Instability Criteria and Growth of Baroclinic Disturbances

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Abstract

The instability criterion and the rate of growth of long baroclinic disturbances in horizontally uniform zonal flow is derived for a sloping streamsurface. Both quantities are expressed by one explicit and simple function. Unstable disturbances are characterized by upward motion toward the cold air and downward motion toward the warm air. The slope of the streamsurface is eliminated, and the growth equation is compared with the results of other studies. The maximum rate of growth is small enough to indicate that numerical prediction of the development of baroclinic disturbances is practicable. Other results are that long waves may occur in a baroclinic atmosphere above a flat earth as well as above a spherical earth and that on a spherical earth unstable disturbances occur only north of a critical latitude.

1. Historical Perspective

The fundamental importance of differential heating in creation of the large scale atmospheric motions was stated long ago by HADLEY (1735—36). But only in recent years have observations been available to show that the actual motions differ markedly from the simple convective circulation envisaged by Hadley. Theory has been, if anything, even slower; the hydrodynamics of planetary atmospheres has been developed only in the past two decades, and it is still far from complete.

Observations in low latitudes show that the zonal speed increases with latitude in rough conformity with conservation of angular momentum, and the dominant motion is relatively slow and steady (UNIVERSITY OF CHICAGO, 1947). By contrast, in middle and high latitudes the atmosphere is subject to large amplitude disturbances, and the zonal speed decreases with latitude. These features of the atmospheric circulation pose several problems; viz., (1) why are the regimes of low and high latitude different? (2) how is the middle latitude maximum of zonal wind speed maintained? (3) under what conditions does this zonal current "break down" into large scale meridional currents? Clearly, these problems are interrelated, but their complexity makes it desirable to treat them separately. This paper is concerned mainly with the third problem and to a lesser extent with the first.

Observations suggest that the transformation from zonal to meridional currents is an instability phenomenon; that is, one in which small displacements are amplified with time. The standard method for determining the conditions necessary for instability (THOMSON, 1871) consists of discovering the conditions under which the speed of periodic disturbances in the fluid is a complex function. The imaginary part of the complex function yields an exponential increase of the amplitude with time.

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Studies of instability in the atmosphere stem from LORD RAYLEIGH'S (1878, 1880, 1887) investigations of the conditions under which small displacements at the boundary between two air streams grow into larger displacements and from HELMHOLTZ' (1889) determination that unstable gravitational waves may form on a surface separating two layers of different density and velocity. These two discoveries led to further studies of the effects of velocity shear at the boundary between fluids, and the term *shearing instability* has been used to describe instability of this type. This series of investigations culminated in the polar front theory of cyclone formation (V. BJERKNES et al., 1933). As a result of the widespread interest in this theory, attention was focused on the region near a boundary between two fluids rather than on the behavior of a single extensive fluid.

A second series of investigations has stemmed from HELMHOLTZ' (1888) and RAYLEIGH'S (1913) studies of the relation of vorticity distribution to instability. SOLBERG (1936) showed that disturbances within a zonal atmospheric current are unstable if the meridional velocity shear along isentropic surfaces exceeds the value of the Coriolis parameter. Subsequent studies by ERTEL (1940), KLEINSCHMIDT (1941) and others and a more complete criterion derived by KUO (1949) represent attempts to explain the more or less chronic instability of mid-latitude zonal flow on this basis. Instability of this type often is called *dynamic instability*, a particularly unfortunate choice of terms because the label *dynamic* implies that other types of instability are unrelated to forces. Despite the unquestioned validity of the instability criterion, observations gradually have made it clear that the criterion is so severe that the instability of zonal flow cannot be explained primarily by the quasi-horizontal shear of the zonal wind.

2. Baroclinic Instability

The development of the hydrodynamics of planetary atmospheres began with ROSSBY'S (1939) demonstration that large-scale disturbances may exist in a barotropic atmosphere. SUTCLIFFE (1947) and CHARNEY (1947) extended the theory to a baroclinic atmosphere; their results showed that if the vertical shear of the undisturbed zonal current (here called baroclinity) exceeds a certain critical value which depends on wavelength and latitude small disturbances will grow in amplitude. According to this criterion, waves of less than a critical length are unstable. The importance of this discovery quickly was recognized, so that since 1947 primary attention has shifted from instability arising from quasi-horizontal shear or from shear at boundaries between homogeneous air masses to instability arising from vertical wind shear. The term *baroclinic instability* has come to be used to describe this sort of instability, and extensions of the theory have been made.

Several investigators, notably FJØRTOFT (1950), BERSON (1951), KUO (1952), and THOMPSON (1953), have succeeded in extending the theory into the domain of unstable waves. The results of these studies show that maximum instability exists for waves of intermediate length. However, there is considerable difference in these results. Kuo finds short waves unstable, whereas the others find them to be stable. Fjørtoft attributes short-wave stability to non-geostrophic velocity at short wavelengths, the others do not. Various boundary conditions have been used, but their effects on the solutions are not clear.

3. Status of the Problem

The foregoing brief survey of the history of baroclinic instability serves to illuminate the very rapid progress which has been made toward the solution of the problem. It also reveals that at present the role of non-geostrophic wind in short waves is uncertain, and that understanding of the effects of various simplifying assumptions is incomplete. One must recognize, too, that the most complete solution to the problem (KUO, 1952) was possible only with mathematical complexity so great as to defy clear understanding of the role played by the various parameters of the problem.

Finally, it is appropriate to point out that each of the instability criteria contain parameters like vertical wind shear, hydrostatic stability, and undisturbed zonal wind speed which are, in part at least, products of the instability phenomenon and which, therefore, may depend on other parameters of the problem. For this reason, it is possible that disturbances which are indicated as unstable by a theoretical criterion in reality may be stable, or vice-versa. To put it differently, arbitrary choice of the parameters of the problem may not be proper.

There is, therefore, only limited importance in a discussion of whether one instability criterion or another is superior. The problem of greatest interest and importance at the present time is to incorporate more of the parameters into the solution or at least to gain additional insight into the relation of parameters to solution. Eventually, of course, one hopes to consider a model in which the only arbitrary parameter is the distribution of solar energy absorbed at the earth's surface. Then the problem of the general circulation may be said to have been solved.

4. Purpose

The purpose of this paper is to clarify the influence of the various parameters in the development of long baroclinic disturbances, and, thereby, to gain further insight into the mechanism of the general circulation.

5. Method

The central difficulty encountered in developing a frequency equation for a compressible baroclinic atmosphere appears when the solution or solutions are substituted into the boundary conditions. Typically, the resulting equation is a complex function and is hideously transcendental. However, this difficulty may be avoided if it is possible to incorporate the boundary conditions or the results of observation in the equations at the outset in such a way that the number of dependent variables and their derivatives is equal to the number of independent equations. If this can be done, the frequency equation emerges promptly upon algebraic reduction.

The normal component of velocity is required to vanish at rigid impervious surfaces; therefore, immediately above the horizontal ground surface the streamlines are horizontal. Computations by FLEAGLE (1947) and others indicate that above the ground the streamlines slope upward toward the pole. In a broad zonal current these streamlines are approximately plane so that planes may be passed through individual streamlines to form surfaces as shown in Fig. 1. These surfaces are referred to hereafter as streamsurfaces. The maximum slope of the streamsurfaces is reached in the middle troposphere, not always at the same height. If the streamsurface of greatest slope is selected as the x, y plane of a Cartesian coordinate system, then the velocity normal to the x, y plane and the normal gradient of the normal velocity vanishes everywhere on this plane. It follows that, so long as the equations are applied to the streamsurface of maximum

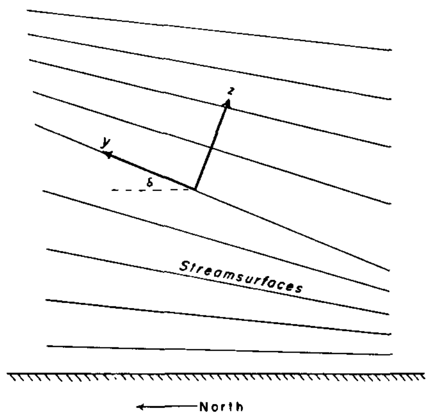

Fig. 1. Vertical cross section showing sloping streamsurfaces and orientation of the coordinate system.

slope, the vertical component of velocity and its space derivatives do not appear in the hydrodynamic equations.

Upon adopting the coordinate system described above, the five perturbation equations which describe a compressible baroclinic atmosphere contain the dependent variables: pressure, density, vertical pressure gradient, and the velocity components in the x and y directions.

The enormous simplification of the problem which is introduced by the elimination of the vertical velocity and its height derivative makes it possible to eliminate all five variables by algebraic reduction. This procedure, of course, leads only to the frequency equation; the vertical structure of the disturbance must be developed from the complete set of equations including the vertical velocity terms. Observations indicate that growth of disturbances occurs simultaneously and at about the same rate throughout a large part of the troposphere.

Therefore, although the frequency equation derived for the streamsurface of greatest slope applies directly only to that surface, it also applies indirectly to a layer of considerable thickness.

6. The Frequency Equation

In the sloping Cartesian coordinate system described above the vectors representing the acceleration of gravity and the angular velocity of the earth may be resolved into components as follows:

$$\mathbf{g} = -g\,(\mathbf{j}\sin\delta + \mathbf{k}\cos\delta) \quad (1)$$

$$\boldsymbol{\Omega} = \Omega\,(\mathbf{j}\cos(\phi-\delta) + \mathbf{k}\sin(\phi-\delta)) \quad (2)$$

The symbols used here and throughout the paper are defined in the Appendix.

The perturbation method as described by Haurwitz (1951) is accurate for infinitesimal disturbances and therefore is the natural method for determination of instability criteria. The undisturbed motion will be assumed to be characterized by unchanging zonal flow which is constant in the x and y directions and which increases linearly with height (normal to the earth's surface). The equations of undisturbed motion given by Haurwitz (1951) then become

$$V = 0 \quad (3)$$

$$fU = -Q^{-1}P_y - g\sin\delta \quad (4)$$

$$g\cos\delta = -Q^{-1}P_z \quad (5)$$

$$W_z = 0 \quad (6)$$

$$dQ/dt = 0 \quad (7)$$

From (4) and (5) it follows that

$$Q^{-1}Q_y = \frac{fU_z}{g\cos\delta} - g\Gamma\sin\delta - fU\Gamma \quad (8)$$

where $Q^{-1}Q_z$ is represented by $-g\Gamma\cos\delta$.

Because the undisturbed current is assumed to be independent of y, it seems reasonable to impose the arbitrary restriction that the velocity perturbations are independent of y. The only justification for this restriction is the observation that velocity disturbances which are approximately independent of y do occur, and the conditions under which they develop do not appear to be markedly different from the conditions under which velocity disturbances develop which are dependent on y. The perturbation equations given by Haurwitz (1951) may be written for this case in the form

$$L(u) + \underline{wU_z} - fv = -Q^{-1}p_x \quad (9)$$

$$L(v) + fu = -Q^{-1}p_y + q\,Q^{-2}P_y \quad (10)$$

$$\underline{L(w)} + g\,q\,Q^{-1} = -Q^{-1}p_z \quad (11)$$

$$L(q) + vQ_y + Qu_x + \underline{(Qw)_z} = 0 \quad (12)$$

$$L(q) + vQ_y + \underline{wQ_z} - \gamma\,[L(p) + vP_y + \underline{wP_z}] = 0 \quad (13)$$

Upon applying these equations to the streamsurface of maximum slope, the underlined terms vanish. Then, if p_y is eliminated by cross differentiation of (9) and (10)

$$L(v_x) + fu_x + v\beta = Q^{-2}(q_xP_y - p_xQ_y) \quad (14)$$

where β represents $\partial f/\partial y$.

Equations (9)—(14) are linear equations so that solutions may be superimposed. Therefore, no loss of generality occurs if the disturbances are assumed to have the simple form given by

$$u = A(z)\,e^{i\alpha(x-ct)} \quad (15)$$

$$v = B(z)\,e^{i\alpha(x-ct)} \quad (16)$$

$$p = D(z)\,e^{i\alpha(x-ct)} \quad (17)$$

$$q = F(z)\,e^{i\alpha(x-ct)} \quad (18)$$

Substitution of (15)—(18) in (9), (11), (12), (13), (14) yields five simultaneous algebraic equations in A, B, D, D_z, and F. In order that non-trivial solutions exist for A, B, D, F, and D_z, the determinant of the coefficients must vanish. It follows directly that

$$-\gamma(c-U)^3[\alpha^2(c-U)+\beta] + (c-U)^2(\alpha^2 + f^2\gamma) + (c-U)[\beta + fQ^{-1}(Q_y - 2\gamma P_y)] - Q^{-2}P_y(Q_y - \gamma P_y) = 0 \quad (19)$$

The first term in (19) is small compared to the second for all values of $c-U$ much smaller than the speed of sound; it therefore is neglected. If the meridional velocity had been assumed to obey the geostrophic equation, this term would not have appeared; but equation (19) would otherwise be unchanged. So, it must be concluded that the quasi-geostrophic assumption does not lead to significant error *for disturbances in which the velocity perturbations are independent of y*.

Elimination of the first term in (19) leads directly to

$$c - U = -\frac{1}{2(\alpha^2 + f^2\gamma)}\{\beta + fQ^{-1}(Q_y - 2\gamma P_y) \mp$$
$$\mp[\beta^2 + 2\beta f Q^{-1}(Q_y - 2\gamma P_y) + 4\alpha^2 Q^{-2} P_y (Q_y -$$
$$- \gamma P_y) + f^2 Q^{-2} Q_y^2]^{1/2}\} \qquad (20)$$

Substitution of (4) and (8) in (20) gives a cumbersome equation which, for long baroclinic disturbances, may be simplified by the following approximations.

$$\alpha^2 + f^2\gamma \approx \alpha^2 \qquad \sin\delta \approx \delta$$
$$2\gamma - \Gamma \approx \gamma \qquad \cos\delta \approx 1$$
$$U_z - 2Us_z \approx U_z$$
$$\beta + 2f^2 U\gamma + 2f^2 U_z/g \approx \beta$$

It follows that (20) may be written

$$c - U = -\frac{\beta + fg\gamma\delta}{2\alpha^2} \mp \frac{1}{\alpha}\left\{\frac{1}{4\alpha^2}(\beta + fg\gamma\delta)^2 -\right.$$
$$\left. - \delta(fU_z - gs_z\delta)\right\}^{1/2} \qquad (21)$$

For unstable disturbances the wave speed given by (21) is complex; the real part describes the speed and the imaginary part describes the rate of growth of the unstable waves. Thus

$$c_r - U = -\frac{\beta + fg\gamma\delta}{2\alpha^2} \qquad (22)$$

$$\alpha c_i = \left\{\delta(fU_z - gs_z\delta) - \frac{1}{4\alpha^2}(\beta + fg\gamma\delta)^2\right\}^{1/2} \qquad (23)$$

The preceding derivation may be performed in spherical coordinates as readily as in Cartesian coordinates. The result is very nearly identical with (22) and (23) except very near the poles. Equation (23) shows that instability depends in a direct sense on U_z, the baroclinity. However, instability is possible only if δ is positive; therefore, upward motion to the east and downward motion to the west of pressure troughs is essential in order that disturbances grow. This relationship is reflected in the fact that cloudiness and precipitation occur to the east of low pressure systems and clear skies prevail to the west. The first term in (23) indicates that instability is possible for positive $gs_z\delta$ only if $gs_z\delta$ is less than fU_z; that is, if the slope of the streamsurface is less than the slope of the isentropic surface. The second term in (23) indicates that very long waves are stable and shorter waves are unstable. This conclusion is valid, of course, only if δ, as well as other parameters, is reasonably independent of wavelength.

For the case of horizontal motion, δ vanishes; only stable disturbances are possible, and (21) reduces to the trough formula for a barotropic atmosphere. This reduction demonstrates that, although horizontal velocities in large disturbances are three orders of magnitude greater than the vertical velocities, vertical velocity plays a role of crucial importance in the conversion of the potential energy of the baroclinic atmosphere to kinetic energy.

7. Elimination of the Slope of the Streamsurface

The slope of the streamsurface may be eliminated from (21) by evaluating the ratio of the vertical to the meridional velocity components at the surface of its maximum value. This requires manipulation of the perturbation equations (9), (11), (12), (13), (14) including the vertical velocity terms. Upon eliminating from these equations the perturbation pressure and perturbation velocity in the x direction, the following equations result.

$$fL(q_x) - Q^{-1}Q_y L^2(q) + Q^{-1}P_y q_{xx} + Q_y U_z w_x -$$
$$- Q^{-1}Q_y^2 L(v) - QL(v_{xx}) - Q\beta v_x + f(Qw)_{zx} -$$
$$- Q^{-1}Q_y L[(Qw)_z] = 0 \qquad (24)$$

$$L(q_{xx}) - \gamma L^3(q) + Q_z w_{xx} + \gamma U_z Q L(w_x) +$$
$$+ g\gamma Q w_{xx} + Q_y v_{xx} - f\gamma Q L(v_x) - \gamma Q_y L^2(v) -$$
$$\gamma P_y v_{xx} - \gamma L^2[(Qw)_z] = 0 \qquad (25)$$

$$U_zL(q_x) + g\,q_{xx} + L^2(q_z) + Q_yL(v_z) + + L[(Qw)_{zz}] + QL(w_{xx}) + U_zQ_yv_x + fQ_zv_x + + fQv_{zx} = 0 \qquad (26)$$

Then, upon substituting solutions (15)—(18) there result three algebraic equations. Small terms may be eliminated at this point if care is taken to ensure that the remaining large terms do not cancel at a subsequent step. The relative magnitudes of all small terms but two are independent of wavelength. The exceptions are negligible for all disturbances more than several hundred kilometers in length. Elimination of the small terms leads to the equations

$$\alpha cfF + if(QC)_z - i[\alpha^2(c-U) + \beta]QB = 0 \qquad (27)$$

$$i\alpha(c-U)F + s_zQC - f[U_z/g + \gamma c - \Gamma U]QB = 0 \qquad (28)$$

$$\alpha gF - if(QB)_z = 0 \qquad (29)$$

where C represents the amplitude of the vertical velocity perturbation. It is easy to show that if quasi-geostrophic and quasi-hydrostatic equilibrium had been assumed at the outset, equations (27), (28), and (29) would have resulted without the final elimination of small terms. So, *for disturbances in which the velocity perturbations are independent of the y coordinate*, these assumptions are valid, at least down to wavelengths of a few hundred kilometers.

If F is eliminated from (27), (28), and (29)

$$gf(QC)_z + f^2c(QB)_z = g[\alpha^2(c-U) + \beta]QB \qquad (30)$$

$$gs_zC + f(cs_z - U_z)B = f(c-U)B_z \qquad (31)$$

At the streamsurface of greatest slope $(QC)_z = \delta(QB)_z$. This condition serves the function of boundary conditions in the complete integration of the hydrodynamic equations and makes it possible to eliminate $(QC)_z$ and $(QB)_z$ between (30) and (31). If, as before, only terms of the largest order of magnitude are retained, there results

$$\delta = \frac{f[U_z - g\Gamma(c-U)]}{gs_z} \qquad (32)$$

If only unstable disturbances are considered, $(c-U)$ may be eliminated from (32) by the use of (22). This gives

$$\delta = \frac{2fU_z\alpha^2 - \beta fg\Gamma}{2gs_z\alpha^2 + f^2g^2\gamma\Gamma} \qquad (33)$$

Substitution of (33) in (22) and (23) gives

$$c_r - U = -\frac{f^2\gamma U_z + \beta s_z}{2s_z\alpha^2 + f^2g\gamma\Gamma} \qquad (34)$$

$$\alpha c_i = \frac{f\Gamma}{2s_z\alpha^2 + f^2g\Gamma\gamma} \{\alpha^2(f^2U_z^2 - \beta^2 s_z^2/f^2\Gamma^2) - \beta(gf^2\Gamma U_z + g\beta s_z)\}^{1/2} \qquad (35)$$

These relations can be obtained without the prior derivation of (21), but this would leave somewhat obscure the role of vertical motion in the release of baroclinic instability.

Inspection of (35) shows that short disturbances (large α) are slightly unstable, longer disturbances are more unstable, but very long disturbances are stable. The wavelength of greatest instability is given by

$$\lambda_m = 2\pi\left\{\frac{2s_z(f^2U_z^2 - \beta^2s_z^2/f^2\Gamma^2)}{gf^2\Gamma U_z(f^2\Gamma U_z + 4\beta s_z) + 3g\beta^2s_z^2}\right\}^{1/2} \qquad (36)$$

8. Comparison with Results of Other Studies

The critical curves and the amplification per 24 hours computed from (35) is shown in Fig. 2. The parameters chosen are those used by CHARNEY (1947), FJØRTOFT (1950), and KUO (1952). The critical curve found by Charney and Kuo is indicated for comparison. The critical curve for a different lapse rate also is shown.

The difference between the results shown in Fig. 2 and KUO's (1952) results must reflect differences in the atmospheric models used. The advantage of Kuo's treatment lies in its generality; his frequency equation was obtained from the perturbation equations by introducing special and rather simple boundary conditions. The advantage of the treatment given here lies in its simplicity; the result was obtained by using the fact of the existence of a streamsurface of maximum slope. It may be noted that Kuo's solution requires the slope of the streamsurfaces to increase upward without limit, presumably, as a result of an unrealistic upper boundary condition. It is

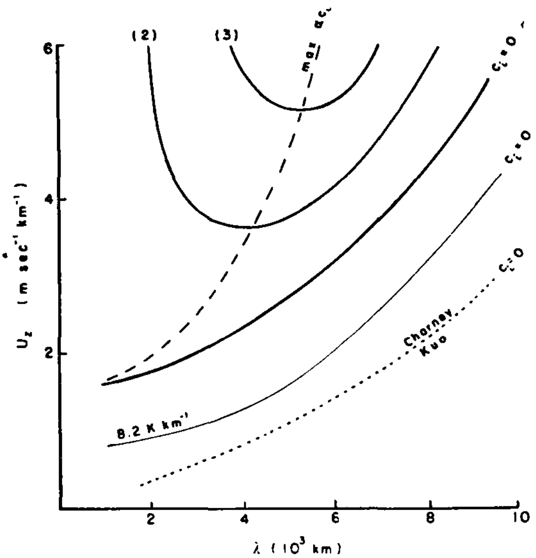

Fig. 2. Critical curves (labelled $c_i = 0$) and the 24 hour amplification (labelled (2) and (3)) computed from (35) for latitude of 45° N, and temperature lapse rate of 6.5 K km^{-1} compared with *the critical curves found by* CHARNEY (1947) and KUO (1952). The critical curve for a temperature lapse rate of 8.2 K km^{-1} is shown by the thin line.

not possible to conclude which frequency equation is the more accurate description of the real atmosphere, but this is a matter of little importance because other simplifications in the models and in the mathematical treatment probably exert greater influences on the final accuracy. One need only mention the assumption of horizontally uniform unchanging mean flow with infinitesimal perturbations, an assumption which is never fulfilled by the atmosphere.

In order to gain insight into the short-wave stability found by FJØRTOFT (1950), BERSON (1951), and THOMPSON (1953), the stream-surface may be eliminated from (22) and (23) by integrating (30) from sea level to the surface of greatest slope. If the height of this surface is chosen independently of wavelength, short waves are markedly stable. Similar arbitrary choices of characteristic heights were made by Fjørtoft, Berson, and Thompson with very similar results. This comparison indicates that the short-wave stability arose from the assumption that a characteristic height was independent of wavelength; the stability probably is not inherent in the real atmosphere.

9. The Rate of Growth

Fig. 2 indicates that the amplitude of disturbances cannot grow at a rate exceeding a factor of from one to three per day for the vertical shear and hydrostatic stability commonly observed. It is difficult, therefore, to believe that major disturbances ever develop by the process described here from very small more or less random disturbances; rather, the disturbance in its initial stage must be only about an order of magnitude less than in its fully developed stage. The initial disturbance presumably could be created by topography, by heating, or by the effects of dispersion acting on disturbances in a remote part of the zonal current. In any case, the fairly deliberate growth of these unstable disturbances makes prediction of their amplitude and movement a practicable possibility. This must be carried out, of course, by numerical integration of nonlinear equations. However, insights gained from this study of linear equations may be of value in the selection of nonlinear atmospheric models.

10. Effect of the Spherical Shape of the Earth

Equation (35) shows that for any combination of baroclinity and wavelength, instability increases with latitude. In Fig. 3 the critical curves are illustrated for several baroclinities.

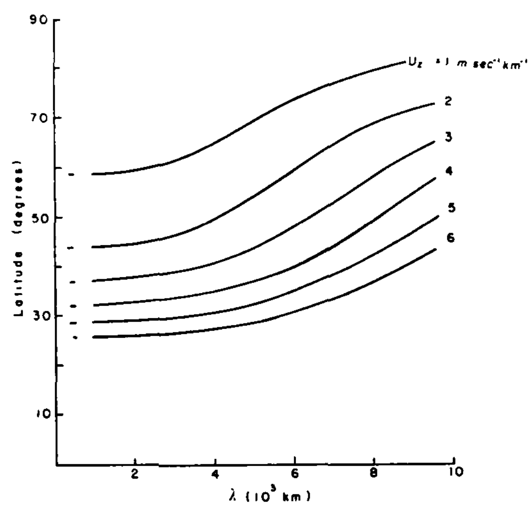

Fig. 3. Critical latitude south of which disturbances are stable for a temperature lapse rate of 6.5 K km^{-1} ((computed from (35)).

Instability is possible for reasonable baroclinities only north of about 30 degrees. This is consistent with the observation that large scale disturbances are characteristic of middle and high latitudes and do not penetrate to low latitudes.

On a flat earth, f is constant and β vanishes. Equation (34) shows that disturbances characteristic of observations are still possible, so long as the atmosphere is baroclinic. These considerations make understandable the observations of long waves in a rotating dishpan, described by Long.[2] The generation of the waves in this case is to be attributed to the baroclinity rather than to conservation of potential vorticity.

11. Conclusions and Inferences

The frequency equation for long baroclinic disturbances has been expressed in very simple form (equations 34 and 35). The results are in substantial agreement with those of KUO (1952), but the simpler form derived here makes it easy to determine the dependence of the wave speed and the rate of growth of unstable disturbances on baroclinity, hydrostatic stability, wavelength, and latitude.

Equations (34) and (35) show that disturbances of about four thousand kilometer length may be expected to develop in middle latitudes where the baroclinity reaches 3 to 4 m $\sec^{-1}$ km^{-1}. On the other hand instability is not to be expected south of about 30° latitude. It follows that a relatively steady large scale circulation controlled primarily by differential heating and conservation of angular momentum is more likely in the tropics than at higher latitude.

The results suggest a qualitative insight into the problem of the meridional distribution of zonal wind speed. Although (35) indicates that greatest instability should occur at the pole, it is intuitively plausible that the atmosphere selects the mode of instability which permits the greatest conversion of potential to kinetic energy. This would occur sufficiently far from the pole that a substantial part of the atmosphere could take part in the instability process but still north of the critical latitude. Viewed in this way, the maximum zonal speed observed in middle latitudes is a consequence of the conversion of potential energy of the baroclinic atmosphere to kinetic energy. It may be analogous to the localized vertical convection currents associated with hydrostatic instability.

[2] LONG, R. R., 1951: Research on experimental hydrodynamics in relation to large-scale meteorological phenomena. Prog. Rep. No. 5, 55 pp.

Appendix

List of Symbols:

g: acceleration of gravity
δ: slope of the streamsurface
ϕ: latitude
Ω: angular velocity of earth
U: undisturbed velocity component in x direction
u, v, w: perturbation velocity components in x, y and z directions, respectively
f: $2\,\Omega \sin(\phi - \delta)$ and $2\,\Omega \sin \phi$
t: time
Q, q: undisturbed and perturbation densities, respectively
P, p: undisturbed and perturbation pressures, respectively
j, k: unit vectors in y and z directions, respectively
A,B,C,D,F: amplitudes of u, v, w, p, q, perturbations, respectively
Subscripts: partial differentiation with respect to that variable
γ: coefficient of piezotropy
Γ: $-(g\mathrm{Q})^{-1}\,\mathrm{Q}_z \sec\delta$ and $-(g\mathrm{Q})^{-1}\,\mathrm{Q}_z$
L: operator, $(\partial/\partial t + U\,\partial/\partial x)$
β: $\partial f/\partial y$
α: wave number
λ: wavelength
c: complex wave speed
c_r: real part of wave speed of unstable wave
ic_i: imaginary part of wave speed
s_z: hydrostatic stability

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