A Comparison between Observed and Computed Winds with Respect to their Applicability for Vorticity Computations[1]

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Abstract

The insufficiency of computed height values for computations of derivatives such as vorticity and convergence, at least for the 300-mb and higher levels, is illustrated by theoretical and practical examples, and it is recommended to make more extensive use of observed winds. This gives a possibility to take into account the cyclostrophic and other non-geostrophic wind components for the vorticity computation which is the first step in the numerical forecasting methods (CHARNEY 1951).

1. Introduction

The approach towards numerical forecasting methods during the last few years has emphasized the need for ample and exact observations from the upper air, for a detailed and skilful map analysis, and for empirical support of the theoretical assumptions on which the methods are based.

One of the assumptions usually made is that the absolute (or, in three-dimensional models, the potential) vorticity of an air parcel may be considered as a constant, or at least as a property which is sufficiently conservative to allow a forecast of 24 hours ahead or more. Even if this assumption holds true, there are several reasons why we cannot expect the resulting prognostic charts — e. g., maps showing the contours of the 500-mb surface — to be absolutely correct:

(a) The aerological observations are always too few to permit a really detailed analysis of the whole map, and the errors in the observations cannot all be corrected with a sufficient degree of certainty.

(b) The use of grid points means that the vorticity is computed only at a relatively small number of points, and for these points a rather crude method of computing the relative vorticity (by means of finite differences) is used.

(c) Additional theoretical assumptions concerning the transport of absolute vorticity are necessary to permit the construction of a 500-mb prognostic chart, namely: The vorticity is transported with the horizontal component of the geostrophic wind; the vertical transport of vorticity may be neglected; the horizontal divergence is assumed to be zero, which means that the effect of a three-dimensional field of deformation on the horizontal distribution of vorticity is neglected.

The complex character of the whole problem makes it difficult to separate the effect of any individual source of error. Thus, for instance, it is hardly possible to draw conclusions from the failure of one or a few

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forecasts as to the validity or non-validity of the basic theoretical assumptions. It has been tried, however, (NEWTON et AL., 1951) to investigate the variation of the absolute vorticity with time in developing kinematic systems, and it has been indicated that a mechanism producing absolute vorticity may be of importance in certain situations.

The present study deals with the problem of computing the vorticity from actual observations. It arose from an attempt to construct vorticity charts for a selected period over a limited area (the British Isles) where a dense network of aerological stations provide for a material which is, qualitatively as well as quantitatively, probably superior to any other material now available for an area of equal size.

2. Computation of the geostrophic wind field by means of temperature soundings only

If the topography of a constant-pressure surface is completely known, the geostrophic wind can be derived for any point of this surface by determination of the slope, taking into account the Coriolis parameter as a function of latitude. By means of observations made at three adjacent points A, B, and C (not in one line), we may determine the mean slope and, approximately, the mean geostrophic wind within the triangle ABC. By means of observations at A and B only, we may determine a mean value of the geostrophic wind component perpendicular to the direction of AB. Occasionally such calculations are made, mainly in connection with the construction of vertical sections.

Let us assume that the height of the 1000-mb surface at stations A and B is known with sufficient accuracy. The actual (harmonic) mean of the absolute temperature of the layer between 1 000 and p mb may be T_A and T_B respectively, but, due to errors in the soundings, we have obtained instead $T_A' = T_A + \Delta T_A$ and $T_B' = T_B + \Delta T_B$. This gives us for the height of the isobaric surface p above 1000 mb

$$\begin{aligned} Z_A' &= Z_A + \Delta Z_A = Z_A \left(1 + \frac{\Delta T_A}{T_A}\right), \\ Z_B' &= Z_B + \Delta Z_B = Z_B \left(1 + \frac{\Delta T_B}{T_B}\right), \end{aligned} \tag{1}$$

where Z_A' and Z_B' are computed, Z_A and Z_B actual values.

For the difference $Z_A' - Z_B'$, we get

$$\begin{aligned} Z_A' - Z_B' &= Z_A \left(1 + \frac{\Delta T_A}{T_A}\right) - \\ &- Z_B \left(1 + \frac{\Delta T_B}{T_B}\right), \end{aligned} \tag{2}$$

which deviates from the actual difference $Z_A - Z_B$ by

$$\begin{aligned} \Delta (Z_A - Z_B) &= \Delta Z_A - \Delta Z_B = \\ &= Z_A \frac{\Delta T_A}{T_A} - Z_B \frac{\Delta T_B}{T_B}; \end{aligned} \qquad (\ )$$

and as the quotients $\frac{Z_A}{Z_B}$ and $\frac{T_A}{T_B}$ are not very different from 1, this may be approximated by

$$\begin{aligned} \Delta (Z_A - Z_B) &\approx \frac{Z_A}{T_A} (\Delta T_A - \Delta T_B) = \\ &= \Delta (T_A - T_B) \frac{Z_A}{T_A}; \end{aligned} \tag{4}$$

the relative error is

$$\begin{aligned} \frac{\Delta (Z_A - Z_B)}{Z_A - Z_B} &\approx \frac{(\Delta T_A - \Delta T_B) Z_A}{(Z_A - Z_B) T_A} = \\ &= \frac{\Delta (T_A - T_B) Z_A}{(Z_A - Z_B) T_A}. \end{aligned} \tag{5}$$

If the distance between A and B is d, we have for the geostrophic wind component $(v_g)_n$ perpendicular to AB

$$(v_g)_n = \frac{g}{f} \frac{Z_A - Z_B}{d}; \tag{6}$$

by using Z_A' and Z_B' we get, however,

$$(v_g)_n' = \frac{g}{f} \frac{Z_A' - Z_B'}{d}, \tag{7}$$

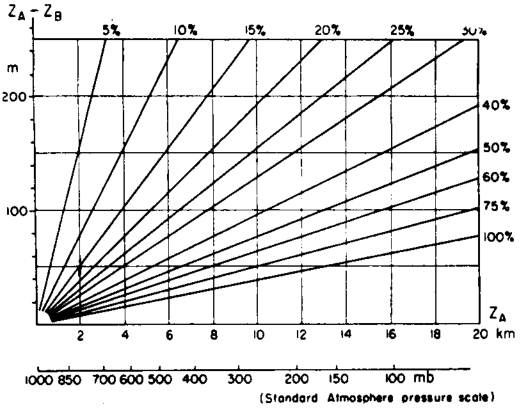


Fig. 1. Error in height difference $Z_A - Z_B$ between two radiosonde stations, A and B, caused by an error of 1° C in mean temperature difference $T_A - T_B$. The height error is given as a function of the height Z_A ($\approx Z_B$) and the actual height difference $Z_A - Z_B$, and expressed as a percentage of $Z_A - Z_B$. The figure also gives the error in the computed geostrophic wind component normal to AB in per cent of the corresponding component of the actual geostrophic wind.

and the difference

$$(\Delta v_g)_n = (v_g)_n' - (v_g)_n =$$

$$= \frac{g}{f}\left(\frac{Z_A' - Z_B'}{d} - \frac{Z_A - Z_B}{d}\right)$$

$$= \frac{g}{fd}(\Delta Z_A - \Delta Z_B)$$

$$\approx \frac{g}{fd}(\Delta T_A - \Delta T_B)\frac{Z_A}{T_A}; \qquad (8)$$

$$(\Delta v_g)_n \approx \frac{g}{fd}\Delta(T_A - T_B)\frac{Z_A}{T_A}. \qquad (9)$$

The relative error $\frac{(\Delta v_g)_n}{(v_g)_n}$ is given by

$$\frac{(\Delta v_g)_n}{(v_g)_n} \approx \frac{\frac{g}{fd}(\Delta T_A - \Delta T_B)\frac{Z_A}{T_A}}{\frac{g}{fd}(Z_A - Z_B)} =$$

$$= \frac{\Delta T_A - \Delta T_B}{Z_A - Z_B}\frac{Z_A}{T_A}, \qquad (10)$$

$$\frac{(\Delta v_g)_n}{(v_g)_n} \approx \frac{\Delta(T_A - T_B)}{Z_A - Z_B}\cdot\frac{Z_A}{T_A} \qquad (11)$$

Fig. 1 gives an evaluation of this expression under the following assumptions:

$\Delta(T_A - T_B)$ is taken to be 1° C. As the error in v_g is proportional to the error in temperature difference, the diagram may of course be used for any value of $\Delta(T_A - T_B)$ if the percentage found from the diagram is multiplied by $\Delta(T_A - T_B)$.

For T_A the value 260° has been chosen. The deviation from this value is in most practical cases less than 10 %, hence the values in the diagram are reasonably correct for practically any temperature.

The figure shows, to take a realistic example, that at the 300-mb level (z_A approximately 9,000 m) the error amounts to 25 % of the comparable component of the actual wind, if the true height difference between the 300-mb level at the two stations is 140 m and the error in temperature difference is 1°.

Evidently, as far as the geostrophic wind field of the upper troposphere and the stratosphere is concerned, it is not advisable to draw any conclusions from height differences based on unsmoothed height values. On the other hand, any kind of smoothing will tend to efface details which may be both real and significant. As an incorporation of observed winds would introduce non-geostrophic wind components of which our knowledge is very deficient, it must be stated that *no general method exists by which it is possible to determine the detailed distribution of geostrophic wind in the upper troposphere and the stratosphere.* Consequently, it is not possible to obtain (from daily maps) more than a very general picture of the geographical distribution of non-geostrophic wind components in these levels. Even a considerable increase in the number of observations would not bring about any material change in this state of affairs, whereas a further improvement of the quality of the soundings would be more important from this particular point of view.

It may be noted, that besides temperature errors, pressure errors too occur in the soundings. In the troposphere these are almost equivalent to the temperature errors, the "scale-value" being dependent upon the actual lapse-rate but, at first approximation, independent of the height. In the stratosphere, the relative importance of pressure errors for computations of height (and geostrophic winds) decreases with altitude, as the absolute height error remains virtually constant above the tropopause level. The existence of pressure errors thus tends to modify, for levels above 300 mb, the normal increase of errors with height shown in Fig. 1. On the other hand, temperature errors often increase, and seldom decrease, with height, which accentuates the tendency shown in the figure.

In order to test the accuracy of geostrophic winds which may actually be obtained by use of uncorrected height values, a random example (May 29, 1951, 15z) was chosen among a number of cases in which complete observations of temperature and wind were available from all existing British radiosonde stations up to 100 mb. A triangular net was introduced (Fig. 2), and the geostrophic wind was computed for each triangle, using uncorrected height values for 500, 300, 200, and 100 mb respectively. In the polar diagrams shown in Fig. 3a–d each of the computed winds is represented by a cross, and each actual wind observation by a dot. The figures show, for 200 and above all for 100 mb, in a striking manner the erratic distribution of the crosses and the denser "clouds" formed by the dots. It should be stressed that the scattering of the dots is not necessarily due to lack of exactness or representativity of the wind data, as there is no reason to assume that the actual wind, or the representative wind, i. e., the actual wind corrected for the (unknown) effect of turbulence is exactly the same at all stations. The analagous statement might be made as far as the computed winds are concerned, though the real differences in this case might be supposed to be smaller because the centers of gravity of the triangles are distributed over a smaller area than the stations. In fact, some of the "absurdly" placed crosses (*c* and *g* on Figs. 3 c—d) belong to the only triangles without "outer walls". The real differences in geostrophic winds are, in the stratosphere at least, one order of magnitude less than the fictitious differences which result from a computation based on height differences. This result, which has been derived from a situation with relatively quiet conditions in the stratosphere, is not directly applicable to situations where a strong horizontal shear is present, but it is seen that the actual shear must be extremely large to match the "false shear" normally introduced by temperature errors.

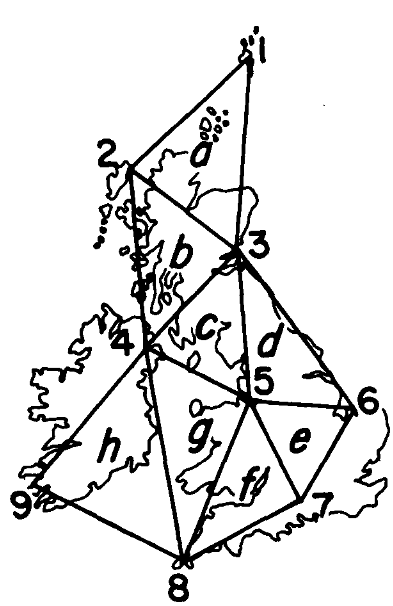
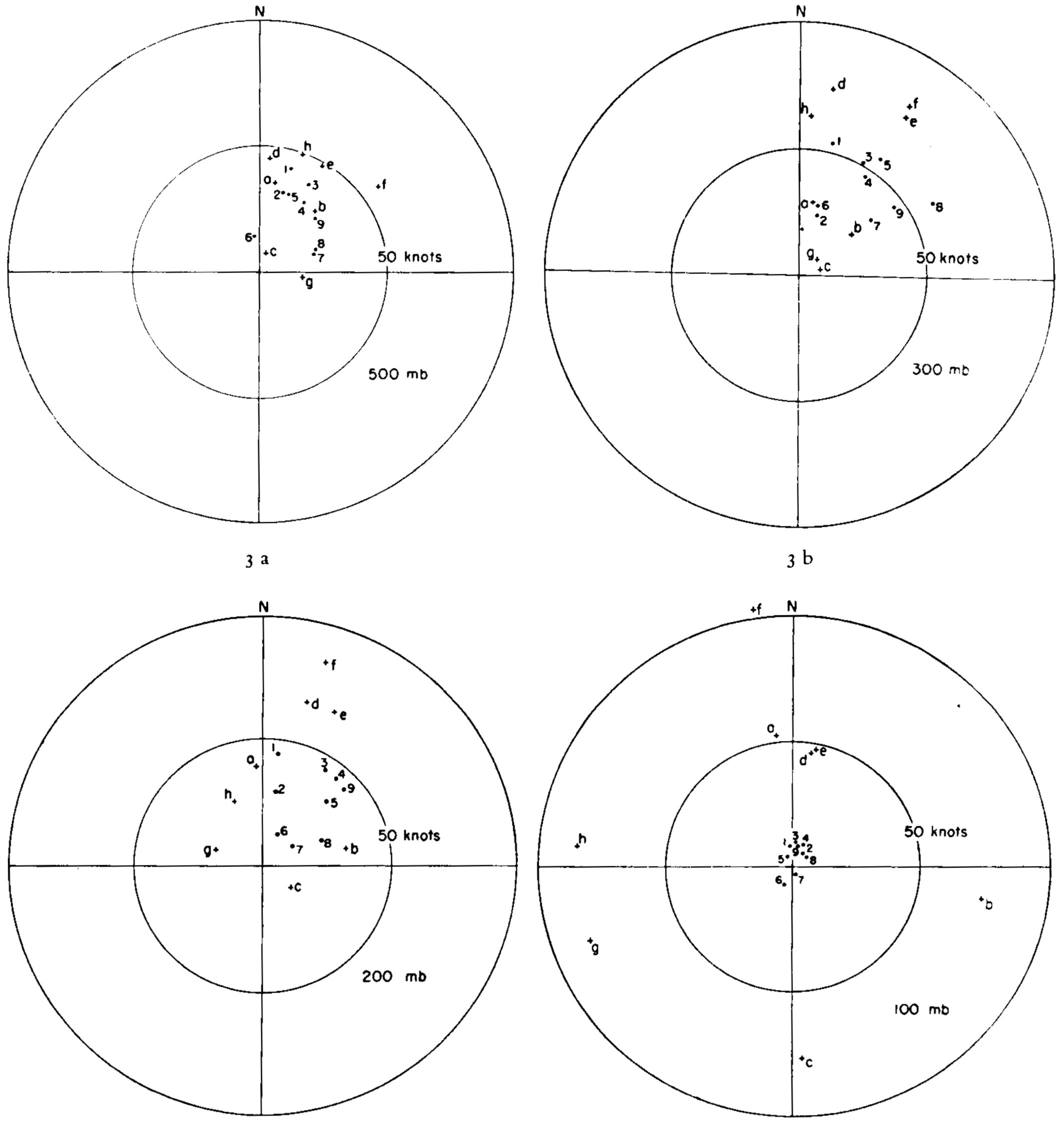


Fig. 2. Triangular net used in comparing computed geostrophic winds and observed winds.

For levels above 300 mb the observed wind (if obtained by radar or an approximately equally good method) normally is a better approximation to the geostrophic wind than a value derived from computed heights, in spite of the systematic difference existing between geostrophic and actual wind. For the 300 and 500 mb levels, the same statement probably is true in many cases, but not generally.

3. Computations of vorticity by means of geostrophic winds based on temperature soundings only

If height values computed from temperature soundings are not sufficiently exact to allow one differentiation, giving the geostrophic wind, they must, evidently, be quite unsuitable for computations based on a *second* differentiation, such as computations of horizontal shear or vorticity. (In the case of horizontal convergence, the value obtained by means of computed heights necessarily must be equal to zero, which is obviously wrong. Nevertheless, the absolute error in the case of this computation may be considerably smaller than in the case of vorticity computations.)



Figs. 3 a—d. Polar diagrams showing the computed geostrophic winds and the observed winds in a case (May 29, 1951, 15z) where complete temperature and wind observations were available for 9 British stations as far up as to 100 mb. Each cross denotes a geostrophic wind computed for one of the triangles in Fig. 2, each dot an observed wind, the letters and numbers referring to Fig. 2.

To give an example showing the magnitude of the errors, the "fictitious vorticity" emerging from one erroneous temperature sounding surrounded by correct soundings shall be discussed.

Suppose that the computed height at the station A is $Z' = Z + \Delta Z$, where Z is the actual height and ΔZ the error. The height values at the distance r are supposed to be correct. Whether the vorticity in the point A is computed by comparing Z' with the average height along the circle or at a few fixed points at the distance r, the error ΔZ will introduce a fictitious vorticity $\Delta \zeta$ which is added to the actual vorticity. To compute $\Delta\zeta$ we have to determine the "false geostrophic wind" along the circle, which is

$$\Delta v = \frac{g}{f} \cdot \frac{\Delta Z}{r}; \qquad (12)$$

this gives for the fictitious angular velocity

$$\Delta\omega = \frac{\Delta v}{r} = \frac{g}{f} \frac{\Delta Z}{r^2}, \qquad (13)$$

and for the fictitious vorticity

$$\Delta\zeta = 2\,\Delta\omega = 2\frac{g}{f}\frac{\Delta Z}{r^2}. \qquad (14)$$

Here we may insert the value of ΔZ which can be obtained from (4) by putting $\Delta(Z_A - Z_B) = \Delta Z$ and, consequently, $\Delta(T_A - T_B) = \Delta T_A$:

$$\Delta\zeta \approx 2\frac{g}{f}\frac{\Delta T_A}{T_A} \cdot \frac{Z_A}{r^2}. \qquad (15)$$

As $\Delta\zeta$ is proportional to Z_A, it is normally much greater in the stratosphere than in the middle troposphere, even when the temperature error does not increase with height. The variation with T_A is unimportant; the variation

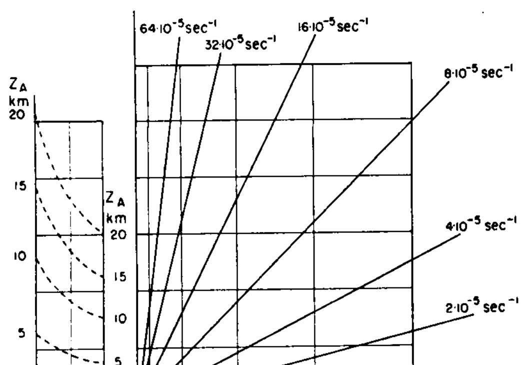


Fig. 4. Fictitious vorticity resulting from an error of 1° in the mean temperature at A in the model described in text to Fig. 4. The vorticity is expressed as a function of the radius r, the height Z_A, and the latitude. For instance, if the height is 10 km, the latitude 45°, and the radius (repesenting the distance between adjacent radiosonde stations) 300 km, the fictitious vorticity is $8 \cdot 10^{-5}$ sec^{-1} per degree Centigrade.

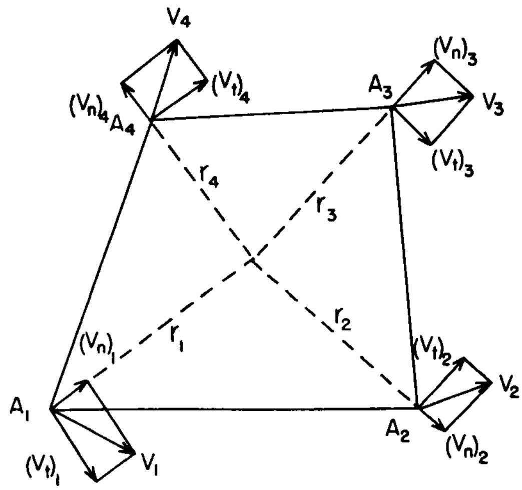


Fig. 5. Diagram illustrating the polygon method for computations of the vorticity field.

with latitude is, of course, the same as for the geostrophic wind itself, which means that the error is particularly large at low latitudes.

Fig. 4 shows the numerical value of $\Delta\zeta$ at 30—60° latitude as a function of Z_A and r, if $\Delta T = 1°$.

4. Computation of vorticity by means of observed winds only

On a few occasions, observed winds have been used for a computation of horizontal convergence (BYERS and RODEBUSH, 1948, BYERS and HULL, 1949), or vorticity (BELLAMY, 1949). This may be done by combining the observed winds in groups of three or four and determining the radial, respectively tangential, component of the wind relative to the center of gravity of the polygon (Fig. 5). The mean vorticity, e. g., for the quadrangle $a_1a_2a_3a_4$ is then, approximately,

$$\zeta = \frac{2}{n}\,\Sigma_p \left(\frac{v_t}{r}\right)_p,$$

v_t being the tangential component (positive, if the motion is counterclockwise), r the radial distance, and p the index number of the point ($1 \leqq p \leqq n$).

This method, of course, is rather crude, as each computation is based on a very limited number of observations without any warrant

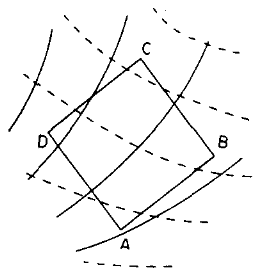


Fig. 6. Diagram illustrating an alternative method of computing vorticity values, using an analysed field of wind components and a grid of semi-squares.

that these characterize sufficiently well the wind distribution within the polygon. Little, if anything, could be gained by giving the different terms $\frac{v_t}{r}$ different weights (to compensate for the non-regular *form* of the polygon). A minor difficulty arises from the finite and unequal *size* of the polygons: each vorticity value computed in the manner just described is an approximate mean value for the polygon; in the final analysis, the field distribution of vorticity therefore should be such that the mean value for each polygon should be equal to that found by the computation. This would be of importance mainly in the vicinity of marked vorticity extremes.

The polygon method is inconvenient if the total number of stations reporting upper winds is considerable. This is true partly because of the arbitrariness of the structure of the polygonal net, but mainly because of the large work involved in measuring angles and distances and making trigonometrical computations. An arbitrary method which is, from this point of view, slightly more convenient, is the following:

As the relative vorticity is defined by the formula

$$\zeta_r = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \qquad (16)$$

it can be derived by means of maps showing the horizontal and meridional wind component separately. Such maps can easily be constructed, if a reasonably large number of well-distributed observations is available within the area to be investigated. Probably it is justified, at least as a general rule, to use only plain interpolation principles for the construction of u- and v-isotachs, but nothing will prevent the use of more refined methods of interpolation in special cases.

Although the picture which the wind component maps give of the wind distribution is based on rather few observations and a not too well-founded hypothesis regarding the "smoothness" of the wind field, it is thought that vorticity computations based on such maps are at least as reliable as computations made by any other method. As to the question whether the wind observations should be accepted, without any smoothing, in drawing the curves, no general answer can be given. If the quality of the observations is very good, it is probable that nothing is gained by a smoothing unless the network of stations is unusually dense.

The procedure by which the vorticity itself is computed is shown by Fig. 6, representing a section of a map for an arbitrary pressure level. The solid lines show the west component, the broken lines the south component as obtained by the interpolation described above. The mean value of vorticity within the square shown on the figure can now be approximated by

$$\zeta = k[(v_B - v_D) - (u_C - u_A)], \qquad (17)$$

the value of the proportionality factor k depending upon the size of the square (but not on the latitude).

In practical analysis, it is convenient to use a network of squares of the type shown in Fig. 6. Because of the curvature of the earth, this involves making a compromise with respect to the form of each square. If the dimensions of the area are of the order of magnitude of 10° or 20° of latitude and longitude, a reasonable compromise may be seen from Fig. 7. At first, a meridian AB dividing the area in nearly equal parts is chosen as a base-line. Along this base-line, division marks are made at equal intervals, a convenient interval being 1° of latitude or 100 km. Next, the latitude circles through the division marks are drawn, and along each of these circles new division marks are made starting from the original base line and using the same length unit as a base. Of the two systems of main diagonal lines which may be constructed in this network, every second diagonal is actually drawn; the points which are not on any of these diagonals are used as grid points for which the vorticity is computed in the manner described above (see Fig. 6). The fact that the figures are not ideal squares is of little or no importance.

The method outlined above has been used in an investigation of the life history of areas of excessive vorticity over the British Isles. The result of this investigation was meager, mainly because the area having a sufficiently dense network was not sufficiently extensive, but it is thought that the method of computing the vorticity proved to be advantageous, or at least practicable.

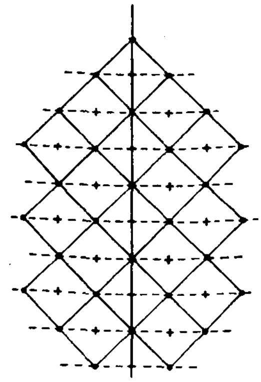


Fig. 7. Model of a network which is convenient for computation of vorticity values from maps showing distribution of zonal and meridional wind components.

5. Computation of vorticity by use of maps based on all available material

In most cases the vorticity is actually computed from contour maps of the ordinary type. These maps are constructed by means of both computed heights and observed winds, and it is generally accepted as a rule for the analysis that at least for the 300-mb and higher levels the observed winds give a better idea of the gradient than the height differences. However, if we use the contour values to compute, by differerentiating twice, the vorticity, we use the *geostrophic* wind components only, as this is what can be derived from the maps. It is not possible, generally, to construct a chart of the "potential of the wind", from which the actual wind might be taken in a manner analagous to that which is used by determining geostrophic winds from a contour map. Even the simple transition from geostrophic to gradient wind cannot be achieved by replacing the contour lines by another, similar set of curves.

Thus, the possible (or, rather, probable) importance of non-geostrophic wind components for the distribution of vorticity is left out of consideration in any case where the vorticity is computed by differentiating twice the contour values. As the additional vorticity which depends on non-geostrophic winds is likely to mix up with the geostrophic vorticity, we may see in this limitation of the standard methods one reason, though possibly not a very important one, why the result of numerical forecasting cannot be absolutely correct.

It might, however, be worth while to try numerical computation based on a vorticity map which is constructed by using observed winds wherever available, and geostrophic winds only where the wind data are insufficient. The procedure might be this: from an ordinary contour map, approximate components of the geostrophic wind are determined and plotted for a sufficiently large number of points (not necessarily the same points for each component) — the zonal components by means of height differences along the meridians, the meridional components by means of height differences along the latitude circles. It will be advantageous to plot the numbers on two sheets of transparent paper, one for each component. On the same sheets the components of the actual winds are plotted, most conveniently in another color, and by the analysis more stress is laid upon these values than on the geostrophic components (which are necessary, however, to get a reasonable picture of the wind distribution over areas where wind observations are missing or too scanty). The two components of relative vorticity are then readily computed, and by numerical or graphical addition of these components plus the Coriolis parameter the vorticity field is determined.

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