# Investigation of OCT Images Descriptions on the Base of Representational MDL Principle

Igor Gurov Alexey Potapov
Saint Petersburg State University of Information Technologies, Mechanics and Optics
49 Kronverksky ave., Saint Petersburg, 197101, Russia
E-mail: gurov@mail.ifmo.ru; pas.aicv@gmail.com

## Abstract

*Investigation of images representations in Optical Coherence Tomography (OCT) is carried out based on the objective quality criterion introduced with the use of the novel representational minimum description length principle. Several image segmentation algorithms are proposed that recover layered structure of the OCT images and can be used for morphological analysis of biological tissues in disease diagnostics. It is shown that the value of the quality criterion corresponds to adequacy of segmentation results, so it can be used for further investigation and improvement of OCT image analysis algorithms.*

## 1. Introduction

Optical coherence tomography (OCT) is the modern high-resolution method for analyzing internal microstructure of biological tissues. This method is very promising in disease diagnostics, because of it is both highly informative and non-invasive [1].

OCT images differ from ordinary optical images, which human visual system is adapted for. Therefore, it is difficult for human to interpret OCT images for their analyses relying on common sense. Existent methods of OCT-image analysis (see, *e.g.*, [2, 3]) are mostly based on spectral features heuristically introduced for certain type of tissues and diseases. A strict general approach and unbiased criteria are required to overcome these difficulties in order to develop optimal OCT image analysis methods.

The main task in OCT imaging is to detect different layers and insertions in bio-tissues and to evaluate their properties, that is, to describe their structure. Image description is always performed within a certain representation, thus the urgent problem is to introduce objective criterion of OCT images representations adequacy in order to choose the best of them.

The notion of representation of images is one the fundamental notions in machine vision. Unfortunately, there are almost no general approaches with formal definition and selection criterion for them, which can be used for arbitrary new type of images such as OCT images. One such approach was proposed in [4], where representation is defined as such the program for Universal Turing Machine that can reproduce any image from the given ensemble using its appropriate description. The simplest (and useless) representation is the program that accepts an image as its own description and outputs it. The representational minimum description length principle (RMDL) is introduced for estimating how efficiently content of images of a certain type can be described within the given representation:

1. The best representation for the given set of images is the representation, for which the following sum is minimized:
   - the length of the representation;
   - sum of lengths of the best descriptions of images within the representation.

Usually, there is a variety of descriptions of a single image within the given representation. To choose the best description (consisting, *e.g.*, of the regular part, or the model, and random part) one can use the second part of the RMDL principle:

2. The best model of the image within the given representation is the model, for which the following sum is minimized:
   - the length of the model;
   - the length of the image described within the representation and the model.

If one representation gives shorter descriptions of every image of a set than another one, it can be called superior.

It was shown [5], that the RMDL principle can be used to choose between representations of different ensembles of ordinary optical images. Here, we adopt this principle to investigate the OCT images representations. As the main result, we state the principle possibility to apply the RMDL principle for estimation of representations quality. We also propose two simplified representations of OCT images based on some segmentation models. These representations recover layered structure of OCT images and can have practical utility in biomedicine.

## 2. The Representational MDL Principle

The RMDL principle is the extension of the known MDL principle that can formally be introduced on the base of Kolmogorov algorithmic complexity [6]. Let $U$ be the Universal Turing Machine (UTM). Let prefix algorithmic complexity of the binary string $\beta$ be

$$K_U(\beta) = \min_{\alpha}[l(\alpha) \mid U(\alpha) = \beta],$$

where $l(\alpha)$ is the length of the program $\alpha$. Index $U$ will be omitted for simplification of notation, when it is clear from context. Program for UMT can be considered as a model of the source, which generated data $\beta$ (it is not necessarily an image).

The string $\alpha$ can be represented as concatenation of two strings $\alpha=\mu\delta$, where $\mu$ is interpreted as the program itself (the model or regular component), and $\delta$ is the initial data for this program (random component). It can be written

$$K(\beta)=\min_{\mu\delta}\left[l(\mu)+l(\delta)\mid U(\mu\delta)=\beta\right]=$$

$$\min_{\mu}\left[l(\mu)+\min_{\delta}\left[l(\delta)\middle|U(\mu\delta)=\beta\right]\right].$$

Thus, the description length of data can be calculated as

$$K(\beta)=\min_{\mu}\left[l(\mu)+K(\beta\mid\mu)\right],$$

where quantity $K(\beta|\mu)$ is the conditional algorithmic complexity of the string $\beta$ with the given string $\mu$.

This gives the criterion for choosing the best model:

$$\mu^{*}=\arg\min_{\mu}\left[K(\beta\mid\mu)+l(\mu)\right].$$

However, this criterion cannot be directly applied for image analysis problems, because it does not incorporate prior information contained in a representation, and image analysis is applied to different images independently.

Consider an ensemble of images $f_1,\ldots,f_n$. Optimal description of these images can be found only for the whole ensemble, because the following inequality holds:

$$\sum_{i=1}^{n}K(f_i)\geq K(f_1f_2\ldots f_n).$$

One can overcome this difficulty by extracting mutual information from images:

$$K(f_1f_2\ldots f_n)\approx\min_{S}\left[\sum_{i=1}^{n}K(f_i\mid S)+K(S)\right].$$

Here, $S$ is the program for UMT, for which descriptions $\mu_i\delta_i$ of every image exist: $U(S\mu_i\delta_i)=\beta_i$. This equation corresponds to the RMDL principle stated above.

The description length of a single image within the given representation will be

$$L_S(\beta)=K(\beta\mid S)=\min\left[l(\mu)+K(\beta\mid S\mu)\right]$$

that gives the second part of the RMDL principle.

As the result, the RMDL principle gives an opportunity to correctly compare lengths of descriptions obtained within different representations. These lengths are ordinarily considered as obtained with the use of different (incommensurable) criteria.

## 3. Image Representations Based on the Segmentation Models

Consider the following representations, which can be used within the same segmentation algorithm.

1. The base representation $S_0$, in which values of pixel brightness are supposed to be independent and identically distributed. The description length of an image $f(x,y):G\to R$ can be calculated as

$$L_{S_0}(f)=\|G\|H(f)+N_b\log_2 N_b,$$

where $\|G\|$ is an area of the image region $G$, $H(f)$ is the entropy of brightness, $N_b$ is the number of different brightness values. The first term in the sum stands for the code length of all the brightness values, and the second one stands for the length of code words table.

2. Representation $S_1$, in which the image region $G$ is supposed to be divided into some number of sub-regions $G_1,\ldots,G_d$. Brightness values in each region are supposed to have their own distribution. In addition to brightness values one should also describe borders of regions $\delta G_i$. The description length of an image within this representation can be calculated as

$$L_{S_1}(f)=\sum_{i=1}^{d}\left(\|G_i\|H(f_i)+N_b\log_2 N_b+\|\delta G_i\|\log_2 N_d\right),$$

where $f_i$ is the constriction of the image on the region $G_i$, $N_d$ is the number of directions from a border point to its neighbor points (we used $N_d$=8).

The representation $S_1$ admits arbitrary division of the given image into regions. However, the best division should be determined on the base of the description length criterion. If a division corresponds to the real regions on the image, entropy values will be smaller. Oversegmentation is prohibited, because of model complexity (coding tables and borders description) is also taken into account.

3. The representation $S_2$, in which images are also divided into regions, but content of each region is described with some regression model, i.e. some regular functions $g_i(x,y,\mathbf{w}_i)$ with parameter vectors $\mathbf{w}_i$ are subtracted from the brightness values $f_i(x,y)$. Parameter estimation is performed in the way to minimize entropy value in the corresponding region $G_i$. The description length can be evaluated as

$$L_{S_2}(f)=\sum_{i=1}^{d}\left(\|G_i\|H(r_i)+N_b\log_2 N_b+\right.$$

$$\left.L(\mathbf{w}_i)+\|\delta G_i\|\log_2 N_d\right),$$

where

$$r_i(x,y)=f_i(x,y)-g_i(x,y,\mathbf{w}_i)$$

are residuals, and

$$L(\mathbf{w}_i)=\frac{m_i}{2}\log_2\|G_i\|$$

are the description lengths of the parameter vectors $\mathbf{w}_i$ that consist of $m_i$ elements.

Here we adopted linear regression models, however selection of the most adequate class of regression models is the problem that requires further investigations (the RMDL principle can be used to perform such a selection).

Optimization of the length of image description within one of representations should be performed by a certain segmentation algorithm. We used region growing algorithm that starts with a lot of small regions and consequently unite pairs of regions, which yield the best profit in the description length calculated within the representation $S_1$ or $S_2$. Because description lengths differ depending on representation, the same segmentation algorithm will find different solutions using different representations. Relative quality of segmentation can be evaluated on the base of achieved description lengths.

It should be pointed out that segmentation algorithms based on the basic MDL principle are well-known [7, 8]. Novelty of our results consists in two parts. Firstly, the RMDL principle allows to compare representations (i.e. different segmentation algorithms themselves) in contrast to the basic MDL principle, which allows only to select best description of a single image. Secondly, the representation $S_2$ (that accounts for smooth variations of brightness within single image region) has not been used yet in the MDL-based segmentation algorithms and in application to the OCT images.

Mentioned algorithm does not find the best (relatively involved representation) descriptions of images, but it gives somewhat satisfactory results. Investigation of search algorithms exceeds the bounds of the present work.

## 4. OCT-image Segmentation Results

Consider application of the representations $S_1$ and $S_2$ to the task of segmentation of OCT images. Two OCT images of bio-tissues (cervix of the uterus [9]) are shown in fig. 1. These tissues consist of different number of layers. Results of their segmentation are shown in fig. 2.

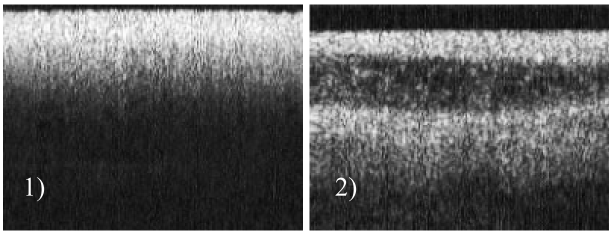


Figure 1. Example of layered OCT images.

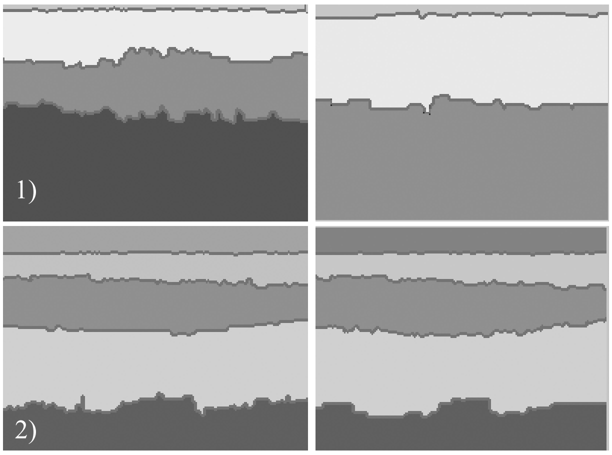


Figure 2. Results of segmentation of OCT images.

Segmentation results on the left are obtained within the representation $S_1$, and results on the right are obtained within the representation $S_2$. One can find corresponding description lengths in the table 1 ($L_k$ stands for the description length within the representation $S_k$).

Table 1. Description lengths.

| N | $L_0(f)$, bit | $L_1(f)$, bit | $L_2(f)$, bit |
|---|---|---|---|
| 1 | 212204 | 184672 | 175096 |
| 2 | 231201 | 212268 | 207864 |

The description length is reduced using the representation $S_1$, in which images are divided into regions, in comparison with the representation $S_0$ without image segmentation. That is, the representation $S_1$ is superior in accordance with the RMDL criterion. This result confirms that such the division is adequate (at least, as a first approximation) to the real structure of the OCT images. Moreover, detected regions have the horizontal orientation intrinsic to bio-tissues that was not put into segmentation algorithm as prior information. Found regions are also visually plausible.

The representation $S_2$ appeared to be even more efficient (in terms of RMDL criterion). This result is also plausible, since brightness values of deeper pixels in OCT images are reduced due to absorption of back-reflected light in tissues. This fading causes such the regular variations of brightness values inside image regions that can be described by regression models within the representation $S_2$. Results of application of the representation $S_1$ can be oversegmented, because there is a bias towards regions with constant brightness, so regions with regular brightness variations can be divided into false sub-regions. One can see that the principle problem, which should be solved in order to achieve correct results of layers detection, is the problem of the image representation selection, and the RMDL principle gives the objective criterion for it.

The representation $S_2$ can give some useful results in the case of clear layered structure, however it is still too simplified and it does not incorporate some specific information about structure of OCT images. Let us consider some examples of images with insertions (see fig. 3). These insertions cannot be correctly described within linear regression models and assumption of independently and identically distributed brightness values (or residuals after subtraction of regression models).

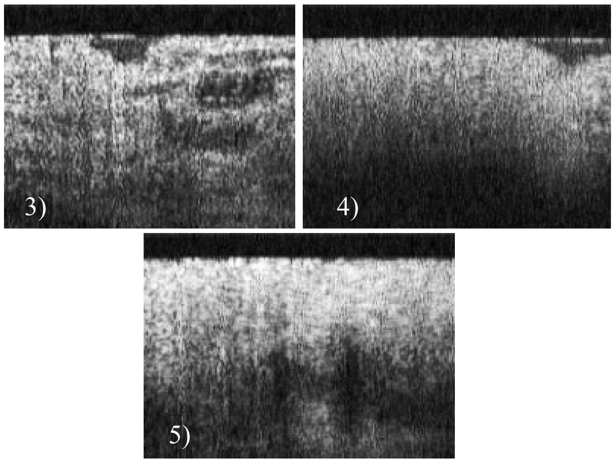


Figure 3. Example of OCT image with insertions.

Segmentation results are shown in the fig. 4 (left and right columns are obtained within the representations $S_1$ and $S_2$ correspondingly).

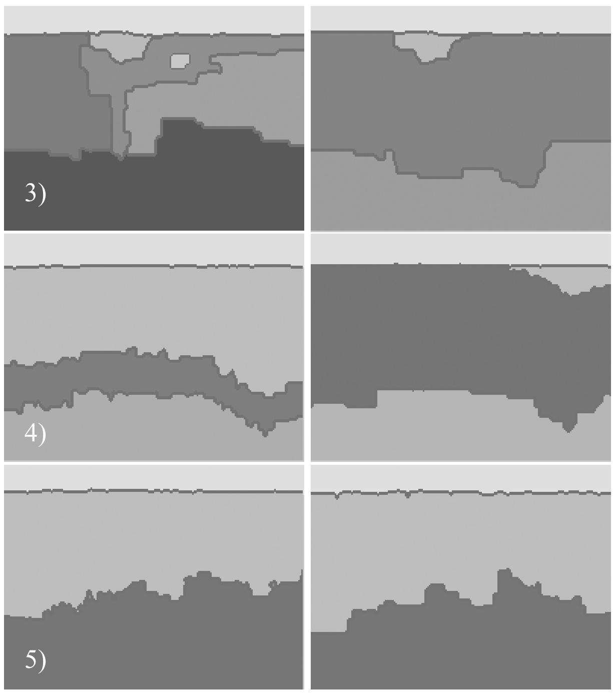


Figure 4. Results of segmentation of OCT images.

Image descriptions within the representation $S_1$ are visually worse in sense of detected layers and inclusions, and reduction of the description lengths within the representation $S_2$ indicates the same fact (see table 2).

Table 2. Description lengths.

| N | $L_0(f)$, bit | $L_1(f)$, bit | $L_2(f)$, bit |
|---|---|---|---|
| 3 | 235566 | 219641 | 215066 |
| 4 | 236421 | 213105 | 206204 |
| 5 | 225355 | 199267 | 194743 |

One should form some full sample of images, and to take into account the length of representations in order to correctly compare different representations on the base of the RMDL principle. However, we tested 20 different OCT images and found out that the representation $S_2$ always gives shorter (by the factor 1.03 in average) image descriptions in comparison with the representation $S_1$ that is always better (by the factor 1.10 in average) than the representation $S_0$. Consequently, the representation $S_2$ is the superior one. Lengths of considered representations differ not very much, so the superior representation is better in terms of the RMDL principle and allows getting better descriptions of OCT images structure. However, this representation probably can be further improved.

Situation can be different for some other representations or image samples: one representation can be better for one image from a sample, and other representation can be better for another image. For example, if an image contains only white noise, the representation $S_0$ will be the best one, because it doesn't require inclusion of some additional information (such as regions border) into image descriptions.

This implies that different representations can appear to be more efficient for describing OCT images of different types of bio-tissues. In this case, the description length criterion can help to recognize the type of bio-tissue by its OCT image on the base of selection of the most efficient representation from a set of representations corresponding to different types of tissues.

To discover more efficient OCT image representations further investigations are needed. The main result of this paper consists in conclusion that such investigations can be performed with guidance of the RMDL principle. No objective criteria to compare results of analysis of OCT images by different algorithms were available recently.

## 5. Conclusions

We considered three representations, within which OCT images were described.

One of the representations did not imply segmentation of images and was used as a base for comparison. The other two representations contained division of images into regions, which were supposed to correspond to layers in bio-tissues. These representations differed in the way of description of content of regions. Texture models for content description have not been utilized yet, but there is a principle possibility to incorporate them into the developed representations.

It was proposed to use the RMDL criterion for comparison of representations adequacy. It was shown that this criterion gives plausible comparison results. The values of the criterion indicated that layers of bio-tissues correspond to regions with different properties, and they also show that regular variations of brightness inside regions (caused by light absorption) should be taken into account in order to improve quality of descriptions.

Further improvements of the developed representations of OCT images should be carried out using the objective criterion based on the RMDL principle.

## References


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