Sensitivity Analysis for Direction of Arrival Estimation using a Root-MUSIC Algorithm

Zekeriya Aliyazicioglu, H.K. Hwang, Marshall Grice, Anatoly Yakovlev

Abstract— An array antenna system with innovative signal processing can enhance the resolution of a signal direction of arrival (DOA) estimation. Super resolution algorithms take advantage of array antenna structures to better process the incoming signals. They also have the ability to identify multiple targets. This paper explores the eigen-analysis category of super resolution algorithm. A class of Multiple Signal Classification (MUSIC) algorithms known as a root-MUSIC algorithm is presented in this paper.

The root-MUSIC method is based on the eigenvectors of the sensor array correlation matrix. It obtains the signal estimation by examining the roots of the spectrum polynomial. The peaks in the spectrum space correspond to the roots of the polynomial lying close to the unit circle.

Statistical analysis of the performance of the processing algorithm and processing resource requirements are discussed in this paper. Extensive computer simulations are used to verify the performance of the algorithm for imperfect system calibration.

Index Terms— **Array antenna, Direction of arrival estimation, Signal processing.**

I. Introduction

Accurate estimation of a signal direction of arrival (DOA) has received considerable attention in communication and radar systems of commercial and military applications. Radar, sonar, seismology, and mobile communication are a few examples of the many possible applications. For example, in defense application, it is important to identify the direction of a possible threat. One example of commercial application is to identify the direction of an emergency cell phone call in order to dispatch a rescue team to the proper location.

DOA estimation using a fixed antenna has many limitations. Antenna mainlobe beamwidth is inversely proportional to its physical size. Improving the accuracy of angle measurement by increasing the physical aperture of the receiving antenna is not always a practical option. Certain systems such as a missile seeker or aircraft antenna have physical size limitations; therefore they have relatively wide mainlobe beamwidth. Consequently, the resolution is quite poor and if there are multiple signals falling in the antenna mainlobe, it is difficult to distinguish between them.

Instead of using a single antenna, an array antenna system with innovative signal processing can enhance the resolution of a signal DOA. An array sensor system has multiple sensors distributed in space. This array configuration provides spatial samplings of the received waveform. A sensor array has better performance than the single sensor in signal reception and parameter estimation. A sensor array also has applications in interference rejection [1], electronic steering [2], multi-beam forming [3], etc.

There are many different super resolution algorithms including spectral estimation, model based, and eigen-analysis to name a few [4, 5, 6]. In this paper, we discuss the application of estimating the DOA of multiple signals using uniform linear array (ULA) antenna with a class of Multiple Signal Classification (MUSIC) algorithms known as root-MUSIC. It does not require using a scan vector; resolution improvement does not necessarily require additional processing power. Detailed simulation results for the algorithm to demonstrate the performance are presented in this paper.

Although an ideal array antenna has uniform spacing between the array elements, there will be always some small deviation of spacing from an array element to adjacent elements. This small deviation causes an error in the electrical angle of each element. Additionally, when the array system is not perfectly calibrated, there will be a random phase error. Statistical analysis is carried out in this paper to investigate the effect of random element position variation and random phase variation on DOA estimation.

Computer simulation programs using MATLAB were developed to evaluate the direction finding performance of an array processor. Statistical analysis for element position deviation and phase error due to imperfect system calibration are included in this simulation study.

II. Sensor Array System

A sensor array system has multiple sensors distributed in space. This array configuration provides spatial sampling of the received waveform. We use an array antenna with an *M* element uniform linear array (ULA) in this paper. Fig.1

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H. K. Hwang and Zekeriya Aliyazicioglu are with the Electrical and Computer Engineering Department, California State Polytechnic University, Pomona, CA 91768 USA, (e-mail: hkhwang@ csupomona.edu and zaliyazici@csupomona.edu).

Marshall Grice was a graduate student at California State Polytechnic University, Pomona, CA. He is now with the Boeing, CA. (e-mail: grice@csupomona.edu)

Anatoly Yakovlev was an undergraduate student at California State Polytechnic University, Pomona, CA. He is now with Western Digital, San Jose, CA 95138, USA (e-mail: Anatoly.Yakovlev@wdc.com)

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shows the general configuration for a ULA antenna having M elements arranged along a straight line with the distance between adjacent sensor elements, be $d = \lambda/2$, where λ is the incoming signal wavelength. The angle of the incoming signal, θ, is measured relative to the antenna bore sight.

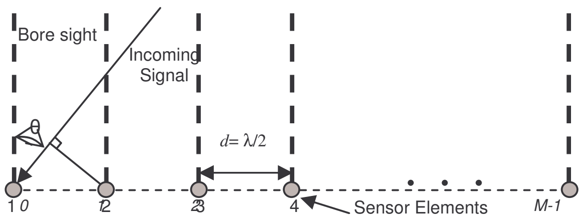

Figure 1. A uniformly spaced linear array antenna uniformly spaced

For a conventional antenna, the main lobe beam width (MLBW) is given by, in radians

$$MLBW = m\frac{\lambda}{D} \tag{1}$$

where D is the length of the antenna array and m is a proportionality constant, in most cases $m \approx 1$ [6].

Consider a uniformly spaced linear array of M sensors as shown in Fig. 1. The coordinate of the i^{th} sensor is $i = 0, 1, \ldots, M$-1. Suppose a plane target signal waveform comes from the direction of $k = \sin\theta$. The difference of the propagation path of this wave between the origin i=0 and the i^{th} sensor Δd_i is $\Delta d_i = d_i \sin\theta_i$. The corresponding propagation time delay τ_i is

$$\tau_i = \frac{\Delta d_i}{c} = \frac{d_i \sin\theta_i}{c} \tag{2}$$

where c is the speed of light.

If the bandwidth of signals is sufficiently narrow, then the data picked up by different sensor elements are related by a pure phase factor. The relative phase shift of the i^{th} sensor with respect to the reference sensor at the origin is

$$\beta_i = -\frac{2\pi}{\lambda} d_i \sin\theta_i . \tag{3}$$

To avoid the effect of grating lobes, the distance between the two neighboring sensors has to be no more than one half of the wavelength. If the reference sensor is located at the origin, and a waveform received by the reference sensor due to signal coming from direction of k is x(t), then the received waveform at i^{th} sensor is $x_i(t) = x(t-\tau_i)$.

For the sensor array with M elements, we can define the array input vector $\mathbf{x}$(t) and the array weighting vector $\mathbf{w}$ as

$$\mathbf{x}(t) = \left[x_0(t), x_1(t), \ldots, x_{M-1}(t)\right]^T, \tag{4}$$

$$\mathbf{w} = \left[w_0, w_1, \ldots, w_{M-1}\right]^T. \tag{5}$$

where $x_i(t)$ is the data input to the i^{th} sensor and w_i^* is the weight of the i^{th} sensor. The sensor array output $\mathbf{y}(t)$ is

$$\mathbf{y}(t) = \mathbf{w}^H \mathbf{x}(t) \tag{6}$$

where the superscript H represents the complex conjugate transpose (Hermitian) of the matrix.

Suppose there are L independent signal sources impinging on the antenna and we want to use a sensor array system to identify their directions of arrival (DOA). The input signal to each individual sensor is the combination of L independent signals. Every sensor in the array also receives random environmental ambient noise. This noise is modeled as Additive White Gaussian Noise (AWGN). The input waveform of the i^{th} sensor element $x_i(t)$ is given by

$$x_i(t) = \sum_{k=1}^{L} s_{k,i}(t) + n_i(t), \quad i = 0,1,\ldots M-1 \tag{7}$$

where $s_{k,i}(t) = s_{k,o}(t-\tau_{k,i})$, and $s_{k,o}(t)$ denote the k^{th} signal picked up by the sensor at the origin, $n_i(t)$ is the noise at i^{th} sensor, and $\tau_{k,i}$ is the relative delay of k^{th} signal at the i^{th} sensor.

For the narrowband input signals, signal $s_{k,i}$(t) is related to the signal $s_{k,o}(t)$ by a phase shift factor of $\beta_{k,i}$. If the input signals have a wide bandwidth, the delay time of signal at i^{th} sensor from reference signal at the origin may not be an integer multiple of the sampling time; additional interpolation filtering is required to emulate their delay. The weighted sum of samples of all sensors forms the array output. To estimate the DOA of wideband signals, each single weight is replaced by a tape delay lines filter. Such a processor is referred to as the Space Time Adaptive Processor (STAP) [7]. In this paper, only narrowband signal is considered.

III. Root-MUSIC Algorithm

The eigen-analysis method assumes that the received data can be decomposed into two mutually orthogonal subspaces. One is the signal plus noise subspace, the other is the noise only subspace. There are several important eigen-analysis methods; the Pisarenko Harmonic Decomposition [8], Multiple Signal Classification (MUSIC) [9] and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [10]. The MUSIC algorithm, using temporal averaging and spatial smoothing, is briefly presented for convenience.

Assume the number of signals impinging on the array antenna is L, and the array antenna has M elements, usually $M > L$. For finite number of data sequences, a sequence of received data can be considered as a vector in the sample space. If the noise is assumed to be white, it spans over the entire sample space. For signals with sufficient narrowband, they span over L dimensional subspace. For example, radar signal of a moving target is a sinusoid with a frequency equal to the Doppler frequency shift. If there are L targets, the received waveform at the reference sensor at the origin $x_0(n)$ can be expressed as

$$x_0(n) = \sum_{k=1}^{L} \alpha_k e^{j2\pi f_k n} + n_0(n) \qquad (8)$$

where α_k, f_k, and $k = 1, 2, \ldots, L$ are the complex amplitude and frequency of the k^{th} sinusoid, and $n_0(n)$ is the additive white noise with variance σ^2. Since the signal at the other sensors has a relative phase shift, the waveforms of the other sensors are

$$x_i(n) = \sum_{k=1}^{L} \alpha_k e^{j2\pi f_{k,i} n} e^{j\beta_{k,i}} + n_i(n), \; i = 1, 2, \ldots, M-1 \qquad (9)$$

where $\beta_{k,i} = -\frac{2\pi}{\lambda} x_i \sin\theta_k, \; k = 1, 2, \ldots, L$ for one dimensional array.

One of the eigen-analysis methods is the "Multiple Signal Classification" (MUSIC). MUSIC is especially practical in handling radar signals.

Define the signal matrix **S** as

$$\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_L] = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{j\beta_{1,1}} & e^{j\beta_{2,1}} & \cdots & e^{j\beta_{L,1}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j\beta_{1,M-1}} & e^{j\beta_{2,M-1}} & \cdots & e^{j\beta_{L,M-1}} \end{bmatrix} \qquad (10)$$

The correlation matrix of received data sequence **R** is

$$\mathbf{R} = \mathbf{SDS^H} + \sigma^2 \mathbf{I} \qquad (11)$$

where $\mathbf{D} = diag[P_1, P_2, \ldots P_L]$ is the signal power matrix, **I** is the identity matrix, and σ^2 is the variance of the Gaussian white noise,

The eigenvalues λ_i and eigenvectors $\mathbf{q}_i$ of the matrix **R** satisfy the following equation.

$$\mathbf{R}\mathbf{q}_i = \lambda_i \mathbf{q}_i \qquad (12)$$

where

$$\lambda_i = \begin{cases} P_i + \sigma^2 & i = 1, 2, \ldots, L \\ \sigma^2 & i = L+1, \ldots, M \end{cases} \qquad (13)$$

and P_i, i = 1, 2, … , L are the eigenvalues of matrix $\mathbf{SDS}^H$.

Eigenvectors $\mathbf{q}_i$, i = 1, …, L span over the signal and noise subspace and eigenvectors $\mathbf{q}_i$, $i = L+1, \ldots, M$ span over the noise only subspace. Noise only subspace eigenvectors satisfy the following equation.

$$\mathbf{R}\mathbf{q}_i = \sigma^2 \mathbf{q}_i, \; i = L = 1, \ldots, M \qquad (14)$$

The signal vectors $\mathbf{s}_i$ i = 1, 2, … , L belong to the subspace span by $\mathbf{q}_i$, i = 1, ... , L, thus they are orthogonal to noise only subspace eigenvector $\mathbf{q}_i$, $i = L+1, \ldots, M$. If the signal to noise ratio (SNR) is reasonably high, then there should be an obvious gap between the largest L eigenvalues to the rest. Thus the number of signals can be easily identified by using the L eigenvectors associated with the L largest eigenvalues. We can compute the eigenvectors associated with the smallest M-L eigenvalues of the matrix **R** in the following form of matrix

The MUSIC algorithm can be summarized as

1. Compute the eigenvectors associated with the smallest M-L eigenvalues of the matrix **R**.
2. Form the matrix

$$\mathbf{V}_N = [\mathbf{q}_{L+1} \quad \mathbf{q}_{L+2} \quad \ldots \quad \mathbf{q}_M] \qquad (15)$$

3. Compute the MUSIC spectrum $S_{MUSIC}(\theta)$ as

$$S_{MUSIC}(\theta) = \frac{1}{\mathbf{s}^H(\theta)\mathbf{V}_N\mathbf{V}_N^H\mathbf{s}(\theta)} \qquad (16)$$

   where $\mathbf{s}(\theta)$ is a scan vector scans over all possible elevation angles.
4. Whenever the MUSIC spectrum reaches peak value, the corresponding angle θ must be the signals DOA

To compute the MUSIC spectrum, we must first estimate the correlation matrix of the received data. After the estimated correlation matrix is obtained by using the temporal averaging or the spatial smoothing method, the conventional MUSIC algorithm based on Eq.(16) has to compute MUSIC spectrum by using the scan vector $\mathbf{s}(\theta)$ to scan over all possible directions. A more efficient method to estimate the DOA of the signal is to compute the roots of the polynomial $J(z)$ defined by the following Equation.

$$J(z) = \mathbf{a}^H \mathbf{V}_N \mathbf{V}_N^H \mathbf{a} \qquad (17)$$

where

$$\mathbf{a} = \begin{bmatrix} 1 & z & z^2 & \ldots & z^{M-1} \end{bmatrix}^T \qquad (18)$$

and z for ULA can be given

$$z = e^{j\frac{2\pi d}{\lambda}\sin\theta}. \qquad (19)$$

Then the roots of $J(z)$ contain the directional information of the incoming signals. Ideally, the roots of $J(z)$ would be on the unit circle at locations determined by the directions of the incoming signals; however, due to the presence of noise, the roots may not necessarily be on the unit circle. In this case, the L closest roots to the unit circle are the roots that correspond to the L incoming signals [11]. These selected roots, by themselves, do not directly represent the incoming angle. For each root, the incoming angle is found by solving Eq.(20).

$$\theta_k = \sin^{-1}\left[\frac{\lambda}{2\pi d}\arg(z_k)\right], \; k = 1, 2, \ldots, L. \qquad (20)$$

Obviously, when the root-MUSIC algorithm is implemented, there is no prior knowledge of the incoming signal directions or signal powers needed to construct the correlation matrix using Eq.(11). Therefore the correlation matrix must be estimated using only the information available from the sensor array. There are several methods commonly used to perform this estimation such as temporal averaging, spatial smoothing, or a hybrid combination of both temporal averaging and spatial smoothing [8]. In this paper, we use only the temporal averaging method.

IV. Computer Simulation Results

Consider a 16 element ULA with inter-element spacing equals half wavelength. In a conventional fixed antenna, the mainlobe beamwidth would be around 7°. This antenna array would not be able to resolve multiple signals if their angle separation is less than 7°. Using the root MUSIC algorithm, this ambiguity can be easily resolved.

Let $x_i(1), x_i(2), \ldots, x_i(N)$ represent the received sample data from i^{th} element, where $i = 0, 1, \ldots, M$-1. The incoming data matrix **A** can be given

$$\mathbf{A}^H = \begin{bmatrix} x_0(1) & x_0(2) & \cdots & x_0(N) \\ x_1(1) & x_1(2) & \cdots & x_1(N) \\ \vdots & \vdots & \ddots & \vdots \\ x_{M-1}(1) & x_{M-1}(2) & \cdots & x_{M-1}(N) \end{bmatrix} \qquad (21)$$

The estimated correlation matrix $\mathbf{\Phi}$ is computed by

$$\Phi = \mathbf{A}^{\mathrm{H}}\mathbf{A} \quad . \qquad (22)$$

We can find the eigenvalues from the estimated correlation matrix $\mathbf{\Phi}$,. The columns of matrix $\mathbf{V}_{\mathrm{N}}$ are the eigenvectors associate with the M-L smallest eigenvalues of matrix $\mathbf{\Phi}$. Once this matrix is available, the signals' DOA can be derived from L roots of polynomial $J(z)$ closest to the unit circle.

Fig.2 shows the roots computed from Eq.(16) when two signals impinging on a 16 element ULA from angles of 40° and 46°. In this simulation, the number of snapshot N is 32 and the signal to noise ratio (SNR) is 20 dB.

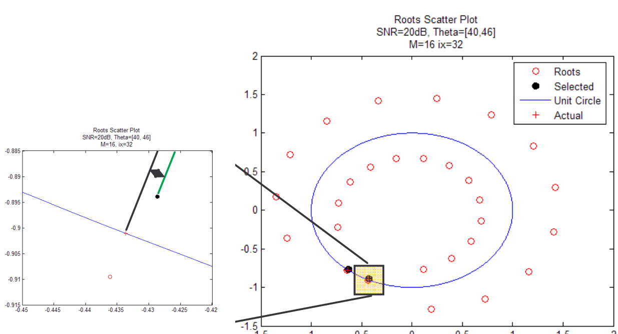

Figure 2. Roots of Polynomial J(z)

Fig.2 shows when the angle separation between two signals smaller than the mainlobe beamwidth, two distinct pair of roots closest to the unit circle can easily be identified. The zoom area shows one of the pair roots that they are very close to the theoretical root of the signal's DOA. The results are very similar to Fig.2 when the angle separation is further reduced to 3°, and 1.5°. Thus, the spatial resolution is improved by root MUSIC algorithm.

Eq.(19) converts the roots of polynomial $J(z)$ to the signals' DOA. Assume there are two signals impinging on a 16 element ULA with 20 dB SNR and taking 32 snapshots, the estimated signals DOA with different angle separations are shown in Fig.3. The result shows that two incoming signals are clearly identified even as the separation between the two signals is well below the conventional main lobe beam width. The average estimation error is nearly zero in all cases.

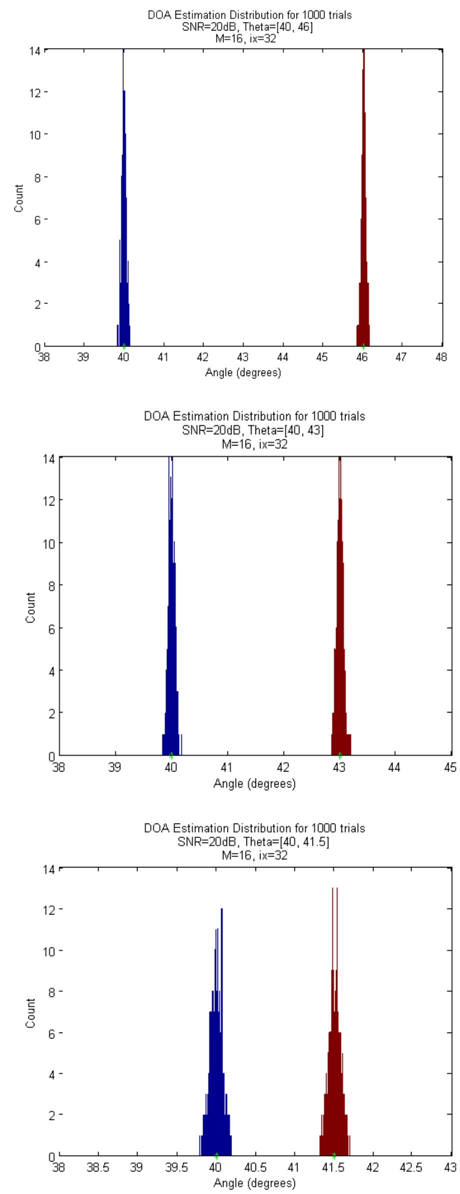

Figure 3. Histogram of the Estimated Signals' DOA for Angle Separation equal 6°, 3°, and 1.5°

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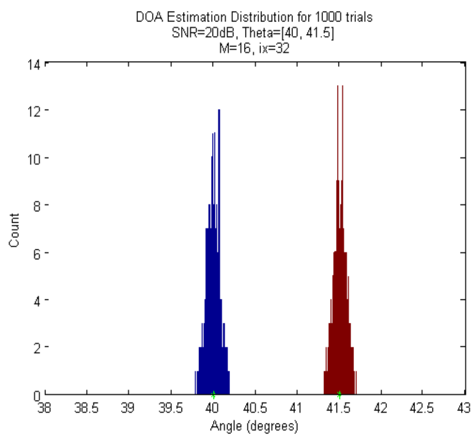
The estimated means and variances based on 1000 trials are summarized in Table I. Fig. 3 and Table-1 show that the estimation variance increases as the angle separation becomes smaller.

Table I The Estimated Mean and Variance of DOAs for SNR = 20 dB

Angle Separation	6°		3°		1.5°	
True Angles	40°	46°	40°	43°	40°	41.5°
Estimated Mean	39.997°	46.004°	39.999°	42.999°	39.999°	41.498°
Variance	0.0009	0.0012	0.0023	0.0025	0.0047	0.0042

Increasing the estimated correlation matrix from 32 snapshots to 96 snapshots reduces the estimated variance. Fig. 4 compares the histogram of the estimated signals' DOA for 20 dB SNR, and the signals' DOA are 40° and 41.5° for 32 and 96 snapshots. The estimated mean values and variances based on 1000 trials are listed in Table II.



(a)

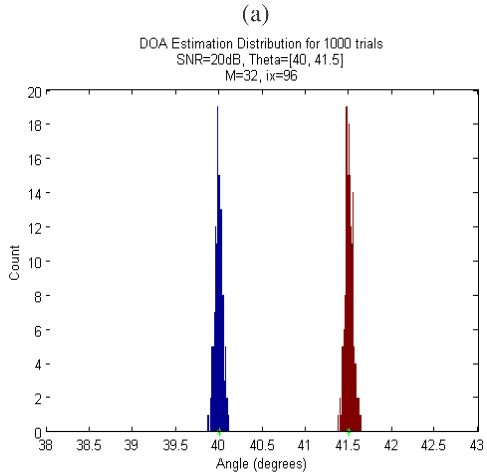


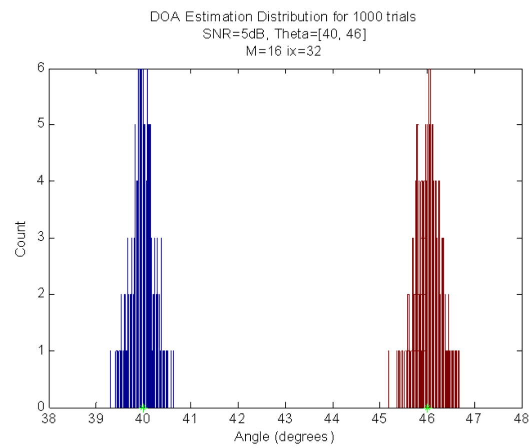
Figure 4. Histogram of the Estimated Signals' DOA with SNR = 20 dB and Angel Separations 40° and 41.5°
(a) 32 Snapshots, (b) 96 Snapshots

Table II The Estimated Mean and Variance of DOAs for SNR = 20 dB

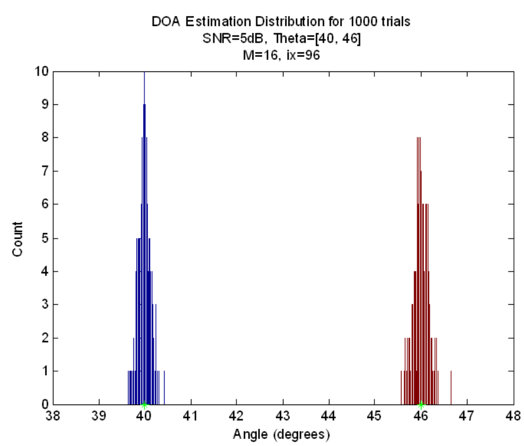
Number of Snapshots	32		96	
True Angles	40°	41.5°	40°	41.5°
Estimated Mean	39.9999°	41.4987°	39.9992°	41.5008°
Variance	0.0047	0.0042	0.0015	0.0016

The above simulation results assume that the system operates in a high SNR environment such as SNR value at 20 dB. If the SNR is only 5 dB, the simulation result yields a larger estimation variance. Fig.5 shows the histogram of the estimated signals' DOA for 5 dB SNR, and the signals' DOA are 40° and 46°. This result is based on 1000 independent simulations where the number of snapshots of each simulation is 32 and 96.

The estimated mean values and variances based on 1000 trials for 5 dB SNR and the two different numbers of snapshots are listed in Table III. This simulation result shows that as we increase the number of snapshots, the estimation variance decreases.



(a)



(b)

Figure 5 Histogram of the Estimated Signals' DOA with SNR = 5 dB. (a) 32 Snapshots (b) 96 Snapshots, Signals' DOA are 40° and 46°

Table III. The Estimated Mean and Variance of DOAs for SNR = 5 dB

Number of Snapshots	32		96	
True Angles	40°	46°	40°	46°
Estimated Mean	39.9708°	46.0279°	39.9893°	46.0094°
Variance	0.0358	0.0430	0.0112	0.0140

V. Sensor Spacing and Phase Sensitivity

The root-MUSIC algorithm in the previous section assumes that the positions of the elements ULA are exactly uniformly spaced and the weights of elements are perfect. However, the real system always has some variation in element position and phase. Fig. 6 shows the sensor element position has a slight deviation from the ideal element position.

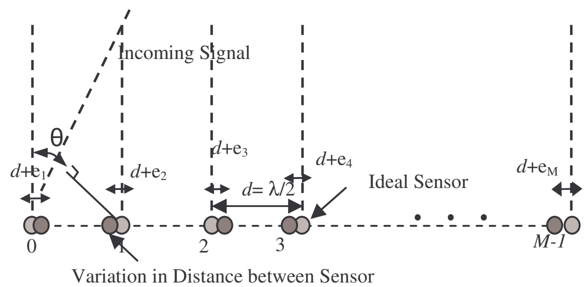


Figure 6. Non-ideal ULA sensor spacing

The spacing error of each sensor, e_i, is a Gaussian random variable added to the ideal spacing. Taking this error into account, Eq.(3) is used to create the phase shift between sensor elements for the incoming signals and becomes

$$\beta(\theta_i) = \frac{2\pi(d + e_i)}{\lambda}\sin(\theta_i) \qquad (23)$$

where $d_k = (k\text{-}1)d$, and e_k represents the ideal positions and the position error of the k^{th} element, and θ is the signal's DOA. Increasing the position error increases the estimation variance. Suppose the position error has a Gaussian distribution with standard deviation equal to 1% and 5% of the theoretical inter-element spacing d. Using 20 dB SNR and two signals impinging on the ULA at $\theta = 40^o$ and 46^o, the histograms based on 1000 simulations are shown in Fig. 7. The number of snapshots is assumed to be 32.

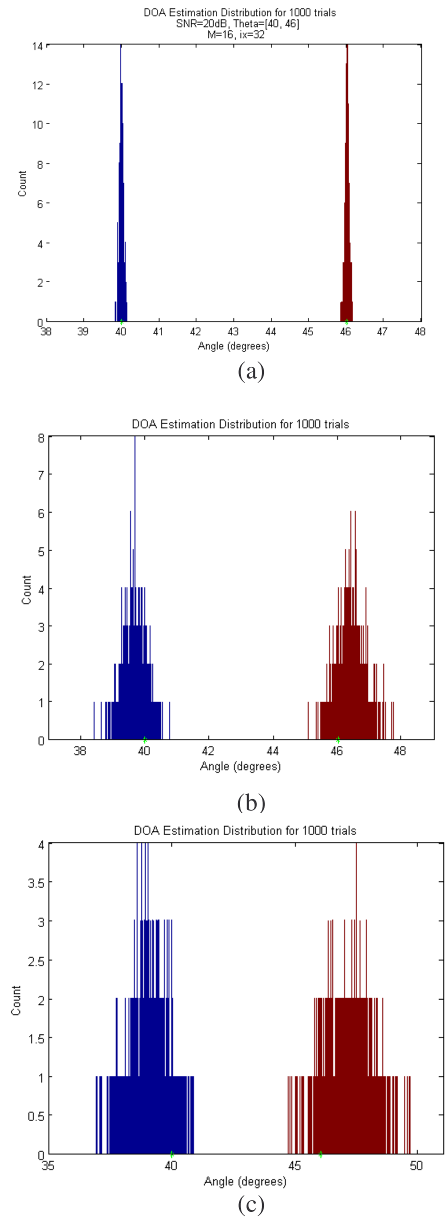
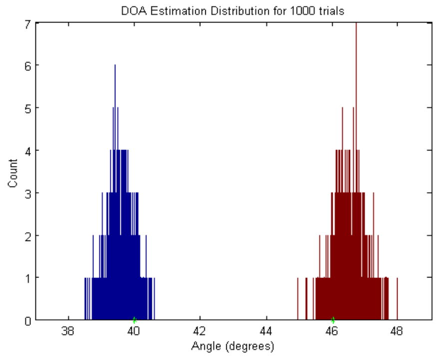
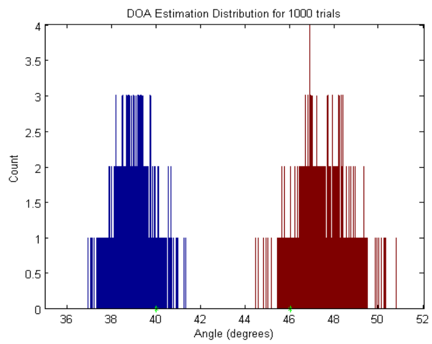


Figure 7. Histograms of Two Signals with DOA = 40° and 46°, SNR = 20 dB with (a) Ideal Element Spacing d, (b) Gaussian Random Spacing with Standard Deviation equal .01d, (c) Gaussian Random Spacing with Standard Deviation equal .05d

If the phase error of each received signal increases, the estimation variance also increases. Example of a random Gaussian phase error with standard deviation equal to 1° and 5° and histograms based on 1000 simulations are shown in Fig.8. The number of snapshots is assumed to be 32, and SNR = 20 dB.



(a)



(b)

Figure 8. Histograms of Two Signals with DOA = 40o and 46o, SNR = 20 dB with
(a) Gaussian Random Phase Error with Standard Deviation = 1°,
(b) Gaussian Random Phase Error with Standard Deviation = 5°

Combining the effect of random phase error and position error further degrades the performance of the ULA. Fig. 9 shows the histogram of two signals impinging on the ULA with DOA equal = 40° and 46°, SNR = 20 dB. The position error is a Gaussian random variable with a standard deviation equal to 1% of the ideal spacing between elements. The phase error is a Gaussian random variable with standard deviation equal to 1°. This result is based on 1000 simulations with 32 snapshots in each simulation.

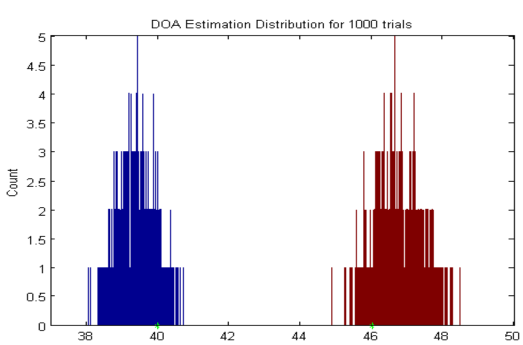


Figure 9. Histogram with Combine Phase Error and Position Error

Fig.10 shows the estimation variance of different Gaussian random phase error versus signal's DOA for 20 dB SNR. The estimated variance increases rapidly as the signal's DOA approaches the end fire of the ULA. If the signal's DOA is greater than 75°, the estimation variance become so large that reliable DOA estimation is difficult to achieve.

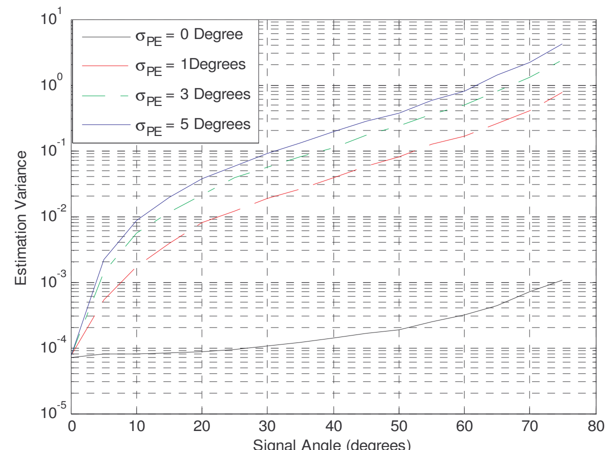


Figure 10. Estimation Variance of Different Gaussian Random Phase Error versus signal's DOA for 20 dB SNR

Fig.11 compares the variance of different Gaussian random phase error versus signal's DOA for 10 dB and 20 dB SNR. This simulation result shows that whenever the SNR reduce the estimation variance increase. Fig.10 and Fig.11 are simulation results when the correlation matrices are computed based on 32 snapshots. With finite snapshots, the estimated correlation matrix is not very reliable, thus if the signal impinging the ULA with large angle, the estimated signal's DOA has very large variance. To reduce the estimation variance, the estimated correlation matrix should be based on averaging large number of snapshots.

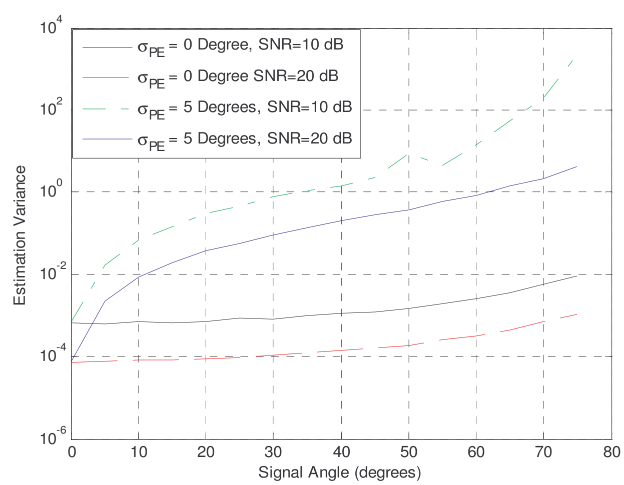


Figure 11. Estimation Variance of Different Gaussian Random Phase Error versus signal's DOA for 10 and 20 dB SNR

VI. Conclusion

This paper investigates the possibility of combining the array antenna and advanced signal processing techniques to enhance the estimation of the direction of signal sources.

A conventional method to detect the direction of signal source is to use a fixed antenna to scan over a certain searching region. This primitive estimation technique has many limitations. First, its resolution is limited by the antenna mainlobe beamwidth. Also, if there are multiple signal sources, a conventional fixed antenna has difficulty in detecting them simultaneously.

Using the advanced signal processing techniques, the DOA estimation can be improved. The root-MUSIC method presented in this paper is based on the eigenvector of the sensor array correlation matrix to estimate angle of incoming signals. Extensive computer simulation is used to demonstrate the performance of the algorithms, which enhance the DOA estimation.

- The simulation results of the root-MUSIC algorithm suggests the following:
- The capability to resolve multiple targets with separation angles smaller the main lobe beam width of the array antenna.
- The estimation variance can be reduced by increasing the number of snapshots in correlation matrix estimation
- The estimation variance increases as the angle separation between signals becomes smaller
- The estimation variance depends on the direction of the signal. A signal coming from the bore sight has minimum estimation variance.

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