The Spectrum of Large-scale Turbulent Transfer of Momentum and Heat

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Abstract

A method for obtaining the spectrum of turbulent transfer of a fluid property is presented based on cross-correlations and Fourier transforms. An application to large-scale eddy transfers of momentum and heat at 850 mb over the southeastern part of the United States shows that most of the transfers may be attributed to disturbances of periods from 0 to 10 days, the most important transfers occurring in the neighborhood of 4 days. Some implications of these results on the problem of the general circulation are discussed.

1. Introduction

One aspect of the general circulation which has attracted considerable interest in recent years concerns the nature of the processes which accomplish the required global balance of momentum and energy of the atmosphere. The necessity for the meridional transfer of angular momentum from tropical regions to middle latitudes was first pointed out by JEFFREYS (1926). He noted that since the prevailing winds at middle latitudes are westerlies, there is a constant abstraction of angular momentum from the atmosphere by frictional forces on the surface of the earth at these latitudes. On the other hand, the tropics, being regions of easterlies, are areas where angular momentum is supplied to the atmosphere. Since the zonal wind systems are quasi-stationary in the mean, there must be no progressive accumulation nor reduction of angular momentum at any one latitude belt. Hence, a continuous poleward flow of angular momentum from the source regions in the tropics to the sink at middle latitudes must exist. In a similar manner, if one considers the global heat balance, there is an excess of incoming solar radiation over outgoing radiation over equatorial regions; over the polar regions there is a net loss of heat. Therefore, in order to maintain the mean temperature difference between the poles and the equator relatively constant for long periods of time, there must exist a poleward transfer of heat.

For many years the mechanism whereby the atmosphere accomplishes these required transfers had been conceived along lines which depend upon convectively driven meridional circulation models originally proposed by HADLEY (1735). For a number of theoretical reasons, the validity of these models has been questioned. Consequently, emphasis is now being placed upon the theory that horizontal mixing processes are the mechanism which maintains the global balance of momentum and energy. Quantitative studies by WHITE (1951) and STARR and WHITE (1951) show that the

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meridional transfers of heat and momentum due to turbulent processes are capable of maintaining the required balance in middle latitudes. Thus, it is of prime importance in understanding the overall problem of the general circulation to have a quantitative knowledge of the distribution of the intensity of the turbulent transfers among the different scales of disturbances which comprise the entire spectrum of atmospheric turbulence. This paper is the result of investigations toward this direction. In the next section a method for determining the spectrum of turbulent transfer from observed data will be presented. The remaining sections give a numerical application and a discussion of some of the meteorological implications of the results.

Although the present study represents perhaps the first quantitative determination of the partition of turbulent flux among the different scales of eddies in the large-scale circulations of the atmosphere, a qualitative picture is readily obtained from synoptic considerations. An important study by DEFANT (1921) suggested that the traveling cyclones and anticyclones of middle latitudes might be considered as the individual "turbulence elements" in an intensive, quasi-horizontal process of exchange between air masses in high and low latitudes. In the work of Jeffreys cited, he also concluded that turbulence associated with these disturbances constitute the individual eddies which bring about a poleward flux of angular momentum. Along this line, STARR (1948) suggested that troughs in the westerlies which are oriented along a northeast-southwest direction produce the necessary velocity correlations to accomplish a net poleward flux of momentum. Aside from eddies of the scale of cyclones and anticyclones, there are fluctuations of much longer periods, such as those associated with variations in the zonal index, principally "blocking action", which may also produce net poleward transfers of momentum and energy. According to ELLIOT and SMITH (1949) the period of "blocking action" is from 15 to 30 days. While the role of "blocking action" in contributing to the meridional transport of momentum is difficult to visualize, studies by NAMIAS (1950) and ELLIOT and SMITH (op. cit.) emphasize their importance in the poleward transfer of heat. Based upon these considerations one would, therefore, anticipate the presence of maximum poleward eddy transfers in regions of the spectrum corresponding to a period of about 4 to 6 days which is approximately the mean interval between successive frontal or cyclone passages in middle latitudes. Another region of large transfers should be expected at longer periods to account for variations in the zonal index. Of course, the seasonal variations might also be of importance.

2. Theoretical Analysis for the Spectrum

The analysis which will be presented here will be developed through the use of cross-correlations and Fourier transforms. It must be mentioned that the results of such a development may be inferred from studies of the properties of general stationary random functions from the viewpoint of probability theory (BATCHELOR, 1953). However, the physical significance of a turbulent transfer spectrum is difficult to grasp solely with the aid of probability considerations. A more heuristic approach, although perhaps less rigorous, will be employed here by adopting a device introduced by WIENER (1930) of replacing an actual function by another function which differs from the former only in physically trivial aspects and extending ideas used by TAYLOR (1936) in obtaining the energy spectrum of velocity fluctuations.

Consider then a fluid element which has a velocity component V in the meridional direction and a property F. In the numerical application which will be given, F may either be a measure of heat or angular momentum such as absolute temperature, T, or the zonal velocity, U, respectively. Hence, the product VF is the meridional flux of the fluid property through an area normal to V. From a long and continuous record of observations of V and F over a station, let us choose two simultaneous intervals, one of each in V and F, having lengths, $2L$, sufficiently long to include a large number of fluctuations. The assumption regarding the length of the interval is necessary in order that one may obtain reasonably accurate information about statistical characteristics of the fluctuations. Let the origin of time be chosen at the midpoints of the intervals. Then it is evident from the manner in which the length of the intervals are determined that the average values, $\bar{V}$ and $\bar{F}$, evaluated over the intervals must be very nearly equal to the corresponding averages over infinitely long periods of time, i.e.

$$\bar{V} = \frac{1}{2L}\int_{-L}^{L} V(t)dt \approx \lim_{A\to\infty} \frac{1}{2A}\int_{-A}^{A} V(t)dt \quad (1)$$

$$\bar{F} = \frac{1}{2L}\int_{-L}^{L} F(t)dt \approx \lim_{A\to\infty} \frac{1}{2A}\int_{-A}^{A} F(t)dt \quad (2)$$

Denoting the deviations from the means by the primed quantities

$$V'(t) = V(t) - \bar{V} \quad (3)$$

$$F'(t) = F(t) - \bar{F} \quad (4)$$

the mean turbulent or eddy transfer is

$$\overline{V'F'} = \frac{1}{2L}\int_{-L}^{L} V'(t)F'(t)dt = \overline{VF} - \bar{V}\bar{F} \quad (5)$$

which shows that the mean transfer, $\overline{VF}$, may be considered as the sum of the mean turbulent transfer and a quantity, $\bar{V}\bar{F}$, which depends upon the existence of a mean meridional flow, $\bar{V}$. Recent calculations indicate that the contribution of $\bar{V}\bar{F}$ to the global momentum and heat balance is small compared to that of $\overline{V'F'}$, at least in the middle latitudes. An attempt will now be made to decompose the mean eddy transfer, $\overline{V'F'}$, into its spectral components.

For the purpose of analysis, let us replace the original set of fluctuations, $V'(t)$ and $F'(t)$, by a corresponding set denoted by $V^\star(t)$ and $F^\star(t)$ which would still retain the general statistical characteristics of the original fluctuations. Such a set will be defined for all values of time by the equations

$$V^\star(t) = \begin{cases} V'(t) & -L \le t \le L \\ 0 & t \ge |L| \end{cases} \quad (6)$$

$$F^\star(t) = \begin{cases} F'(t) & L \le t \le L \\ 0 & t \ge |L| \end{cases} \quad (7)$$

The implication of this artifice is that the original fluctuations, which may persist for an infinite period of time, are replaced by the finite segments of the original fluctuations which occur over the intervals, $-L \le t \le L$, and flanked on both sides by infinitely long intervals with fluctuations which are identically zero. It may be shown that the primary effect of this mathematical expediency is a tendency to "smear" the spectrum. For example, an actual line at $\omega = \omega_0$ in the spectrum would appear like an error curve, or more precisely, the function $\frac{\sin(\omega - \omega_0)}{\omega - \omega_0}$ in the analysis. The "resolving power" of the analysis is directly proportional to the length, $2L$, of the interval.

Let us express the fluctuations given by equations (6) and (7) as the Fourier integrals,

$$V^\star(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g_v(\omega)e^{i\omega t}d\omega \quad (8)$$

where

$$g_v(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} V^\star(t)e^{-i\omega t}dt \quad (9)$$

and

$$F^\star(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g_F(\omega)e^{i\omega t}d\omega \quad (10)$$

$$g_F(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F^\star(t)e^{-i\omega t}dt \quad (11)$$

Using Parseval's theorem for Fourier integrals, one obtains

$$\int_{-\infty}^{\infty} V^\star(t)\hat{F}^\star(t)dt = \int_{-\infty}^{\infty} g_v(\omega)\hat{g}(\omega)d\omega \quad (12)$$

the circumflex indicating that the quantities are complex conjugates. Noting that $V^\star(t)$ and $F^\star(t)$ are real

$$\int_{-\infty}^{\infty} V^\star(t)\hat{F}^\star(t)dt = \int_{-\infty}^{\infty} V^\star(t)F^\star(t)dt =$$

$$= \int_{-L}^{L} V'(t)F'(t)dt \approx 2L\overline{V'F'} \quad (13)$$

where $\overline{V'F'}$ is the average value of the turbulent transfer taken over an infinitely long record of the original set of fluctuations. Therefore, with the aid of equation (12)

$$\overline{V'F'} = \frac{1}{2L}\int_{-\infty}^{\infty} g_v(\omega)\hat{g}_F(\omega)d\omega = \int_{-\infty}^{\infty} Q(\omega)d\omega \quad (14)$$

$$Q(\omega) \equiv \frac{g_v(\omega)\hat{g}_F(\omega)}{2L} \quad (15)$$

which shows that the turbulent transfer may be expressed as contributions of a continuous spectrum where ω is the frequency. The quantity, $Q(\omega)$, contains a measure of the intensity of eddy transfer per unit bandwidth of frequency.

In order to express $Q(\omega)$ in terms of the original fluctuations, one may proceed in the following manner. Using the relation between the Fourier transform of the product of two functions and the "Faltung" of their Fourier transforms,

$$\int_{-\infty}^{\infty} g_v(\omega)\hat{g}_F(\omega)e^{i\omega\tau}d\omega = \int_{-\infty}^{\infty} V^\star(t)F^\star(t+\tau)dt = \int_{-L}^{L} V'(t)F'(t+\tau)dt \quad (16)$$

and with the aid of the definition for $Q(\omega)$,

$$\int_{-\infty}^{\infty} Q(\omega)e^{i\omega\tau}d\omega = \frac{1}{2L}\int_{-L}^{L} V'(t)F'(t+\tau)dt = \overline{V'(t)F'(t+\tau)} \quad (17)$$

It is worthy to note again that, since the interval, $2L$, contains a large number of the fluctuations, the mean over $2L$, $\overline{V'(t)\,F'(t+\tau)}$, must be an excellent approximation of the actual value of the cross-correlation with lag τ evaluated for an infinitely long record of the original fluctuations. The preceding equation resembles one member of a set of integrals defining a Fourier integral and its transform. Therefore, by analogy

$$Q(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{V'(t)F'(t+\tau)}e^{-i\omega\tau}d\tau \quad (18)$$

Hence, the spectrum of turbulent transfer can be obtained as a Fourier transform of the cross-correlation function. In general, $Q(\omega)$ is a complex quantity. Its real part, $Q_r(\omega)$, corresponds to the "amplitude spectrum" and represents one-half of what will be defined later as the spectrum of turbulent transfer. The imaginary part, $Q_i(\omega)$, which is physically less significant in our particular study, represents the "phase spectrum". Since the cross-correlation is real, the real part of the spectrum is

$$Q_r(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{V'(t)F'(t+\tau)}\cos\omega\tau d\tau \quad (19)$$

By following exactly the same procedure used in the preceding analysis except for interchanging the role of $V'(t)$ and $F'(t)$, one can also show that

$$Q_r(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{V'(t)F'(t-\tau)}\cos\omega\tau d\tau \quad (20)$$

Adding these two expressions for $Q_r(\omega)$ one derives

$$Q_r(\omega) = \frac{1}{4\pi}\int_{-\infty}^{\infty}\left[\overline{V'(t)F'(t-\tau)} + \overline{V'(t)F'(t+\tau)}\right]\cos\omega\tau d\tau \quad (21)$$

Since the integrand is now an even function of τ

$$Q_r(\omega) = \frac{1}{2\pi}\int_{0}^{\infty}\left[\overline{V'(t)F'(t-\tau)} + \overline{V'(t)F'(t+\tau)}\right]\cos\omega\tau d\tau \quad (22)$$

Note that $Q_r(\omega)$ is also an even function, i.e., $Q_r(\omega) = Q_r(-\omega)$. The sum of $Q_r(\omega)$ and $Q_r(-\omega)$ representing the total contribution due to any frequency, ω, regardless of sign, will be termed the spectrum of turbulent transfer, $\Phi(\omega)$. Thus,

$$\Phi(\omega) = \frac{1}{\pi}\int_{0}^{\infty}\left[\overline{V'(t)F'(t+\tau)} + \overline{V'(t)F'(t-\tau)}\right]\cos\omega\tau d\tau \quad (23)$$

One may again write

$$\left.\begin{aligned}\overline{V'(t)F'(t+\tau)}+\overline{V'(t)F'(t-\tau)}&=\\=2\int_0^\infty \Phi(\omega)\cos\omega\tau d\omega&\end{aligned}\right\}\quad(24)$$

by observing that the resulting forms of the equations are analogous to a Fourier transform and its inverse. In particular, if τ is zero, the preceding equation reduces to an expression for the mean eddy transfer in terms of its spectrum,

$$\overline{V'F'}=\int_0^\infty \Phi(\omega)d\omega \quad (25)$$

showing that the total area under the spectral curve is equal to the mean turbulent transfer.

In practical applications, it is sometimes more convenient to evaluate directly an integral of the spectral curve, representing the total transfer due to particular frequency bands, and then derive the actual spectrum by differentiating the integrated spectrum. For this purpose, the contribution of all eddies whose frequencies range from ω_1 to ω_2 may be represented by the expression

$$S(\omega_1\rightarrow\omega_2)=\int_{\omega_1}^{\omega_2}\Phi(\omega)d\omega \quad (26)$$

or substituting the expression for $\Phi(\omega)$ in (23)

$$S(\omega_1\rightarrow\omega_2)=$$
$$=\frac{1}{\pi}\int_0^\infty\left[\overline{V'(t)F'(t+\tau)}+\overline{V'(t)F'(t-\tau)}\right]\cdot$$
$$\cdot\frac{\sin\omega_2\tau-\sin\omega_1\tau}{\tau}d\tau \quad (27)$$

3. The Integrated Spectrum of Turbulent Transfer for Heat and Momentum.

An application of the theoretical analysis in the preceding section was made based on radiosonde soundings obtained twice daily over Greensboro (36° N, 80° W) in the southeastern part of the United States for the year 1949. Since the numerical work was fairly extensive it was possible to treat only the data at the 850 mb level. It was also convenient to use geostrophic winds instead of the actual observed winds because the former were readily available in forms suitable for computation. In order to avoid unnecessary numerical work, constants involving density, specific heat at constant pressure, and the distance of the station from the earth's axis are omitted. Heat and angular momentum flux are denoted simply by VT and UV, respectively. To convert them into the proper units the appropriate physical constants for the set of observations must be applied.

At the start of the computations it was believed that the annual eddy would contribute sizable transfers. Consequently, it was decided to minimize the possible "smearing effect" of the annual cycles in U, V, and T by estimating their components using harmonic analysis. These components, together with the corresponding annual means, were subsequently subtracted from the original observations to obtain the fluctuations, U', V', and T'. The average cross-correlations,

$$\frac{\overline{V'(t)T'(t+\tau)}+\overline{V'(t)T'(t-\tau)}}{2}$$

and

$$\frac{\overline{V'(t)U'(t+\tau)}+\overline{V'(t)U'(t-\tau)}}{2}$$

were then computed from the fluctuations for convenient lags. The results up to a lag of 100 days are shown in Fig. 1. It can be seen that both curves are similar for small values of lags. The correlations decrease very rapidly from their values at zero lag indicating that the most effective eddies in transporting heat and momentum are short period disturbances. At larger lag periods the curves have no resemblance to each other. It had been expected that the correlations would become extremely small at long lag periods of the order of 100 days. Actually this was not realized on account of sampling errors introduced by the relatively smaller number of cases from which the means were evaluated at these lags. Although the correlations do become quite small near lags of 100 days, beyond this, the correlations become more and more unstable. This is a reflection of the fact that, for a given set of observations, the number of cases used in computing a cross-correlation decreases in direct proportion to the length of the lag period. It is, therefore, quite possible that for the longer lag periods,

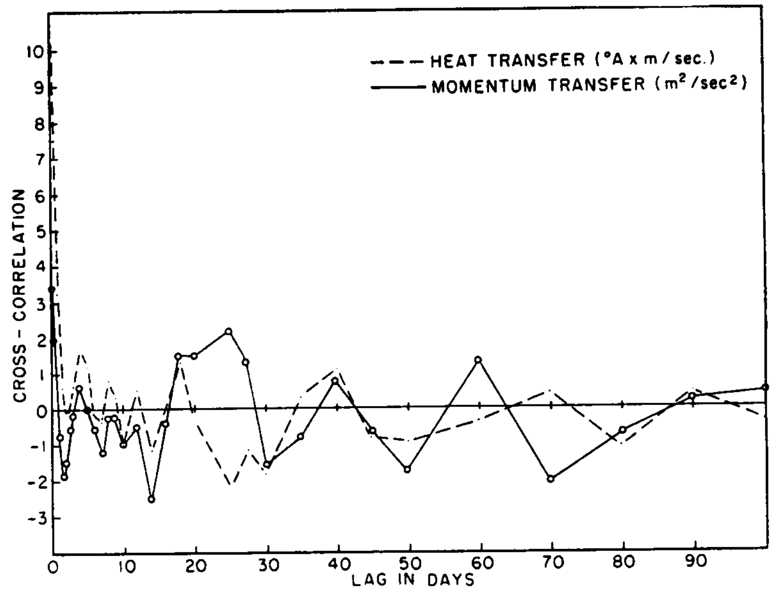

Fig. 1. The average cross-correlations for heat and momentum.

the correlations are highly unrepresentative; this will result in an inaccurate determination of the spectrum at longer periods. In view of these observations on the uncertainties of these particular correlations, it was decided to minimize their effects as much as possible by evaluating the integrated spectrum representing the total contribution of all eddies having frequencies from 0 to ω, instead of the actual spectrum. Thus, using equation (27) and expressing the frequency in terms of the period, $p = \frac{2\pi}{\omega}$

$$S(0 \rightarrow \omega) =$$

$$= \frac{2}{p} \int_0^\infty \left[\overline{V'(t)F'(t+\tau)} + \overline{V'(t)F'(t-\tau)}\right] \cdot$$

$$\cdot \frac{\sin \frac{2\pi\tau}{p}}{\frac{2\pi\tau}{p}} d\tau \qquad (28)$$

For large values of τ the factor $\sin \frac{2\pi\tau}{p} \Big/ \frac{2\pi\tau}{p}$ becomes small, having a value of at most $\frac{p}{2\pi\tau}$; the whole integrand becomes small and its contribution to the value of the integral is negligible. Therefore, the unreliability of the cross-correlations at large lags are minimized.

Figures 2 and 3 represent the integrated spectra for the turbulent transfers of heat and

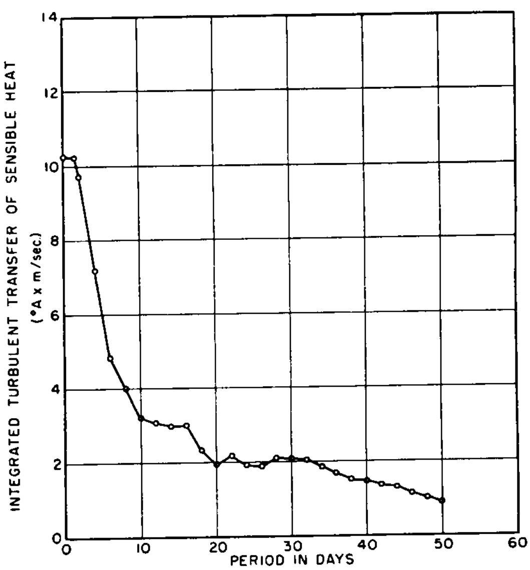

Fig. 2. The integrated spectrum for turbulent heat transfer. The ordinate represents the total transfer due to all disturbances with periods longer that than given by the corresponding abscissa.

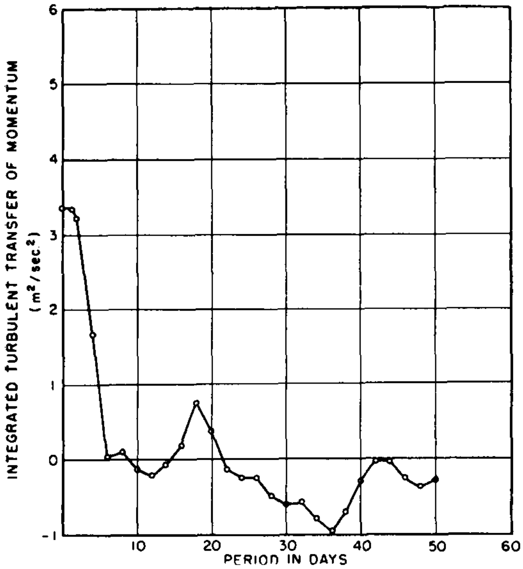

Fig. 3. The integrated spectrum for turbulent momentum transfer. The ordinate represents the total transfer due to all disturbances with periods longer than that given by the corresponding abscissa.

momentum obtained with the aid of equation (28) after replacing the integration by a summation. In order to show more clearly the relative importance of the different scales of disturbances in the turbulent transfer of heat and momentum, the integrated spectra were smoothed and then graphically differentiated. The results given in Figure 4 show the intensity of transfer as a function of period.

The most striking characteristic of both spectra is the extreme importance of short-period disturbances in the eddy transfer of heat and momentum. The high peaks at about 4 days are undoubtedly the expected contributions of frontal disturbances and the associated anticyclones. The total amount of transfers due to eddies with periods larger than 10 days is relatively small in the case of heat flux; and for momentum transfer the net contribution of these long-period eddies is entirely negligible for this particular set of data due to the presence of approximately equal amounts of northward and southward transfers. This effect is extremely slight for heat transport and one might infer that the tendency of all kinds of atmospheric disturbances, irrespective of size, is to effect poleward heat transfers. The differences in the characteristics of the two spectra might be a result of the different effectiveness of zonal index variations with regard to the ability to transfer heat and momentum. It must be emphasized again that the results of the spectrum analysis for longer periods are not entirely reliable so that these should be interpreted with caution.

The integrated spectra show that the total turbulent transfer due to all eddies excluding the yearly eddy are 3.35 m^2 sec^{-2} and 10.21 $A x m$ sec^{-1} for momentum and heat, respectively. Since the corresponding total transfers including the yearly eddy as evaluated from the original data are 3.54 $m^2 sec^{-2}$ and 10.80 $A x m$ sec^{-1}, the contributions of the yearly eddy are approximately 3 per cent and 6 per cent for momentum and heat, respectively. This comparatively small contribution of the yearly eddy in the meridional flux of these quantities is also true for the turbulent transfer of latent heat as shown by recent studies (BENTON et al., 1953). Consequently, the evaluation of the local eddy flux with the aid of averaging methods (using an equation of the form, $\overline{V'F'} = \overline{VF} - \bar{V}\bar{F}$) for periods of at least one month would include the transfers of most of the disturbances of the entire spectrum. The dominant role of short-period disturbances is more clearly illustrated in Table I which shows the ratio of the transfers due

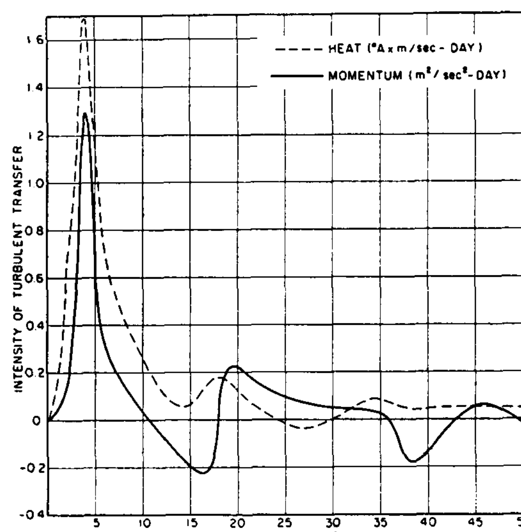

Fig. 4. The intensity of heat and momentum transfers plotted against period.

to different bands of the spectrum to the total turbulent transfer.

Table 1. Turbulent transfers due to different regions of the spectrum expressed as percentage of the total transfer. Negative percentages indicate southward transfers.

Period (days)	0—10	10—20	20—30	30—40	40—50	over 50
Momentum (per cent)	98	—14	28	—9	0	— 3
Heat (per cent) ..	65	12	—1	6	5	13

The prominent regions in the momentum spectrum showing appreciable transfers at longer periods are quite interesting. These characteristics of the spectrum are difficult to associate definitely with any large-scale disturbances in the atmosphere and may possibly be peculiar to the particular set of data analyzed or due to the unreliability of results at the longer periods. A more complete analysis of other cases should indicate whether these peculiarities in the spectrum are real and local in character.

4. Summary and Conclusions

Perhaps an important result of the present study is the fact that the method of spectrum analysis based on correlations is practicable and useful in deducing the nature of large-scale turbulent transfer processes such as those involved in the general circulation. At this point it is worthwhile to mention some points which will serve as guides in future applications of the method. Our numerical work revealed that the most critical aspect of the computations involved the determination of representative cross-correlations. It was found that there is difficulty in obtaining reliable values of the correlations for lags of the order of 100 days by using only one year of data consisting of two observations per day. The only solution to this difficulty is to make either more frequent observations or use longer periods of observation. The former is preferred because the analysis would then enable one to assess readily the transporting ability of disturbances with periods of fractions of a day. In order to minimize "smearing", the removal of the components of the annual cycle from the original set of observations is necessary if its contribution is not negligible as compared to the total turbulent transfer. A rough idea of the transfer due to the annual cycle is given by the formula $\frac{AB}{2} \cos \Theta$, where A and B are the amplitudes of the annual waves in V and F obtained from monthly means and Θ is the difference of their phase angles. Finally, it is not advisable to compute the spectrum directly using equation (23) unless the reliability of the cross-correlations is completely assured. A better procedure is to evaluate the integrated spectrum first and to derive the spectrum by differentiation.

It is certainly not appropriate to make definite conclusions about the nature of horizontal mixing processes which accomplish the balance of momentum and heat required by the general circulation problem from the very limited application of the spectrum analysis presented above. Our work must be extended first to include observations at upper levels of the troposphere as well as at other latitudes. A spectrum analysis at 300 mb over the same station considered here is particularly desirable since it is the level of maximum northward transport of angular momentum and hence may indicate the nature of the principal mechanism of momentum balance. It is also extremely interesting to apply the analysis over tropical latitudes where the atmospheric processes accomplishing poleward eddy transfers of heat and momentum are least understood. Recent work by STARR and WHITE (1952) indicates that the local eddy flux over these regions are not at all negligible as it was presumed to be.

In spite of the limited scope of our numerical application, however, it is possible to offer some comments regarding the nature of atmospheric processes which accomplish the required poleward transfers of momentum and heat energy. In the lower portions of the troposphere, and possibly including the 500 mb level, the dominant mechanisms are those associated with frontal disturbances and migratory cyclones and anticyclones. This fact lends support to the theory that frontal disturbances and the associated cyclones and anticyclones are indispensable processes in the maintenance of the general circulation. For the local eddy flux of enthalpy and latent heat, these disturbances are the most effective throughout the entire vertical extent of the atmosphere at middle latitudes since heat and moisture flux above 500 mb is relatively weak. The same statement cannot, of course, be made in the case of momentum flux. The level of maximum transfers occurs at about 300 mb, much higher than the level treated in the present study. It is possible that at this level the region of most intense transfers, which were found near periods of 4 days at 850 mb, may be shifted to longer periods as a result of the effect of long waves in the upper westerlies.

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