Note on the Release of Kinetic Energy in Tropical Cyclones

By E. PALMÉN and C. L. JORDAN, University of Chicago[1]

(Manuscript received September 27, 1954)

The rate of change of horizontal kinetic energy in a given volume can be expressed by

$$\frac{\partial}{\partial t}\int\limits_V K dV = -\int\limits_S K v_r dS -$$

$$-\int\limits_V g\varrho\left(u\frac{\partial H}{\partial x}+v\frac{\partial H}{\partial y}\right)dV-\int\limits_V F dV, \quad (1)$$

where K denotes the horizontal kinetic energy per unit volume, v_r is the outward component of velocity at the boundary, g is the acceleration of gravity, ϱ is the density, u and v denote the horizontal components of velocity and H is the height of a given isobaric surface. In Eq. (1) dV further denotes an element of volume and dS an element of the boundary surface. The third term on the right represents the rate at which all kinds of friction are decreasing the kinetic energy per unit time. Since the contribution of the vertical kinetic energy to the total kinetic energy is in general small, K can be considered as the total kinetic energy per unit volume.

Eq. (1) states that the rate of change of kinetic energy in a given volume V is determined by the flux of kinetic energy across the boundary, the generation of kinetic energy within the volume itself and the dissipation of kinetic energy in the volume due to frictional forces. In a steady state the latter must be equal to the generation reduced by the outflow of kinetic energy.

A tropical cyclone in the mature stage represents closely a steady state in a coordinate system moving at the mean speed of the cyclone. Considering a cyclone with symmetrical distribution of wind and pressure about a vertical axis in the center of the eye, the second term on the right of (1) can be written:

$$-A\int\limits_0^{p_0}\overline{v_r\frac{\partial H}{\partial r}}\,dp. \quad (2)$$

Here p_0 denotes the pressure at the surface, v_r is the velocity in the direction of the radius r, and A is the area limited by the circle considered. The bar indicates a "mean value" of the product $v_r\frac{\partial H}{\partial r}$ in the total area A. Expression (2) represents the generation of kinetic energy per unit time in a cylinder with the horizontal area A extending from the earth's surface to the upper limit of the atmosphere. As the integration is extended over larger volumes the contribution from the first term on the right in Eq. (1) decreases rapidly and over a sufficiently large volume probably becomes negligible. Then the total dissipation of energy can be considered approximately equal to the generation computed from (2).

The generation of kinetic energy in a cyclone of the symmetric type discussed here can be evaluated from the integral (2). Since v_r is negative at lower levels and positive at higher levels the condition for generation of kinetic energy is that the slope of the isobaric surfaces in the cyclone must, on the average, be steeper in the layers of inflow than in the layers of outflow. This is possible only if the interior of the cyclone with ascending motion is warmer than the outer region. Such a distribution of temperature is possible as long as the surface temperature remains sufficiently high, as has been shown by PALMÉN (1948).

[1] Prepared under a research contract between the Office of Naval Research and the University of Chicago.

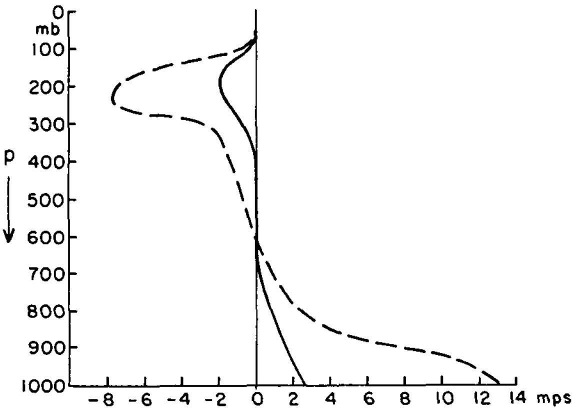

Fig. 1. Vertical distribution of the negative radial velocity component in the "mean tropical cyclone" for the 0.25—2.0° lat. ring (dashed curve) and the 5.0—6.0° lat. ring (solid curve).

In the following an attempt will be made to compute the integral (2). Since the data are not adequate for any individual case, the application will be made to a "mean tropical cyclone" as defined by the mean wind and pressure-height data presented by HUGHES (1952) and E. S. and C. L. JORDAN (1952, 1954). The use of mean data raises some difficulties, but since our computation only intends to give an idea of the order of magnitude of the mean release of kinetic energy, these difficulties should not be critical. In fact, since individual tropical cyclones over the warm ocean areas are quite similar, a storm of the size defined by the above investigations could be expected to show wind and pressure distributions close to those given for the mean case.

The distribution of v_r and $\frac{\partial H}{\partial r}$ given by the basic data of the "mean tropical storm" was not complete enough in all respects for the evaluation of (2). Certain extrapolations of these data were necessary; for example, the winds at the lowest level—1,000 ft—extended to a distance of 0.5° lat. from the storm center, but at upper levels only to a distance of 2° lat. Likewise, no data were available for the uppermost 100 mb of the atmosphere. The mean wind data were fitted together in a consistent manner by imposing the mass continuity requirement for each concentric ring. Greater reliance was placed in the strong inflow near the surface and the strong outflow in the upper troposphere since the individual data in these areas showed good consistency. Data for the layer 600—300 mb were not considered very reliable since the individual wind reports here ranged from strong inflow to strong outflow.

Mean values of v_r and $\frac{\partial H}{\partial r}$ were determined at each level[1] for the rings with the following radii: 0.25—2° lat.,[2] 2—3°, 3—4°, 4—5°, and 5—6°. These values were plotted against a linear pressure scale and smooth curves were drawn. The curves of v_r and the product $v_r \frac{\partial H}{\partial r}$ for the inner and outer rings are shown in Figs. 1 and 2. From the latter figure the production of kinetic energy per unit area in the different layers can be computed graphically. The total production for each ring can then be determined by summing the individual layers and multiplying by the appropriate area.

In the inner zone the layer of strong inflow coincides with the regions of highest values of $\frac{\partial H}{\partial r}$, hence the major contribution to the kinetic energy increase appears here. At upper levels the outflow is toward higher

[1] Wind data were available at 1000, 4000, 7000, 10000, 18000, 30000, 40000 and 45000 ft and pressure-height data at 1000, 850, 700, 500, 400, 300, 200 and 100 mb.

[2] The eye was assumed to have a diameter of 0.5° lat.

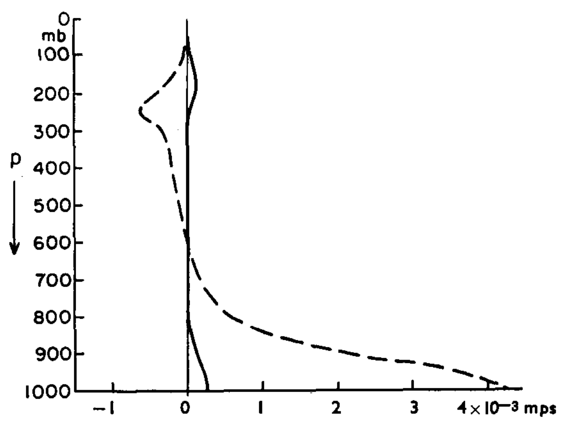

Fig. 2. Vertical distribution of the product $-\overline{v_r \frac{\partial H}{\partial r}}$ in the "mean tropical cyclone" for the 0.25—2.0° lat. ring (dashed curve) and 5.0—6.0° lat. ring (solid curve).

Table I. Production of kinetic energy (unit: 10^{24} ergs/day) in different rings of a mean tropical cyclone compared to the release of latent heat in the interior of the storm

Ring	Release of kinetic energy	Release of latent heat
0.25—2.0° lat.	4.1	419
2.0 —3.0	2.5	17
3.0 —4.0	2.5	29
4.0 —5.0	2.2	—
5.0 —6.0	1.6	—
0.25—6.0	12.9	about 500

pressure near the storm center and therefore energy consuming, but in the outer regions is toward lower pressure and thus contributes to an increase in the kinetic energy. However, even after considering the greater area in the outer ring this contribution is small in comparison to the production of kinetic energy in the regions of strong inflow near the surface in the interior of the cyclone.

The net increase of kinetic energy in ergs per day for the various rings, when frictional dissipation and flux of energy through the boundaries are neglected, is shown in Table I. For comparison the release of latent heat, as computed by HUGHES (1952) is presented in the last column. This release of latent heat is almost entirely limited to the inner region of the cyclone (the rain area). A rough calculation of the radial transport of kinetic energy at the 6° lat. ring showed essentially no net transfer. Thus the accumulative total generation of kinetic energy inside the radius of 6° lat., roughly 13×10^{24} ergs/day, also approximates closely the frictional dissipation in the same volume, since in our mean cyclone the energy can be considered constant. If the total release of latent heat inside the ring at 6° lat. is roughly estimated to be 500×10^{24} ergs/day by extrapolating the computations made by Hughes, the generation of kinetic energy in a typical tropical cyclone can be estimated as about 1/40 the release of latent heat energy due to ascent of moist air near the storm center. This result is in agreement with estimates made by RIEHL (1954) and also with computations of the dissipation of kinetic energy due to friction in the surface layer (HORIGUTI, 1928; HUGHES, 1952). Since these authors did not consider the total frictional dissipation it seems likely that they have somewhat overestimated the surface friction. However, considering the vertical distribution of wind velocity it is probable that a larger proportion of the frictional dissipation occurs in the surface layer in tropical cyclones than in extratropical storms where the wind maximum is usually near the tropopause level.

The total kinetic energy in the "mean tropical cyclone" was computed by determining the areal mean value of $v_s^2 + v_r^2$ at each level and performing a graphical integration from the surface to the upper limit of the atmosphere. The computed kinetic energy was 9×10^{24} ergs inside the 4° ring and 12×10^{24} ergs inside the 6° ring. The latter value is roughly equal to the daily generation of kinetic energy in the mean cyclone according to Table I. Therefore the half-life of the tropical cyclone—the time the cyclone would require to lose half its kinetic energy if its energy source were to disappear—is somewhat less than one day. Although the use of mean values in such a computation must in general lead to a kinetic energy total which is too small, the half-life obtained is certainly reasonable considering the rapid dissipation of intense tropical cyclones which occurs with the movement of colder air into the storm circulation at lower levels (LA SEUR and JORDAN, 1952). This result seems also to be in agreement with the ideas recently expressed by BERGERON (1954) concerning the filling of tropical storms moving in over land.

REFERENCES

BERGERON, T., 1954: The problem of tropical hurricanes. *Quart. J. R. Meteor. Soc.*, **80**, 131—164.

HORIGUTI, Y., 1928: On the typhoon of the Far East. *Mem. imp. Observ., Kobe*, **3**, 23—65.

HUGHES, L. A., 1952: On the low-level wind structure of tropical storms. *J. Meteor.*, **9**, 422—428.

JORDAN, E. S., 1952: An observation study of the upper wind-circulation around tropical storms. *J. Meteor.*, **9**, 340—346.

JORDAN, C. L., and JORDAN, E. S., 1954: On the mean thermal structure of tropical cyclones (To be published in the *Journal of Meteorology*).

LA SEUR, N. E., and JORDAN, C. L., 1952: A typical weather situation of the typhoon season. *Dept. Meteor., Univ. Chicago*, 24 pp.

PALMÉN, E., 1948: On the formation and structure of tropical hurricanes. *Geophysica*, **3**, 26—38.

RIEHL, H., 1954: *Tropical Meteorology*, New York, McGraw-Hill Book Co., 392 pp.