Modelling and Prediction of the MXNUSD Exchange Rate Using Interval Singleton Type-2 Fuzzy Logic Systems

María de los Angeles Hernandez Medina, Gerardo M. Mendez

***Abstract*—This paper presents an application of the interval singleton type-2 fuzzy logic system (FLS) to one step ahead prediction of the daily exchange rate between Mexican Peso and US Dollar (MXNUSD) using recursive least-squared (RLS) - back-propagation (BP) hybrid learning. Experiments show that the exchange rate is predictable and according to a simple short-term investment strategy, a good annual profit rate can be obtained. A singleton type-1 FLS and an interval singleton type-2 FLS, both using only BP learning method, are used as a benchmarking systems to compare the results of the RLS-BP hybrid interval singleton type-2 FLS forecaster. The interval singleton type-2 FLS using RLS-BP hybrid learning presents a better performance than both singleton type-1 FLS and interval singleton type-2 FLS.**

I. Introduction

INTERVAL type-2 fuzzy logic systems (FLS) constitute an emerging technology [1]. As in hot strip mill process [2], [3], in autonomous mobile robots [4], and in plant monitoring and diagnostics [5], [6], financial systems are characterized by high uncertainty, nonlinearity and time varying behavior [7], [8]. This makes very difficult to forecast financial variables such as exchange rate and closing prices of stock indexes. The ability of comprehending as well as predicting the movements of economic indices could result in significant profit, stable economic environments and careful planning. Neural networks are very popular in financial applications and a lot of work has been done in exchange rate and stock markets predictions described elsewhere [9]–[11].

Type-2 fuzzy sets let us model the effects of uncertainties, and minimize it by optimizing the parameters of the type-2 fuzzy sets during a learning process [12]–[14]. Although some econometricians claim that the raw data used to train and test the fuzzy logic systems (FLSs) forecasters should not be directly used in the modelling process, since it is time varying and contains non-stationary noise, they were used directly to train the interval type-2 FLSs system. The inputs were modeled as singletons, incorporating the uncertainties of the training data only in the antecedents and consequents of the fuzzy rule base.

Maria de los Angeles Hernandez Medina is with the Administration and Economics Science Department, Instituto Tecnológico de Nuevo León, Av Eloy Cavazos #2001, Cd. Guadalupe Nuevo Leon, CP 67170 MEX (e-mail: ahernandez@ itnl.edu.mx).

Gerardo M. Mendez is with the Electrical and Electronics Engineering Department, Institutor Tecnológico de Nuevo León, Av Eloy Cavazos #2001, Cd. Guadalupe Nuevo Leon, CP 67170 MEX (phone: +52-81-83-370330; fax: +52-81-83678834; e-mail: gmmendez@ itnl.edu.mx).

II. MXNUSD Exchange Rate Prediction

A. Input Output Data Pairs

The data used to train the three forecasters cover a period of eight years and a half from 01/01/97 to 25/07/05 whereas the test data covers six months from 26/07/05 to 20/12/05. The daily closing price of MXNUSD exchange rate was found on the web site: http://pacific.commerce.ubc.ca/xr/. It was a set of $N=$ 2252 noisy data, $x(1)$, $x(2)$, …, $x(2252)$. It is assumed that the noise free sampled MXNUSD exchange rate, $s(k)$, is corrupted by uniformly distributed stationary additive noise $n(k)$, so that

$$x(k)=s(k)+n(k) \qquad k=1,2,...,2252 \tag{1}$$

and that signal to noise ratio (SNR) is equal to 0 dB. Fig. 1, shows the trend of the raw data. The first 2150 noisy data were used for training, and the remaining 102 data were used for testing the forecasters**.** Four antecedents were used as inputs, $x(k-3)$, $x(k-2)$, $x(k-1)$ and $x(k)$, to predict the output, $y=x(k+1)$.

B. Type-2 Fuzzy Logic System Design

The architecture of the interval singleton type-2 FLS (type-2 SFLS) was established in such away that parameters were continuously optimized. The number of rule-antecedents was fixed to four. Each antecedent-input space was divided in three fuzzy sets, fixing the number of rules to eighty one. Gaussian primary membership functions of uncertain means were chosen for the antecedents. Each rule of the each interval type-2 FLS was characterized by twelve antecedent membership function parameters (two for left-hand and right-hand bounds of the mean and one for standard deviation, for each of the three antecedent Gaussian membership functions) and two consequent parameters (one for left-hand and one for right-hand end points of the consequent type-1 fuzzy set), giving a total of 14 parameters per rule.

The resulting interval type-2 FLS used type-1 singleton fuzzification, join under maximum t-conorm, meet under product t-norm, product implication, and center-of-sets type-reduction.

The interval type-2 SFLS was trained using two main learning mechanisms: the back-propagation (BP) method for

(Advance online publication: 15 August 2007)

both antecedent and consequent parameters tuning, and the hybrid training method using recursive least-squared method (RLS) for consequent parameter tuning and BP method for antecedent parameters tuning. In this work, the former is named as interval type-2 SFLS (BP), and the latter as hybrid interval type-2 SFLS (RLS-BP).

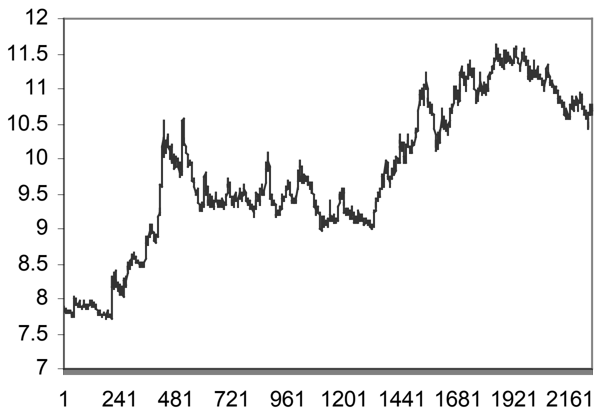

Fig. 1. The daily closing price of MXNUSD

C. Fuzzy Rule Base

The interval type-2 SFLS fuzzy rule base consists of a set of IF-THEN rules that represents the model of the system. The interval type-2 SFLS has four inputs $x_1 \in X_1$, $x_2 \in X_2$, $x_3 \in X_3$, $x_4 \in X_4$ and one output $y \in Y$. The rule base has $M = 81$ rules of the form:

$$R^i : IF \quad x_1 is \widetilde{F}_1^i \quad and \quad x_2 is \widetilde{F}_2^i, \quad x_3 is \widetilde{F}_3^i \quad and \quad x_4 is \widetilde{F}_4^i,$$

$$THEN \quad y \quad is \quad \widetilde{G}^i \tag{2}$$

where $i = 1,2,3,\ldots,81$ rules; $\widetilde{F}_1^i, \widetilde{F}_2^i, \widetilde{F}_3^i$ and $\widetilde{F}_4^i$ are antecedent type-2 fuzzy sets, and $\widetilde{G}^i$ is the consequent type-2 fuzzy set, of rule i.

D. Antecedent Membership Functions

The primary membership functions for each antecedent are interval type-2 fuzzy sets described by Gaussian primary membership functions with uncertain means:

$$\mu_k^i(x_k) = \exp\left[-\frac{1}{2}\left[\frac{x_k - m_k^i}{\sigma_k^i}\right]^2\right] \tag{3}$$

where $m_k^i \in \left[m_{k1}^i, m_{k2}^i\right]$ is the uncertain mean, with $k = 1,2,3,4$ (the number of antecedents) and $i = 1,2,\ldots,81$ (the number of M rules), and σ_k^i is the standard deviation. Table I shows the values of the three fuzzy sets established for each antecedent.

TABLE I
INTERVALS OF UNCERTAINTY FOR ALL INPUTS

	m_{k1}	m_{k2}	σ_k^i
1	7.58	7.78	1.04
2	9.68	9.88	1.04
3	11.68	11.88	1.04

Fig. 2 shows the initial membership functions the antecedent fuzzy sets being the same for all inputs.

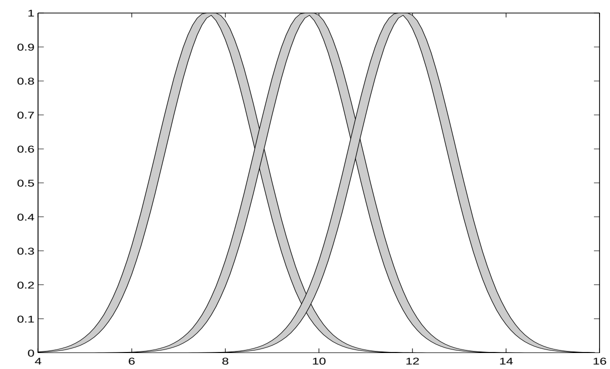

Fig. 2. Membership functions for the antecedent fuzzy sets.

E. Consequent Membership Functions

The architecture of the interval singleton type-2 FLS (type-2 SFLS) was established in such away that parameters were continuously optimized. The primary membership function for each consequent is a Gaussian with uncertain means, as defined in (8). Because the centre-of-sets type-reducer replaces each consequent set $\widetilde{G}^i$ by its centroid, then y_l^i and y_r^i (the M left-points and right points of interval consequent centroids) are the consequent parameters.

Because only the input-output data training pairs $\left(x^{(1)} : y^{(1)}\right)$, $\left(x^{(2)} : y^{(2)}\right), \ldots, \left(x^{(N)} : y^{(N)}\right)$ are available at the starting point, the initial values of the centroid parameters y_l^i and y_r^i were chosen arbitrarily from the output space. In this work the initial values of y_l^i were set equal to 9.7 and the initial values of y_r^i equal to 9.9, for $i = 1,2,3,..,81$ and $k = 1,2,3,4$.

III. MODELLING RESULTS

An interval type-2 SFLS system was trained and used to predict the daily exchange MXNUSD, applying the previous four values as inputs. For each of the two training methods, BP and RLS-BP, we ran 25 epoch computations; 1134 parameters were tuned using 2150 input-output training data pairs per epoch.

The performance evaluation for the learning methods was based on root mean-squared error (RMSE) benchmarking criteria as in [1]:

$$RMSE_{S,2}(*) = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left[Y(k) - f_{S,2}\left(\mathbf{x}^{(k)}\right)\right]^{2}} \qquad (4)$$

where $Y(k)$ is the output from the 102 testing pairs, $RMSE_{S,2}(*)$ stand for $RMSE_{S,2}(BP)$, the RMSE of the interval type-2 SFLS (BP), and for $RMSE_{S,2}(RLS-BP)$, the RMSE of the interval type-2 SFLS (RLS-BP). In order to calculate the RMSE of type-1 SFLS, (4) was used.

Fig. 3, shows RMSEs of the two interval type-2 SFLS systems used and the one of the type-1 SFLS with 25 epochs' computations. It can be appreciated that after one epoch, the hybrid interval type-2 SFLS (RLS-BP) forecaster has better performance than both type-1 SFLS and interval type-2 SFLS (BP).

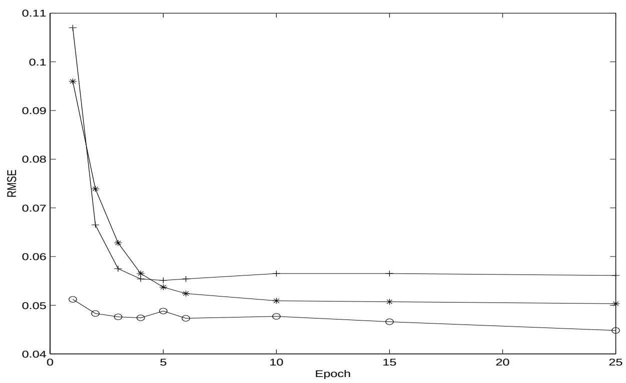

Fig. 3. Type-2 SFLS (*) (BP) - (o) (RLS-BP) - Type-1 SFLS (+) (BP).

The comparison between the predicted exchange rate after one epoch using type-1 SFLS and the real price is depicted in Fig. 4. The type-1 SFLS prediction looks smooth and linear after one epoch of training. Fig. 5 shows the type-1 SFLS prediction after four epochs. Fig. 6 shows that after epoch four, the interval type-2 SFLS outperforms the type-1 SFLS. Fig. 7, shows the comparison between the predicted exchange rate using hybrid interval type-2 SFLS (RLS-BP) and the historical price after one epoch of training. The hybrid interval type-2 SFLS outperforms the interval type-2 SFLS at each epoch as shown in Fig. 3.

IV. CONCLUSION

An interval type-2 SFLS using hybrid RLS-BP training method was tested and compared its forecast of the daily exchange rate between Mexican Peso and US Dollar MXNUSD among the results of singleton type-1 and interval singleton type-2, trained using BP algorithm. The results show that the interval type-2 SFLS system using RLS-BP hybrid learning presented the best performance, and that the type-1 SFLS presented the worst performance, therefore we conclude that we can use the daily data of exchange rate, directly to train both interval type-2 FLS system, (BP) and (RLS-BP), in order to predict MXNUSD one day in advance. It was observed that interval type-2 SFLS forecasters managed efficiently the uncertainties presented in the raw historical data.

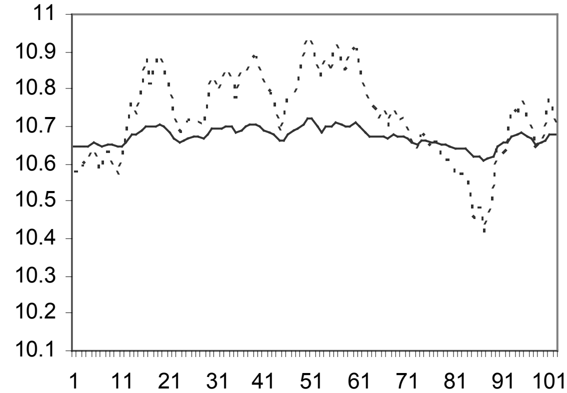

Fig. 4. (--) Historical and (–) predicted prices of MXNUSD. Type-1 SFLS (BP) predictions after one epoch of training.

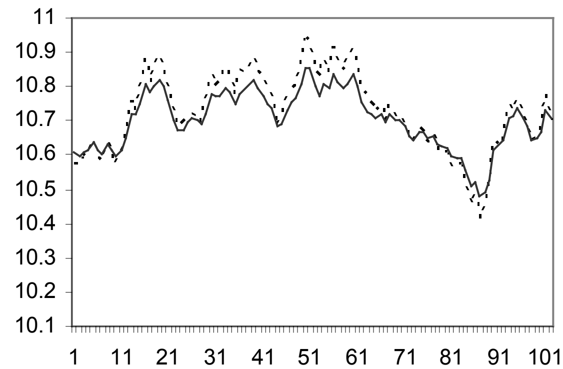

Fig. 5. (--) Historical and (–) predicted prices of MXNUSD. Type-1 SFLS (BP) predictions after four epochs of training.

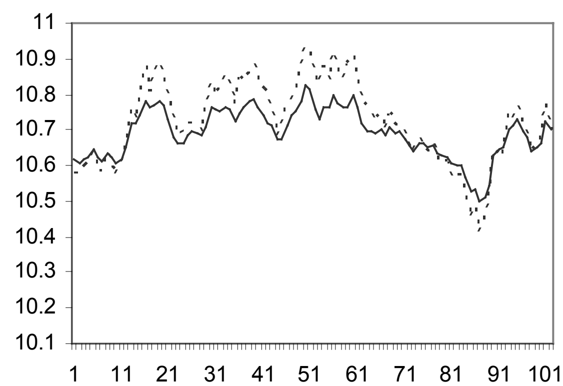

Fig. 6. (--) Historical and (–) predicted prices of MXNUSD Interval type-2 SFLS (BP) predictions after one epoch of training.

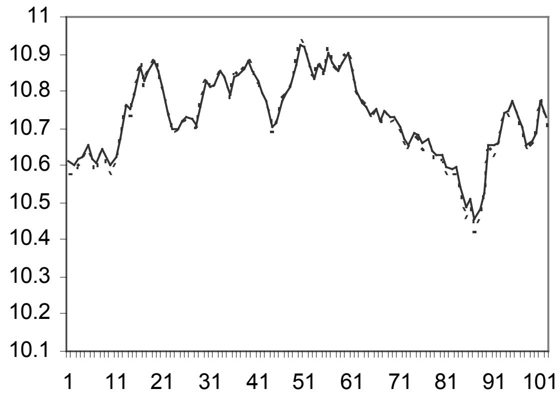

Fig. 7. (--) Historical and (–) predicted prices of MXNUSD. Hybrid interval type-2 SFLS (RLS-BP) predictions after one epoch of training.

REFERENCES

[1] J. M. Mendel, "*Uncertain rule-based fuzzy Logic systems: Introduction and New Directions,"* Upper Saddle River, NJ: Prentice-Hall, 2001

[2] G. M. Mendez, A. Cavazos, L. Leduc, R. Soto, "Hot strip temperature prediction using hybrid learning for interval singleton type-2 FLS," *Proceedings of the IASTED International Conference on Modelling and Simulation*, February 2003, pp 380-385.

[3] G. M. Mendez, "Orthogonal-back propagation hybrid learning algorithm for type-2 fuzzy logic systems," *IEEE Proceedings of the NAFIPS 04 International Conference on Fuzzy Sets*, June 2004, vol. 2, pp. 899-902.

[4] H. A. Hagras, "A hierarchical type-2 fuzzy logic control architecture for autonomous mobile robots," *IEEE Transactions on Fuzzy Systems*, vol. 12, No 4, 2004, pp 524-539.

[5] O. Castillo, P. Melin, "A new hybrid approach for plant monitoring and diagnostics using type-2 fuzzy logic and fractal theory," *Proceedings of the International Conference FUZZ'2003*, May 2003, pp 102-107.

[6] O. Castillo, G. Huesca, F. Valdez, "Evolutionary computing for optimizing type-2 fuzzy systems in intelligent control of non-linear dynamic plants," *IEEE Proceedings of the NAFIPS 05 International Conference*, June 2005, pp. 247-251.

[7] L. Anastasakis, N. Mort, "Prediction of the GSPUSD exchange rate using statistical and neural network models," *Proceedings of the IASTED International Conference on Artificial Intelligence and Applications*, September 2003, pp 493-498.

[8] A. Lendasse, E. de Boot, V. Wertz, M. Verleysen, "Non-linear financial time series forecasting – application to the bel 20 stock market index,", *European Journal of Economics and Social Systems*, vol. 14, 2000, pp. 81-91.

[9] Y. Ito, "Neuro-fuzzy phillips-type stabilization policy for stock process and exchange-rates models," *Proceedings of the IASTED International Conference on Artificial Intelligence and Applications*, September 2003, pp 499-504.

[10] E. W. Saad, D. V. Prokhorov, D. C. Wunch II, "Comparative study of stock trend prediction using time delay, recurrent and probabilistic neural networks," *IEE Trans. On Neural Networks*, vol. 9, 1998, pp. 1456-1470.

[11] W. Leigh, M. Paz, R.Purvis, "An analysis of a hybrid neural network and pattern recognition technique for short term increases in the NYSE composite index," *The International Journal of Management Science,* vol. 30, 2002, pp. 169-76.

[12] Q. Liang, J. M. Mendel, "Interval type-2 fuzzy logic systems: theory and design," *Trans. on Fuzzy Systems*, vol. 8, October 2000, pp 535-550.

[13] R. I. John, "Embedded interval valued type-2 fuzzy sets," *IEEE Transactions on Fuzzy Systems*, 2002.

[14] J.M. Mendel, R. I. John, "Type-2 fuzzy sets made simple," *IEEE Transactions on Fuzzy Systems*, vol. 10, No. 2, April 2002.

María de los Angeles Hernández Medina received the B.Sc. degree in Accountant Engineering from the Universidad de Nuevo Leon, Mexico in 1986, an MBA from the UANL, México in 1996. She is currently professor at the Administration and Finance Science Department of the Instituto Tecnológico de Nuevo Leon - ITNL. Her research interest is in the fields of finance indices modeling and prediction.

Gerardo Maximiliano Mendez received the B.Sc. degree in Electronics from the Tecnológico de Monterrey, Mexico in 1985, an MBA from the UANL, México in 1996 and Dr. Ing. in Mechatronics from CIDESI-CONACYT, México in 2005. He is currently professor at the Electronics and Electromechanical Engineering Department of the Instituto Tecnológico de Nuevo Leon - ITNL. His research interest is in the fields of heat transfer, type-2 intelligent systems, uncertain process modeling and control.