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Torques exerted by a shallow fluid on a non-spherical, rotating planet

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ABSTRACT

A general derivation is given for the torque exerted on a non-spherical planet by a shallow fluid, presenting in a unified form a number of results which are currently scattered through the literature, and deriving those results in a new way. This shows how the sum of gravitational and pressure torques can be rewritten as a 'centrifugal' torque plus a topographic torque due to pressure acting on topography measured relative to the geoid. This clarifies the physics behind the use of spherical coordinate atmosphere and ocean models for calculating torques on the earth. It also shows why the total torque due to the earth's equatorial bulge can be calculated as if it were a pressure torque on an 'effective bulge' of approximately 11 km, the gravitational torque partially offsetting the actual pressure torque on the earth's 21 km bulge.

1. Introduction

The interaction between the atmosphere–ocean system and the rotation of the solid earth is a subject which is commanding increasing interest. In particular, more attention is now being devoted to motion of the earth's pole of rotation. This results from changes in the equatorial components of atmospheric and oceanic angular momentum, as opposed to changes in length of day which are due to changes in the axial angular momentum component. Recent studies include Bell (1994), Dehant et al. (1996), Marcus et al. (1998), Gegout et al. (1998), Egger and Hoinka (1999, 2000), Gross (2000), and de Viron et al. (2001).

Anyone coming new to this subject encounters an apparent paradox. The atmospheric and oceanic models used are written in spherical coordinates, whilst the most significant deviation of the earth from a sphere, the equatorial bulge, is not represented in the models. Yet the models do produce apparently good estimates of angular momentum variations. Delving deeper into the literature, one finds calculations of the torque on the earth's bulge using an apparent bulge (difference between equatorial and polar radii) of about 11 km, while the actual equatorial bulge of the earth is close to 21 km.

The solution to this conundrum can be found in the literature, but it is rather scattered. For example, Bell et al. (1991) show briefly (in their Appendix C) how the pressure torque on an equatorial bulge of height $\Omega^2 r_0^2/(2g_0) \approx 11$ km is accurately described by a spherical coordinate model, assuming the earth's surface is a geopotential, and that the gravitational field of the earth is spherically symmetrical (here, Ω is the angular rate of rotation of the earth about its axis, r_0 is a mean earth radius, and g_0 is the mean strength of surface gravity). Bell (1994) generalises this result to the case in which the earth's gravitational field is not spherically symmetrical (this implicitly explains the difference between the 21 and 11 km bulges, although that is not described explicitly). Egger and Hoinka (1999) again assume a bulge of 11 km, and ignore gravitational torques and

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other topography. Dehant et al. (1996) show that the torque due to the atmospheric S_1 tide has the potential to excite significant prograde annual nutation, but ignore the associated gravitational torque.

Probably the most general derivation is due to Wahr (1982, Section 5.1), who includes gravitational and pressure torques and shows how the two combine and partly cancel. Wahr also includes topography relative to the geoid, but his representation of the torques on this topography is rather obscure, and hard to interpret in terms of atmosphere or ocean model dynamics. Gegout et al. (1998) refine the work of Dehant et al. (1996) by using a more recent atmospheric model, and by recognising the importance of the gravitational torque. They quote Wahr (1982) to derive a formula [their eq. (9)] for the pressure plus gravitational torque, which is equivalent to eq. (14) below, valid for a near-spherical geoid.

All of these results are correct and self-consistent, but it can be difficult to work out which approximations have been made in which case, and which are valid. For example, in Bell (1994), the torque on the atmosphere due to the earth's bulge is described as a 'mountain torque' or 'pressure torque', whereas the formula given in that paper [Bell's eq. (16)] is for the total torque due to both pressure and gravitation (the difference between pressure torque and total torque being the difference between a 21 km and an 11 km bulge).

The purpose of this note is to give a clear, physically motivated derivation of the most general relevant case, that of a shallow fluid on a rotating planet of arbitrary geometry. Specialization to a near-spherical planet is then easy and shows clearly the approximations which have been made. This then shows why the total torque due to the earth's bulge looks like a pressure torque on an 11 km bulge, rather than on the actual 21 km bulge.

In order to make the derivation clear, it is worth describing the terminology in more detail, since some of this is unfamiliar to most atmosphere and ocean modellers. The term *gravitation* ($\mathbf{g}'$) represents the gravitational attraction of the planet due to its mass distribution. This is distinct from *gravity* ($\mathbf{g}$) which is the sum of gravitation and the centrifugal acceleration due to the planet's rotation (the rotation assumed to be effectively constant). Gravity, gravitation, and centrifugal acceleration may each be written as (minus) the gradient of a potential:

$$\mathbf{g}' = -\nabla\Phi', \tag{1}$$

$$\mathbf{g} = -\nabla\Phi, \tag{2}$$

$$\mathbf{g} - \mathbf{g}' = -\nabla c, \tag{3}$$

where $c = -(\Omega^2 r^2 \cos^2\phi)/2$, with r the radius measured from the earth's centre of mass, and ϕ the geocentric latitude (angle measured at the earth's centre, between the point considered and the equator). *Geopotential surfaces* are surfaces of constant Φ, with the *geoid* defined as the geopotential surface closest to mean sea level. *Gravitational potential surfaces* are surfaces of constant $\Phi' = \Phi - c$.

In addition to the above terms, which are conventional, a distinction will be made between *hydrostatic balance*, and *hydrostatic equilibrium*. *Hydrostatic balance* is taken to mean a balance in the vertical between pressure gradient and force due to gravity: $\partial p/\partial\Phi = \rho$, where ρ is density and the partial derivative is along the vertical direction (parallel to $\mathbf{g}$). *Hydrostatic equilibrium* is used to describe a situation in which the fluid is in a state of rest relative to the rotating planet, with pressure and density both purely a function of Φ. Thus, a fluid can be in hydrostatic balance locally, while not being in a state of hydrostatic equilibrium.

2. Approximations

Before proceeding to the derivation, it is useful to consider the accuracy of the various approximations to be made (although the derivation given here is for any shape of planet, numerical values for the earth will be given when assessing the accuracy of approximations). For the general result, the only approximation is that the fluid be 'shallow' in the sense that the depth of fluid is small compared to the distance from the centre of the earth. For a mean earth radius of approximately 6371 km, and an ocean depth of 5 km, this approximation is accurate to better than 0.1%. For the atmosphere the errors are larger. Although most of the mass of the atmosphere is within 10 km of the geoid, errors due to the stratosphere may be as big as 1%. The shallow fluid approximation is also necessary for hydrostatic balance to be a good approximation, although it is harder to assess the affect of this approximation as it depends on the correlation between non-hydrostatic pressure and topography at all length scales.

For comparison with spherical coordinate models, two further approximations are made. The centrifugal potential along the geoid is approximated by that on a spherical surface, incurring errors of order (21 km)/(6371 km) or about 0.3% (the numerator given by the equatorial bulge), and gravity is taken to be constant along the geoid. Taking from Moritz (1992) an equatorial gravity of about 9.780 m s^{-2}, and a polar gravity of about 9.832 m s^{-2}, this leads to an error of about 0.5%.

Compared to the earth's topography of order ± 5 km (measured relative to the geoid), the 21 km equatorial bulge is large, and the undulations of the geoid relative to a reference ellipsoid are significant at ± 100 m (2%). The potential errors due to misinterpreting the model topography as being measured relative to a spherical or spheroidal surface rather than relative to the geoid are thus larger than the errors due to approximations.

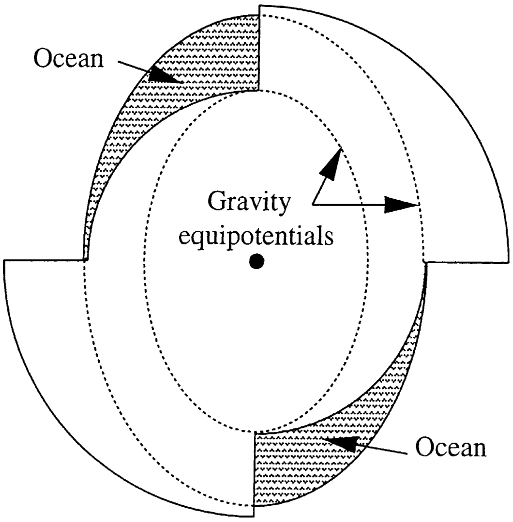

Fig. 1. A planet whose surface consists of parts of two concentric spheres joined by radial walls. Geopotential surfaces are such that the oceans are deeper on the western side than the eastern side, leading to a pressure torque on the planet.

3. Why combine pressure and gravitational torques?

There are three kinds of torque (ignoring electromagnetic torques) which a fluid can exert on a planet: a pressure torque $\mathbf{\Gamma}_p$, a gravitational torque $\mathbf{\Gamma}_g$, and a frictional torque $\mathbf{\Gamma}_f$. Some useful physical insight can be obtained by considering the torques due to a fluid in hydrostatic equilibrium. Consider the non-rotating planet illustrated (viewed from above the north pole) in Fig. 1. This planet has an ocean floor at constant radius from its centre, and a land surface at a larger constant radius, the two joined by radial walls. However, the internal mass distribution of the planet is such that the geoid is further from the centre on western sides of the ocean basins than on eastern sides, meaning that the ocean is deeper on the west than on the east, and so exerts a pressure torque on the planet towards the west. It looks as if the planet should start to rotate as a result, but there is clearly no energy source to permit this. In fact, no such acceleration will occur because the pressure torque is balanced by a gravitational torque. The distribution of mass in the oceans exerts a gravitational attraction on the planet's internal mass distribution, which must be radially asymmetrical to produce the non-spherical geoid. The resulting torque exactly balances the pressure torque. From this we can see that it is useful to consider the pressure and gravitational torques in combination.

More generally, for a fluid in hydrostatic balance, the vertical component of the pressure force exerted on the planet is balanced by the gravitational force, so the sum can exert no torque on the planet, and any torque is due to the horizontal component of the pressure force on the planet (i.e. the pressure force on topography measured relative to the geoid, which defines horizontal). For a rotating planet this is no longer true however, since hydrostatic balance is then a balance between gravity and pressure forces, not between gravitation and pressure forces. The difference, due to the centrifugal force, permits an extra torque to act.

We therefore see that hydrostatic balance strongly constrains the torques which a fluid can exert on a planet. Although the pressure torque can be large if the planet is not spherical, it will be largely balanced by the gravitational torque if the planet's surface is close to a gravitational equipotential surface. For a rotating planet, the surface is more likely to be close to a geopotential surface, in which case the pressure torque will be largely balanced by the sum of gravitational and 'centrifugal' torques. In either case, it is clear that the sum of gravitational and pressure torques will not behave in the same manner as either gravitational or pressure torque alone.

4. Torques on a non-spherical planet

The pressure, gravitational, and viscous torques on a rotating planet of any shape are given by

$$\boldsymbol{\Gamma}_p = -\oint_{\substack{\text{solid}\\\text{surface}}} p_h \mathbf{r} \times \mathrm{d}\mathbf{S}, \qquad (4)$$

$$\boldsymbol{\Gamma}_g = -\int_{\substack{\text{fluid}\\\text{volume}}} \rho \mathbf{r} \times \mathbf{g}' \,\mathrm{d}V, \qquad (5)$$

$$\boldsymbol{\Gamma}_f = -\oint_{\substack{\text{solid}\\\text{surface}}} \mathbf{r} \times \mathbf{F} \,\mathrm{d}S, \qquad (6)$$

where $\mathrm{d}\mathbf{S}$ is an area element of the planet's surface (directed along the outward normal), p_h is the pressure at that surface, and $\mathbf{F}$ is the viscous stress per unit area on the surface (perpendicular to $\mathrm{d}\mathbf{S}$). The gravitational torque on the planet is most easily derived by calculating the gravitational torque exerted on the fluid by the planet, since the two must balance.

If we assume that the fluid is shallow (the fluid is everywhere much closer to the geoid than to the planet's centre), the volume integral (5) can be separated into an area integral over the geoid of $\mathbf{r} \times$ (the vertical integral of $\rho\mathbf{g}'$), and $\mathbf{g}'$ and $\mathbf{g}$ can be taken as constants in the vertical integral. Thus, writing $m = \int_h^\infty \rho \,\mathrm{d}z$, where z is the vertical coordinate measured relative to the geoid and $z = h$ is the height of the solid surface above the geoid, hydrostatic balance leads to $p_h = mg$ where $g = |\mathbf{g}|$. Equation (5) then becomes

$$\boldsymbol{\Gamma}_p = -\oint_{\substack{\text{geoid}\\\text{surface}}} m\mathbf{r} \times \mathbf{g}' \,\mathrm{d}A$$

$$= -\oint_{\substack{\text{geoid}\\\text{surface}}} \frac{p_h}{g} \mathbf{r} \times \mathbf{g}' \,\mathrm{d}A. \qquad (7)$$

The two integrals (7) and (4) can be related by noting that the area elements $\mathrm{d}A$ on the geoid and $\mathrm{d}\mathbf{S}$ on the solid surface are related by

$$\mathrm{d}\mathbf{S} = -\left(\frac{\mathbf{g}}{g} + \nabla_g h\right) \mathrm{d}A, \qquad (8)$$

where ∇_g is the two-dimensional gradient operator along the geoid (this result is derived in the Appendix). Writing $\boldsymbol{\Gamma}_{pg} = \boldsymbol{\Gamma}_p + \boldsymbol{\Gamma}_g$, eqs. (4) and (7) then give

$$\boldsymbol{\Gamma}_{pg} = \oint_{\substack{\text{geoid}\\\text{surface}}} \frac{-p_h}{g} \mathbf{r} \times \mathbf{g}' + p_h \mathbf{r} \times \left(\frac{\mathbf{g}}{g} + \nabla_g h\right) \mathrm{d}A. \qquad (9)$$

Substituting $\mathbf{g} = \mathbf{g}' - \nabla c$ from eq. (3), a cancellation occurs leaving

$$\boldsymbol{\Gamma}_{pg} = \oint_{\substack{\text{geoid}\\\text{surface}}} p_h \mathbf{r} \times \left(\nabla_g h - \frac{\nabla c}{g}\right) \mathrm{d}A. \qquad (10)$$

The sum of pressure and gravitational torques can thus be represented as a pressure torque on the topography h measured (upwards) relative to the geoid, plus a centrifugal torque due to the imbalance between pressure and gravitational torques on the geoid. This is a general result for a shallow fluid on any shape of planet. The fluid will only be shallow if the surface of the planet does not depart far from the geoid, but the geoid need not be nearly spherical for this equation to hold.

Rather counterintuitively, this shows that the combined torque is quite insensitive to departures of the planet's surface topography from a sphere. The topographic torque is due to departures of topography from the geoid, and the remaining centrifugal torque, while not exactly the same for spherical and non-spherical geoids, differs only by a fractional amount of the same order as the ratio of geoid topography (i.e. departure of the geoid from spherical) to mean planetary radius (0.3%), or the ratio of variations in g to mean g (0.5%). This contrasts strongly with the pressure torque alone, which would be zero for a perfectly spherical planet, and with the gravitational torque alone, which would be zero for a planet with perfectly spherical gravitational equipotential surfaces (and therefore an ellipsoidal geoid with an equatorial bulge of about 11 km, for the earth's mass, radius and rotation rate).

The physics behind the centrifugal torque can be somewhat clarified by substituting $p_h = mg$ and $\nabla c = \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$, from which we may write the centrifugal torque as

$$\mathbf{\Gamma}_c = \oint_{\substack{\text{geoid}\\ \text{surface}}} -m\mathbf{r} \times [\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})] \, \mathrm{d}A. \tag{11}$$

Noting that $\mathbf{r} \times [\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})] = \mathbf{\Omega} \times [\mathbf{r} \times (\mathbf{\Omega} \times \mathbf{r})]$, and defining the 'matter' component of fluid angular momentum as

$$\mathbf{M} = \oint_{\substack{\text{geoid}\\ \text{surface}}} m\mathbf{r} \times (\mathbf{\Omega} \times \mathbf{r}) \, \mathrm{d}A, \tag{12}$$

the shallow fluid equivalent of Bell's (1994) eq. (2), we can then write

$$\mathbf{\Gamma}_c = -\mathbf{\Omega} \times \mathbf{M}, \tag{13}$$

confirming that Bell's (1994) eq. (16) holds generally (note that Bell's Γ_1 and Γ_2 represent torques on the atmosphere, and therefore take the opposite sign to $\mathbf{\Gamma}_c$ which is a torque on the earth).

The need for this extra torque can be seen by imagining a rotating planet with an ocean at rest relative to the planet. The angular momentum of the fluid is then all due to the matter term, so the angular momentum vector $\mathbf{M}$ will rotate about the rotation axis, requiring a torque $\mathbf{\Omega} \times \mathbf{M}$ to be exerted on the fluid by the planet. This torque is seen in the atmosphere (Egger and Hoinka, 2000) as a large 'torque on the equatorial bulge' due to the smaller mass of atmosphere found in mountainous regions. The torque is constant in the rotating reference frame of the earth, and therefore rotates once per day in an inertial reference frame. Its effect is therefore not a continuous increase in the earth's angular momentum, but to permit the earth to rotate about an axis which is not one of its principal axes of inertia (rather, it is a principal axis of the earth-plus-atmosphere system). A moving fluid will also have a contribution to its angular momentum from the 'wind' term, but if this is not aligned with the rotation axis it must be balanced either by topographic or frictional torques, or by a rate of change of the wind and matter terms.

5. A near-spherical planet

Turning now to the case of a near-spherical earth, we will substitute $c = -(\Omega^2 r^2 \cos^2\phi)/2$ into eq. (10). Making the order 0.5% approximations that $g = g_0$, a constant, and $r = r_0$, a constant (for values on the geoid), then $\mathbf{r} \times \nabla c \approx \mathbf{r} \times \nabla_g c$ to the same order of approximation, and eq. (10) becomes

$$\mathbf{\Gamma}_{pg} \approx \oint_{\substack{\text{geoid}\\ \text{surface}}} p_h \mathbf{r} \times \nabla_g \left(h + \frac{\Omega^2 r_0^2 \cos^2\phi}{2g_0} \right) \mathrm{d}A. \tag{14}$$

The centrifugal torque can therefore be treated in this approximation as the effect of pressure on an extra piece of topography which consists of an equatorial bulge of height $\Omega^2 r_0^2/(2g_0)$. Taking Ω and the area-averaged values of r and g for the earth from Moritz (1992) ($r_0 = 6371.008\,771\,4$ km, $g_0 = 9.797\,644\,656$ m^2 s^{-2}, $\Omega = 7.292\,115 \times 10^{-5}$ rad s^{-1}) gives an equatorial bulge height of approximately 11.014 km.

As far as the total torque is concerned, it is therefore valid to consider a model in spherical coordinates as a close approximation to a model in geopotential coordinates, as long as the centrifugal acceleration term is omitted in recognition of the fact that is is cancelled by the horizontal part of the gravitational acceleration. If there is a need to distinguish between the pressure and gravitational torques, however, the interpretation is slightly more involved. In a spherical coordinate model, interpreted literally, geopotential surfaces are spherical and the earth's figure has no equatorial bulge. This means that gravitational equipotential surfaces are prolate spheroids, and the 'torque on the equatorial bulge' is a purely gravitational torque on the earth's internal mass distribution which leads to the prolate equipotentials. In form, this gravitational torque looks like a pressure torque on an 11 km equatorial bulge. The dynamics, however, are a close approximation (ignoring only small metric terms) to the dynamics on the real earth, which has an equatorial bulge of about 21 km in the geopotential, and gravitational equipotentials which are approximately oblate spheroids with an equatorial bulge of 10 km (Bell (1994) shows this explicitly for a planet with an ellipsoidal geoid). The pressure torque on the actual 21 km bulge is thus partially offset by a gravitational torque on the internal mass distribution, leaving a total torque again resembling the pressure torque on an 11 km bulge.

It is worth noting, however, that this partial balancing of the pressure torque by a gravitational torque assumes that the pressure is associated with a mass anomaly via hydrostatic balance. Non-hydrostatic pressure anomalies, with no associated mass anomaly, simply act on the topography of the earth relative to a sphere, including the full 21 km equatorial bulge and any other geoid topography, as does the actual (uncompensated) pressure torque in eq. (4). Use of a non-hydrostatic model in spherical coordinates would not, therefore, improve calculations of torques on the earth's bulge, since the non-hydrostatic pressure torque is misrepresented in such models. Since most non-hydrostatic processes occur on short length scales, the corresponding torques will be on small-scale topography rather than on the equatorial bulge, and this is unlikely to be a major problem. An exception may be the disturbance to hydrostatic balance caused by the vertical component of the Coriolis force, which has length scales determined by the atmospheric and oceanic currents, although a typical scaling of the planetary scale currents produces an associated pressure anomaly of less than 1 mbar.

6. Summary

In summary, although the pressure torque on the earth is sensitive to any deviation of the earth's surface from a perfect sphere, the torque due to hydrostatic pressure is partially compensated by a gravitational torque. This compensation makes it possible to consider the total (hydrostatic pressure plus gravitational) torque as due to a topographic torque on the topography measured relative to the geoid, plus a centrifugal torque. For a geoid of general shape, these torques are given by eq. (10). When the geoid is close to spherical, this reduces to eq. (14), in which the centrifugal torque is equivalent to a pressure torque on an 'effective bulge' which is not the same as the actual equatorial bulge. In the case of the earth's 21 km equatorial bulge, the compensation reduces the torque due to pressure alone by almost a half, resulting in an effective bulge of 11 km.

7. Acknowledgments

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8. Appendix

Equation (8) is derived as follows. Consider the lower triangle in Fig. 2. The base is of length dl along the (horizontal) geoid, and the hypoteneuse lies along the earth's solid surface and has length dl/cos θ. The plane of this triangle is perpendicular to the line of intersection of geoid and solid surface. The vertical elevation of the solid surface above the geoid is denoted h.

An area element dA on the geoid, given by dA = dl dn, *where* dn is a distance perpendicular to the triangle, corresponds to a vector area element on the solid surface **dS**, where $|\mathbf{dS}| =$ dl dn/cos θ, and **dS** is directed perpendicular to the hypoteneuse of the lower triangle, as shown in the upper triangle. The upper triangle is therefore similar to the lower triangle. The length of the short side of the upper triangle is then given by dA tan θ, but tan $\theta = \mathrm{d}h/\mathrm{d}l = |\nabla_g h|$, the gradient along the geoid of the solid surface's height above the geoid. Since the short side of the upper triangle is parallel to the geoid, it can be represented by the vector d$A\nabla_g h$. The vertical side of the upper triangle can then be represented as a

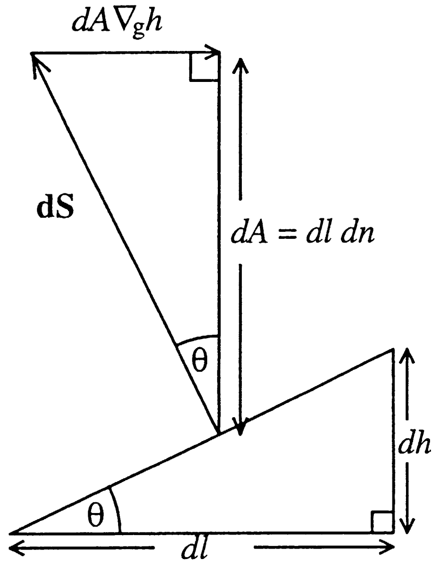

Fig. 2. Schematic showing a line segment dl along the geoid, and a section of the earth's solid surface sloping at angle θ relative to the geoid, and the resulting relationship between area elements on the geoid and on the solid surface.

downward-pointing vector $\mathrm{d}A\,\mathbf{g}/|g|$. Summing the three vectors around the upper triangle, we have

$$\mathrm{d}\mathbf{S} + \mathrm{d}A\nabla_g h + \mathrm{d}A\,\frac{\mathbf{g}}{|g|} = 0, \quad (15)$$

or, the same as eq. (8):

$$\mathrm{d}\mathbf{S} = -\mathrm{d}A\left(\frac{\mathbf{g}}{|g|} + \nabla_g h\right). \quad (16)$$

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