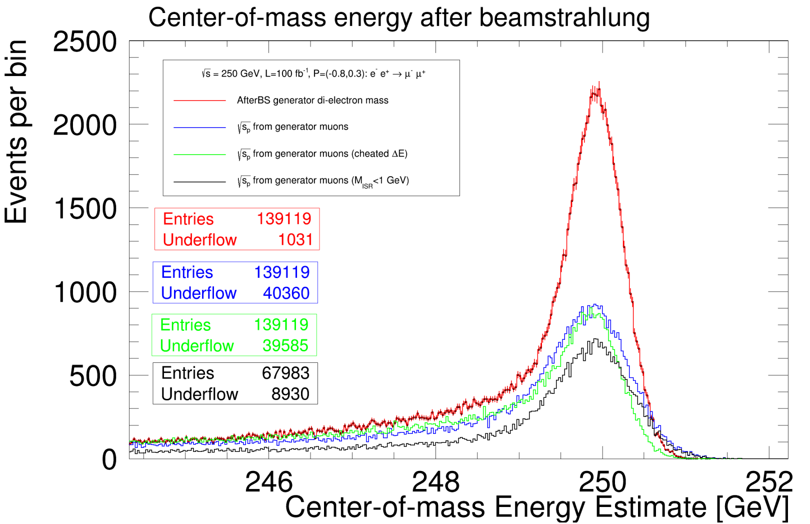
Dimuon Estimate of $\sqrt{s}$ (Low m_3) (After BS)



- This is the generator-level $\sqrt{s_p}$ calculated from the 2 muons
- For events with ISR photon system mass < 1 GeV
- Looks like the p_z issue dominates

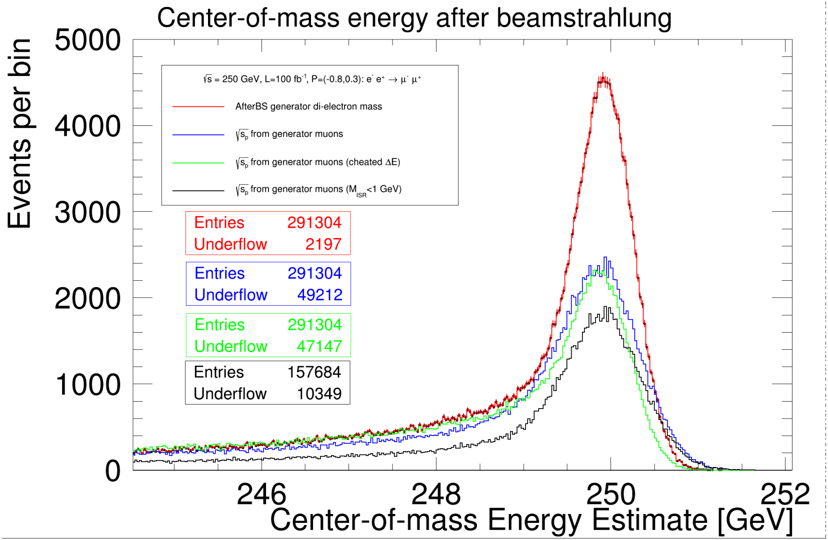
$\sigma_{\rm R}/\sqrt{s} = 0.1698 \pm 0.0007\%$ (cf 0.1232% with true $\sqrt{s}$)

Comparisons III Low Dimuon Mass (After BS) Zoomed



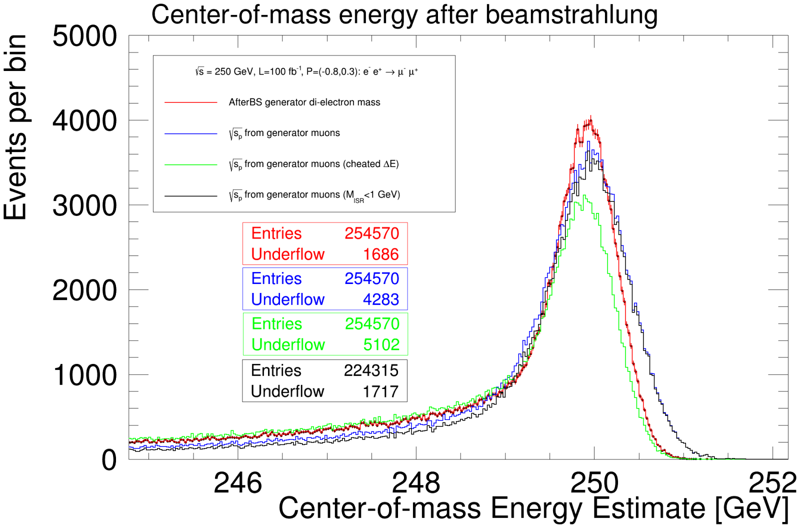
Note: Underflow statistics still refer to < 220 GeV.

Comparisons III Medium Dimuon Mass (After BS) Zoomed



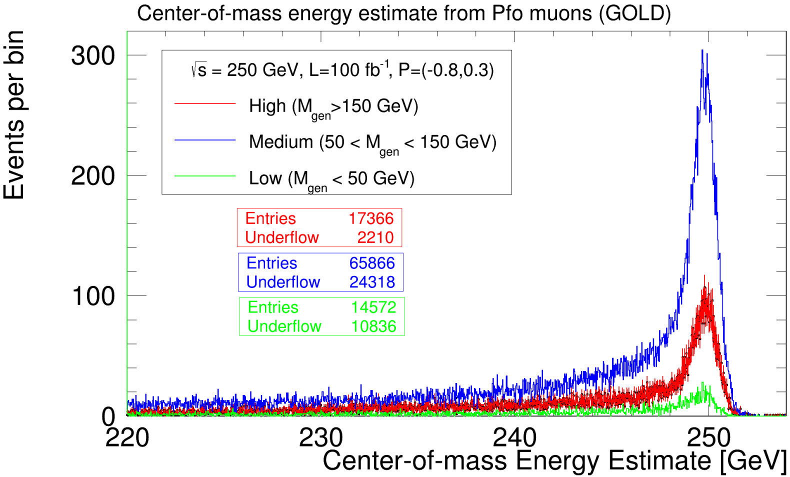
Note: Underflow statistics still refer to < 220 GeV.

Comparisons III High Dimuon Mass (After BS) Zoomed



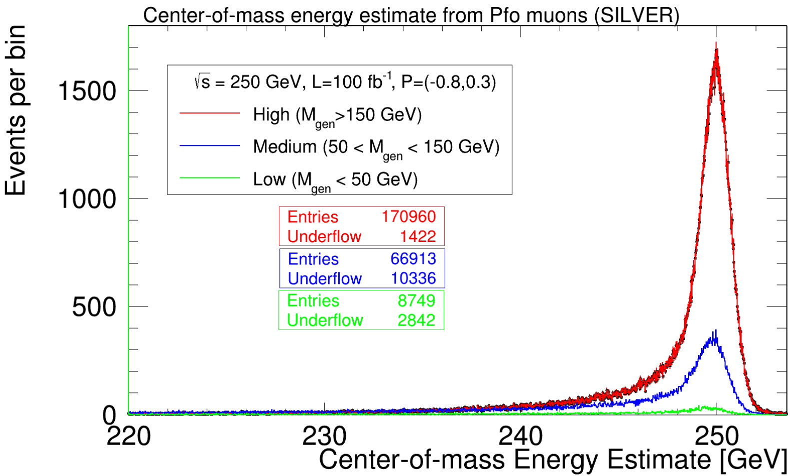
Note: Underflow statistics still refer to < 220 GeV.

Gold Quality Dimuon PFOs (After BS)



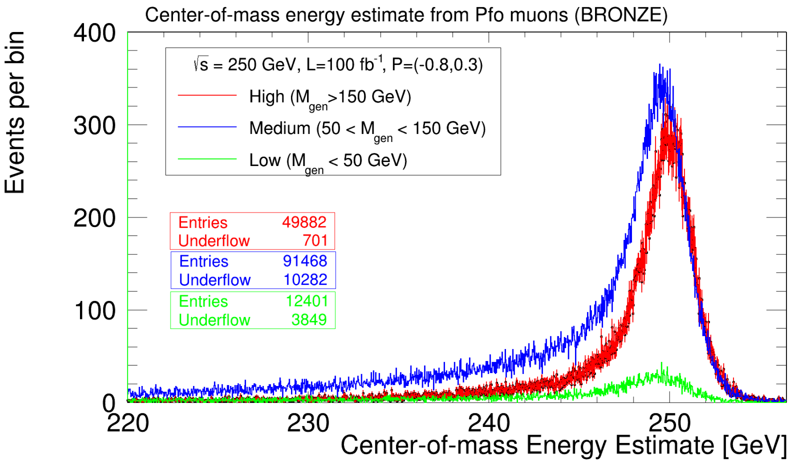
Mostly Z-like

Silver Quality Dimuon PFOs (After BS)



Mostly high mass

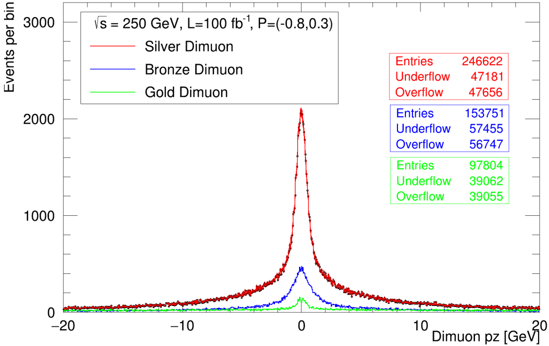
Bronze Quality Dimuon PFOs (After BS)



Mix of high mass and Z-like. Z-like with one forward muon?

Measuring the z-imbalance

Likely can use both p_z and acolinearity (for high mass events).



Will be sensitive to energy asymmetries. The suggestion by Tim Barklow in 2005 (which I now understand) is to measure

$$E_{\mu^+\mu^-} + p_z(\mu^+\mu^-) = (E_+ + E_-) + (E_- - E_+) = 2E_-$$

$$E_{\mu^+\mu^-} - p_z(\mu^+\mu^-) = (E_+ + E_-) - (E_- - E_+) = 2E_+$$

$\sqrt{s}$ Sensitivity Estimate at $\sqrt{s} = 250$ GeV

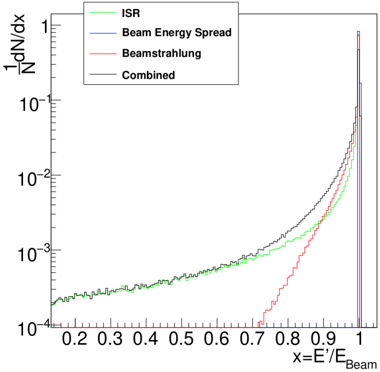
Statistical uncertainties in ppm on $\sqrt{s}$ for $\mu^+\mu^-$ channel

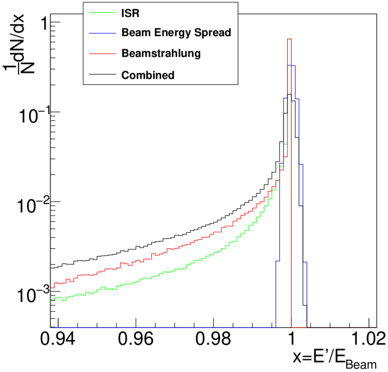
$L_{\rm int}$ [ab^{-1}]	Poln [%]	Gold	Silver	Bronze	G+S+B
0.9	$-80, +30$	11.1	4.8	16	4.3
0.9	$+80, -30$	12.0	5.5	18	4.8
0.1	$-80, -30$	43	19	64	16
0.1	$+80, +30$	46	21	68	18
2.0	–	7.9	3.5	11.7	**3.1**

Fractional errors on μ_0 parameter (mode of peak) when fitting with 4-parameter symmetric Crystal Ball function with all four parameters floating.

This is more conservative and likely too pessimistic. It does degrade from the pure statistical uncertainty of perfectly known shape parameters given the need to determine the shape parameters.

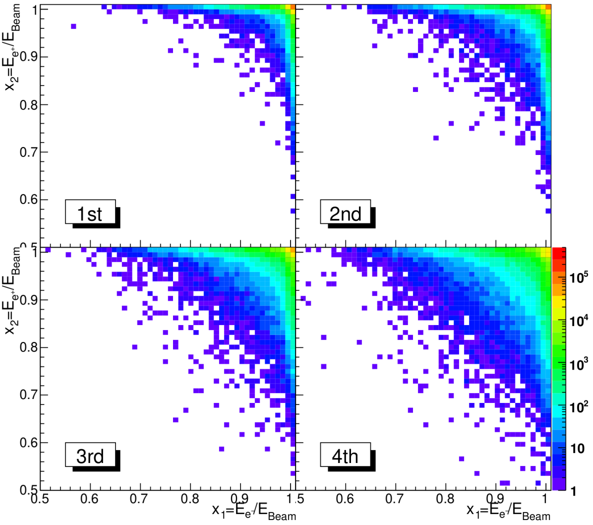
ISR and Beamstrahlung





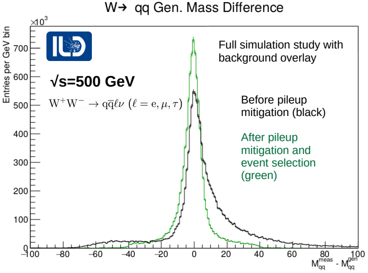
This is for ILC $\sqrt{s} = 500$ GeV TDR parameters from Andre Sailer's diploma thesis. ISR is the dominant effect in the far tail.

Beamstrahlung

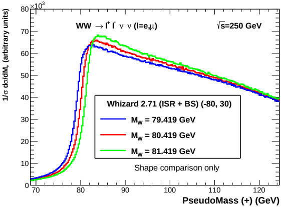


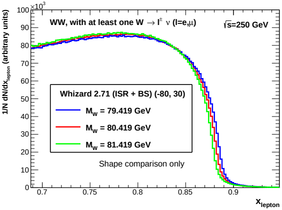
This is for ILC $\sqrt{s} = 500$ GeV TDR parameters from Andre Sailer's diploma thesis. Each plot is a consecutive collision time quartile.

$M_{\rm W}$, $\Gamma_{\rm W}$ measurements concurrent with Higgs program



- **Hadronic mass study**, J. Anguiano (KU).
- Stat. $\Delta M_{\rm W} = 2.4$ MeV for $1.6\ {\rm ab}^{-1}$ (-80%, +30%).
- Can be improved, but $m_{\rm had}$-only measurement likely limited by JES systematic
- Expect improvements with **constrained fit** and $\sqrt{s} = 250$ GeV data set





Sensitivity to $M_{\rm W}$ with lepton distributions: **dilepton pseudomasses**, lepton **endpoints**

- Stat. $\Delta M_{\rm W} = 4.4$ MeV for $2\ {\rm ab}^{-1}$ (45,45,5,5) at $\sqrt{s} = 250$ GeV
- **Leptonic observables** (shape-only): M_+, M_-, $x_\ell \equiv E_\ell/E_{\rm b}$. Exptl. systematics small.

New approach to tracker momentum scale

See LCWS2021 talk for details. Use Armenteros-Podolanski kinematic construction for 2-body decays (AP).

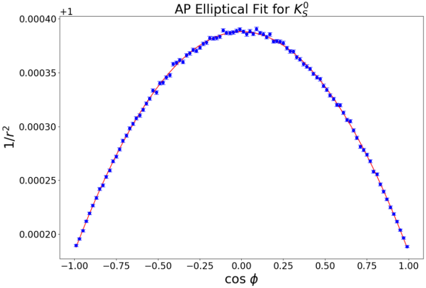
1. Explore AP method using mainly $\mathrm{K^0_S} \to \pi^+\pi^-$, $\Lambda \to \mathrm{p}\pi^-$ (inspired by Rodríguez et al.). **Much higher statistics than J/ψ alone.**
2. If proven realistic, **enables precision Z program** (polarized lineshape scan)
3. Bonus: potential for **large improvement in** parent and child particle **masses**

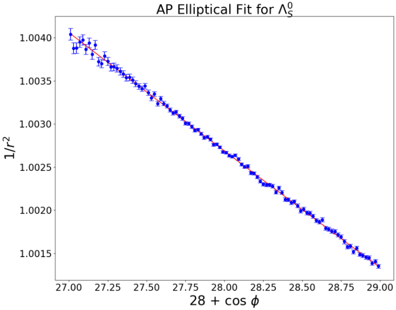
For a "V-decay", $M^0 \to m_1^+ m_2^-$, decompose the child particle lab momenta into components transverse and parallel to the parent momentum. The distribution of (child p_T, $\alpha \equiv \frac{p_L^+ - p_L^-}{p_L^+ + p_L^-}$) is a semi-ellipse with parameters relating the CM decay angle, θ^*, β, and the masses, (M, m_1, m_2), that determine, p^*.

By obtaining sensitivity to both the parent and child masses, and positing improving ourselves the measurements of more ubiquitous parents ($\mathrm{K^0_S}$ and Λ), can obtain high sensitivity to the momentum scale

Proving the feasibility of sub-10 ppm momentum-scale uncertainty needs much work when typical existing experiments are at best at the 100 ppm level

Used sample of 250M hadronic Z's at $\sqrt{s} = 91.2$ GeV. Fit $\mathrm{K_S^0}, \Lambda, \overline{\Lambda}$ in various momentum bins.





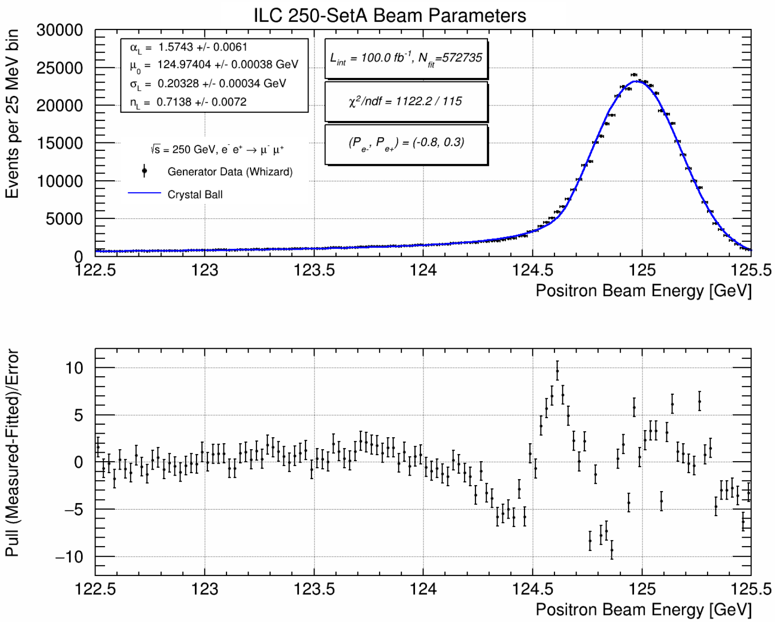
1. $m_{\mathrm{K_S^0}}$: 0.48 ppm
2. m_Λ: 0.072 ppm
3. m_π: 0.46 ppm
4. S_p: 0.57 ppm

- Fit fixes proton mass
- Factors of (54, 75, 3) improvement over PDG for $(\mathrm{K_S^0}, \Lambda/\overline{\Lambda}, \pi^\pm)$
- Momentum-scale to **2.5 ppm stat.** per 10M hadronic Z, ILC Z run has 400 such samples.

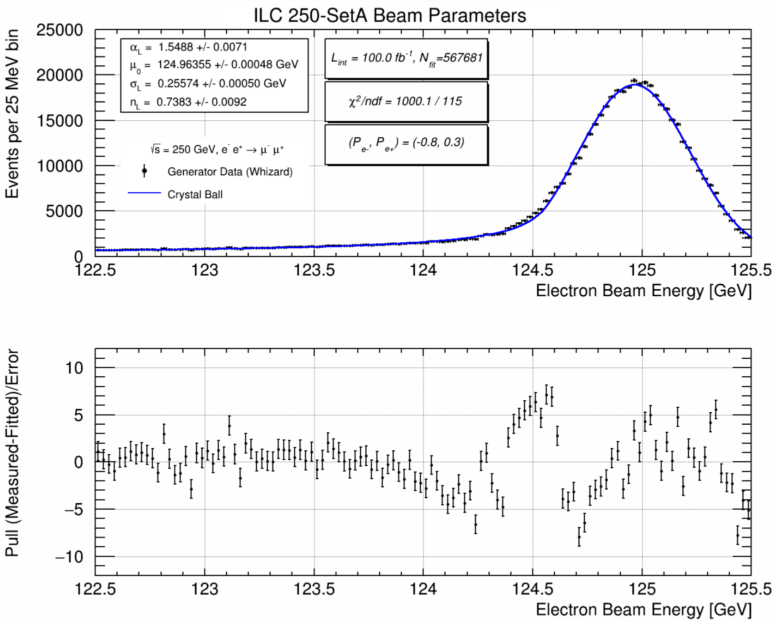
Fit Considerations

- Most of these are 4-parameter Crystal Ball fits. Particularly for those with more sharply resolved features, the χ^2 is substantially worse than the 5-parameter asymmetric fits shown earlier.
- The fits generally need the additional σ_{R} parameter to describe the beam energy spread feature while σ_{L} accommodates the convolution of beam energy spread with soft beamstrahlung.
- On the other hand these 4-parameter fits may better represent the statistical error on the mode parameter when able to better constrain the shape of the distributions such as with external knowledge of the beam energy spread.

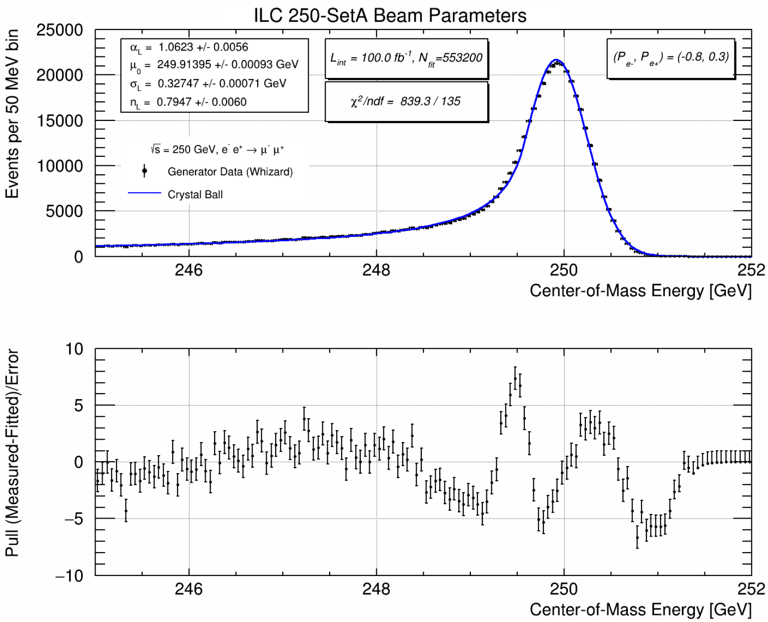
Positron Beam Energy (After Beamstrahlung)



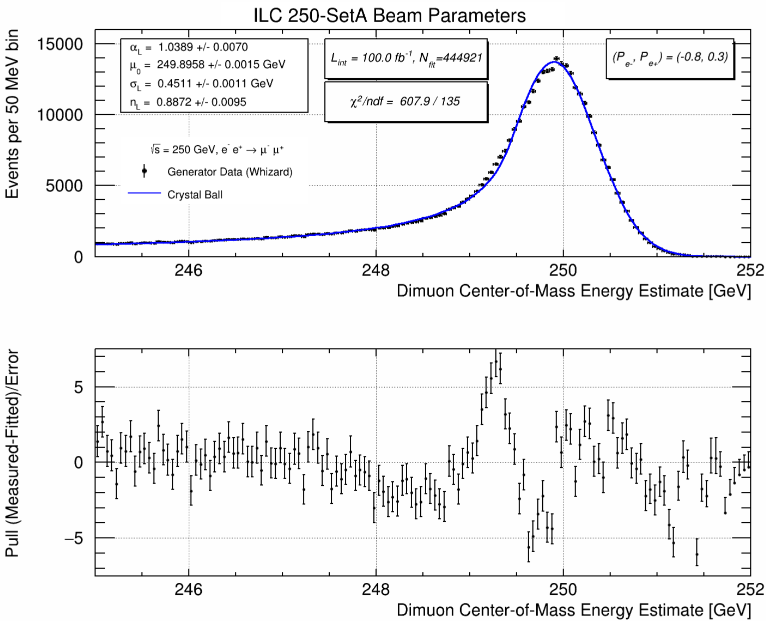
Electron Beam Energy (After Beamstrahlung)



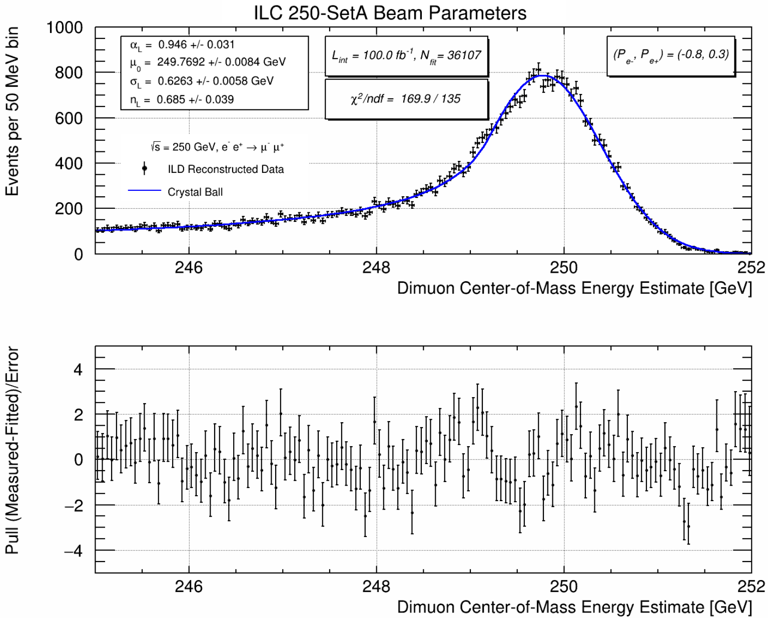
Center-of-Mass Energy (After Beamstrahlung)



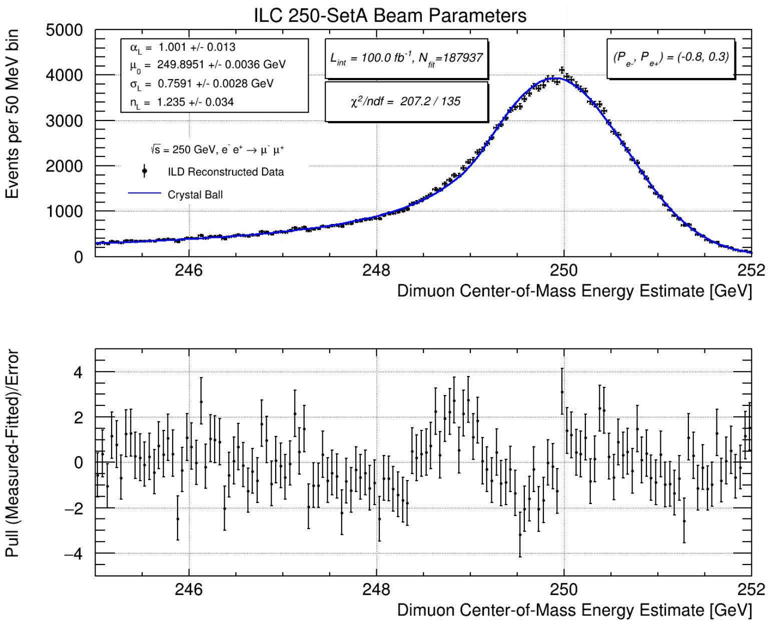
Dimuon Estimate of Center-of-Mass Energy (After BS)



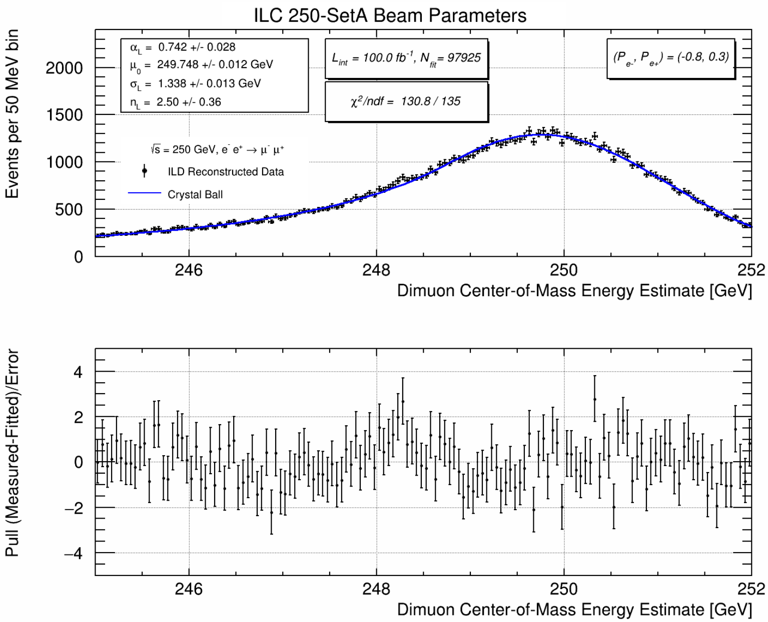
Gold Quality Dimuon PFOs (After BS)



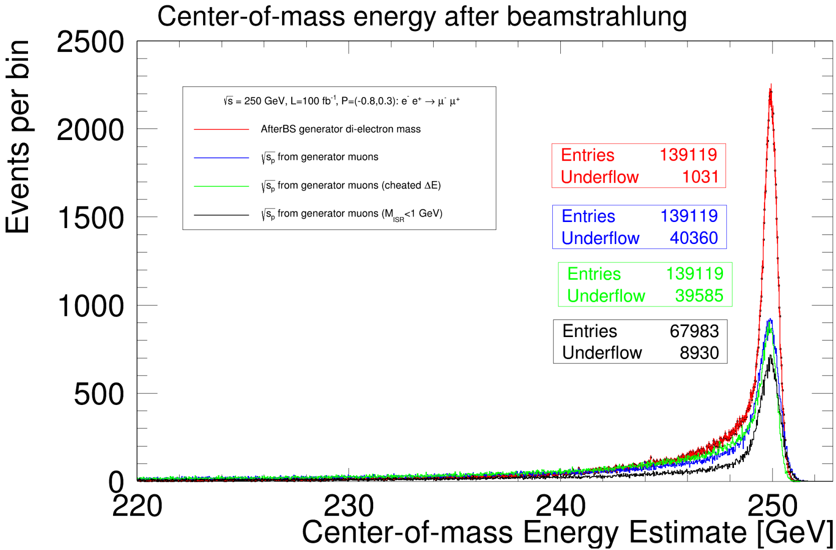
Silver Quality Dimuon PFOs (After BS)



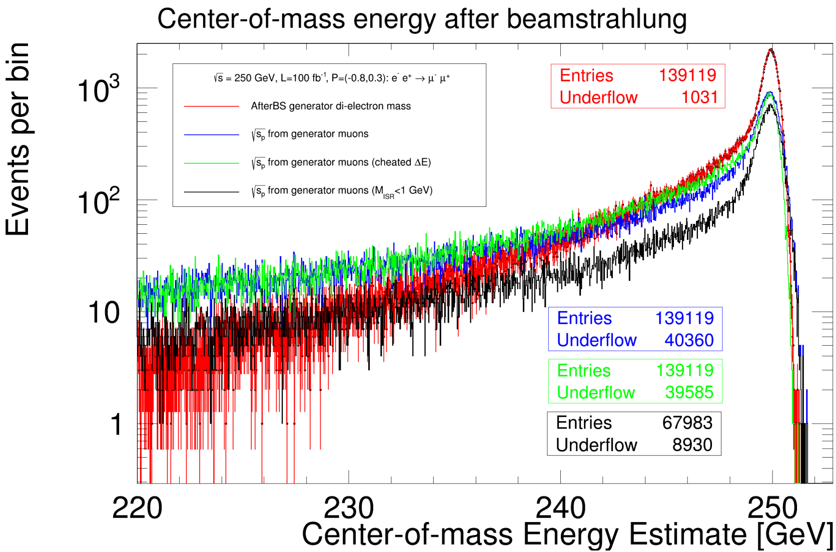
Bronze Quality Dimuon PFOs (After BS)



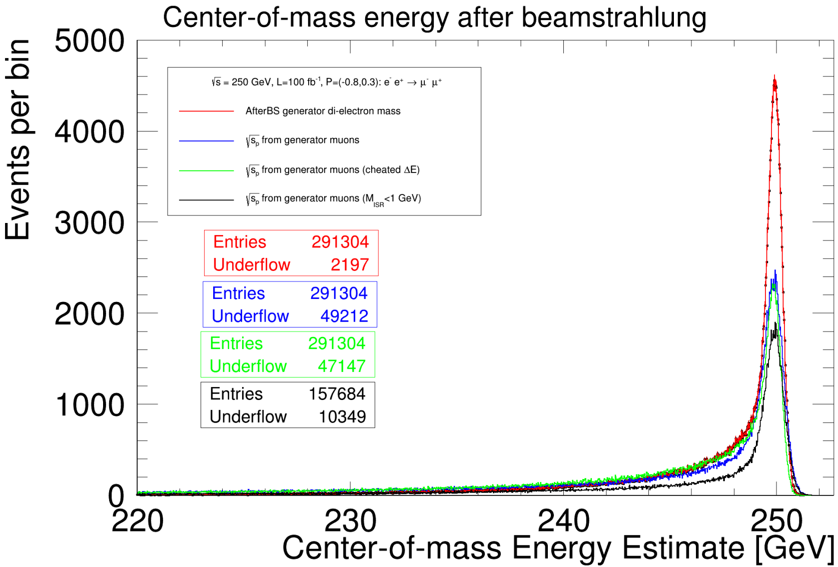
Comparisons I Low Dimuon Mass (After BS)



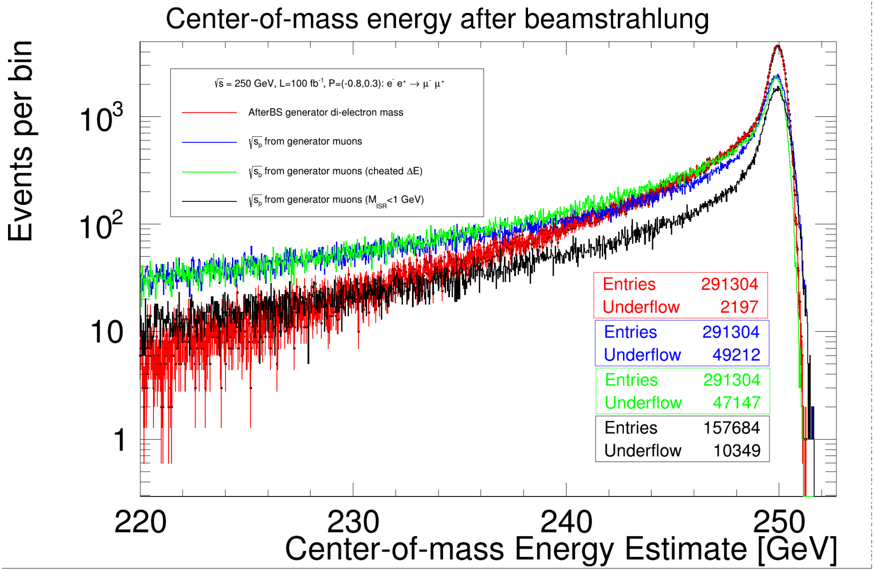
Comparisons II Low Dimuon Mass (After BS)



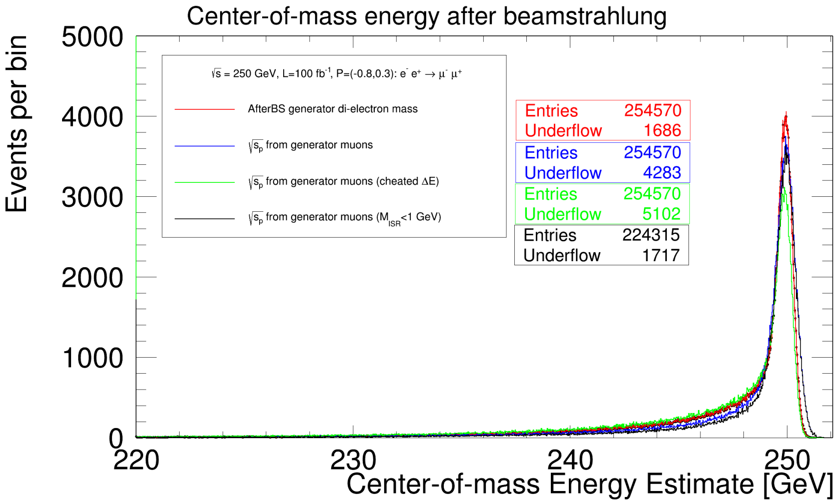
Comparisons I Medium Dimuon Mass (After BS)



Comparisons II Medium Dimuon Mass (After BS)



Comparisons I High Dimuon Mass(After BS)



Comparisons II High Dimuon Mass (After BS)