Numerical evaluation of the scatter of principal magnetic susceptibilities from the uncertainties in experimental measurements

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ABSTRACT

Explicit equations are derived which specify the total eigenvector uncertainties for a tensor of second rank in terms of the matrix of experimental measurements and the estimated measurement errors appropriate to the experiments. Only the minimum but sufficient number of measurements necessary to specify the representative ellipsoid are considered in this analysis. The specific application in mind is the measurement of magnetic anisotropy of rocks, and the calculation of the statistical scatter of the total susceptibility ellipsoids. The result of the calculations provide direct proof that the errors in the triad of principal axes calculated from a meaned set of measurements for a sample should be less than the errors in the triad of axes calculated from a single measurement of the sample. This provides the investigator with the option of using a formal statistical approach to the analysis of anisotropy data based directly upon the raw measurements and estimates of instrumental accuracy, rather than upon the scatter of spatial orientations of the total susceptibility ellipsoids estimated from the measurements. The occasionally poor agreement of a set of individual specimen measurements with matrix theory is likely due to uncorrelated errors in the raw measurements and not necessarily to operational error, instrumental defects, or the specimen itself.

1. Introduction

The consideration of errors for the magnetic susceptibility tensor is more complicated than for the magnetic remanence vector problem, although the instrumental errors of measurement should be nearly the same in both cases. The values given in the Appendix of this paper to represent estimates of errors in magnetic anisotropy measurements are based upon the results obtained from the anisotropy and remanence statistics of a spinner magnetometer (Noltimier, 1967*a*, *b*) which showed that if the positioning of the rock cylinders in the spinner specimen holder was accurate, the overall results were consistent with the known uncertainties in calibration and applied field. Any error of sample orientation angle is doubled in the anisotropy signal phase output of a spinner magnetometer, and errors in alignment of the reference signals with the specimen axes can introduce troublesome systematic error (Noltimier, 1971*a*).

An estimate can be made of the errors in all of the measured quantities which determine the total susceptibility ellipsoid, represented by the $[k_{ij}]$ matrix. This includes the error in the measurement of the mean total susceptibility, $\langle k \rangle$, on an a.c. total susceptibility bridge (Collinson, Stone & Molyneux, 1963), and its influence upon the values of k_{11}, k_{22}, and k_{33}. Knowing these errrors, they are transformed into the uncertainties in the magnitude and directions of the principal magnetic susceptibilities. The principal susceptibilities must always be mutually orthogonal, regardless of errors. A spheroid of uncertainty of direction and magnitude about one principal axis implies related uncertainties in the position and magnitude of the remaining two axes.

The purpose of this analysis was to check the validity of using a meaned susceptibility tensor for a set of rock cylinders calculated from the mean of the corresponding tensor measurements, k_{ij}, and not from the mean of the calculated principal axes as done by Stone (1967). The error analysis discussed by Granar (1958), and referred to by King (1967), for magnetic susceptibility ellipsoids had torsion balance measurement in mind and involved a scheme of measurement which overdetermines the ellipsoid matrix components. Hext (1962) also discusses a statistical analysis which involves an abundance of measurements. In this analysis, only the minimum but sufficient number of measurements necessary to determine the ellipsoid are considered.

2. Numerical solution to the eigenvector scatter problem

A few conventions will be stated here for future reference. They are:

$[k_{ij}]$ = symmetric matrix of presumed true total susceptibility values;
$[K_{ij}]$ = matrix of cofactors of the k_{ij};
$[\delta k_{ij}]$ = uncertainties in k_{ij}, assumed known, small, and uncorrelated;
$[\delta K_{ij}]$ = uncertainties in the cofactors of the k_{ij};
$[s_{ij}]$ = $[k_{ij}+\delta k_{ij}]$, matrix of measurements, the best estimate of k_{ij};
(λ) = eigenvalues of k_{ij};
(λ') = $(\lambda+\delta\lambda)$, eigenvalues of s_{ij}.

We wish to solve the equation $s_{ij}X_j=\lambda' X_i$ for λ' and X_k, the three eigenvectors corresponding to the three roots for λ', λ'_k. This equation written out is $(s_{ij}-\lambda' I)X_j=0$, where I is the unitary matrix. The determinant of the coefficients vanishes, giving us the following cubic equation in λ', with coefficients a_i',

$$(\lambda')^3-a_1'(\lambda')^2+a_2'(\lambda')-a_3'=0 \qquad (1)$$

where $a_1'=(a_1+\delta a_1)$, $a_2'=(a_2+\delta a_2)$, and $a_3'=(a_3+\delta a_3)$. The δa_i are the uncertainties in the cubic coefficients due to the estimated uncertainties in the tensor elements s_{ij}, measured directly, or calculated directly from measurements. The s_{ij} are the best estimate of the true k_{ij}, and will be used as k_{ij} in the numerical evaluations which result from the analysis, along with the estimated uncertainties, δk_{ij}. The coefficients, a_j, and their uncertainties, δa_i, are functions of k_{ij} and δk_{ij} as follows:

$$a_1=\text{Trace}\,[k_{ij}]=\sum_{i=1}^{3}k_{ii}$$

$$\delta a_1=\text{Trace}\,[\delta k_{ij}]=\sum_{i=1}^{3}\delta k_{ii}$$

$$a_2=\text{Trace}\,[K_{ij}]=\sum_{i=1}^{3}K_{ii}$$

$$\delta a_2=\text{Trace}\,[\delta K_{ij}]=\sum_{i=1}^{3}\delta K_{ii}$$

$$a_3=|k_{ij}|=\sum_{i=1}^{3}k_{ji}K_{ji}$$

$$\delta a_3=\delta|k_{ij}|=\sum_{i=1}^{3}\delta(k_{ji}K_{ji}) \qquad (2)$$

The Trace (German "Spur") of the above square matrices is the sum of their diagonal elements, as indicated. While the meaning of the preceding equations in (2) for a_1, δa_1, a_2, and a_3 should be clear, the expressions for δa_2 and δa_3 are given below to prevent any confusion with notation.

$$\delta a_2=\left[\begin{vmatrix}\delta k_{22} & \delta k_{23}\\ k_{32} & k_{33}\end{vmatrix}+\begin{vmatrix}k_{22} & k_{23}\\ \delta k_{32} & \delta k_{33}\end{vmatrix}\right]+\left[\begin{vmatrix}\delta k_{11} & \delta k_{13}\\ k_{31} & k_{33}\end{vmatrix}+\begin{vmatrix}k_{11} & k_{13}\\ \delta k_{31} & \delta k_{33}\end{vmatrix}\right]+\left[\begin{vmatrix}\delta k_{11} & \delta k_{12}\\ k_{21} & k_{22}\end{vmatrix}+\begin{vmatrix}k_{11} & k_{12}\\ \delta k_{21} & \delta k_{22}\end{vmatrix}\right] \qquad (3)$$

$$\begin{aligned}\delta a_3&=\delta(k_{11}K_{11}+k_{12}K_{12}+k_{13}K_{13})\\&=(k_{11}\delta K_{11}+\delta k_{11}K_{11})+(k_{12}\delta K_{12}+\delta k_{12}K_{12})+(k_{13}\delta K_{13}+\delta k_{13}K_{13})\\&=k_{11}\left[\begin{vmatrix}\delta k_{22} & \delta k_{23}\\ k_{32} & k_{33}\end{vmatrix}+\begin{vmatrix}k_{22} & k_{23}\\ \delta k_{32} & \delta k_{33}\end{vmatrix}\right]-k_{12}\left[\begin{vmatrix}\delta k_{21} & \delta k_{23}\\ k_{31} & k_{33}\end{vmatrix}+\begin{vmatrix}k_{21} & k_{23}\\ \delta k_{31} & \delta k_{33}\end{vmatrix}\right]\\&\quad+k_{13}\left[\begin{vmatrix}\delta k_{21} & \delta k_{22}\\ k_{31} & k_{32}\end{vmatrix}+\begin{vmatrix}k_{21} & k_{22}\\ \delta k_{31} & \delta k_{32}\end{vmatrix}\right]+\delta k_{11}K_{11}+\delta k_{12}K_{12}+\delta k_{13}K_{13}\end{aligned} \qquad (4)$$

The solution to Eq. (1), with the above expressions (Eq. (2), (3), and (4) for the uncertainties in the coefficients, a_1', a_2', a_3', produces three roots with associated uncertainties, $(\lambda+\delta\lambda)_k = \lambda'_k$, $k=1, 2, 3$.

3. The problem without uncertainties in measurement

We must solve the determinent $(k_{ij}-\lambda I)X_j = 0$, and in doing so we get a cubic equation in λ (like Eq. (1) for λ'), which has roots λ_k, $k=1, 2, 3$. Because the determinent is homogeneous (equals zero), the components of the eigenvectors corresponding to the kth eigenvalue, λ_k, are undetermined to an arbitrary constant, but their ratio may be obtained for each λ_k by the relation,

$$X_j = NR_{ij} \tag{5}$$

where R_{ij} are the cofactors of the r_{ij} which are given by the symmetric matrix,

$$r_{ij} = \begin{bmatrix} (k_{11}-\lambda) & k_{12} & k_{13} \\ k_{12} & (k_{22}-\lambda) & k_{23} \\ k_{13} & k_{23} & (k_{33}-\lambda) \end{bmatrix} \tag{6}$$

The value of N for each λ is determined by the relationship $X_iX_j=1$, the Schmidt process, which orthogonalizes and normalizes the X_i, fixing the undetermined constant in Eq. (5). This constant, N, might be labeled N_k since a value of N corresponds to each of the eigenvalues λ_k as determined by the solution to the cubic in λ. Eq. (5) for the X_j may be written out as follows, using Eq. (6) to indicate the cofactors R_{ij};

$$X_1 = NR_{11} = N\begin{vmatrix} (k_{22}-\lambda) & k_{23} \\ k_{23} & (k_{33}-\lambda) \end{vmatrix} = N[K_{11}-\lambda(k_{22}+k_{33})+\lambda^2] \tag{7}$$

$$X_2 = NR_{12} = (-N)\begin{vmatrix} k_{12} & k_{23} \\ k_{13} & (k_{33}-\lambda) \end{vmatrix} = N[K_{12}+k_{12}\lambda] \tag{8}$$

$$X_3 = NR_{13} = N\begin{vmatrix} k_{12} & (k_{22}-\lambda) \\ k_{13} & k_{23} \end{vmatrix} = N[K_{13}+k_{13}\lambda] \tag{9}$$

Since $X_iX_j=1$, a convenient equation for N is $(1/N^2) = (R_{11}^2+R_{12}^2+R_{13}^2)$. Using Eqs. (7), (8), and (9), this expression for $(1/N^2)$ becomes,

$$(1/N^2) = [K_{11}-\lambda(k_{22}+k_{33})+\lambda^2]^2 + [K_{12}+k_{12}\lambda]^2 + [K_{13}+k_{13}\lambda]^2 \tag{10}$$

4. The problem with uncertainties in measurement

The uncertainties in k_{ij} give rise to uncertainty in λ, N, and X_j. Thus, λ becomes $(\lambda+\delta\lambda)$, N becomes $(N+\delta N)$, and X_i becomes $(X_j+\delta X_j)$. For example, Eq. (5) may be used to express $(X_j+\delta X_j)$ and to derive δX_j.

$$(X_j+\delta X_j) = (N+\delta N)\,(R_{ij}+\delta R_{ij}) = X_j + N\delta R_{ij} + \delta N R_{ij} + \delta N\delta R_{ij} \tag{11}$$

Ignoring the products in $\delta N\delta R_{ij}$ which will all be of second order,

$$\delta X_j = (\delta N/N)\,X_j + N\,\delta R_{ij} \tag{12}$$

Eqs. (7), (8), and (9) are available for the calculation of δR_{11}, δR_{12}, and δR_{13}. Before writing out the complete expressions for δX_j, $(\delta N/N)$ and $\delta\lambda$ must be known.

$\delta\lambda$ may be obtained by differentiating Eq. (1), expressing λ' in terms of λ and $\delta\lambda$. Thus, from (1),

$$3\lambda^2\delta\lambda - \lambda^2\delta a_1 - 2a_1,\ \lambda\delta\lambda + a_2\delta\lambda + \lambda\delta a_2 - \delta a_3 = 0,$$

$$\delta\lambda = (a_1,\ \lambda^2\delta a_1 - \lambda\delta a_2 + \delta a_3)/(3\lambda^2 - 2a_1\ \lambda + a_2) \tag{13}$$

$(\delta N/N)$ may be obtained from the relation that $(1/N^2) = R_{11}^2 + R_{12}^2 + R_{13}^2$.

$$\delta(1/N^2) = -2\delta N/N^3 = 2R_{11}\delta R_{11} + 2R_{12}\delta R_{12} + 2R_{13}\delta R_{13} \tag{14}$$

Therefore,

$$(\delta N/N) = -N^2(R_{11}\delta R_{11} + R_{12}\delta R_{12} + R_{13}\delta R_{13}) = -(R_{11}\delta R_{11} + R_{12}\delta R_{12} + R_{13}\delta R_{13})/(R_{11}^2+R_{12}^2+R_{13}^2) \tag{15}$$

Referring to Eqs. (5) and (12), δX_j may be written down, remembering that $N = 1/R_{11}^2 + R_{12}^2 + R_{13}^2)^{1/2}$, in this example.

$$\delta X_1 = -(R_{11}\,\delta R_{11} + R_{12}\,\delta R_{12} + R_{13}\,\delta R_{13})\,X_1/(R_{11}^2+R_{12}^2+R_{13}^2) + \delta R_{11}/(R_{11}^2+R_{12}^2+R_{13}^2)^{\frac{1}{2}}$$

$$= [-(R_{11}\delta R_{11} + R_{12}\delta R_{12} + R_{13}\delta R_{13})R_{11} + \delta R_{11}]/(R_{11}^2 + R_{12}^2 + R_{13}^2)^{\frac{1}{2}} \quad (16)$$

and thus

$$\delta X_2 = [-(R_{11}\delta R_{11} + R_{12}\delta R_{12} + R_{13}\delta R_{13}) \times R_{12} + \delta R_{12}]/(R_{11}^2 + R_{12}^2 + R_{13}^2)^{\frac{1}{2}}$$

$$\delta X_3 = [-(R_{11}\delta R_{11} + R_{12}\delta R_{12} + R_{13}\delta R_{13}) \times R_{13} + \delta R_{13}]/(R_{11}^2 + R_{12}^2 + R_{12}^2)^{\frac{1}{2}}$$

Knowing the three R_{ij}, (R_{11}, R_{12}, R_{13}), and the corresponding δR_{ij}, we may numerically evaluate the δX_j for each eigenvalue, λ_k. If δX_k represents the total uncertainty in the X_kth eigenvector.

$$\delta X_k = \delta X_1 + \delta X_2 + \delta X_3 \quad (17)$$

the δX_j being given by Eq. (16). The magnitude of δX_k is given by

$$|\delta X_k|^2 = \delta X_1 \delta X_1 + \delta X_2 \times \delta X_2 + \delta X_3 \delta X_3 \quad (18)$$

The mean scatter of the three eigenvectors, X_k, is $\langle |\delta X|^2 \rangle$, and may be obtained directly from Eq. (18) if all of the δX_j's are known numerically for each λ_k. The general form for the mean scatter is

$$\langle |\delta X|^2 \rangle = C_{ij\,kl}\, \delta k_{ij}\, \delta k_{kl} = Q_{ik} \langle |\delta k_{ik}|^2 \rangle \quad (19)$$

Where the Q_{ik} are functions of k_{ij}, and the δk_{ik}'s are assumed uncorrelated. Terms of the form $\delta k_{ij}\, \delta k_{kl}$ average to zero. While the Q_{ik} can be written down, and they are complex, the Eq. (16) are sufficient to numerically evaluate the δX_j and $\langle |\delta X|^2 \rangle$.

5. Review of the formulae determining R_{ij} and δR_{ij}

In order to specify the mean scatter of the eigenvectors, X_k, we must have the uncertainties, δX_j, for each X_k. These uncertainties are expressed by Eqs. (16), in terms of the cofactors and uncertainties in the cofactors of the r_{ij} matrix as given by Eq. (6). The R_{ij} are apparent in Eqs. (7), (8), and (9), and the δR_{ij} are obtained in a manner similar to δK_{ij}, as seen in Eqs. (3) and (4). For sake of clarity, the R_{ij} and δR_{ij} are summarized as follows. The R_{ij} are;

$$R_{11} = \begin{vmatrix} (k_{22}-\lambda) & k_{23} \\ k_{23} & (k_{33}-\lambda) \end{vmatrix} = K_{11} - \lambda(k_{22}+k_{33}) + \lambda^2$$

$$R_{12} = -\begin{vmatrix} k_{12} & k_{23} \\ k_{13} & (k_{33}-\lambda) \end{vmatrix} = K_{12} + k_{12}\lambda$$

$$R_{13} = \begin{vmatrix} k_{12} & (k_{22}-\lambda) \\ k_{13} & k_{23} \end{vmatrix} = K_{13} + k_{13}\lambda \quad (20)$$

The δR_{ij} are;

$$\delta R_{11} = \delta[K_{11} - \lambda(k_{22}+k_{23}) + \lambda^2] = \begin{vmatrix} \delta k_{22} & \delta k_{23} \\ k_{32} & k_{33} \end{vmatrix} + \begin{vmatrix} k_{22} & k_{23} \\ \delta k_{32} & \delta k_{33} \end{vmatrix} - \lambda(\delta k_{22} + \delta k_{33}) - \delta\lambda(k_{22}+k_{33}) + 2\lambda\delta\lambda,$$

$$\delta R_{12} = \delta[K_{12} + k_{12}\lambda] = -\begin{vmatrix} \delta k_{21} & \delta k_{23} \\ k_{31} & k_{33} \end{vmatrix} - \begin{vmatrix} k_{21} & k_{23} \\ \delta k_{31} & \delta k_{33} \end{vmatrix} + \lambda\delta k_{12} + \delta\lambda k_{12},$$

$$\delta R_{13} = \delta[K_{13} + k_{13}\lambda] = \begin{vmatrix} \delta k_{21} & \delta k_{22} \\ k_{31} & k_{32} \end{vmatrix} - \begin{vmatrix} k_{21} & k_{22} \\ \delta k_{31} & \delta k_{32} \end{vmatrix} + \lambda\delta k_{13} + \delta\lambda k_{13} \quad (21)$$

Eqs. (20) involve the $s_{ij} \simeq k_{ij}$ known from measurements, and $\lambda_k' \simeq \lambda_k$ obtained directly from the solution of $|s_{ij} - \lambda' I| X_j = 0$. Eqs. (21) involve the k_{ij} and δk_{ij}, known from measurement and the estimated measurement uncertainties, the λ_k as before, and $\delta\lambda_k$, which is given by Eq. (13). The coefficients of Eq. (13), the a_i, and their uncertainties, the δa_i, are given in terms of the k_{ij} and δk_{ij} in Eqs. (2), (3), and (4). Therefore, in order to numerically specify the δX_j, we must have a solution for the eigenvalues of the s_{ij} matrix, the λ_k, as well as the coefficients and their uncertainties (a_i and δa_i) for the cubic in λ. The basis of all these calculations is knowledge of the experimentally determined k_{ij} and δk_{ij}.

A suitable computer program would seem the best way to handle the calculations, and the input data will be the $[k_{ij}]$ and $[\delta k_{ij}]$ matrices. The output would presumably be the eigenvectors (principal axes), their direction cosines, the δX_j, and the scatter, $\langle |\delta X|^2 \rangle$.

6. Conclusion

The obvious conclusion is that the scatter of the eigenvectors of a second rank tensor property may be expressed numerically as functions of the errors of the individual measurements. Given the results of a single set of measurements for a specimen, the scatter of each eigenvector may be determined, and the mean scatter of all three eigenvectors may also be specified. If a set of repeat measurements is made on a single specimen and the results, $s_{ij} \cong k_{ij}$, meaned, the magnitude of the uncertainties in the meaned measurements, $\langle k_{ij} \rangle$, will be smaller than the uncertainties in the individual measurements. Hence, there will be a smaller scatter of the eigenvectors calculated from the meaned measurements. What has been attempted here is to justify using the set of meaned measurements for a set of similar rock specimens, and then to calculate the eigenvectors for the population of similar measurements directly from the meaned values, rather than averaging the eigenvectors calculated individually from each sample (Noltimier, 1965).

A test of the accuracy of a discreet set of measurements for one specimen is how closely the sum of the diagonal terms (Trace) of the raw data matrix approaches zero. (This also applies to torsion balance measurements.) In most cases tested by the author (over 1 000 anisotropy calculations) the meaned spinner raw data matrices for related specimens gave a Trace much nearer the theoretical value (zero) than the raw results for an individual set of measurements. (See Appendix, Eq. (22).)

The eigenvectors calculated from the meaning procedures described here are mutually perpendicular, and the spheroids of uncertainty about the terminii of each eigenvector, X_k, defined by the δX_j, do not imply that each eigenvector may wander at will throughout their volumes, independently of the other eigenvectors. The results also indicate that the occasional poor agreement of a set of individual specimen measurements with theoretical matrix behavior is likely due to uncorrelated scatter of the raw measurements and not necessarily due to operational errors, or instrumental defects.

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Appendix

An example of the estimation of the measurement uncertainties for magnetic susceptibility tensors determined with a spinner magnetometer and an a.c. total susceptibility bridge.

The spinner magnetometer modified for anisotropy measurement gives an output data matrix of the following form, defined as Δk_{ij}:

$$\begin{bmatrix} (k_{11}-k_{22}) & k_{12} & k_{13} \\ k_{12} & (k_{22}-k_{33}) & k_{23} \\ k_{13} & k_{23} & (k_{33}-k_{11}) \end{bmatrix} = \Delta k_{ij} \quad (22)$$

The matrix elements, $(k_{ii}-k_{jj})$ and k_{ij}, obtained from spinner anisotropy measurements are of the form,

$$(k_{ii}-k_{jj}) = (2|E|/vH)\cos 2\phi$$
$$k_{ij} = (|E|/vH \sin 2\phi \quad (23)$$

(Noltimier, 1967b). $\langle k \rangle$, the mean specimen total susceptibility, is measured separately on an a.c. bridge, and using the Δk_{ij} diagonal terms, $(k_{ii}-k_{jj})$, with $\langle k \rangle$ gives the total susceptibility matrix of measurements, s_{ij}, which may then be written down. $|E|$ is the spinner signal magnitude, v the specimen volume, H the applied magnetic field in oersteds, and 2ϕ is the signal output phase angle. The uncertainties of measurement are the uncertainties in $|E|$, v, H, $\langle k \rangle$, and 2ϕ.

(1) The basic amplitude calibration should be good to about 2 %, and this error is ignored in this analysis since it is systematic. The instrumental noise level is taken as $\pm 0.3 \times 10^{-6}$ gauss cm³.

(2) The error in volume, v, is determined by the errors in drilling and slicing the rock cylinders if a constant volume is assumed for each specimen. If each cylinder is measured directly for height and diameter, this error is probably negligible. The cylinder height to diameter ratio, (h/a), may or may not introduce a systematic shape effect in the results, depending upon the rocks themselves, but if the (h/a) ratio of $(3/4)^{1/2} = 0.865$ is used, this effect will be minimised (Noltimier, 1971b). The nominal value of v in this example is 5 cm³.

(3) If $|E|$ is greater than three times the rms noise level, the error in the signal phase angle is estimated to be 5° in reading the meter, and 5° due to the noise fluctuations, or 7° (0.122 radians) rms phase error in 2ϕ.

(4) The error in H is determined mainly by the uncertainties in regulatine the Helmholtz coil current, and should be known to within ± 0.5 oersteds.

(5) The error in $\langle k \rangle$ is $\pm 0.5 \times 10^{-6}$ gauss cm³, using a total susceptibility bridge as described by Collinson, Stone & Molyneux (1963).

The total error in each term, Δk_{ij}, may be expressed as,

$$(\delta \Delta k_{ij})^2 = (\partial \Delta k_{ij}/\partial E)^2 \, \Delta E^2 + (\partial \Delta k_{ij}/\partial H)^2 \Delta H^2 + (\partial \Delta k_{ij}/\partial \phi)^2 \, \Delta(2\phi)^2 \quad (24)$$

Using Eqs. (22), noting that Δk_{ij} is symmetric, using $\langle k \rangle$ to determine k_{ij} from Δk_{ii}, and using the effective range of 2ϕ as π radians, the rms uncertainties in the k_{ii} are $\pm 0.6 \times 10^{-6}$ gauss cm³. The rms uncertainties in the non-diagonal k_{ij} are $\pm 0.4 \times 10^{-6}$ gauss cm³. These values compare favorably with the basic instrumental noise level of $\pm 0.3 \times 10^{-6}$ gauss cm³, and indicate that the error in measuring the mean total susceptibility is very significant in determining the total rms uncertainties in the diagonal terms of the total susceptibility matrix, $[k_{ij}]$. In general, it would appear that the uncertainties will be greatest for the diagonal terms, and the uncertainties in all of the k_{ij} will be somewhat greater than the nosie level of the spinner magnetometer itself. The uncertainties in the principal axes, X_k, will be greater still.

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ЧИСЛЕННАЯ ОЦЕНКА РАЗБРОСА ДАННЫХ ОПРЕДЕЛЕНИЯ ГЛАВНЫХ КОМПОНЕНТ МАГНИТНОЙ ПРОНИЦАЕМОСТИ ПО НЕОПРЕДЕЛЕННОСТЯМ ИХ ЭКСПЕРИМЕНТАЛЬНЫХ ИЗМЕРЕНИЙ

Выводятся уравнения, определяющие полную величину ошибок собственного вектора для тензора второго ранга с помощью матрицы данных экспериментальных измерений и соответствующих им оценок ошибок этих измерений. При этом рассматривается лишь минимальное число измерений, которое, однако, является достаточным для определения характерного эллипсоида. Специальной областью применений являются измерения магнитной анизотропии скальных пород и вычисления статистического рассеяния результатов измерений полных эллипсоидов магнитной проницаемости. Результаты вычислений дают прямое доказательство тому, что ошибки в определении положения тройки главных осей для некоторого образца, найденные из осредненного набора измерений, должны быть меньше ошибок определения положений главных осей лишь при одном

измерении. Это дает исследователю возможность использования формального статистического подхода к анализу данных по магнитной анизотропии, основанного непосредственно на материале первичных наблюдений и оценках инструментальной точности, что предпочтительнее анализа рассеяния пространственных ориентаций полных эллипсоидов проницаемости, полученных из измерений. Встречающееся иногда плохое совпадение ряда индивидуальных измерений с результатами матричной теории, наиболее вероятно, происходит благодаря некоррелированным ошибкам в первичных измерениях, а не обязательно благодаря ошибкам вычислений, приборным дефектам или особенностям самого образца.