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A Computer Study of 3-Element Groupoids

Joel Berman and Stanley Burris

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In [2] it is noted that current computing power may be inadequate to answer some basic questions about a three-element algebra, e.g., does it generate a Mal'cev variety? or a congruence distributive variety? With this challenge in mind a study of *all* three-element groupoids was undertaken.

Two-element groupoids are well known (just think of the logical connectives "and", "or", etc.), but 3-element groupoids offer a much bigger challenge. There are 19,683 groupoid tables on $\{0, 1, 2\}$; and up to isomorphism there are 3,330 such tables. We started by making a catalog of these groupoids by selecting from each isomorphism class (of groupoids on $\{0, 1, 2\}$) the lexicographically first member, treating each table as the 9 letter word:

row1 row2 row3.

Such a catalog of isomorphism representatives took only a couple of seconds to generate, and was stored in a 3,330-by-9 array.

In initial work on the 3-element groupoids the 3,330 isomorphism representatives were used and thirteen properties were analyzed, namely the properties on page 7 except numbers 3 and 4, together with the cardinalities of the free algebras on 0, 1 and 2 generators in *some* of the varieties generated by these groupoids. (For 0 generators the number of constant unary term functions was used.)

Using this information the pre-order given by $\mathbf{A} \leq \mathbf{B}$ *iff the clone of* $\mathbf{A}$ *is a subset of the clone of some isomorphic copy of* $\mathbf{B}$ was determined.[1] The induced equivalence relation, called *clone equivalence*, had 411 equivalence classes (actually the classes were determined before the ordering $\leq$). Using representatives of these 411 classes the cardinalities of the free algebras on

[1] In [6] Anne Fearnley has carried out a detailed study of the clones on three-elements which are of the form $\mathrm{Pol}(\rho)$ for ρ a unary or binary relation. This study does not bear directly on our work, but it is certainly a valuable companion.

0,1 and 2 generators in *all* the varieties generated by these groupoids were determined. This completed the analysis of the 13 properties mentioned above. Then, with the help of the partial ordering $\leq$ on the 411 clone equivalence representatives, the groupoids that generate congruence distributive or congruence modular varieties were determined. And finally the set of types of each of the 411 groupoids was determined. These three additional properties gave a total of 16 properties analyzed; the results are presented on pages 9 – 15.

Many properties, such as Mal'cev conditions and the sixteen properties considered, are invariant within each clone equivalence class. Consequently it was decided to make the presentation of data more compact by giving information in terms of the 411 clone equivalence representatives.

Now let us look in more detail at the sequence of steps followed. The main tool to analyze a given groupoid $\mathbf{A}$ was the computation of various subuniverses $S(\mathbf{A}, n, X)$ of powers $\mathbf{A}^n$ of $\mathbf{A}$, generated by suitable X — this tool for computer analysis was pioneered in the paper [4] of Berman and Wolk. From a programming point of view it was easiest to simply fix n as the largest power needed, and for smaller powers the coordinates considered were restricted.

Because of the theoretical work in [2] mentioned above the first project selected was to determine which $\mathbf{A}$'s had a Mal'cev term. Thus we concentrated on $S(\mathbf{A}, 15, X)$ where X consisted of the three 15-tuples:

px	0	0	0	0	1	1	1	1	2	2	2	2	0	1	2
py	0	0	1	2	0	1	1	2	0	1	2	2	0	1	2
pz	1	2	1	2	0	0	2	2	0	1	0	1	0	1	2 ;

$\mathbf{A}$ had a Mal'cev term iff the 15-tuple

pm	1	2	0	0	1	0	2	1	2	2	0	1	0	1	2

was in $S(\mathbf{A}, 15, X)$. The method for computing $S(\mathbf{A}, n, X)$ was a straightforward application of the groupoid operation to pairs of previously found elements, proceeding in successive sweeps, each sweep yielding terms with tree depth one greater than the previous sweep. The program made about 85 million applications of the groupoid operation per hour (on a Sun Sparc II).

Early attempts to find the Mal'cev terms soon led to the realization that attempting to compute the full $S(\mathbf{A}, 15, X)$ for each of the 3,330 isomorphism representatives to determine if a Mal'cev term existed would likely take too long. This led to the development of shortcuts via the multiphase attack described next.

Phase 1: When **A** does not have a Mal'cev term one can often show this by working with a small subset of the 15 coordinates, by showing that the restriction of pm to this subset does not lie in the closure under the groupoid operation of the restrictions of px, py, pz to this same subset. This was the first phase of the attack, to try several small subsets of the 15 coordinates in hopes of proving there is no Mal'cev term. If the coordinates are labelled 0 through 14 then the following choices were tried in the first phase: (0 1 2), (1 2 3),. . . , (9 10 11), (0 2 4 5 12 13), (1 3 8 10 12 14), (6 7 9 11 13 14), (evens), (odds). The sets of coordinates (0 2 4 5 12 13), (1 3 8 10 12 14), and (6 7 9 11 13 14) were used to determine if there is a Mal'cev term on $\{0,\ 1\}$, $\{0,\ 2\}$, and $\{1,\ 2\}$ respectively.

An important time saving observation was that if the Cayley table of **A** is the transpose of the Cayley table of **B**, i.e., $x \cdot_{\mathbf{A}} y = y \cdot_{\mathbf{B}} x$, then both or neither have Mal'cev terms. A subroutine to check if a new table to be considered was actually a representative of a transpose of a previous one was incorporated. Since there are 1,596 pairs related by transposes in the catalog of 3,330 isomorphism representatives there is indeed a considerable amount of work saved. For the cases which were not isomorphism representatives of transposes of previous ones, the test in Phase 1 usually required only a second or two per groupoid; and in this phase 1,792 of the groupoids were identified as not having Mal'cev terms.

Phase 2: If the attempt to refute the existence of a Mal'cev term for a groupoid **A** by using few coordinates failed then a quick proof of the existence of a Mal'cev term was sought by looking for

- a binary term $b(x, y)$ and unary terms $u_1(x), u_2(x)$ such that the unary terms have 2-element ranges and the equation $b(u_1(x), u_2(x)) \approx x$ holds in the groupoid.

Such a binary term $b(x, y)$ is said to be *invertible*. Suppose such a term exists. As **A** has passed Phase 1, there must exist terms $m_1(x, y, z)$ and $m_2(x, y, z)$ which give Mal'cev terms on the ranges of u_1, resp. u_2. Then it follows that

$$b(m_1(u_1(x), u_1(y), u_1(z)), m_2(u_2(x), u_2(y), u_2(z)))$$

is a Mal'cev term for **A**. This test, along with the test for transposes, identified 1,474 of the groupoids as having Mal'cev terms.

Phase 3: This left 64 of the 3,330 groupoids to be cataloged. For the final phase we turned to the generation of $S(\mathbf{A}, 15, X)$. In all but 7 of the remaining cases *pm* was found in the generated subuniverse.

The last seven cases (591, 746, 774, 951; and transposes 978, 1487, 1512) generate the same clone. This was discovered after trying a direct computation on #591 lasting over 60 hours, without completion or resolution of the existence of a Mal'cev term. Then it was found that the groupoid operation of #599 is a term function of #591; and #599 had a Mal'cev term. Thus all seven groupoids had a Mal'cev term.

With this a complete classification of which 3-element groupoids have Mal'cev terms[2] was obtained. In total only a few hours of CPU time were needed to run this part of the program.

Having given a fairly detailed account of the part of the project devoted to Mal'cev terms we will now simply outline some of the ideas behind the programs used for the study of the other properties, and for the determination of $\leq$.

- The test for a 3-element groupoid to be *quasiprimal* comes from [7], namely *it must be hereditarily simple*[3]*, have a Mal'cev term, and nontrivial subalgebras must be nonabelian*; thus a 3-element groupoid is quasiprimal iff it satisfies 7, 10 and 11 on page 7.

- The following result from Berman and McKenzie [3] was used to find the *Abelian* and *strongly Abelian* groupoids (by Abelian, respectively strongly Abelian, we mean the condition TC, respectively TC*, holds for all terms):

  Let $\mathbf{A}$ *be an algebra. Let* T *be the subuniverse of* $\mathbf{A}^4$ *generated by*
  $$\{(a,a,b,b) \mid a,b \in A\} \cup \{(a,b,a,b) \mid a,b \in A\}$$
  and let S *be the subuniverse of* $\mathbf{A}^4$ *generated by*
  $$\{(a,b,a,b) | a,b \in A\} \cup \{(a,b,c,c) \mid a,b,c \in A\}.$$
  (i) $\mathbf{A}$ *is Abelian if and only if whenever* $(a,a,b,c) \in T$ *or* $(a,b,a,c) \in T$*, then* $b = c$.

  (ii) $\mathbf{A}$ *is strongly Abelian if and only if* $(a,a,b,c) \in S$ *implies* $b = c$.

[2]In a previous version of this study we worked with *near* Mal'cev terms, defined as terms which act like Mal'cev terms provided $x \neq y$ or $y \neq z$. This allowed us to reduce the number of coordinates to 12 in the above work. However it was erroneously claimed that such terms would guarantee the existence of a Mal'cev term. We, and independently Ralph McKenzie, discovered this flawed reasoning. It turns out that 74 of the groupoids have near Mal'cev terms, but not Mal'cev terms (e.g., #565).

[3]Of course simple 3-element algebras are hereditarily simple.

- *Affine* is given by 7 and 8 on page 7.

- Those *generating decidable varieties* were determined as follows. For a 3-element groupoid **A** the McKenzie and Valeriote theorem says that $V(\mathbf{A})$ is decidable iff one of the following holds:

  - **A** is quasiprimal;

  - **A** is affine and the associated variety of modules is decidable;

  - **A** is strongly Abelian, essentially unary, and the monoid of the free algebra on one generator is linear.

  By inspecting the 5 affine algebras, namely the groupoids with numbers

  2124 2155 2302 2346 2934,

  one quickly sees that the associated rings (of idempotent binary term functions) have size 3, and hence the associated rings are $\mathbf{Z}_3$. Thus the varieties of modules associated with the affine algebras are actually vector spaces over a finite field, and hence they are decidable.

  And one also easily checks that each of the 13 strongly Abelian algebras, namely the groupoids with numbers

  1 14 27 275 366 394 1045 2029 2243 2466 3161 3242 3302,

  is essentially unary[4], and that the monoids of the free algebras on one generator are 1-generated, hence linear.

  Thus a 3-element groupoid generates a decidable variety iff it is quasiprimal or affine or strongly Abelian.

- For the pre-order $\leq$ defined on page 1 note that if $\mathbf{A} \leq \mathbf{B}$ and $\mathbf{B} \leq \mathbf{A}$ then the clone determined by **A** is the same as the clone of some isomorphic copy of **B**; if this is the case **A** and **B** are said to be *clone equivalent*, written $\mathbf{A} \sim \mathbf{B}$. A simple straightforward algorithm to determine if $\mathbf{A} \leq \mathbf{B}$ would be to generate $F(2)$, the elements of the free algebra on two generators, for the variety generated by **B** and check if the operation of any of the groupoids on $\{0, 1, 2\}$ isomorphic to **A** appears in $F(2)$. To do this for the more than 10 million pairs (**A**, **B**) would likely have required a prohibitive amount of time.

  So an alternative strategy was adopted. Before determining the pre-order $\leq$ the clone equivalence relation $\sim$ was determined. It was noted that the thirteen properties first studied were invariant under $\sim$; these properties were

[4] Actually any 3-element strongly Abelian algebra is essentially unary by 0.17iii of [8].

used to obtain an upper bound to $\sim$ with 132 equivalence classes. Then using time-limited calculations to obtain some of the $F(2)$'s a lower bound to $\sim$ was established with 440 equivalence classes. Next representatives from each of these 440 classes were selected, and using the induced equivalence relation from the 132 classes, certain $F(2)$'s were calculated to refine these induced equivalence classes. The final result was the collection of 411 clone equivalence classes. Then, taking a representative from each of the 411 classes (the lexicographically first elements) we returned to calculating appropriate $F(2)$'s to determine the partial order $\leq$ on the 411 representatives.

- The *free spectra* $|F(0)|$, $|F(1)|$, $|F(2)|$ of the varieties generated by the 411 clone equivalence representatives were determined by straightforward computations of the closure of suitable generators in $\mathbf{A}^3$ and $\mathbf{A}^9$.

- Using the partial ordering the 411 clone equivalence representatives were searched for those which *generate congruence distributive varieties* as follows. First the 12 representatives *with a majority term* were determined (see the figure on page 19), starting with the 10 quasiprimal representatives and considering subcovers. Next the subcovers of these 12 were examined to see which generated a congruence distributive variety; and this procedure was iterated if necessary. As it turns out, each of the groupoids so encountered which did not generate a congruence distributive variety had either a **1**, **2** or **5** in its set of types, or had a two-element subalgebra which did not generate a congruence distributive variety. For the others a $S(\mathbf{A}, 21, X)$ program was used to search for Jónsson terms. Whenever a new element was generated the program tried to find Jónsson terms incorporating a term corresponding to the new element.

- A similar approach was used for *congruence modularity*, starting with the classification of the congruence distributive and Mal'cev cases. Again the typeset of $\mathbf{A}$ and the two-element subalgebras of $\mathbf{A}$ sufficed to eliminate the negative cases; and a $S(\mathbf{A}, 21, X)$ program to search for Gumm terms made the verifications in the positive cases.

The Jónsson and Gumm terms presented on page 16 are simplest possible in the sense that the number is minimal in each case, and within that constraint the tree depth is smallest possible; and the Mal'cev terms have minimal tree depth.

The following table lists twelve of the properties considered, and the number of groupoids with each property (out of 19,683), the number up to isomorphism (out of 3,330), and the number up to clone equivalence (out of 411):

	Property	Number of Groupoids	Number of Groupoids up to isomorphism	Number of Groupoids up to clone equivalence
1.	generates a decidable variety	8,914	1,503	20
2.	is quasiprimal	8,851	1,485	10
3.	generates a congruence distributive variety	12,199	2,050	57
4.	generates a congruence modular variety	13,117	2,207	76
5.	is affine	12	5	3
6.	is strongly Abelian	51	13	7
7.	has a Mal'cev term	9,145	1,538	20
8.	is Abelian	117	27	16
9.	has an invertible binary term	11,442	1,907	32
10.	Abelian subalgebras are trivial	14,259	2,399	156
11.	is simple	16,009	2,693	191
12.	is rigid (i.e., trivial automorphism group)	19,422	3,237	372

The detailed data on the 411 clone equivalence representatives is presented in several tables. First there is a table of sixteen properties, followed by a summary of this table. There is a picture of an upper segment of this poset given by the 20 representatives with a Mal'cev term; and also one for the 12 representatives with a majority term. To relate the 411 clone equivalence representatives to the 3,330 isomorphism representatives the clone equivalence class of each of these 411 is presented. Instead of the ordering $\leq$ a table of covering elements is given, namely for each of the 411 representatives the subcovers and covers are listed. Next comes a ranking of the 411 representatives by the length of the maximal chain from the smallest element. At the end is a catalog of the 3,330 isomorphism representatives with the numbering used here. There are two types of entries, namely consider

301 #161

$$\begin{array}{|ccc} \hline 0 & 0 & 0 \\ 1 & 2 & 1 \\ 1 & 0 & 0 \end{array}$$

and

#305

$$\begin{array}{|ccc} \hline 0 & 0 & 0 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{array}$$

The first gives the groupoid table for isomorphism representative #301 and says that its clone equivalence representative is #161. The second gives the groupoid table for isomorphism representative #305 and says that it is one of the 411 clone equivalence representatives. The tables are abbreviated — the usual table for #305 would look like

$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & 0 & 0 \\ 1 & 1 & 2 & 1 \\ 2 & 1 & 1 & 1 \end{array}$$

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Some Properties

The next 6 pages discuss the following 16 items for each of the 411 clone equivalence representatives:

- **Types** — the set of types realized in the groupoid (in the sense of tame congruence theory).
- **Dec.** — the variety generated by the groupoid has a decidable first order theory.
- **QP** — the groupoid is quasiprimal.
- **CD** — the groupoid generates a congruence distributive variety.
- **CM** — the groupoid generates a congruence modular variety.
- **Aff.** — the groupoid is affine.
- **Str. Abel.** — the groupoid is strongly Abelian.
- **Mal'cev term** — the groupoid has a Mal'cev term.
- **Abel.** — the groupoid is Abelian.
- **Inv. terms** — the groupoid has an invertible binary term $b(x, y)$.
- **HNA** — nontrivial subalgebras of the groupoid are not Abelian.
- **Simple** — the groupoid is simple.
- **Rigid** — the groupoid is rigid.
- $|F(0)|$ — the number of constant unary term functions.
- $|F(1)|$ — the size of the free algebra on 1 generator in the variety generated by the groupoid.
- $|F(2)|$ — the size of the free algebra on 2 generators in the variety generated by the groupoid.

	Types	Dec.	QP	CD	CM	Aff.	Str. Abel.	Mal'cev term	Abel.	Inv. terms	HNA	Simple	Rigid	$\lvert F(0)\rvert$	$\lvert F(1)\rvert$	$\lvert F(2)\rvert$
1	1	•					•		•					1	2	3
2	1												•	1	3	6
3	1 5												•	0	2	5
4	1												•	1	2	5
5	1												•	1	3	8
6	1 5												•	0	2	7
8	5											•	•	1	4	17
9	1 5												•	0	2	7
10	1												•	1	3	12
11	1												•	1	3	12
12	1 5												•	0	2	11
13	1												•	1	3	7
14	1	•					•		•				•	1	3	5
15	1 5												•	0	2	7
16	3											•	•	1	4	53
18	5											•	•	0	2	13
19	1 3												•	1	3	16
21	1												•	0	2	6
22	3											•	•	1	3	24
24	3											•	•	0	2	8
25	1 3												•	1	3	8
26	1 3												•	1	4	17
27	1	•					•		•				•	0	2	4
30	1												•	1	2	6
31	1												•	1	3	9
32	1 5												•	0	2	8
33	5											•		1	2	5
34	5											•	•	1	4	27
35	5											•	•	0	2	11
36	1												•	1	3	18
37	1												•	1	3	18
38	1 5												•	0	2	15
39	1												•	1	3	12
40	1												•	1	3	10
41	1 5												•	0	2	11
42	3											•	•	1	4	74
43	3											•	•	1	4	70
44	4											•	•	0	2	20
45	3											•	•	1	3	24
46	3											•	•	1	4	65
47	3											•	•	0	2	10
48	3											•	•	1	3	36
49	3											•	•	1	4	71
50	3											•	•	0	2	14
51	1 3												•	1	3	12
52	1 3												•	1	4	23
53	1												•	0	2	6
59	5											•	•	1	4	28
60	1 5												•	0	2	8
61	3											•	•	1	4	55
63	3											•	•	0	2	29
65	5											•	•	1	4	23
66	4											•	•	0	2	19
67	3											•	•	1	4	137
69	4											•	•	0	2	29
70	1 3												•	1	3	26
72	1												•	0	2	10
73	3											•	•	1	3	50
75	3											•	•	0	2	18
78	1												•	0	2	6
79	1 5										•		•	0	3	11
80	5										•			0	1	3
81	1 5										•		•	0	3	13
82	5										•		•	0	1	5
83	5										•	•	•	0	3	24
85	1 3										•		•	0	3	30
87	3 5										•		•	0	1	14
88	1 5										•		•	0	3	19
89	1 5										•		•	0	3	17
90	5										•		•	0	1	10
91	3										•	•	•	0	3	102
93	3										•	•	•	0	1	34
94	3 5										•		•	0	2	18
96	1 5												•	0	1	6
97	3										•	•	•	0	2	30

	Types	Dec.	QP	CD	CM	Aff.	Str. Abel.	Mal'cev term	Abel.	Inv. terms	HNA	Simple	Rigid	$\|F(0)\|$	$\|F(1)\|$	$\|F(2)\|$
99	3											•	•	0	1	10
100	3 5										•		•	0	2	10
101	3 5										•		•	0	3	42
102	1 5												•	0	1	4
104	1 5												•	0	2	5
105	5										•		•	0	1	3
106	3 5										•		•	0	2	10
107	1 5													0	1	4
111	1 5										•		•	0	3	22
112	1 5												•	0	2	7
113	5										•		•	0	1	5
115	3										•	•	•	0	2	26
116	4											•	•	0	1	8
117	3										•	•	•	0	2	34
119	3											•	•	0	1	10
120	3										•	•	•	0	2	44
121	3											•	•	0	2	9
122	3											•	•	0	1	4
123	3 5										•		•	0	2	18
124	3 5										•		•	0	2	14
125	1 5												•	0	1	4
129	2 5												•	0	2	8
130	3										•	•	•	0	3	70
132	3										•	•	•	0	1	38
134	5										•	•	•	0	2	16
135	4											•	•	0	1	10
136	3										•	•	•	0	3	141
137	3											•	•	0	2	24
138	4										•	•	•	0	1	7
139	3 5										•		•	0	2	28
141	1 5												•	0	1	10
142	3										•	•	•	0	2	52
143	3										•	•	•	0	2	34
144	3											•	•	0	1	6
147	1 5													0	1	4
148	5										•	•		1	3	9
149	5										•	•	•	1	7	57
151	3			•	•						•	•	•	1	7	241
153	3			•	•					•	•	•	•	0	3	459
155	5										•	•	•	1	7	49
157	3			•	•						•	•	•	1	7	313
160	1 3										•		•	1	4	29
161	3			•	•					•	•	•	•	1	9	1377
162	1												•	0	3	12
163	3			•	•					•	•	•	•	1	6	480
165	3											•	•	0	3	132
166	1 3										•		•	1	4	31
168	1												•	0	3	8
169	3 5										•		•	0	4	24
170	1 5													0	2	8
171	3			•	•						•	•	•	1	9	497
175	5										•	•	•	0	4	56
176	3				•							•	•	0	2	68
178	3											•	•	0	2	68
179	3			•	•						•	•	•	0	2	70
180	3			•	•						•	•	•	1	4	64
182	3											•	•	0	3	60
183	3			•	•					•	•	•	•	1	6	594
184	3			•	•						•	•	•	0	4	272
185	3											•	•	0	2	24
186	3			•	•						•	•	•	1	4	52
188	5											•	•	0	2	12
194	5											•	•	0	2	16
195	3			•	•						•	•	•	0	2	102
198	4											•	•	0	2	13
199	1 3										•		•	1	5	114
201	1												•	0	3	22
203	3											•	•	0	2	36
204	3											•	•	0	2	32
207	1 3										•		•	1	3	8
209	3 5										•		•	0	2	60
213	3			•	•						•	•	•	1	6	408
215	3			•	•						•	•	•	0	2	136
216	1 3										•			1	2	7
218	3											•	•	0	2	24

	Types	Dec.	QP	CD	CM	Aff.	Str. Abel.	Mal'cev term	Abel.	Inv. terms	HNA	Simple	Rigid	$\lvert F(0)\rvert$	$\lvert F(1)\rvert$	$\lvert F(2)\rvert$
219	3			•	•						•	•	•	1	2	40
221	3											•	•	0	2	48
222	3			•	•						•		•	1	2	14
223	3			•	•						•		•	1	3	48
224	1 3												•	0	2	16
235	1 3												•	0	2	16
239	3			•	•						•			1	2	6
241	1 3												•	0	2	8
244	3			•	•						•	•	•	0	2	160
250	3			•	•						•	•	•	0	2	198
252	4											•	•	0	2	18
253	3			•	•						•	•		1	2	18
255	3											•	•	0	2	72
257	1												•	0	3	26
258	1												•	0	3	28
259	1 5												•	0	1	18
260	1												•	0	3	10
261	1												•	0	3	12
262	1 5												•	0	1	10
263	3											•	•	0	3	90
265	3											•	•	0	1	30
266	3											•	•	0	3	90
267	4											•		0	1	4
268	3											•	•	0	3	54
269	3											•	•	0	1	10
270	1 3												•	0	3	36
271	1												•	0	1	4
272	1												•	0	3	14
273	1 5													0	1	6
274	3											•	•	0	3	54
275	1	•					•		•					0	1	2
278	3											•	•	0	1	44
280	1 3												•	0	2	20
281	1												•	0	1	6
282	3											•	•	0	3	162
283	1 2												•	0	2	16
284	1 5												•	0	1	6
286	1 3												•	0	2	24
287	1													0	1	4
298	3			•	•						•	•		1	3	45
305	1 3										•		•	0	4	128
306	1 2												•	0	2	32
308	1												•	0	2	20
309	1 3										•		•	0	2	32
311	1													0	2	16
316	1												•	0	2	12
317	1 3										•		•	0	2	48
320	1												•	0	2	6
321	1													0	2	8
322	1 3										•			1	3	13
341	3			•	•						•	•		1	3	25
347	3			•	•						•	•		0	1	15
349	3			•	•						•	•	•	0	1	153
353	1												•	0	2	10
354	1 3										•		•	0	1	10
356	1													0	1	4
359	1													0	1	8
366	1	•					•		•					0	2	4
376	1												•	1	3	13
377	1												•	1	3	13
378	1 5												•	0	2	15
379	1												•	1	3	14
380	1												•	1	3	14
381	1 5												•	0	2	14
382	3											•	•	1	4	134
384	3											•	•	0	2	46
385	3											•	•	1	4	107
387	3											•	•	0	2	37
388	3											•	•	1	4	170
390	3											•	•	0	2	58
391	1 3												•	1	4	35
405	1								•				•	1	3	6
406	1												•	1	3	6
407	1 5												•	0	2	8
410	4											•	•	0	2	27

	Types	Dec.	QP	CD	CM	Aff.	Str. Abel.	Mal'cev term	Abel.	Inv. terms	HNA	Simple	Rigid	$\|F(0)\|$	$\|F(1)\|$	$\|F(2)\|$
417	1 3												•	1	4	23
434	3											•	•	1	4	164
436	4											•	•	0	2	33
437	3											•	•	1	4	245
439	3											•	•	0	2	83
454	1 3										•		•	0	3	31
455	1 3										•		•	0	3	31
456	3 5										•		•	0	1	15
457	1 3										•		•	0	3	33
458	1 3										•		•	0	3	33
459	3 5										•		•	0	1	17
460	3										•	•	•	0	3	138
462	3										•	•	•	0	1	46
463	3										•	•	•	0	3	174
465	3										•	•	•	0	1	58
469	3										•		•	0	3	78
483	1 5										•		•	0	3	23
484	1 5												•	0	2	8
485	5										•		•	0	1	6
487	3										•	•	•	0	2	36
488	4											•	•	0	1	12
493	3											•	•	0	2	12
494	3										•	•	•	0	1	10
495	3 5										•		•	0	3	51
496	3 5										•		•	0	2	20
512	3										•	•	•	0	3	168
513	3											•	•	0	2	28
514	4										•	•		0	1	5
515	3										•	•	•	0	3	249
517	3										•	•	•	0	1	83
519	3										•	•	•	0	2	58
520	3											•	•	0	1	20
522	3											•	•	0	2	40
532	3			•	•						•	•	•	1	9	849
534	3	•	•	•	•			•		•	•	•	•	0	3	2187
538	3	•	•	•	•			•		•	•	•	•	1	9	6561
562	4			•	•						•	•	•	0	4	82
563	3				•			•				•	•	0	2	324
565	3											•	•	0	2	324
566	3			•	•						•	•	•	0	2	486
571	3	•	•	•	•			•			•	•	•	0	4	1296
600	1 3										•		•	1	4	49
602	3 5										•		•	0	2	100
603	1 3										•		•	1	5	138
604	1 3										•		•	1	5	140
606	3			•	•						•	•	•	1	4	104
608	3			•	•						•	•	•	0	2	208
609	1 3										•		•	1	5	136
612	1 3										•		•	1	5	140
613	1 3										•		•	1	5	144
615	3			•	•						•	•	•	1	6	624
618	3			•	•						•	•	•	1	6	768
620	3			•	•						•	•	•	0	2	256
624	3			•	•						•		•	1	3	72
629	1 3										•		•	1	5	140
630	1 3										•		•	1	5	144
632	3			•	•						•	•	•	1	4	78
638	1 3										•		•	1	5	145
639	1 3										•		•	1	5	154
652	3			•	•					•	•	•	•	1	6	1008
654	3			•	•						•	•	•	0	2	336
658	3			•	•					•	•	•	•	1	6	1458
677	1												•	0	3	18
678	1								•				•	0	3	10
679	1 5												•	0	1	10
680	1												•	0	3	34
681	1 5													0	1	10
682	3											•	•	0	3	108
684	3											•	•	0	1	60
687	3											•	•	0	3	180
690	1 3												•	0	3	36
691	3											•	•	0	3	336
693	3											•	•	0	1	112
695	3											•	•	0	2	80
696	3											•	•	0	1	24

	Types	Dec.	QP	CD	CM	Aff.	Str. Abel.	Mal'cev term	Abel.	Inv. terms	HNA	Simple	Rigid	$\|F(0)\|$	$\|F(1)\|$	$\|F(2)\|$
697	3											•	•	0	3	486
698	3											•	•	0	2	72
704	3											•	•	0	2	48
705	4											•	•	0	1	14
707	3											•	•	0	2	48
710	3											•	•	0	1	162
712	3											•	•	0	2	72
755	3			•	•						•	•	•	0	4	896
756	3				•							•	•	0	2	224
758	3											•	•	0	2	224
780	3											•	•	0	2	144
792	3			•	•					•	•	•	•	1	7	409
870	3	•	•	•	•			•			•	•	•	0	1	729
885	3	•	•	•	•			•			•	•		0	1	27
898	3	•	•	•	•			•			•	•		0	1	9
984	3											•	•	0	2	36
1012	1 2												•	1	3	18
1014	3											•	•	1	3	34
1038	3											•	•	1	3	38
1040	3											•	•	0	2	13
1065	1 2								•				•	1	3	6
1066	1 2												•	1	4	12
1084	2 5												•	0	2	20
1086	3											•	•	0	2	36
1107	3											•	•	0	2	40
1108	3										•	•		0	1	3
1132	2 5												•	0	2	8
1133	2 5												•	0	2	13
1151	1 2												•	1	5	130
1153	3				•					•		•	•	1	6	672
1176	3				•			•		•		•	•	1	6	972
1200	1 2												•	1	5	34
1202	3			•	•					•	•	•	•	1	4	164
1205	3			•	•					•	•	•	•	1	4	240
1219	3				•					•		•	•	1	4	160
1221	3				•					•		•	•	1	4	216
1225	3			•	•					•	•		•	1	4	68
1227	1 3												•	0	2	20
1231	3			•	•					•	•		•	1	4	96
1233	1 3												•	0	2	32
1242	2 3				•					•			•	1	4	64
1249	3			•	•					•	•		•	1	6	432
1268	2 3				•					•			•	1	4	40
1269	2 3				•					•	•		•	1	6	252
1271	1 3												•	0	3	66
1277	1 3												•	0	3	144
1281	1 2												•	0	2	12
1321	2 3				•					•			•	1	6	288
1433	2 5												•	0	2	68
1437	2 5												•	0	2	20
1481	1 2								•				•	1	5	18
1700	1 2								•				•	1	3	6
1708	1 2												•	1	3	14
1791	3 5										•		•	0	4	264
1793	1 5												•	0	2	28
1799	1 5												•	0	2	52
1818	1 5												•	0	2	20
1829	1 2										•		•	1	5	49
1837	1 2										•		•	1	4	15
1962	1 5												•	0	2	12
2088	1 2													1	2	6
2090	3				•			•				•	•	1	2	36
2102	2				•			•					•	1	2	9
2104	3				•			•				•		1	2	12
2116	2				•			•					•	1	3	42
2124	2	•			•	•		•	•			•		1	3	9
2135	1 5													0	2	20
2144	1 2										•			1	3	21
2159	3	•	•	•	•			•			•	•		1	3	81
2171	1 5													0	2	12
2346	2	•			•	•		•	•			•		0	1	3
2353	1 3										•		•	2	5	18
2354	1 3										•		•	2	9	514
2357	3			•	•					•	•	•	•	2	12	2688
2369	3			•	•					•	•		•	2	12	1152

	Types	Dec.	QP	CD	CM	Aff.	Str. Abel.	Mal'cev term	Abel.	Inv. terms	HNA	Simple	Rigid	$\|F(0)\|$	$\|F(1)\|$	$\|F(2)\|$
2393	3	•	•	•	•			•		•	•	•	•	2	12	3888
2407	3	•	•	•	•			•		•	•	•	•	3	27	19683
2428	2 3				•			•		•	•		•	2	12	672
2430	1 2												•	0	6	68
2436	3											•	•	0	6	972
2460	1												•	0	5	34
2461	1												•	0	5	38
2462	1												•	0	5	34
2463	1								•				•	0	5	18
2464	3											•	•	0	6	672
2466	1	•					•		•				•	0	3	6
2467	1												•	0	5	14
2472	1												•	0	3	14
2476	1 3												•	0	6	132
2478	1												•	0	5	74
2479	1												•	0	5	78
2480	1												•	0	5	66
2483	1												•	0	5	26
2486	1 3												•	0	6	288
2487	3											•	•	0	6	432
2493	3											•	•	0	6	108
2529	3	•	•	•	•			•			•	•		0	3	27
2539	1 3												•	0	6	108
2545	1 3												•	0	6	72
2552	1										•		•	0	6	52
2558	1										•		•	0	5	18
2636	1 3												•	0	6	36
2654	1 3									•	•		•	2	6	56
2686	1 3									•	•		•	2	6	72
2698	1 3										•		•	3	10	83
2702	1 3									•	•		•	3	12	207
2739	1												•	0	5	78
2799	1 3									•	•		•	3	15	333
2803	1 3									•	•		•	3	15	525
2934	2	•			•	•		•	•			•		0	3	9
3242	1	•					•		•			•		0	3	6

Some Information on the Poset of 411 Clone Equivalence Representatives

DECIDABLE: 1 14 27 275 366 534 538 571 870 885 898 2124 2159 2346 2393 2407 2466 2529 2934 3242

number = 20

QUASIPRIMAL: 534 538 571 870 885 898 2159 2393 2407 2529

number = 10
minimals: 571 885 898
maximal non quasiprimal cases: 161 532 658 792 1176 2124 2357 2428 2436 2803 2934

MAL'CEV: 534 538 563 571 870 885 898 1176 2090 2102 2104 2116 2124 2159 2346 2393 2407 2428 2529 2934

number = 20
minimals (with Mal'cev terms):
563: $((xy)(z((xz)(xz))))(((z(x(zz)))((z(xx))((xx)(xy))))(((z(xx))((xx)(zz)))(((xx)(zz))((yz)(zz)))))$
2102: $(x(yz))(z(x(xz)))$
2104: $(xy)z$
2346: $y(xz)$
maximal non Mal'cev cases: 161 532 658 792 2357 2436 2803 3242

CD: 151 153 157 161 163 171 179 180 183 184 186 195 213 215 219 222 223 239 244 250 253 298 341 347 349 532 534 538 562 566 571 606 608 615 618 620 624 632 652 654 658 755 792 870 885 898 1202 1205 1225 1231 1249 2159 2357 2369 2393 2407 2529

number = 57
minimals (with Jónsson terms):
179: $p1 = (x(((xx)y)z))(((xz)y)(x(xz)))$,
$p2 = ((z((zz)x))(xx))(((z((zz)y))((z(xx))((yx)x)))((((zz)x)z)(((zx)(zy))(xy))))$
186: $p1 = x((xx)(((xx)(xy))(xz)))$, $p2 = (x((xx)z))((xz)((xy)(xx)))$, $p3 = (z((xx)z))((zx)(y(zz)))$,
$p4 = z((zz)((yy)(xx)))$
215: $p1 = (x((xy)((xy)(zx))))(((xy)(yx))((z(yx))(zx)))$, $p2 = (z((zx)((zy)(zz))))(((zz)(xx))((yx)y))$
239: $p1 = x((xy)z)$, $p2 = z(yx)$
347: $p1 = (x(yx))((x(zx))(z(yx)))$, $p2 = (z(yz))((yz)x)$
562: $p1 = (xy)(z((xx)(yy)))$
898: $p1 = (((xy)x)((zx)((yx)(zx))))$
maximal non CD cases: 149 155 175 515 1176 1791 2124 2428 2436 2803 2934

CM: 151 153 157 161 163 171 176 179 180 183 184 186 195 213 215 219 222 223 239 244 250 253 298 341 347 349 532 534 538 562 563 566 571 606 608 615 618 620 624 632 652 654 658 755 756 792 870 885 898 1153 1176 1202 1205 1219 1221 1225 1231 1242 1249 1268 1269 1321 2090 2102 2104 2116 2124 2159 2346 2357 2369 2393 2407 2428 2529 2934

number = 76
minimals (with Gumm terms if not CD or Mal'cev):
176: $p1 = x$, $p2 = ((xx)(xy))((x(xz))((xz)(yx)))$, $p3 = (((zz)x)((zz)(yy)))((zx)(((zx)(zy))(xx)))$

179 186 215 239 347 562

756: $p1 = x$, $p2 = (x(yz))(yz)$, $p3 = (z(xz))((xz)(xy))$
1242: $p1 = x$, $p2 = x((x(yz))(z(yx)))$, $p3 = (z(xy))((yx)(xy))$
1268: $p1 = x$, $p2 = x((x(yz))(z(yx)))$, $p3 = z(yx)$

2102 2104 2346
maximal non CM cases: 149 155 175 305 437 515 1791 2354 2436 2803 3242

AFFINE: 2124 2346 2934

number = 3
minimals: 2346
maximals: 2124 2934

STRONGLY ABELIAN: 1 14 27 275 366 2466 3242

number = 7
maximals: 14 366 2466 3242
minimal non strongly abelian cases: 2 3 4 13 21 33 53 78 80 102 104 105 107 122 125 147 168 267 271 287 320 356 405 406 678 1065 1108 1700 2346

ABELIAN: 1 14 27 275 366 405 678 1065 1481 1700 2124 2346 2463 2466 2934 3242

number = 16
maximals: 1481 2124 2463 2934
minimal non Abelian cases: 2 3 4 13 21 33 53 78 80 102 104 105 107 122 125 147 168 267 271 287 320 356 406 1108

INVERTIBLE: 153 161 163 183 534 538 652 658 792 1153 1176 1202 1205 1219 1221 1225 1231 1242 1249 1268 1269 1321 2357 2369 2393 2407 2428 2654 2686 2702 2799 2803

number = 32
minimals (with binary/unary pairs):
- 153: b(x,y) = xy, u1 = xx, u2 = x(xx)
- 163: b(x,y) = yx, u1 = (xx)x, u2 = x((xx)x)
- 1225: b(x,y) = xy, u1 = x(xx), u2 = (xx)x
- 1242: b(x,y) = yx, u1 = (xx)x, u2 = x((xx)x)
- 1268: b(x,y) = yx, u1 = (xx)x, u2 = x((xx)x)
- 2654: b(x,y) = yx, u1 = xx, u2 = x(xx)

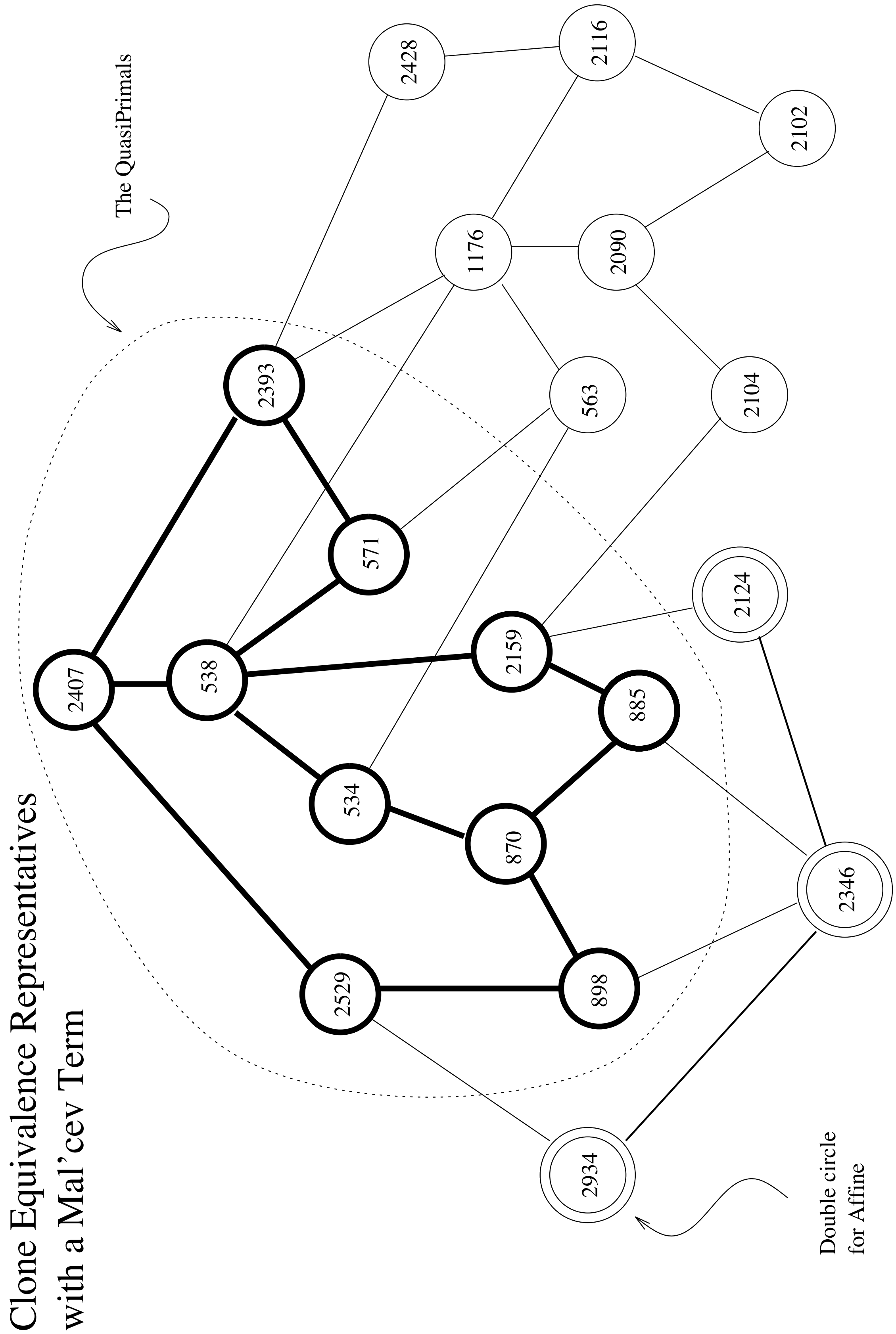
maximal non invertible cases: 157 437 515 532 571 618 870 1829 2090 2116 2159 2354 2436 2529 2552 2698

SIMPLE: 8 16 18 22 24 33 34 35 42 43 44 45 46 47 48 49 50 59 61 63 65 66 67 69 73 75 83 91 93 97 99 115 116 117 119 120 121 122 130 132 134 135 136 137 138 142 143 144 148 149 151 153 155 157 161 163 165 171 175 176 178 179 180 182 183 184 185 186 188 194 195 198 203 204 213 215 218 219 221 244 250 252 253 255 263 265 266 267 268 269 274 278 282 298 341 347 349 382 384 385 387 388 390 410 434 436 437 439 460 462 463 465 487 488 493 494 512 513 514 515 517 519 520 522 532 534 538 562 563 565 566 571 606 608 615 618 620 632 652 654 658 682 684 687 691 693 695 696 697 698 704 705 707 710 712 755 756 758 780 792 870 885 898 984 1014 1038 1040 1086 1107 1108 1153 1176 1202 1205 1219 1221 2090 2104 2124 2159 2346 2357 2393 2407 2436 2464 2487 2493 2529 2934 3242

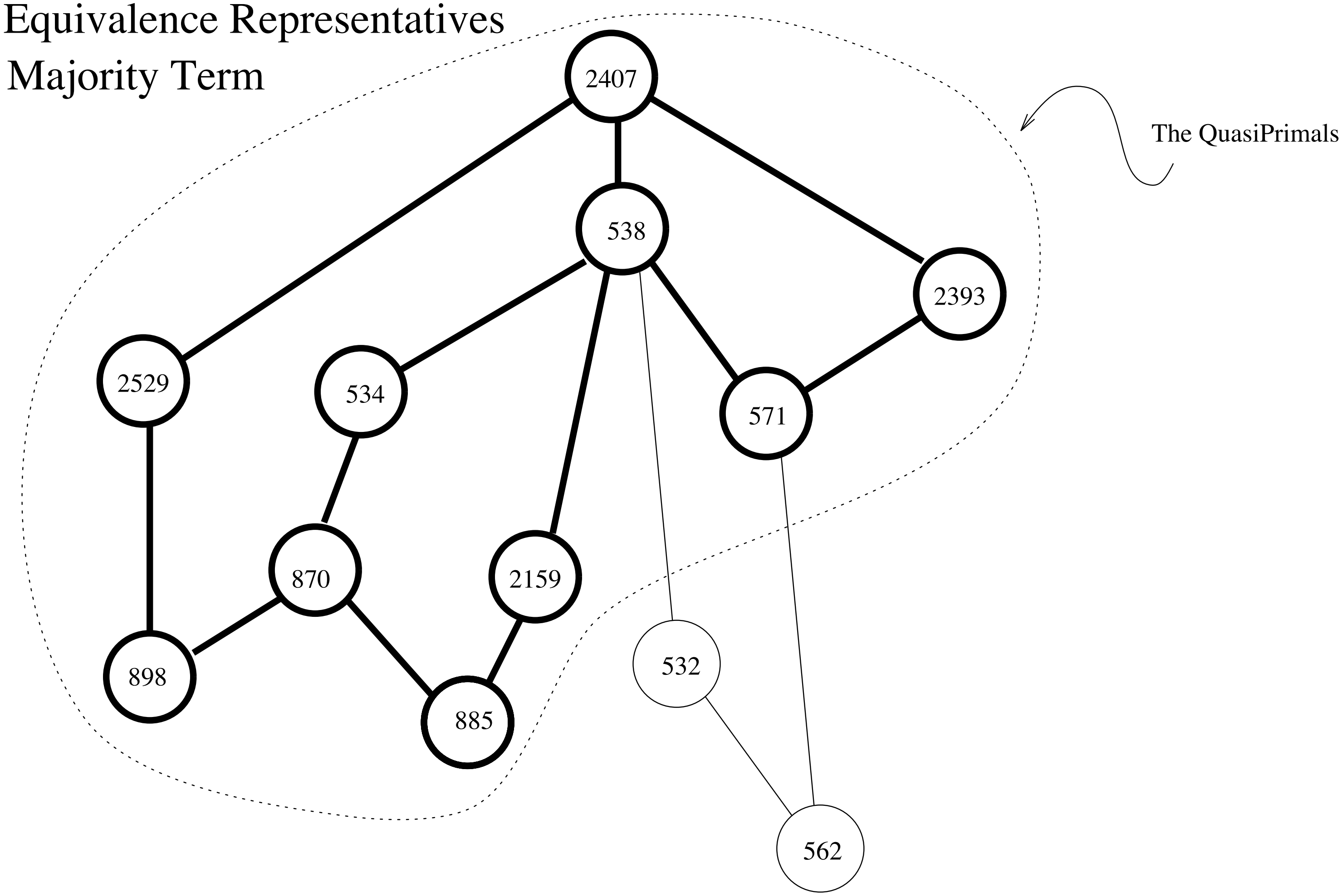
number = 191
minimals: 8 18 24 33 35 47 83 99 116 119 121 122 134 138 144 186 188 194 198 267 514 984 1108 2346 3242
maximal non simple cases: 169 2369 2428 2803

NON-RIGID: 1 33 80 107 147 148 170 216 239 253 267 273 275 287 298 311 321 322 341 347 356 359 366 514 681 885 898 1108 2088 2104 2124 2135 2144 2159 2171 2346 2529 2934 3242

number = 39
maximals = 2159 2529
minimal rigid cases: 4 14 27 82 102 105 122 125 271


Clone Equivalence Representatives
with a Mal'cev Term
The QuasiPrimals
Double circle
for Affine
2428
2116
2102
1176
2090
2393
563
2104
571
2124
2407
538
2159
885
534
870
2346
2529
898
2934

Clone Equivalence Representatives with a Majority Term



Clone Equivalence Classes

This is a listing of the clone equivalence classes among the 3,330 representatives of isomorphism types. Two groupoids are clone equivalent if some isomorphic copy of the first generates the same clone as the second. A boxed number denotes the beginning of an equivalence class. There are 411 such classes below — the first (boxed) element of each is the clone equivalence representative of that class.

1 **2** **3** **4** 7 **5** 28 **6** 29 **8** 54 **9** 55 **10** 367 **11** 368 **12** 369 **13** 393
14 394 **15** 395 **16** 17 20 23 419 420 996 1020 **18** 421 **19** 995 **21** 997 **22** 1019 **24**
1021 **25** 1043 **26** 1044 **27** 1045 **30** **31** **32** **33** 56 **34** 57 **35** 58 **36** 370
37 371 **38** 372 **39** 396 **40** 397 **41** 398 **42** 422 **43** 423 **44** 424 **45** 998 **46** 999
47 1000 **48** 1022 **49** 1023 **50** 1024 **51** 1046 **52** 1047 **53** 1048 **59** **60** **61** 62
64 373 374 399 **63** 375 **65** 400 **66** 401 **67** 68 71 74 77 425 426 1002 1026 1050 **69** 427 **70**
76 1001 1049 **72** 1003 **73** 1025 **75** 1027 **78** 1051 **79** **80** **81** 103 **82** 84 **83** 126
85 86 108 109 445 446 448 449 **87** 110 447 450 **88** 471 **89** 472 **90** 473 **91** 92 95 98 114 118
497 498 500 1068 1071 1092 **93** 499 **94** 1067 **96** 1069 **97** 1091 **99** 1093 **100** 1111 **101**
1112 **102** 1113 **104** **105** **106** 127 **107** 128 **111** 474 **112** 475 **113** 476 **115** 501
116 502 **117** 1070 **119** 1072 **120** 1094 **121** 1064 **122** 1095 **123** 1114 **124** 1115
125 1116 **129** **130** 131 133 451 452 477 **132** 453 **134** 478 **135** 479 **136** 140 503 1074
137 146 504 1118 **138** 505 **139** 145 1073 1117 **141** 1075 **142** 1096 **143** 1097 **144** 1098
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1904 2070 2138 2158 2164 **155** 550 **157** 158 575 576 **160** 1134 **161** 164 167 177 181 196 200 289 290
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2209 2211 2212 2227 **162** 1136 **163** 202 242 243 245 246 1157 1163 1533 1534 1555 1556 **165** 276 277
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1577 1578 1977 1996 2014 **184** 197 206 582 1161 1187 **185** 1162 **186** 1183 **188** 1185 **194** 556
195 557 **198** 583 **199** 205 208 210 211 225 226 228 229 1140 1186 1528 1530 1531 1549 1550 1552 1553
201 1142 **203** 1164 **204** 1165 **207** 1527 **209** 212 227 230 1529 1532 1551 1554 **213** 214 217
220 231 232 234 237 1571 1572 1574 1575 1972 1974 1991 1993 **215** 233 1573 1576 **216** 1971 **218** 1617
219 236 1976 1992 **221** 238 1624 1994 **222** 1973 **223** 240 2010 2012 **224** 1609 **235** 1975
239 2011 **241** 2013 **244** 247 1340 1518 **250** 1193 **252** 1510 **253** 1995 **255** 1332 **257**
1593 **258** 1594 **259** 1595 **260** 1507 **261** 1396 **262** 1441 **263** 264 1605 1606 **265** 1607
266 1616 **267** 1618 **268** 1399 **269** 1444 **270** 2028 **271** 1610 **272** 1596 **273** 1597
274 1608 **275** 2029 **278** 1452 **280** 1250 **281** 1272 **282** 285 1611 1623 **283** 288 1612 2030
284 1125 **286** 1253 **287** 1274 **298** 336 2036 2210 **305** 310 319 364 1653 1678 2052 2242 **306**
360 1654 2108 **308** 1675 **309** 355 1676 2017 **311** 365 2067 2257 **316** 1656 **317** 358 1258 1261
320 1057 **321** 2053 **322** 2172 **341** 361 2194 2226 **347** 1898 **349** 351 1449 1890 **353** 1054
354 1122 **356** 2032 **359** 1278 **366** 2243 **376** **377** **378** **379** 402 **380** 403 **381**

404 **382** 383 408 409 428 429 431 432 **384** 430 **385** 386 411 412 1004 1005 1007 1008 **387** 413 1006 1009 **388** 389 414 415 1028 1029 1031 1032 **390** 416 1030 1033 **391** 392 1052 1053 **405** **406** **407** **410** 433 **417** 418 1055 1056 **434** 435 **436** **437** 438 440 441 443 444 1010 1011 1013 1015 1018 1034 1035 1037 1039 1042 1058 1059 1061 1063 **439** 442 775 979 **454** **455** **456** **457** 480 **458** 481 **459** 482 **460** 461 486 506 507 509 **462** 508 **463** 464 466 467 489 490 492 1076 1077 1079 1080 1099 1100 1102 **465** 468 491 1078 1081 1101 **469** 470 1120 1121 **483** **484** **485** **487** 510 **488** 511 **493** 1016 **494** 1088 **495** 1123 **496** 1124 **512** **513** **514** **515** 516 518 521 1082 1083 1085 1087 1090 1103 1106 1126 1129 **517** 907 **519** 1104 1110 1131 **520** 699 **522** 1127 **532** 533 535 536 558 559 561 **534** 537 540 543 546 560 569 586 594 718 721 724 727 730 733 736 739 744 753 761 769 778 786 794 797 800 803 806 809 812 815 820 823 826 829 832 837 840 845 848 851 854 857 862 865 868 869 871 872 874 875 877 878 880 881 883 884 886 887 889 890 892 893 894 896 897 899 900 902 905 908 909 911 914 916 917 918 921 925 928 931 934 937 944 949 958 973 982 1145 1148 1168 1293 1296 1315 1360 1363 1382 1385 1417 1420 1423 1425 1426 1428 1429 1431 1432 1434 1436 1438 1440 1447 1448 1450 1453 1456 1471 1474 1493 1637 1640 1659 1681 1759 1762 1767 1770 1781 1789 1802 1810 1823 1826 1831 1834 1845 1848 1853 1866 1869 1874 1885 1886 1891 1892 1893 1894 1899 1900 1901 1902 1905 1906 1907 1909 1910 1915 1920 1940 1960 2041 2056 2119 2129 2146 2152 2162 2165 2180 2198 2215 2231 2246 2272 2285 2307 2332 **538** 539 541 542 544 545 547 548 564 567 568 570 573 584 585 587 590 592 593 595 598 716 717 719 720 722 723 725 726 728 729 731 732 734 735 737 738 740 741 742 743 745 748 751 752 754 757 759 760 762 765 767 768 770 773 776 777 779 782 784 785 787 790 795 796 798 799 801 802 804 805 807 808 810 811 813 814 816 817 821 822 824 825 827 828 830 831 833 834 835 836 838 839 841 842 846 847 849 850 852 853 855 856 858 859 860 861 863 864 866 867 926 927 929 930 932 933 935 936 938 939 940 941 942 943 945 946 950 953 956 957 959 962 964 965 966 969 974 977 980 981 983 986 988 989 990 993 1143 1144 1146 1147 1149 1150 1152 1154 1156 1166 1167 1169 1172 1175 1188 1189 1191 1194 1197 1283 1286 1289 1291 1292 1294 1295 1297 1298 1300 1302 1304 1306 1313 1314 1316 1319 1322 1328 1335 1336 1338 1341 1344 1350 1353 1356 1358 1359 1361 1362 1364 1365 1367 1369 1371 1373 1376 1379 1380 1381 1383 1384 1386 1387 1389 1391 1393 1395 1398 1401 1402 1403 1405 1406 1408 1409 1411 1413 1415 1462 1465 1468 1469 1470 1472 1473 1475 1476 1478 1480 1482 1484 1491 1492 1494 1497 1500 1506 1513 1514 1516 1519 1522 1635 1636 1638 1639 1641 1642 1643 1644 1645 1646 1647 1648 1657 1658 1660 1663 1665 1666 1667 1669 1679 1680 1682 1684 1686 1687 1688 1690 1757 1758 1760 1761 1763 1764 1765 1766 1768 1769 1771 1772 1773 1774 1775 1776 1777 1778 1779 1780 1782 1785 1787 1788 1790 1792 1794 1795 1796 1798 1800 1801 1803 1806 1808 1809 1811 1813 1815 1816 1817 1819 1821 1822 1824 1825 1827 1828 1830 1832 1833 1835 1836 1838 1839 1840 1841 1842 1843 1844 1846 1847 1849 1850 1851 1852 1854 1855 1856 1857 1858 1859 1860 1861 1862 1863 1864 1865 1867 1868 1870 1871 1872 1873 1875 1876 1877 1878 1879 1880 1881 1882 1883 1884 1911 1912 1913 1914 1916 1917 1918 1919 1921 1922 1923 1924 1925 1926 1927 1928 1929 1930 1931 1932 1933 1936 1938 1939 1941 1943 1945 1946 1947 1949 1951 1952 1953 1956 1958 1959 1961 1963 1965 1966 1967 1969 2039 2040 2042 2043 2044 2045 2047 2049 2051 2054 2055 2057 2059 2062 2068 2069 2071 2073 2076 2117 2118 2120 2121 2122 2123 2125 2126 2127 2128 2130 2132 2136 2137 2139 2141 2145 2147 2148 2149 2150 2151 2153 2154 2156 2157 2160 2161 2167 2168 2178 2179 2181 2182 2183 2184 2185 2186 2187 2188 2189 2190 2196 2197 2199 2200 2201 2202 2203 2204 2205 2206 2207 2208 2213 2214 2216 2217 2218 2219 2220 2221 2222 2223 2224 2225 2229 2230 2232 2233 2234 2235 2236 2237 2238 2239 2240 2241 2244 2245 2247 2249 2251 2252 2253 2255 2258 2259 2260 2262 2264 2265 2266 2268 2271 2273 2274 2275 2276 2278 2279 2280 2281 2282 2283 2284 2286 2287 2288 2289 2290 2291 2292 2294 2295 2296 2297 2298 2299 2300 2301 2303 2304 2305 2306 2308 2309 2310 2311 2313 2314 2315 2316 2317 2319 2323 2324 2325 2327 2331 2333 2334 2335 2336 2337 2338 2339 2341 2342 2344 2345 2348 2349 **562** **563** 572 747 906 952 1171 1446 **565** 574 588 596 749 771 954 975 1173 1179 1192 1199 1312 1331 1490 1509 **566** 589 597 660 671 674 750 772 903 955 976 1443 **571** 591 599 746 774 951 978 1170 1177 1195 1201 1309 1334 1487 1512 **600** 601 626 627 1349 1394 1461 1505 **602** 605 611 614 628 631 637 640 1416 1422 1439 1445

1537 1540 1559 1562 **603** 1355 **604** 1467 **606** 607 1352 1464 **608** 617 634 643 1419 1442 1581 1584 **609** 610 635 636 1535 1536 1538 1539 **612** 1557 **613** 1558 **615** 616 641 642 1579 1580 1582 1583 **618** 619 621 622 644 645 647 648 1978 1979 1981 1982 1997 1998 2000 2001 **620** 623 646 649 1805 1955 1980 1999 **624** 625 650 651 2015 2016 2018 2019 **629** 1400 **630** 1511 **632** 633 1397 1508 **638** 1560 **639** 1561 **652** 653 655 656 661 662 664 665 1203 1206 1211 1212 1214 1215 1226 1229 1234 1235 1237 1238 1282 1288 1327 1333 1372 1378 1483 1489 1541 1542 1543 1544 1545 1546 1563 1564 1565 1566 1567 1568 **654** 657 663 666 781 789 912 919 985 992 1213 1216 1236 1239 1451 1458 **658** 659 667 668 669 670 672 673 675 676 1209 1217 1218 1220 1222 1224 1232 1240 1241 1243 1245 1247 1255 1262 1263 1265 1267 1285 1308 1330 1375 1486 1585 1586 1587 1588 1589 1590 1591 1592 1983 1984 1986 1988 1990 2002 2003 2005 2007 2009 2020 2021 2023 2025 **677** 1351 **678** 1463 **679** 1418 **680** 685 1357 1600 **681** 686 1424 1601 **682** 683 1354 1466 **684** 689 1421 1615 **687** 688 1613 1614 **690** 2031 **691** 692 694 700 701 703 1284 1290 1374 1485 1602 1603 **693** 702 913 920 **695** 1210 **696** 713 **697** 706 708 709 711 714 1279 1287 1377 1619 1621 1622 1625 2033 **698** 1276 1310 1488 **704** 1204 **705** **707** 715 1280 1620 1626 2034 **710** 904 **712** 1207 1230 1275 **755** 763 783 791 960 967 987 994 1317 1324 1342 1348 1495 1502 1520 1526 1661 1668 1685 1691 1783 1807 1934 1957 2058 2064 2074 2080 2248 2254 2263 2269 **756** 764 915 922 961 968 1318 1435 1454 1457 1460 1496 1662 1738 1741 1746 1784 1935 2083 2096 **758** 766 963 970 1320 1326 1498 1504 1683 1689 1804 1954 2072 2078 2261 2267 **780** 788 991 1339 1346 1524 1664 1670 1786 1937 2060 2066 **792** 793 818 819 843 844 923 924 947 948 971 972 **870** 873 876 879 882 888 891 895 901 910 1427 1430 1887 1895 **885** 2163 **898** 1908 **984** 1517 2250 2256 **1012** 1017 1060 **1014** 1036 1041 1062 **1038** **1040** **1065** **1066** **1084** 1089 1128 **1086** 1105 1109 1130 **1107** **1108** **1132** **1133** **1151** 1155 1196 1366 1410 1414 1477 1521 1525 1692 1693 1703 1704 1714 1715 1717 1718 1722 1723 1725 1726 **1153** 1174 1178 1198 1299 1303 1343 1347 1368 1388 1392 1412 1479 1499 1503 1523 1698 1699 1706 1707 1709 1710 1711 1720 1721 1728 1729 1730 1731 1732 1733 1736 1737 1739 1740 1744 1745 1747 1748 2046 2050 2075 2079 2082 2084 2085 2094 2095 2097 2098 **1176** 1301 1323 1345 1390 1501 1742 1743 1749 1750 1751 1752 1753 1754 1755 1756 2061 2063 2065 2086 2087 2089 2091 2093 2099 2100 2101 2103 2105 2111 2112 2113 2115 **1200** 1370 1695 1701 **1202** **1205** 1208 1228 1254 **1219** 1223 1264 1985 1989 2022 **1221** 1244 1266 2004 2006 2008 **1225** 1248 **1227** 1270 **1231** 1251 **1233** 1273 **1242** 1246 1987 2024 **1249** 1252 1256 1257 1259 1260 1305 1311 1547 1548 1569 1570 **1268** 2026 **1269** 2027 **1271** 1307 **1277** 1604 **1281** 2035 **1321** 1325 1712 1713 1734 1735 2048 2077 2106 2107 2109 2110 **1433** 1455 1459 1694 1705 1716 1719 1724 1727 **1437** 1697 1702 **1481** 1696 **1700** **1708** 2081 **1791** 1797 1814 1820 1942 1948 1964 1970 2131 2134 2142 2169 2318 2321 2328 2350 **1793** 1812 1950 1968 **1799** 1944 2140 2326 **1818** 2133 **1829** 2270 **1837** **1962** 2320 **2088** **2090** 2092 **2102** **2104** 2114 **2116** **2124** 2155 2302 **2135** 2143 2170 2329 **2144** 2277 2330 **2159** 2166 2293 2312 2340 2343 2347 **2171** 2322 2351 2352 **2346** **2353** 2650 **2354** 2355 2356 2359 2360 2361 2362 2371 2372 2373 2374 2377 2378 2379 2380 2643 2644 2645 2646 2649 2651 2652 2659 2660 2661 2662 2665 2666 2667 2668 **2357** 2358 2363 2364 2365 2366 2367 2368 2375 2376 2381 2382 2383 2384 2385 2386 2389 2390 2391 2392 2395 2396 2397 2398 2647 2648 2653 2655 2656 2657 2658 2663 2664 2669 2670 2671 2672 2673 2674 2675 2676 2677 2678 2681 2683 2684 2869 2870 2871 2872 2875 2876 2877 2878 2883 2884 2885 2886 2889 2890 2891 2892 **2369** 2370 2387 2388 2541 2620 2626 2638 2897 2898 2899 2900 2903 2904 2905 2906 **2393** 2394 2399 2400 2401 2402 2403 2404 2405 2406 2434 2440 2454 2535 2614 2632 2679 2680 2685 2687 2688 2689 2690 2710 2716 2722 2728 2782 2860 2873 2874 2879 2880 2881 2882 2887 2888 2893 2894 2895 2896 2901 2902 2907 2908 2909 2910 **2407** 2408 2409 2410 2411 2412 2413 2414 2415 2416 2417 2418 2419 2420 2421 2422 2423 2424 2425 2426 2427 2429 2431 2432 2433 2435 2437 2438 2439 2441 2443 2444 2445 2447 2449 2450 2451 2453 2455 2456 2457 2459 2504 2505 2506 2507 2508 2509 2510 2511 2512 2513 2514 2515 2516 2517 2518 2519 2520 2521 2522 2523 2524 2525 2526 2527 2528 2530 2531 2532 2534 2536 2537 2538 2540 2542 2543

2544 2546 2547 2548 2549 2550 2551 2553 2554 2555 2556 2557 2559 2560 2561 2562 2563 2564 2565 2566 2567 2568
2569 2570 2571 2572 2573 2574 2575 2576 2577 2578 2579 2580 2581 2582 2583 2584 2585 2586 2587 2588 2589 2590
2591 2592 2593 2594 2595 2596 2597 2598 2599 2600 2601 2602 2603 2604 2605 2606 2607 2608 2609 2610 2611 2612
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2693 2694 2695 2696 2697 2699 2700 2701 2703 2704 2705 2706 2707 2708 2709 2711 2713 2714 2715 2717 2719 2720
2721 2723 2724 2725 2727 2729 2731 2733 2734 2735 2736 2761 2762 2763 2764 2765 2766 2767 2768 2769 2770 2771
2772 2773 2774 2775 2776 2777 2778 2779 2781 2783 2784 2785 2787 2788 2789 2790 2792 2793 2794 2795 2796 2797
2798 2800 2801 2802 2804 2805 2806 2807 2808 2809 2810 2811 2812 2813 2814 2815 2816 2817 2818 2819 2820 2821
2822 2823 2824 2825 2826 2827 2828 2829 2831 2832 2833 2834 2835 2836 2837 2838 2839 2840 2841 2842 2843 2844
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3120 3121 3122 3123 3124 3125 3126 3127 3128 3129 3130 3131 3132 3133 3134 3135 3136 3137 3138 3139 3140 3141
3142 3143 3144 3145 3146 3147 3148 3149 3150 3151 3152 3153 3155 3156 3157 3159 3160 3176 3177 3178 3179 3180
3181 3182 3184 3185 3186 3187 3188 3190 3191 3192 3193 3194 3195 3196 3197 3198 3199 3200 3201 3202 3203 3204
3205 3206 3207 3208 3210 3211 3213 3214 3215 3216 3217 3218 3219 3220 3221 3222 3223 3224 3225 3226 3227 3228
3229 3230 3231 3232 3233 3234 3235 3236 3237 3238 3239 3240 3241 3247 3248 3249 3250 3251 3252 3253 3254 3255
3257 3258 3259 3260 3261 3262 3263 3264 3265 3266 3267 3268 3269 3271 3272 3273 3274 3275 3276 32773278 3279
3280 3281 3282 3283 3284 3285 3286 3287 3288 3289 3290 3291 3292 3293 3294 3295 3296 3297 3298 32993300 3301
3303 3304 3305 3306 3307 3308 3309 3310 3311 3312 3313 3314 3315 3316 3317 3318 3319 3320 3322 3323 3325 3326
3327 3329 $\boxed{2428}$ 2448 $\boxed{2430}$ 2446 $\boxed{2436}$ 2442 2452 2458 2491 2492 2497 2498 2499 2500 2501 2502 2503
2533 2616 2630 2712 2718 2726 2732 2737 2750 2751 2756 2757 2758 2759 2760 2780 2858 2929 2938 2943 2946 2950
2952 2969 3030 3061 3066 3067 3068 3084 3089 3154 3174 3175 3246 $\boxed{2460}$ 2468 2469 3039 3046 3047 $\boxed{2461}$
3040 $\boxed{2462}$ 3051 $\boxed{2463}$ 3052 $\boxed{2464}$ 2465 2470 2471 2473 2474 2475 2481 2482 2484 2485 2741 2742 2744
2745 3057 3058 3059 3060 3062 3063 3064 3065 3166 3167 3168 3169 3170 3171 3172 $\boxed{2466}$ 3161 $\boxed{2467}$ 3162
$\boxed{2472}$ 3173 $\boxed{2476}$ 2477 3189 3243 $\boxed{2478}$ 2738 3041 3044 $\boxed{2479}$ 3042 $\boxed{2480}$ 2740 3053 3055 $\boxed{2483}$
2743 3163 3164 $\boxed{2486}$ 2622 3244 3245 $\boxed{2487}$ 2488 2489 2490 2495 2496 2746 2747 2748 2749 2754 2755 2786
2791 2868 2974 2977 3038 3043 3048 3049 3050 3054 3056 $\boxed{2493}$ 2494 2752 2753 2864 3035 3158 3165 $\boxed{2529}$
2854 3106 3256 3328 3330 $\boxed{2539}$ 2642 2947 2948 $\boxed{2545}$ 2949 $\boxed{2552}$ 3212 $\boxed{2558}$ 3183 $\boxed{2636}$ 2951
$\boxed{2654}$ 2682 $\boxed{2686}$ $\boxed{2698}$ 2830 2862 $\boxed{2702}$ 2730 $\boxed{2739}$ 3045 $\boxed{2799}$ 2845 3209 3321 $\boxed{2803}$ 3270
3324 $\boxed{2934}$ $\boxed{3242}$ 3302

Covers and Subcovers in the Poset of 411 Clone Equivalence Representatives

In the following the subcovers of a boxed element are to its left, the covers are to its right.

275 **[1]** 4 14 25 33 207 216 239 1065 1700 2088 2124
14 **[2]** 5 8 11 26 39 1200
27 **[3]** 6 9 15 25 100 1132
1 **[4]** 8 11 19 30 39 222 2654
2 **[5]** 10 31 34 40 59
3 **[6]** 8 12 18 22 32 35 41 83 94 2654
2 4 6 **[8]** 16 34 59 65
3 **[9]** 18 19 35 60 79 123
5 13 **[10]** 16 36 37 61 160 376 379
2 4 13 **[11]** 16 36 37 61 160 377 380
6 15 **[12]** 16 38 63 381
14 **[13]** 10 11 26 39 1200
1 **[14]** 2 13 405 406
3 **[15]** 12 18 26 41 407
8 10 11 12 18 22 26 **[16]** 43 46
6 9 15 **[18]** 16 44 66
4 9 21 25 **[19]** 22 45 70 160 1225 1268
27 **[21]** 19 24 47 72 218 235 260 1281
6 19 24 **[22]** 16 48 73 1202
21 **[24]** 22 50 63 75 704
1 3 **[25]** 19 26 51 2353
2 13 15 25 **[26]** 16 52 417
275 **[27]** 3 21 53 78 104 168 241 320 678 1065 1700 2466
4 **[30]** 43 59 70 180 219 223 1012 2102 2686
5 **[31]** 43 180 199 223 1151 2116 2803
6 **[32]** 44 139 1084 2686
1 **[33]** 34 59 65 73 148 219 253 1014 1202 2104
5 8 33 35 **[34]** 43 61 149
6 9 **[35]** 34 44 59 66 73 117 1086
10 11 **[36]** 67 199 223 382 600 1151 2116 2803
10 11 **[37]** 67 199 223 382 600 1151 2116 2803
12 41 **[38]** 67 209 384 1433 2803
2 4 13 **[39]** 42 199 223 606 1151 2116 2702
5 **[40]** 43 199 223 606 1151 2116 2702
6 15 **[41]** 38 44 2702
39 43 **[42]** 67 213 382
16 30 31 34 40 44 **[43]** 42
18 32 35 41 **[44]** 43 69 215 410
19 47 51 **[45]** 46 48 61 73 180 1202
16 45 52 **[46]** 49 157 385

21 53 **47** 45 50 63 75 182 704
22 45 50 **48** 49 1205 1219
46 48 **49** 67 213 382
24 47 **50** 48 221 384 707 1038
25 53 **51** 45 52 186 1225 1242 2654
26 51 **52** 46 391 2702
27 **53** 47 51 65 66 198 224 283 353 1708 2472
5 8 30 33 35 **59** 67 171
9 **60** 69 70 111 120 180 562 1107
10 11 34 45 65 **61** 67 151 163 382 385
12 24 47 66 **63** 67 157 244 384 387
8 33 53 **65** 61 155
18 35 53 **66** 63 69 410
36 37 38 42 49 59 61 63 69 73 **67** 183 434
44 60 66 78 **69** 67 250 436
19 30 60 72 **70** 73 199 1231
21 78 **72** 70 75 201 252 257 1012 1233
22 33 35 45 70 75 **73** 67 163 1205 1221
24 47 72 **75** 73 165 255 439 695 698 1014
27 **78** 69 72 185 188 198 272
9 104 **79** 81
275 **80** 82 100 148 239 347
79 **81** 83 89
80 **82** 83 90 94 123 149 186 222
6 81 82 107 **83** 91 130
111 **85** 91 130 199 454 455
90 **87** 93 209 223 456
89 **88** 111
81 112 **89** 88
82 113 **90** 87 101
83 85 93 101 115 120 **91** 136 213 460
87 99 116 119 122 **93** 91 132 215
6 82 96 100 **94** 97 117 139 1225
102 **96** 94 99 119 141 186 218 235 262
94 99 123 **97** 120 142 157 1202
96 125 **99** 93 97 221 278 704 1086
3 80 102 **100** 94 123
90 111 123 124 **101** 91 495
275 **102** 96 100 155 241 354 679 1132 1962
27 **104** 79 106 112 129 198 207
275 **105** 106 113 120 138 139 180 219
104 105 107 **106** 115 134 169
275 **107** 83 106 116 117 129 170 216 347
60 88 **111** 85 101 483
104 **112** 89 121 124 134 484
105 **113** 90 124 485
106 116 121 122 124 **115** 91 143 179 487
107 125 **116** 93 115 135 203

35 94 107 119 123 **117** 120 130 142 151 215 1202
96 125 **119** 93 117 180 182 221 265 704 1086
60 97 105 117 122 **120** 91 250 1205
112 **121** 115 137 493
275 **122** 93 115 120 221 494 520 1107
9 82 100 125 **123** 97 101 117 1225
112 113 125 **124** 101 115 496
275 **125** 99 116 119 123 124 134 144 188 194 224 269 284 984
104 107 **129** 137 169
83 85 117 134 **130** 136 163 171 460
93 135 **132** 136 244 349 462
106 112 125 **134** 130 143 175 487
116 **135** 132 137 143 178 488
91 130 132 137 138 142 143 **136** 153 183 512
121 129 135 144 **137** 136 176 513
105 147 **138** 136 179 250 349 517
32 94 105 141 **139** 142 171 209 1231
96 147 **141** 139 182 255 259 517 1084 1233
97 117 139 144 **142** 136 244 1205
115 134 135 144 **143** 136 195 519
125 **144** 137 142 143 178 185 204 255 278 520 696 1086
275 **147** 138 141 188 273 514
33 80 366 **148** 149 155 341
34 82 148 **149** 151
61 117 149 155 194 218 222 235 341 **151** 157 171
136 176 195 250 282 349 **153** 161 534
65 102 148 **155** 151
46 63 97 151 **157** 161 792
10 11 19 **160** 166
153 157 171 183 184 298 **161** 538
260 **162** 182 199 268 270 1151 2461
61 73 130 165 213 244 **163** 183 652
75 204 263 266 270 278 **165** 163 282 691
160 **166** 186 199 600
27 **168** 201 257 270 274 677 2467
106 129 170 **169** 175
107 366 **170** 169 194 322
59 130 139 151 175 180 182 199 **171** 161 532
134 169 188 194 **175** 171 184 562
137 185 198 306 **176** 153 184 563
135 144 203 308 311 **178** 184 565
115 138 188 198 309 **179** 195
30 31 45 60 105 119 186 **180** 171 213 606
47 119 141 162 188 201 224 258 261 **182** 171 263 682
67 136 163 250 253 282 **183** 161 658
175 176 178 195 305 **184** 161 571
78 144 283 **185** 176 282 698
51 82 96 166 224 **186** 180 632

78 125 147 **188** 175 179 182 204 252 274
125 170 **194** 151 175 203
143 179 204 317 **195** 153 184 566
53 78 104 320 **198** 176 179 439
31 36 37 39 40 70 85 162 166 201 207 258 261 **199** 171 213 609
72 168 260 **201** 182 199 268 1151 1277 2739
116 194 316 321 **203** 178
144 188 280 **204** 165 195 695
1 104 **207** 199 2353
38 87 139 259 **209** 215 602
42 49 91 180 199 215 219 223 263 266 270 **213** 163 615
44 93 117 209 221 265 **215** 213 244 608
1 107 **216** 219 253 322 1202 2654
21 96 224 267 **218** 151 221 266 632 682 704
30 33 105 216 222 269 **219** 213 1205
50 99 119 122 218 235 269 **221** 215 255 712
4 82 239 271 **222** 151 219 223 1225
30 31 36 37 39 40 87 222 **223** 213 624
53 125 241 271 **224** 182 186 218 268 274 280 309
21 96 241 271 **235** 151 221 270 606 682 1227
1 80 **239** 222 253 341
27 102 **241** 224 235 690 2636
63 132 142 215 278 **244** 163 250 654
69 120 138 244 255 284 **250** 153 183 566
72 188 **252** 255 268 562 695
33 216 239 267 287 **253** 183 298
75 141 144 221 252 287 **255** 250 282 1221
72 168 260 272 **257** 258 2478
257 **258** 182 199 268 1151 1277 2479
141 262 273 **259** 209 265 266 1277 1433 1799
21 **260** 162 201 257 261
260 **261** 182 199 268 270 1151 2460
96 **262** 259 270 1793
182 265 274 **263** 165 213 687
119 259 271 **265** 215 263 278 684
218 259 268 269 274 **266** 165 213 687
275 **267** 218 253 269 341 347 705 2104
162 201 224 252 258 261 **268** 266 682
125 267 271 **269** 219 221 266 278 684 696 2090
162 168 235 261 262 **270** 165 213 687 1271 2539
275 **271** 222 224 235 265 269 281 316 984 2102
78 **272** 257 274 680 2483
147 **273** 259 274 681 2171
168 188 224 272 273 **274** 263 266 2493
275 1 27 80 102 105 107 122 125 147 267 271 287 356 366 1108 2346 3242
99 144 265 269 281 **278** 165 244 349 693
224 281 **280** 204 286 704 1225 1268
271 **281** 278 280 283 308 705 1227 1281 1708 2472

165 185 255 284 286 **282** 153 183 697
53 281 287 **283** 185 306 707 1242
125 **284** 250 282 354 496 520 1133
280 287 **286** 282 317 712 1231
275 **287** 253 255 283 286 359 696 1233 2088
253 311 341 347 **298** 161 2159
306 308 311 317 **305** 184 755 2369 2428
283 353 359 **306** 176 305 756 1321
281 316 **308** 178 305 758 1829 2430 2476 2552
224 353 354 356 **309** 179 317 620
321 359 **311** 178 298 305 758 2144 2486
271 366 **316** 203 308 780 2539
286 309 359 **317** 195 305 654 1249 1269
27 **320** 198 353 417 1066 1837 2558
356 366 **321** 203 311 780 2545
170 216 **322** 341 2799
148 239 267 322 **341** 151 298
80 107 267 359 **347** 298 349 885
132 138 278 347 354 **349** 153 870
53 320 **353** 306 309 387 391 1829 2552
102 284 **354** 309 349 465 469
275 **356** 309 321 359 624 690 2116
287 356 **359** 306 311 317 347 693 1277
275 **366** 148 170 316 321 1962 2124 2171
10 **376** 382 600 624 1151 2116 2803
11 **377** 382 600 624 1151 2116 2803
381 **378** 384 602 1433 2803
10 **379** 382 600 624 1151 2116 2799
11 **380** 382 600 624 1151 2116 2799
12 407 **381** 378 2799
36 37 42 49 61 376 377 379 380 384 405 406 **382** 388 434 615
38 50 63 378 410 **384** 382 390 608
46 61 387 391 **385** 388 792
63 353 **387** 385 390
382 385 390 **388** 437 618 1153
384 387 **390** 388 439 620 756
52 353 417 **391** 385 1249 1321 2799
14 **405** 382 606 624 638 1066 1481 2116
14 **406** 382 606 624 639 1200 2116 2698
15 **407** 381 410 1437 2698
44 66 407 **410** 384 436
26 320 **417** 391 2698
67 382 436 **434** 437
69 410 **436** 434 439
388 434 439 1038 1066 **437** 658 1176
75 198 390 436 1040 **439** 437 563 566
85 **454** 457
85 **455** 458

87 **456** 459
454 **457** 460 469 532 1269 2354
455 **458** 460 469 532 1269 2354
456 **459** 462 469 602 624
91 130 457 458 462 483 487 **460** 463 512 615
132 459 485 488 **462** 460 465 608
460 465 469 493 **463** 515 618
354 462 494 **465** 463 517 620
354 457 458 459 495 **469** 463 1249
111 **483** 460 495 532 1269 2354
112 **484** 487 493 496 513 562 1133 2354
113 **485** 462 487 494 496 532 602 624
115 134 484 485 488 **487** 460 519
135 **488** 462 487 513 520
121 484 **493** 463 1040
122 485 **494** 465 519
101 483 496 **495** 469
124 284 484 485 **496** 495 519
136 460 513 514 **512** 515
137 484 488 **513** 512 522
147 **514** 512 517 885
463 512 517 519 522 1107 **515** 534 658
138 141 465 514 520 1108 **517** 515 566 870
143 487 494 496 520 1040 1108 **519** 515 566
122 144 284 488 **520** 517 519 522 710
513 520 1040 1133 **522** 515 563
171 457 458 483 485 562 606 639 682 1084 1151 **532** 538
153 515 563 566 697 870 **534** 538
161 532 534 571 658 792 1176 1829 2159 **538** 2407
60 175 252 484 **562** 532 571
176 439 522 698 710 756 1107 **563** 534 571 1176
178 710 758 **565** 571 2436
195 250 439 517 519 654 695 710 **566** 534 571 658
184 562 563 565 566 755 **571** 538 2393
36 37 166 376 377 379 380 **600** 609 632
209 378 459 485 679 681 **602** 608 1249 1791
609 **603** 604 612 629
603 **604** 613 630
39 40 180 235 405 406 632 **606** 532 615
215 384 462 602 684 **608** 615 620
199 600 **609** 603
603 **612** 613 638
604 612 **613** 639
213 382 460 606 608 639 687 **615** 618
388 463 615 620 624 690 **618** 652
309 390 465 608 **620** 618 654
223 356 376 377 379 380 405 406 459 485 **624** 618 1249
603 **629** 630 638

604 629 **630** 639
186 218 600 **632** 606
405 612 629 **638** 639
406 613 630 638 **639** 532 615 1249 1269 2354
163 618 654 691 1205 1249 **652** 658 2357
244 317 620 693 712 **654** 566 652 755
183 437 515 566 652 697 1221 1269 **658** 538 2393
168 678 **677** 680 690 1200 2462
27 **678** 677 1481 2463
102 **679** 602 684 690 1437 1818
272 677 **680** 682 1151 1269 1277 2480
273 **681** 602 684 885 1277 1433 2135
182 218 235 268 680 **682** 532 687
265 269 679 681 **684** 608 687 693 780
263 266 270 682 684 **687** 615 691 2487
241 356 677 679 **690** 618 1277 2545
165 687 693 695 1277 **691** 652 697 1153 2464
278 359 684 696 **693** 654 691 710 756 758
75 204 252 696 704 1233 **695** 566 691 1205 1219
144 269 287 705 **696** 693 695 707 712
282 691 698 710 712 **697** 534 658 1176 2436
75 185 707 **698** 563 697 1221
24 47 99 119 218 280 705 1227 **704** 695 712 1202
267 281 **705** 696 704
50 283 696 1281 **707** 698 756 1219
520 693 **710** 563 565 566 697 870
221 286 696 704 **712** 654 697 1205 1221
305 654 756 758 1791 **755** 571 2357
306 390 693 707 1086 1433 **756** 563 755 1153
308 311 693 780 **758** 565 755 2464
316 321 684 984 1799 1818 2135 **780** 758 2487
157 385 1202 **792** 538
349 517 710 885 898 **870** 534
347 514 681 2346 **885** 870 2159
1108 2346 **898** 870 2529
125 271 1962 2171 **984** 780 2493
30 72 1065 **1012** 1014 1151 1242
33 75 1012 **1014** 1038 1219
50 1014 **1038** 437 1221
493 **1040** 439 519 522
1 27 **1065** 1012 1066 1268 1481 2353
320 405 1065 **1066** 437 1269 2698
32 141 1132 **1084** 532 1086 1242 1433
35 99 119 144 1084 **1086** 756 1107 1219
60 122 1086 **1107** 515 563 1221
275 **1108** 517 519 898
3 102 **1132** 1084 1268 1437
284 484 **1133** 522 1269

31 36 37 39 40 162 201 258 261 376 377 379 380 680 1012 1200 **1151** 532 1321 2354
388 691 756 1219 1321 1708 **1153** 1176 2357
437 563 697 1153 1221 2090 2116 **1176** 538 2393
2 13 406 677 1481 **1200** 1151
22 33 45 97 117 216 704 1225 **1202** 792 1205
48 73 120 142 219 695 712 1202 1231 **1205** 652
48 695 707 1014 1086 1242 1268 **1219** 1153 1221
73 255 698 712 1038 1107 1219 **1221** 658 1176
19 51 94 123 222 280 1227 **1225** 1202 1231
235 281 **1227** 704 1225 1233 1271 2654
70 139 286 1225 1233 **1231** 1205 1249
72 141 287 1227 **1233** 695 1231 1242 1277 2686
51 283 1012 1084 1233 **1242** 1219 1321
317 391 469 602 624 639 1231 1277 **1249** 652 2369
19 280 1065 1132 1281 **1268** 1219 1269
317 457 458 483 639 680 1066 1133 1268 **1269** 658 2428
270 1227 **1271** 1277 2476 2702
201 258 259 359 680 681 690 1233 1271 **1277** 691 1249 1321 2486
21 281 **1281** 707 1268 2430 2654
306 391 1151 1242 1277 1433 **1321** 1153 2369
38 259 378 681 1084 1437 **1433** 756 1321 1791
407 679 1132 **1437** 1433
405 678 1065 1700 **1481** 1200
1 27 **1700** 1481 1708 1837 2353
53 281 1700 **1708** 1153 1829 2654
602 1433 1799 1818 2135 **1791** 755 2369
262 1962 **1793** 1799 2539
259 1793 2171 **1799** 780 1791 2486
679 1962 **1818** 780 1791 2545
308 353 1708 1837 **1829** 538 2799
320 1700 **1837** 1829 2698
102 366 **1962** 984 1793 1818 2636
1 287 **2088** 2104 2144 2686
269 2102 2104 **2090** 1176
30 271 **2102** 2090 2116
33 267 2088 **2104** 2090 2159
31 36 37 39 40 356 376 377 379 380 405 406 2102 **2116** 1176 2428
1 366 2346 **2124** 2159
681 2171 **2135** 780 1791 2159 2486
311 2088 **2144** 2159 2803
298 885 2104 2124 2135 2144 **2159** 538
273 366 **2171** 984 1799 2135
275 **2346** 885 898 2124 2934
25 207 1065 1700 2466 **2353** 2354 2654 2698
457 458 483 484 639 1151 2353 2461 2479 2480 2739 **2354** 2369 2428
652 755 1153 2369 2464 2654 **2357** 2393
305 1249 1321 1791 2354 2486 **2369** 2357
571 658 1176 2357 2428 2436 2686 **2393** 2407

538 2393 2529 2803 **2407**
305 1269 2116 2354 2430 **2428** 2393
308 1281 2461 **2430** 2428 2436 2799
565 697 2430 2464 **2436** 2393
261 2466 **2460** 2461 2478
162 2460 **2461** 2354 2430 2539
677 2463 2467 **2462** 2480 2545
678 2466 **2463** 2462
691 758 2472 2486 2487 **2464** 2357 2436
27 **2466** 2353 2460 2463 2467 2472 2483 2558
168 2466 **2467** 2462 2478 2636
53 281 2466 **2472** 2464 2552 2654
308 1271 2539 **2476** 2486 2799
257 2460 2467 2483 **2478** 2479 2739
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680 2462 2483 **2480** 2354 2486 2487
272 2466 **2483** 2478 2480 2493
311 1277 1799 2135 2476 2479 2480 2545 2739 **2486** 2369 2464 2803
687 780 2479 2480 2493 2539 2545 2739 **2487** 2464
274 984 2483 2636 **2493** 2487
898 2934 **2529** 2407
270 316 1793 2461 2636 **2539** 2476 2487
321 690 1818 2462 2636 **2545** 2486 2487
308 353 2472 2558 **2552** 2799
320 2466 **2558** 2552 2698
241 1962 2467 **2636** 2493 2539 2545
4 6 51 216 1227 1281 1708 2353 2472 **2654** 2357 2686 2702
30 32 1233 2088 2654 **2686** 2393 2803
406 407 417 1066 1837 2353 2558 **2698** 2702
39 40 41 52 1271 2654 2698 **2702** 2799
201 2478 **2739** 2354 2486 2487
322 379 380 381 391 1829 2430 2476 2552 2702 **2799** 2803
31 36 37 38 376 377 378 2144 2486 2686 2799 **2803** 2407
2346 3242 **2934** 2529
275 **3242** 2934

Measuring the longest chain from the bottom in the poset of clone equivalence representatives

height = 0: 275

height = 1: 1 27 80 102 105 107 122 125 147 267 271 287 356 366 1108 2346 3242

height = 2: 3 4 14 21 33 53 78 82 96 104 113 116 138 144 168 170 216 239 241 269 273 281 284 316 320 321 359 514 678 679 898 1065 1700 1962 2088 2124 2466 2934

height = 3: 2 6 9 13 15 24 25 30 47 72 90 99 100 106 112 119 129 135 141 148 188 194 198 207 222 224 235 253 260 262 272 283 308 311 322 347 353 354 405 406 485 677 681 705 1132 1281 1708 1818 1837 2104 2171 2463 2467 2472 2529 2558

height = 4: 5 8 11 12 18 19 26 32 35 39 41 50 51 60 75 79 87 94 121 123 124 134 162 169 185 201 203 218 219 252 257 259 261 274 280 306 309 341 407 484 488 494 680 690 696 885 984 1012 1066 1227 1481 1793 1829 2102 2135 2144 2353 2462 2483 2552 2636

height = 5: 10 22 31 34 38 40 44 45 52 59 65 66 70 81 93 97 115 117 137 139 175 178 204 221 258 265 270 286 298 377 380 381 417 456 493 496 520 704 707 1014 1084 1133 1200 1225 1233 1268 1437 1799 2090 2460 2480 2493 2545 2654

height = 6: 16 36 37 48 61 63 69 73 83 89 120 132 142 143 149 155 160 176 179 182 209 255 268 278 317 376 378 379 391 410 459 487 513 562 684 695 698 712 1038 1040 1086 1202 1231 1242 1271 2159 2461 2478 2686 2698

height = 7: 43 46 88 151 166 195 215 223 263 266 305 349 384 387 436 462 519 522 602 682 693 780 1107 1151 1205 1219 1277 1433 2116 2430 2479 2539 2702 2739

height = 8: 42 49 111 157 165 184 186 244 385 390 465 600 608 624 687 710 758 1221 1321 1791 2476

height = 9: 67 85 101 180 250 282 382 439 483 517 565 620 632 691 756 792 2486 2487 2799

height = 10: 91 130 199 388 434 454 455 495 563 606 654 697 870 2464 2803

height = 11: 136 171 213 437 457 458 566 609 755 1153 2436

height = 12: 153 163 460 469 571 603 1176

height = 13: 183 463 512 604 612 629

height = 14: 161 515 613 630 638

height = 15: 534 639

height = 16: 532 615 1249 1269 2354

height = 17: 618 2369 2428

height = 18: 652

height = 19: 658 2357

height = 20: 538 2393

height = 21: 2407