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# Extension of factorial concept to negative numbers

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**Abstract.** This paper present a comparative study of the various types of positive factorial functions, among which include the conventional factorial, double factorial, quadruple factorial, superfactorial and hyperfactorial. Subsequently, an extension of the concepts of positive $n!$ to negative numbers $-n!$ is introduced. Based on this extension, a formulation of specific generalization cases for different forms of negative factorials are analyzed and presented.

**Keywords:** Factorial, Negative factorial, Conventional factorials, Factorial functions.

**AMS Classification:**

## 1 Introduction

In factorial functions emphasis is generally placed on its application as in combinatorics, calculus, number theory and probability theory. Study of factorial function helps us to understand more about it application in factorial Designs to optimize animal experiments and reduce animal use and Economic factorial analysis. Different types of factorials like: Double factorial $n!!$, quadruple factorial $(2n)!/n!$, superfactorial $Sf(n)$ and hyperfactorial $H(n)$ were derived and extended to factorials of non-positive integers (i.e. negative factorials) under the study of factorial functions.

Multiple scientists worked on this subject, but the principal inventors are [1] who gives the asymptotic formula after some work in collaboration with De Moivre, [2], finally [3] and [4] introduces the actual notation $n!$. Of course other scientists such as Taylor also worked a lot with this notation. The notation $n!$ was introduced by French mathematician Christian Kramp (1760–1826) in 1808 in *Elements d'arithmétique universelle*. The term 'factorial' was first coined (in French as *factorielle*) by French mathematician Louis Francois Antoine Arbogast (1759–1803) and Kramp decided to use the term factorial so as to circumvent printing difficulties incurred by the previous used symbol $\lfloor n$.

# 2 Related work

The factorial of a natural number $n$ is the product of the positive integers less than or equal to $n$. This is written as $n!$ and pronounced '$n$ factorial.' The factorial function is formally defined by $n! = \prod_{k=1}^{n} k$ for all $n \geq 0$, or recursively defined by

$$n! = \begin{cases} 1 & if n = 0, \\ (n-1)! \times n & if n > 0. \end{cases}$$

That is 0! = 1, because the product of no number at all is 1 (i.e. there is exactly one permutation of zero objects) [6].

For example: By definition of factorial; $n! = n(n-1)(n-2)(n-3) \dots 1$

$$1! = 1$$

$$2! = 2(2-1)! = 2.1 = 2$$

$$3! = 3(3-1)(3-2)! = 3.2.1 = 6$$

$$4! = 4(4-1)(4-2)(4-3)! = 4.3.2.1 = 24$$

$$5! = 5(5-1)(5-2)(5-3)(5-4)! = 5.4.3.2.1 = 120$$

## 2.1 Double factorials

A function related to the factorial is the product of all odd (even) values up to some odd (even) positive integer n. It is often called double factorial (even though it involves about half the factors of the ordinary factorial, and its value is closer to the square root of the factorial), and denoted by $n!!$ [6].

For odd positive integer $n = 2k-1, k \geq 1$, it is; $(2k-1)!! = \prod_{i=1}^{k}(2k-1)$.

For even positive integer $n = 2k, k \geq 2$, it is; $(2k)!! = \prod_{i=1}^{k}(2i) = k!\, 2^k$

It is recursively defined by:

$$n!! = \begin{cases} n.(n-2)(n-4) \dots 5.3.1 & n > 0 \quad odd \\ n.(n-2)(n-4) \dots 6.4.2 & n > 0 \quad even \\ 1 & n = -1, 0 \end{cases}$$

Like in the ordinary factorial, $0!! = 1!! = 1$

$$2!! = 2(2-2)!! = 2(0)!! = 2.1 = 2$$

$$3!! = 3(3-2)!! = 3(1)!! = 3.1 = 3$$

$$4!! = 4(4-2)(4-4)!! = 4(2)(0)!! = 4.2.1 = 8$$

$$5!! = 5(5-2)(5-4)!! = 5(3)(1)!! = 5.3.1 = 15$$

In general, a common related notation is to use multiple exclamation points to denote a multifactorial, as the product of integers in steps of two $(n)!!$, three $(n)!!!$, or more. The double factorial is the most commonly used variant, but one can similarly define the triple factorial $(n!!!)$ and so on. One can define the $k^{th}$factorial, detonated by $n!^{(k)}$, recursively for non-negative integers as:

$$n!! = \begin{cases} 1, & if\ 0 \leq n < k, \\ ((n-k)!^{(k)}), & if n \geq k \end{cases}$$

Or $(kn)!^{(k)} = k^n n!$

Here are the computed values for $n!$ and $n!!$ (starting from $n = 0$ to $n = 5$).

| $n!$ | $n!!$ |
|---|---|
| 1 | 1 |
| 1 | 1 |
| 2 | 2 |
| 6 | 3 |
| 24 | 8 |
| 120 | 15 |

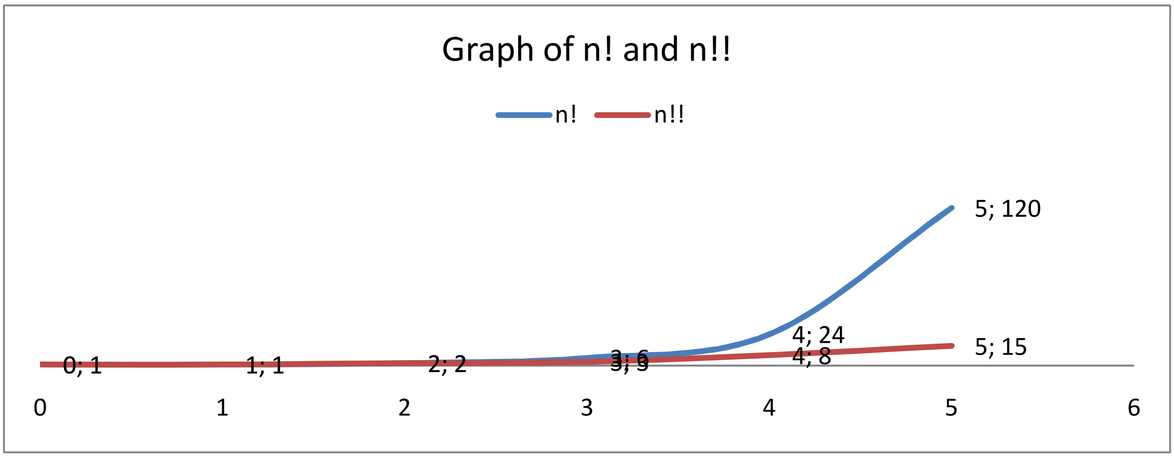


Figure 1. Graph of $n$ factorial ($n!$) and $n$ double factorial ($n!!$) for $n = 0$ to 5.

From Figure 1, we noticed that the graph of $n!$ is the same as the graph of $n!!$ for $n = 0, 1$ and $2$ this is because;$0! = 0!! = 1, 1! = 1!! = 1$ and $2! = 2!! = 2$ and they diverged from $n = 3, 4$ and $5$ as a result of; $3! = 6 \neq 3 = 3!!, 4! = 24 \neq 8 = 4!!$ and $5! = 120 \neq 15 = 5!!$, and so on.

## 2.2 Relations between double factorials and conventional factorials

There are many identities relating double factorials to conventional factorials. Since we can express

$$(2n+1)!!\,2^n n! = [(2n+1)(2n-1)\ldots 1][2n][2(n-1)][2(n-2)]\ldots 2.1$$

$$= [(2n+1)(2n-1)\ldots 1][2n][2n(2n-2)(2n-4)\ldots 2.1$$

$$= (2n+1)(2n)(2n-1)(2n-2)(2n-3)(2n-4)\ldots 2.1$$

$$= (2n+1)!.$$

It follows that

$$(2n+1)!! = \frac{(2n+1)!}{2^n n!} \text{ for } n = 0, 1, 2, \ldots n \qquad (1)$$

Also,

$$(2n)!! = (2n)(2n-2)(2n-4)\ldots 2$$

$$= [2(n)][2(n-1)][2(n-1)] \dots 2$$

i.e.

$$(2n)!! = 2^n n! \text{ for } n \text{ even} \tag{2}$$

$$(2n-1)!! = \frac{(2n)!}{2^n n!} \text{ for } n \text{ odd} \tag{3}$$

It follows that, for $n$ even;

$$\frac{n!}{n!!} = \frac{n(n-1)(n-2)\dots(2)}{n(n-2)(n-4)\dots(2)}$$

$$= (n-1)(n-3)\dots(2)$$

$$= (n-1)!!.$$

For $n$ odd:

$$\frac{n!}{n!!} = \frac{n(n-1)(n-2)\dots(1)}{n(n-2)(n-3)\dots(1)}$$

$$= (n-1)(n-3)\dots(1)$$

$$= (n-1)!!.$$

Therefore, for any $n$:

$$\frac{n!}{n!!} = (n-1)!!$$

$$\Rightarrow n! = n!!\,(n-1)!! \tag{4}$$

## 2.3 Quadruple factorials

The so-called quadruple factorial, however, is not the multiple factorial $n!^{(4)}$; it is a much larger number given by $\frac{(2n)!}{n!}$ [6]. For example, the quadruple factorials $n = 0, 1, 2, 3, 4, 5$ and 6 are:

$$\frac{(2\times0)!}{0!} = \frac{0!}{0!} = 1$$

$$\frac{(2\times1)!}{1!} = \frac{2!}{1!} = \frac{2}{1} = 2$$

$$\frac{(2\times2)!}{2!} = \frac{4!}{2!} = \frac{24}{2!} = 12$$

$$\frac{(2\times3)!}{3!} = \frac{6!}{3!} = \frac{720}{6} = 120$$

$$\frac{(2\times4)!}{4!} = \frac{8!}{4!} = \frac{40320}{24} = 1680$$

$$\frac{(2\times5)!}{5!} = \frac{10!}{51} = \frac{3628800}{120} = 30240$$

$$\frac{(2\times6)!}{6!} = \frac{12!}{6!} = \frac{479,001,600}{720} = 665,280$$

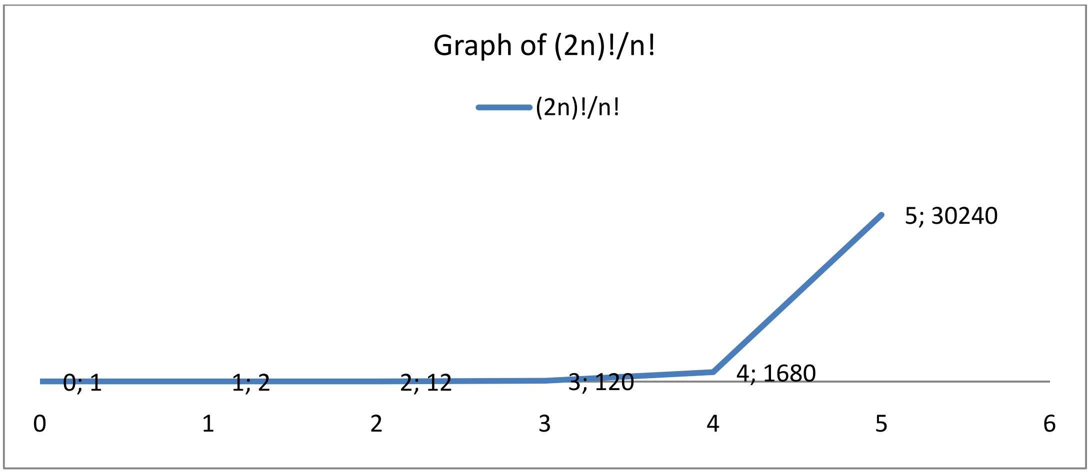


Figure 2. Graph of quadruple factorial for $n = 0$ to 5.

Figure 2 is collinear on $x$-axis from $n = 0$, 1, 2 and 3 and it makes a shaped curve along the $y$-axis at $n = 3$, which projects upward from $n = 4$.

## 2.4 Superfactorials

In [7], the superfactorial is defined as the product of the first $n$ factorials. Superfactorial is defined by;

$$Sf(n) = \prod_{k=1}^{n} k! = \prod_{k=1}^{n} k^{n-k+1} = 1^{\mathrm{n}}.2^{\mathrm{n}-1}.3^{\mathrm{n}-2} \ldots (\mathrm{n}-1)^{2}.\mathrm{n}^{1}$$

The sequence of superfactorials starts (from $n = 0$).

For example, the superfactorials of $n$ =0, 1, 2, 3, 4 and 5 are;

$$Sf(0) = 1$$

$$Sf(1) = 1$$

$$Sf(2) = 1^2.2^1 = 2$$

$$Sf(3) = 1^3.2^2.3^1 = 12$$

$$Sf(4) = 1^4.2^3.3^2.4^1 = 288$$

$$Sf(5) = 1^5.2^4.3^3.4^2.5^1 = 34560$$

$$Sf(6) = 1^6.2^5.3^4.4^3.5^2.6^1 = 24883200.$$

Equivalently, the superfactorial is given by the formula $Sf(n) = \prod_{0 \le i < j \le n}(j - i)$ which is the determinant of a Vandermonde matrix.

## 2.5 Hyperfactorials

Occasionally the hyperfactorial of $n$ considered, it is written as $H(n)$ and defined b

$$H(n) = \prod_{k=1}^{n} k! = 1^1.2^2.3^3 \ldots .(n-1)^{n-1}.n^n.$$

For example the hyperfactorials for $n$= 1, 2, 3, 4 and 5 are:

$$H(1) = 1$$

$$H(2) = 1^1.2^2 = 4$$

$$H(3) = 1^1.2^2.3^3 = 108$$

$$H(4) = 1^1.2^2.3^3.4^4 = 27{,}648$$

$$H(5) = 1^1.2^2.3^3.4^4.5^5 = 86{,}400{,}000$$

Here are the computed values for $S(n)$ and $H(n)$ for $n = 1\ to\ 5$

| $Sf(n)$ | $H(n)$ |
|---|---|
| 1 | 1 |
| 2 | 4 |
| 12 | 108 |
| 288 | 27648 |
| 34560 | 86400000 |

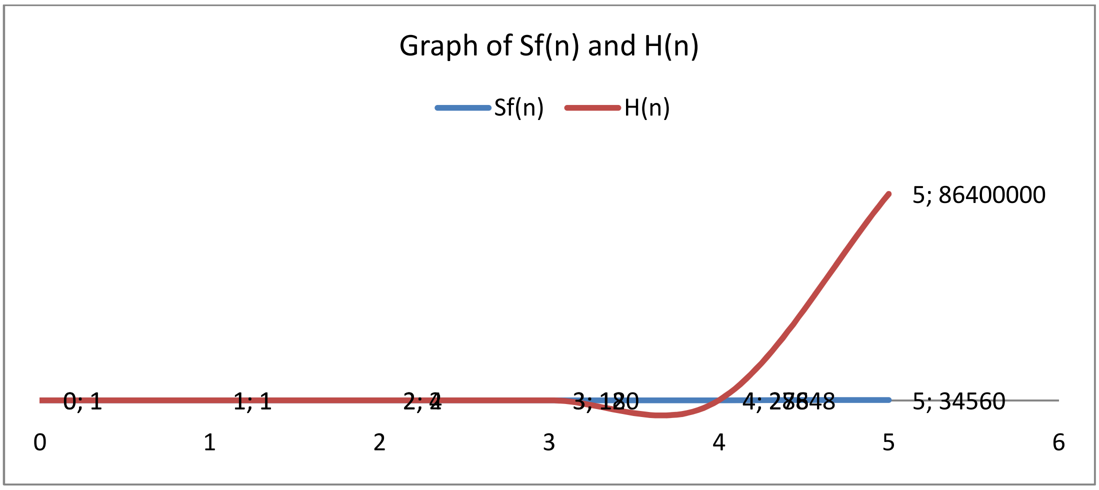


Figure 2. Graph of superfactorial $Sf(n)$ and hyperfactorial $H(n)$ for $n = 0$ to 5.

From Figure 2, we see that for $n = 0, 1, 2$ and 3 the graph of $H(n)$ is the same as the graph of $Sf(n)$ which are collinear on $x$-axis and from $n = 3$ the graph of $H(n)$ diverges by making a curve along the negative $y$ axis and projects vertically along the $y$-axis from $n = 4$ while the graph of $Sf(n)$ maintain its symmetric on $x$-axis for all values of $n$.

## 2.6 Relationship between factorial powers

In [4], a proof on relationship between positive and negative factorial power was discussed. We introduce this concept to support the point that there exist corresponding relationships between negative and positive factorials. The proof on the concept of negative factorials is provided in section 3.

Positive factorial power is defined as:

$$k^n = \frac{k^{(n+1)}}{(k-n)}, \tag{5}$$

we note that

$$k^n = k(k-1)\ldots(k-n+1) \tag{6}$$

by substituting (n+1) for n into (6), we have

$$k^{(n+1)} = k(k-1)\ldots(k-n+1)(k-n) \tag{7}$$

Therefore,

$$k^{(n+1)} = k^n(k-n)$$

Similarly, we define negative factorial power as:

$$k^{(-n)} = \frac{1}{(k+1)^{(n)}} \tag{8}$$

We claim that by mathematical induction, that if (10) is true then so is, when we substitute $-n+1$ for $n$

$$k^{(-n+1)} = \frac{1}{(k+n-1)^{(n-1)}} \tag{9}$$

We recall that,

$$(k+n)^{(n)} = (k+n)(k+n-1)\ldots(k+1) \tag{10}$$

and

$$(k+n-1)^{(n-1)} = (k+n-1)\ldots(k+1) \tag{11}$$

Hence by substituting these in (8) and (9)

$$k^{(-n)} = \frac{1}{(k+n)(k+n-1)\ldots(k+1)} \tag{12}$$

$$k^{(-n+1)} = \frac{1}{(k+n-1)\ldots(k+1)} \tag{13}$$

From (12) and (13) we note:

$$k^{(-n)} = \frac{k^{-n+1}}{(k+n)}$$

This is true by definition (5).

# 3 Concept of negative factorial

The classical case of the integer form of the factorial function $n!$ consists of the product of $n$ and all integers less than $n$, down to 1. This defined the factorial of positive integers on the right side on real line as:

$$n! = n(n-1)(n-2)\ldots 1 \tag{14}$$

We can extend the above definition to the left side of the real line as shown in Figure 3.

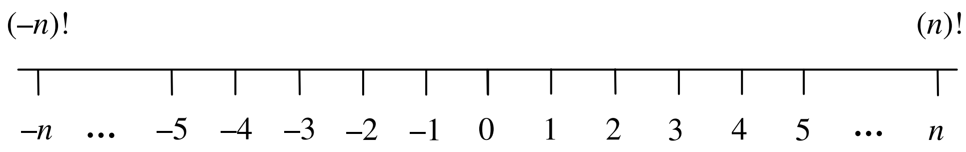


Figure 3: Factorial of negative and positive integers on the left and right side on real line

## 3.1 Kurba's classical case of negative factorials function

The Kurba's classical case of negative integer form of the factorial function $-n!$ consists of the product of $-n$ and all integers greater than $-n$ up to -1. This defined the factorial of negative integers on the left hand side of the real line recursively defined by;

$$(-n)! = -n(-n+1)(-n+2)\ldots(-1) \tag{15}$$

For example: By definition of negative factorial: $-n! = -n(-n+1)(-n+2)\ldots(-1)$

$-1! = -1$

$-2! = -2(-2+1)! = -2.-1 = 2$

$-3! = -3(-3+1)(-3+2)! = -3.-2.-1 = -6$

$-4! = -4(-4+1)(-4+2)(-4+3)! = -4.-3.-2.-1 = 24$

$-5! = -5(-5+1)(-5+2)(-5+3)(-5+4)! = -5.-4.-3.-2.-1 = -120$

Therefore, the negative factorial function is formally defined by $-n! = \prod_{k=1}^{n}(-1)^k \times k$ for all $n \leq -1$, or recursively defined by

$$-n! = \begin{cases} -\mathrm{n}(-n+1)! \times n & if\ \ n = -1, \\ -1 & if\ n < -1. \end{cases} \tag{16}$$

## 3.2 Negative double factorials

A function related to the negative factorial is the product of all odd (even) negative values up to some odd (even) negative integer $-n$. It is often called negative double factorial (even though it involves about half the factors of the ordinary negative factorial.) which is denoted by $-n!!$.

Recursively defined as:

$$-n!! = \begin{cases} -n(-n+2)(-n+4)\ldots(-6)(-4)(-2) for - n\ even \\ -n(-n+2)(-n+4)\ldots(-5)(-3)(-2) for - n\ odd \\ -1 \qquad\qquad\qquad\qquad\qquad\qquad for - n = -1 \end{cases}$$

Simplified as follows:

$$-n!! = \begin{cases} -\mathrm{n}(-n+2)!! \times n & if\ \ n = -1, \\ -1 & if\ n < -1. \end{cases} \tag{17}$$

Like in the negative factorial, $0! = 0!! = 1$ and $-1! = -1!! = -1$

$-2!! = -2(-2+2)!! = -2.0!! = -2.1 = -2$

$-3!! = -3(-3+2)!! = -3.-1!! = -3.-1 = 3$

$-4!! = -4(-4+2)(-4+4)!! = -4.-2.0!! = -4.-2.1 = 8$

$-5!! = -5(-5+2)(-5+4)!! = -5.-3.-1!! = -5.-3.-1 = -15.$

Here are the computed values for $-n!\ and - n!!$ (starting from –1 to –5)

| $-n!$ | $-n!!$ |
|---|---|
| – 1 | –1 |

| | |
|---|---|
| 2 | –2 |
| –6 | 3 |
| 24 | 8 |
| –120 | –15 |

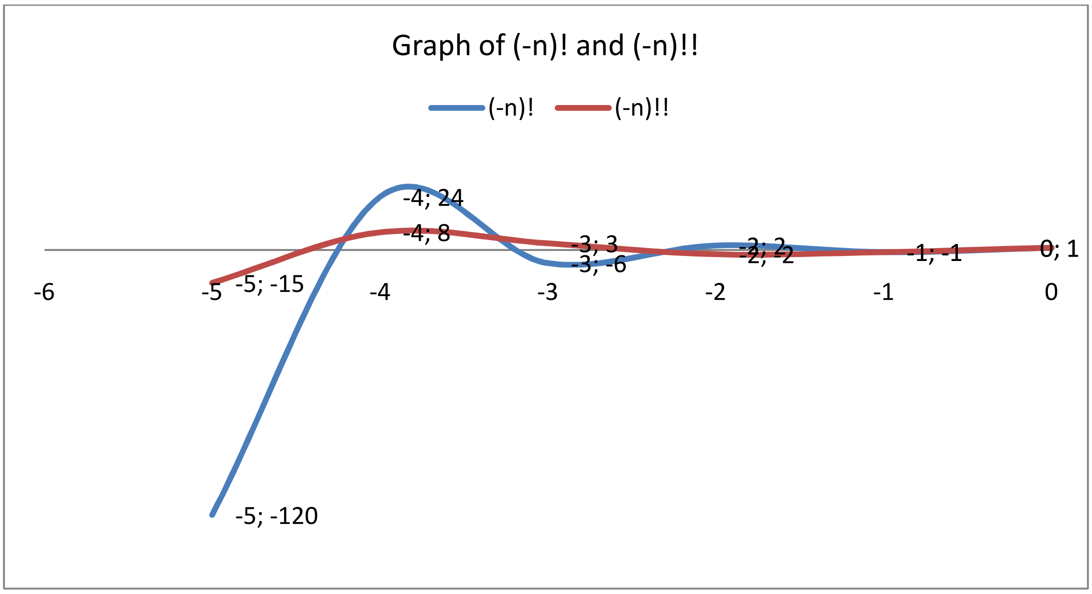


Figure 4. Graph of negative $n$ factorial $-n!$ and negative double $n$ factorial $-n!!$ from 0 to –5.

From Figure 4, we noticed that the graph of $-n!$ is the same as the graph of $-n!!$ only from 0 and –1 this is because; $0! = 0!! = 1$ and $-1! = -1!! = -1$ which are symmetric on the negative $x$-axis and they started oscillating and intersecting by making curves along positive and negative $y$-axis respectively immediately after $-1$ as a result of; $-2! = 2 \neq -2 = -2!!, -3! = -6 \neq 3 = -3!!, -4! = 24 \neq 8 = -4!!$ and $-5! = -120 \neq -15 = -5!!$ and so on.

## 3.3 Relations between negative double factorials to negative factorials

There are many identities relating negative double factorials to negative factorials. Since we can express

$$(-2n-1)!!\,2^n.-n! = [(-2n-1)(-2n+1)\ldots-1][-2n][2(-n+1)][2(-n+2)]\ldots-2.-1$$

$$= [(-2n-1)(-2n+1)\ldots-1][-2n][-2n(-2n+2)(-2n+4)\ldots-2.-1$$

$$= (-2n-1)(-2n)(-2n+1)(-2n+2)(-2n+3)(-2n+4)\ldots-2.-1$$

$$= (-2n-1)!.$$

It follows that

$$(-2n-1)!! = \frac{(-2n-1)!}{2^n-n!} \text{ for } n = 0,1,2,\ldots n \tag{18}$$

Also,

$$(-2n)!! = (-2n)(-2n+2)(-2n+4)\ldots-2$$

$$= [2(-n)][2(-n+1)][2(-n+1)]\ldots-2$$

i.e.

$$(-2n)!! = 2^n - n! \text{ for } n \text{ even} \tag{19}$$

$$(-2n+1)!! = \frac{(-2n)!}{2^n - n!} \text{ for } n \text{ odd} \tag{20}$$

It follows that, for $n$ even;

$$\frac{-n!}{-n!!} = \frac{-n(-n+1)(-n+2)\dots(-2)}{-n(-n+2)(-n+4)\dots(-2)}$$

$$= (n-1)(n-3)\dots(2)$$

$$= (-n+1)!!.$$

For $n$ odd:

$$\frac{-n!}{-n!!} = \frac{-n(-n+1)(-n+2)\dots(-1)}{-n(-n+2)(-n+3)\dots(-1)}$$

$$= (-n+1)(-n+3)\dots(-1)$$

$$= (-n+1)!!.$$

Therefore, for any $n$:

$$\frac{-n!}{n!!} = (-n+1)!!$$

$$\Rightarrow -n! = -n!!\,(-n+1)!! \tag{21}$$

For example:

1. $-1! = -1!!\,(-1+1)!! = -1!! \times 0!! = -1 \times 1 = -1$
2. $-2! = -2!!\,(-2+1)!! = -2!! \times -1!! = -2 \times -1 = 2$
3. $-3! = -3!!\,(-3+1)!! = -3!! \times -2!! = 3 \times -2 = -6$
4. $-4! = -4!!\,(-4+1)!! = -4!! \times -3!! = 8 \times 3 = 24$
5. $-5! = -5!!\,(-5+1)!! = -5!! \times -4!! = -15 \times 8 = -120$

(Using the fact that $0! = 0!! = 1$.)

## 3.4 Negative quadruple factorial

Negative quadruple factorial for negative integers can be defined as $\frac{(-2n)!}{-n!}$ like in the ordinary quadruple factorial. For example, the negative quadruple factorials are;

$$\frac{(2\times 0)!}{0!} = \frac{0!}{0!} = 1$$

$$\frac{(2\times(-1))!}{1!} = \frac{(-2)!}{(-1)!} = \frac{2}{-1} = -2$$

$$\frac{(2\times(-2))!}{(-2)!} = \frac{(-4)!}{(-2)!} = \frac{24}{2} = 12$$

$$\frac{(2\times(-3))!}{(-3)!} = \frac{(-6)!}{(-3)!} = \frac{720}{-6} = -120$$

$$\frac{(2\times(-4))!}{(-4)!} = \frac{(-8)!}{(-4)!} = \frac{40320}{24} = 1680$$

$$\frac{(2\times(-5))!}{(-5)!} = \frac{(-10)!}{(-5)!} = \frac{3628800}{-120} = -30240$$

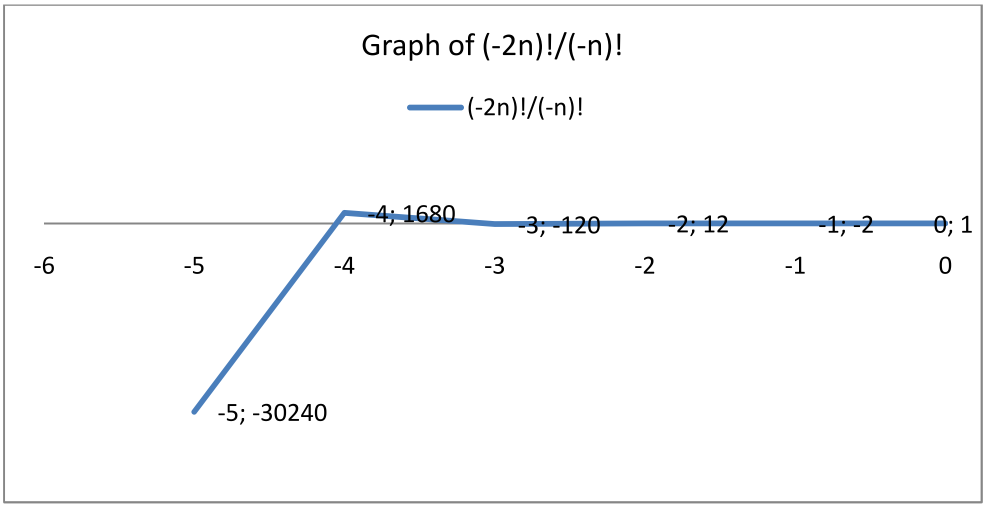


Figure 5. Graph of negative quadruple factorial from 0 to –5.

Figure 5 is collinear on the negative $x$ axis from $0, -1, -2$ and $-3$ and its makes a shaped curve along the positive $y$axis from $-3$ which declined to the negative $y$axis from$-4$.

## 3.5 Negative superfactorial

Negative superfactorial for negative integer can be defined as;

$$Sf(-n) = \prod_{k=1}^{n}(-k)! = \prod_{k=1}^{n}(-k)^{n-k+1} = (-1)^{\mathrm{n}}.(-2)^{\mathrm{n}-1}.(-3)^{\mathrm{n}-2} \dots (-\mathrm{n}+1)^{2}.(-\mathrm{n})^{1}$$

For example, the negative superfactorials from 0 to –5 are given below:

$$Sf(0) = 1$$

$$Sf(-1) = 1$$

$$Sf(-2) = (-1)^2.(-2)^1 = -2$$

$$Sf(-3) = (-1)^3.(-2)^2.(-3)^1 = 12$$

$$Sf(-4) = (-1)^4.(-2)^3.(-3)^2.(-4)^1 = -288$$

$$Sf(-5) = (-1)^5.(-2)^4.(-3)^3.(-4)^2.(-5)^1 = 34560$$

## 3.6 Negative hyperfactorial

Negative hyperfactorial for negative integers $H(-n)$ is given by:

$$H(-n) = \prod_{k=1}^{n}(-k)! = (-1)^1.(-2)^2.(-3)^3 \dots (-n+1)^{n-1}.(-n)^n$$

For example the hyperfactorials for $k$ = 1, 2, 3, 4 and 5 are:

$$H(-1) = -1$$

$$H(-2) = (-1)^1.(-2)^2 = -4$$

$$H(-3) = (-1)^1.(-2)^2.(-3)^3 = 108$$

$$H(-4) = (-1)^1.(-2)^2.(-3)^3.(-4)^4 = 27{,}648$$

$$H(-5) = (-1)^1.(-2)^2.(-3)^3.(-4)^4.(-5)^5 = -86{,}400{,}000$$

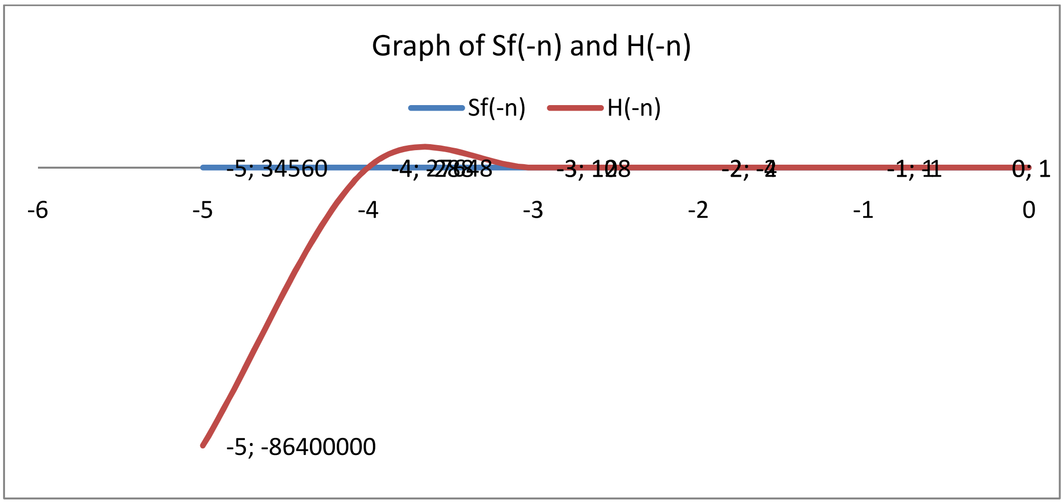


Figure 6. Graph of negative superfactorial and negative hyperfactorial from 0 to –5.

From Figure 6, we see that for $n = 0\,, -1, -\,2$ and $-\,3$ the graph of $H(-n)$ is the same as the graph of $Sf(-n)$ which are both collinear on $negativex$-axis and at $n = -3$ the graph of $H(-n)$ shoots up a curve along the positive $y$-axis which declined to the negative $y$-axis from $n = -4$ while the graph of $Sf(-n)$ maintain its collinearity on negative $x$-axis for all values of negative $n$.

## 4 Conclusion and discussion of the result

Factorials of positive integers i.e. $n!$ is the product of $n$ and all integers less than$n$, down to 1while factorials of negative integers i.e. $-n!$ is the product of $-n$ and all negative integers greater than $-n$ up to -1. Therefore factorials of both positive and negative integers exists.

We noticed that both factorials of positive and negative integers have the same values with same (different) signs as in the case of negative $n$ factorial $-n!$ and negative quadruple factorial $(-2n)!/-n!$ due to whether factorials of negative integers are odd or even. If the negative integer is even then it has the same value with that of positive integers and if the negative integer is odd then it has different sign to that of positive integer. While negative superfactorial $Sf(-n)$ has the same value with that of positive superfactorial $Sf(n)$ if the negative integer is odd.

We finally observed that negative double factorial $-n!!$ and negative hyperfactorial $H(-n)$ follows the same pattern in which they have the same value but different sign to that of their positive integers for the first and second negative odd and even integers and the same value, same sign for the third and fourth odd and even negative integers continuously.

Therefore, $-n!$ and $(-2n)!/-n!$ has the same properties with $n!$ and $(-2n)!/n!$ for $-n$ even, $Sf(-n)$ has the same properties with $Sf(n)$ for $-n$ odd and $-n!!$ and $H(-n)$ has the same properties with $n!!$ and $H(n)$ for the third and fourth odd and even negative integers respectively.

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