

An Easy Technique for Fisheye Camera Calibration

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Abstract

We present an easy fisheye camera calibration technique based on a square in the scene plane in this paper. Firstly, aiming at a generic quadrangle including two parallel line segments with the known length ratio, the planar homography between the scene planar including the generic quadrangle and its image is estimated by images of two parallel line segments, then constraints on fisheye camera basic internal parameter can be obtained from n fisheye images. Secondly, when the generic quadrangle becomes a square in planar scene, two linear constraint equations can be determined from one image of the square. Under the assumption of zero skew, we may obtain four constraint equations on the basic intrinsic parameters (the focal length, the aspect ratio and the principal point) from two fisheye images. Lastly, because the projected line of the scene straight line on the unit sphere is a great circle, the angle between the normal vector of the great circle and the projected point of fisheye image point is estimated by the initial value. An objective function may be established from these angles. The fisheye lens distortion parameters may further be refined on minimizing the objective function from all fisheye image points corresponding on the same scene straight line. Experimental results demonstrate the validity of our method.

1. Introduction

Camera calibration with the geometry of parallel straight lines has been used extensively in computer vision. Recently, there are various applications from robot and virtual reality to surveillance and navigation because omnidirectional camera can provide a wide field of view.

Previous work on pinhole model camera and fisheye camera calibration using the geometry of parallel straight lines is introduced as the following. Under the pinhole camera model, literatures [1,2,3] studied camera calibration method using the geometry of parallel straight lines. Devernay[4] calibrated fisheye lens by using the constraint that the straight lines in fisheye image have to be straight. Scaramuzza[5] proposed a flexible technique for single viewpoint omnidirectional camera calibration. This method required a planar pattern including parallel straight lines. Kannala [6] proposed a technique for fisheye camera calibration using one view of a planar calibration object. Nakano [7] presented a method of calibrating a fisheye camera by a stripe pattern that is composed of a group of parallel straight lines. In these fisheye calibration methods, literatures[5,6] needed to know 3D control point coordinate and literatures[4,7] used the geometry informa-

tion of straight line.

We propose an easy fisheye camera calibration technique based on a square in the planar scene in this paper. Firstly, two linear constraint equations can be determined from one image of the square. And we may obtain four constraint equations on the basic intrinsic parameters (that is, the focal length, the aspect ratio and the principal point) from two fisheye images under the assumption of zero skew.

Because the projected line of the scene straight line on the unit sphere is a great circle, an objective function may be established from the normal vector of the great circle and the projected point of fisheye image point. Thus, The fisheye lens distortion may further be refined on minimizing the objective function from all fisheye image points corresponding on the same scene straight line.

Compare with conventional fisheye calibration methods, advantages of this method include: (i) Square is common in many man-made scenes; (ii) Only image points are used and no 3D space point coordinates are needed; (iii) No conics fitting in fisheye image is required and just taking four image points. These advantages make this calibration method easy to implement.

The structure of the paper is the following. In Section 2, we describe fisheye lens camera model. A calibration algorithm for fisheye camera model is presented in Sections 3. Experiments and conclusion are given in Sections 4 and 5.

2. Fisheye Camera Model

The existing projection models of the fisheye camera are classified into two categories: the first model denotes relation between fisheye image radius r and its corresponding perspective image radius r' . The second model represents relation between fisheye image radius r and incident angle θ . The incident angle θ is the angle between the optical axis and the incoming ray. Fleck [7] has introduced several projections: (1) Stereographic projection: $r = 2f \tan(\theta/2)$; (2) Equidistance projection: $r = f\theta$; (3) Equisolid angle projection: $r = 2f \sin(\theta/2)$; (4) Sine law projection: $r = f \sin(\theta)$.

We choose equidistance projection model as the basic model of fisheye lens in this paper. Suppose the skew s of image is 0, the basic intrinsic parameter matrix of fisheye camera is equal to

$$K = \begin{bmatrix} af & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Where f is the focal length, α is the aspect ratio, (u_0, v_0) is the principal point. We get the pixel coordinates (u, v) of fisheye image point from

$$\begin{cases} u = \alpha f \cdot x + u_0 \\ v = f \cdot y + v_0 \end{cases} \quad (2)$$

(x, y) is the fisheye image point coordinate in the image orthogonal coordinate system.

Using nonlinear distortion of camera depicted as [8,9], we choose the radial distortion of fisheye camera in this paper

$$r \approx f\theta + k_1\theta^3 + k_2\theta^5 \quad (3)$$

Decentering distortion including radial direction error dr and tangential direction error dt is

$$\begin{cases} dr \approx 3p_1r^2 \cos(\varphi) + 3p_2r^2 \sin(\varphi) \\ dt \approx -p_1r \sin(\varphi) + p_2r \cos(\varphi) \end{cases} \quad (4)$$

where r is the radial distance from image point to image plane center. θ is the angle between the optical axis and the incoming ray. $k_i (i=1,2)$ is the value of parameters. φ the angle between the radial direction distortion axis and the x plus direction of image coordinate system, $p_i (i=1,2)$ are the value of parameters. Then, we have

$$\begin{cases} x = (r + dr) \cos(\varphi) - dt \cdot \sin(\varphi) \\ y = (r + dr) \sin(\varphi) + dt \cdot \cos(\varphi) \end{cases} \quad (5)$$

Thus, the distortion parameters to be corrected are $\{u_0, v_0, \alpha, f, k_1, k_2, p_1, p_2\}$

3. Calibration Method

In this section, the basic intrinsic parameters are first estimated using image points in center area of fisheye image. Then, the fisheye lens distortion parameters including radial and decentering distortion may be further refined on using all fisheye image points on the same line.

3.1. Constraints on the Basic Intrinsic Parameters

Suppose the skew s of image is 0, the basic intrinsic parameters are focal length f , aspect ratio α and the principal point (u_0, v_0) . Then constraints on the basic intrinsic parameters may be obtained from this section.

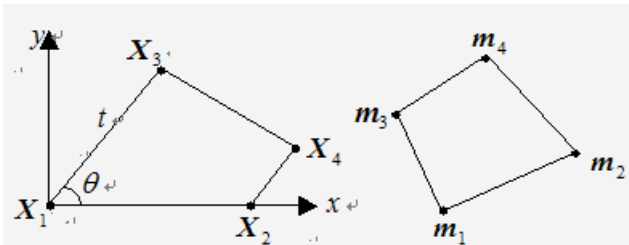


Figure1: Two parallel line segments and their image

If ratio both two parallel line segments X_1X_3 and X_2X_4 is $r = |X_1X_3|/|X_2X_4|$ in the generic quadrangle, we choose two parallel line segments to lie in the xy -plane and set an orthogonal coordinate system on

the two parallel line segments. It's shown as Figure 1. Therefore, four end points of two parallel line segments are given through the following homogeneous coordinates:

$$X_1 = (0, 0, 1)^T, X_2 = (1, 0, 1)^T$$

$$X_3 = (t \cos \theta, t \sin \theta, 1)^T, X_4 = (1 + \frac{t}{r} \cos \theta, \frac{t}{r} \sin \theta, 1)^T \quad (6)$$

Where, θ denotes the angle between line segment X_1X_3 and plus direction of x axis, t is the length of line segment X_1X_3 . Suppose image points of four end points are $m_i = (u_i, v_i, 1)^T (i=1,2,3,4)$. So, a planar homography between the space plane and the image plane is computed by

$$s_i m_i = H X_i \quad (7)$$

where s_i is non-zero factor and H is a 3×3 matrix.

From equation (7), we have

$$H [X_1 X_2 X_3] = [m_1 m_2 m_3] \text{diag}(s_1, s_2, s_3) \quad (8)$$

and

$$H = [m_1 m_2 m_3] \text{diag}(s_1, s_2, s_3) [X_1 X_2 X_3]^{-1} \quad (9)$$

From $s_4 m_4 = H X_4$ and equation (9), we get

$$s_i = \frac{s_4 q_i}{p_i}, (1 \leq i \leq 3)$$

where $[X_1 X_2 X_3]^{-1} X_4 = (p_1, p_2, p_3)^T$,

$$[m_1 m_2 m_3]^{-1} m_4 = (q_1, q_2, q_3)^T$$

Then, the planar homography H is equal to

$$H \approx [m_1 m_2 m_3] \text{diag}\left(\frac{q_1}{p_1}, \frac{q_2}{p_2}, \frac{q_3}{p_3}\right) [X_1 X_2 X_3]^{-1} \quad (10)$$

There has a transformation matrix from equation (6) and (8):

$$T = [X_1 X_2 X_3] = \begin{bmatrix} 0 & 1 & t \cos \theta \\ 0 & 0 & t \sin \theta \\ 1 & 1 & 1 \end{bmatrix} \quad (11)$$

Therefore, the planar homography H can be computed by equation (10)

$$H \approx \tilde{M} \begin{bmatrix} -1 & ctg\theta - \frac{csc\theta}{t} & 1 \\ 1 & -ctg\theta & 0 \\ 0 & \frac{csc\theta}{t} & 0 \end{bmatrix} \quad (12)$$

where $\tilde{M} = [-rq_1 m_1 \quad q_2 m_2 \quad rq_3 m_3]$. If the circle point of the plane including two parallel line segments is $I = (1, i, 0)^T$, its image point under the planar homography H is

$$m_i = H \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} \approx \tilde{M} \begin{pmatrix} -1 + de^{i\theta} \\ -de^{i\theta} \\ 1 \end{pmatrix} \quad (13)$$

If $C = K^{-T} K^{-1}$ represents the image of the absolute conic and the circle point is on the image of the absolute conic, then $m_i^T C m_i = 0$. Let

$$A = \tilde{M}^T C \tilde{M} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, d = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, e = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \quad (14)$$

Then, we know

$$\mathbf{m}_1^T \mathbf{C} \mathbf{m}_1 = \mathbf{d}^T \mathbf{A} \mathbf{d} + t e^{i\theta} (\mathbf{e}^T \mathbf{A} \mathbf{d} + \mathbf{d}^T \mathbf{A} \mathbf{e}) + t^2 e^{i2\theta} \mathbf{e}^T \mathbf{A} \mathbf{e} = 0 \quad (15)$$

Because \mathbf{A} is a symmetric matrix, we get equation (16) from equation (15).

$$\begin{cases} 1 \cdot \mathbf{d}^T \mathbf{A} \mathbf{d} + 2t \cos \theta \cdot \mathbf{d}^T \mathbf{A} \mathbf{e} + t^2 \cos 2\theta \cdot \mathbf{e}^T \mathbf{A} \mathbf{e} = 0 \\ 0 \cdot \mathbf{d}^T \mathbf{A} \mathbf{d} + 2t \sin \theta \cdot \mathbf{d}^T \mathbf{A} \mathbf{e} + t^2 \sin 2\theta \cdot \mathbf{e}^T \mathbf{A} \mathbf{e} = 0 \end{cases} \quad (16)$$

In equation (16), all unknown parameters include the intrinsic parameter of camera, r , θ and t . So, we can't determine the intrinsic parameter from single image of two parallel line segments. Suppose Image points of four end points in two parallel line segments in n images are

$$\mathbf{m}_1^{(j)}, \mathbf{m}_2^{(j)}, \mathbf{m}_3^{(j)}, \mathbf{m}_4^{(j)}, j = 1, 2, \dots, n$$

$$\text{Let } (q_1^{(j)}, q_2^{(j)}, q_3^{(j)})^T = [\mathbf{m}_1^{(j)} \mathbf{m}_2^{(j)} \mathbf{m}_3^{(j)}]^{-1} \mathbf{m}_4^{(j)}$$

$$\tilde{\mathbf{M}}^{(j)} = \begin{bmatrix} -r q_1^{(j)} \mathbf{m}_1^{(j)} & q_2^{(j)} \mathbf{m}_2^{(j)} & r q_3^{(j)} \mathbf{m}_3^{(j)} \end{bmatrix}^T$$

$$\mathbf{A}^{(j)} = \tilde{\mathbf{M}}^{(j)T} \mathbf{C}^{(j)} \tilde{\mathbf{M}}^{(j)}$$

We may obtain equation (12) from equation (11)

$$\begin{cases} 1 \cdot \mathbf{d}^T \mathbf{A}^{(j)} \mathbf{d} + 2t \cos \theta \cdot \mathbf{d}^T \mathbf{A}^{(j)} \mathbf{e} + t^2 \cos 2\theta \cdot \mathbf{e}^T \mathbf{A}^{(j)} \mathbf{e} = 0 \\ 0 \cdot \mathbf{d}^T \mathbf{A}^{(j)} \mathbf{d} + 2t \sin \theta \cdot \mathbf{d}^T \mathbf{A}^{(j)} \mathbf{e} + t^2 \sin 2\theta \cdot \mathbf{e}^T \mathbf{A}^{(j)} \mathbf{e} = 0 \end{cases} \quad (17)$$

In terms of equations (17), we know linear correlation of three column vectors. Thus equation (18) may be obtained.

$$\begin{pmatrix} \mathbf{d}^T \mathbf{A}^{(1)} \mathbf{d} \\ \mathbf{d}^T \mathbf{A}^{(2)} \mathbf{d} \\ \vdots \\ \mathbf{d}^T \mathbf{A}^{(n)} \mathbf{d} \end{pmatrix} = \alpha \begin{pmatrix} \mathbf{e}^T \mathbf{A}^{(1)} \mathbf{e} \\ \mathbf{e}^T \mathbf{A}^{(2)} \mathbf{e} \\ \vdots \\ \mathbf{e}^T \mathbf{A}^{(n)} \mathbf{e} \end{pmatrix}, \begin{pmatrix} \mathbf{d}^T \mathbf{A}^{(1)} \mathbf{e} \\ \mathbf{d}^T \mathbf{A}^{(2)} \mathbf{e} \\ \vdots \\ \mathbf{d}^T \mathbf{A}^{(n)} \mathbf{e} \end{pmatrix} = \beta \begin{pmatrix} \mathbf{e}^T \mathbf{A}^{(1)} \mathbf{e} \\ \mathbf{e}^T \mathbf{A}^{(2)} \mathbf{e} \\ \vdots \\ \mathbf{e}^T \mathbf{A}^{(n)} \mathbf{e} \end{pmatrix} \quad (18)$$

Therefore, we can acquire $2(n-1)$ constraints on the intrinsic parameters from n images.

$$\begin{cases} \mathbf{e}^T \mathbf{A}^{(j)} \mathbf{e} \cdot \mathbf{d}^T \mathbf{A}^{(j)} \mathbf{d} = \mathbf{e}^T \mathbf{A}^{(j)} \mathbf{e} \cdot \mathbf{d}^T \mathbf{A}^{(j)} \mathbf{e} \\ \mathbf{e}^T \mathbf{A}^{(j)} \mathbf{e} \cdot \mathbf{d}^T \mathbf{A}^{(j)} \mathbf{e} = \mathbf{d}^T \mathbf{A}^{(j)} \mathbf{e} \cdot \mathbf{e}^T \mathbf{A}^{(j)} \mathbf{e} \end{cases} j = 2, 3, \dots, n \quad (19)$$

When the generic quadrangle becomes the square, we have the following result.

Result 1: If r and t are equal to 1 and θ is 90° , there is a square in the space plane. Then, constraints on the basic intrinsic parameter of camera are derived from two images of the square. The constraints are shown as equation (20).

$$\begin{cases} \mathbf{d}^T \mathbf{A}^{(j)} \mathbf{d} = \mathbf{e}^T \mathbf{A}^{(j)} \mathbf{e} \\ \mathbf{d}^T \mathbf{A}^{(j)} \mathbf{e} = 0 \end{cases}, j = 1, 2 \quad (20)$$

3.2. The Objective Function

In the distortion parameters to be corrected $\{u_0, v_0, \alpha, f, k_1, k_2, p_1, p_2\}$, we may estimate the basic intrinsic parameters using method in section 3.1, and let initial value of the other parameters be zero. All fisheye image points are mapped on the unit sphere. Therefore, we can establish an objective function from the projected point on the unit spherical image.

Let $\tilde{\mathbf{x}} = (u, v, 1)^T$ be a point in the fisheye image, then the projected point \mathbf{p} on the unit sphere may be computed by the initial value of distortion parameters. Its the spherical coordinate on the unit sphere with the polar angle θ and azimuth angle φ is:

$$\mathbf{p} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)^T$$

A scene straight line, which its image is a conic curve in the fisheye image, may be projected as a great circle on the unit sphere. Let the normal vector of the plane including the great circle be $\mathbf{n} = (A, B, C)^T$, point \mathbf{p} is on the great circle. We may compute angle Φ between the normal vector of the great circle and each the projected point, which belongs to the same scene line, on the unit sphere by equation (21).

$$\cos(\Phi) = \frac{\mathbf{n} \cdot \mathbf{p}}{\|\mathbf{n}\| \|\mathbf{p}\|} \quad (21)$$

Where $\mathbf{n} \cdot \mathbf{p}$ denotes dot product. The angle Φ is usually 90° under zero noise. If the number of scene straight lines is L and the number of points on each line is N , then the objective function is

$$\varepsilon = \sum_{i=1}^L \left[\sum_{j=1}^N \cos(\Phi_{i,j}) \right] \quad (22)$$

3.3. Algorithm

The calibration process is summarized on the following algorithm.

- 1) Detect four image points $\mathbf{m}_i (i=1,2,3,4)$ of a square at the center area of fisheye image;
- 2) Compute $q_i (i=1,2,3)$ and the matrix $\tilde{\mathbf{M}}$ from image points in each image;
- 3) Construct the matrix \mathbf{A} and get constraint equations of the basic intrinsic parameter from equation (14) and (20).
- 4) Because two constraints on the basic intrinsic parameter may be acquired by step3 for each image, the basic intrinsic parameter can be estimated from at least two images;
- 5) Regard the basic intrinsic parameter as an initial value and refine full parameters of fisheye camera from the objective function (22).

4. Experiments

In this section, there are many experiments to validate the fisheye camera calibration method. One of some results from real image is reported as follows.

One of a real fisheye image sequence of a calibration object, captured by a digital camera with fisheye lens, is shown in Figure 2. Its size is 1024×768 pixels. In Figure 2, Feature points marked by red "+" are four corner points of a bigger square and white points "." are four corner point of a square in the center area of fisheye image. These two group feature points are used to estimate the basic intrinsic parameters of fisheye camera, and experimental results are shown in Table 1. Four curves marked white are used to optimize the distortion parameters of fisheye camera. Then, the basic intrinsic parameters are considered as an initial value and we obtain the optimized distortion parameters by minimizing the objective function. In the case of the estimated basic intrinsic parameter using white feature points, one of the distortion parameters of fisheye camera is shown as the following:

$$u_0 = 502.4708, v_0 = 383.4550, \alpha = 1.0706, f = 231.5145, k_1 = 0.1778, k_2 = 0.4168, p_1 = -4.775e-8, p_2 = 4.010e-9$$



Figure 2. One of real fisheye image sequence

Table 1 The Estimated Basic Intrinsic Parameters

	Calibrated the basic intrinsic parameter using feature points marked by white dot	Calibrated the basic intrinsic parameter using feature points marked by red cross
u_0	502.4741	511.0922
v_0	383.4550	357.5259
f	231.5068	294.9616
α	1.0055	0.9924

In order to verify the estimation of fisheye camera intrinsic parameters, we transform the original distorted fisheye image by these estimated intrinsic parameters. The result of some parts is shown Figure 3(a) and (b). Notice that all lines are straight in Figure 3(a) using white feature points in the center area of fisheye image.

From the part transformed image in Figure3 (a) and (b), the distortion correction result of Figure3 (a) is rather better than that of Figure3 (b). This experimental result shows that calibrating fisheye camera from feature point in the center area of fisheye image is better than using feature points in the other image area.

5. Conclusion

In this paper, we describe an easy fisheye camera calibration method to estimate the intrinsic parameters. Constraint equations on fisheye camera basic internal parameter are derived from n fisheye images of a square by using the property of the absolute conic and its image points. Under the assumption of zero skew, we compute the basic intrinsic parameters (the focal length, the aspect ratio and the principal point). Then, the fish-eye lens distortion parameters are further refined on minimizing the objective function. Experimental results show that our method is efficient and reliable.

Acknowledgement

This work was supported by the National Natural Science Foundation of China under grant No.60875023 and No.60872127.

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(a)



(b)

Figure 3. The part correction fisheye image using the distortion parameters

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