Continuous Terrain Guarding with Two-Sided Guards

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Abstract

We consider the continuous two-sided guarding on a 1.5-dimensional(1.5D) terrain T. To our knowledge, this is the first work on this problem. Specifically, we aim at selecting a minimum number of guards such that every point on the terrain can be seen by a guard to its left, and another guard to its right. A vertex v sees a point p on T if the line segment connecting v to p is on or above T. We demonstrate that the continuous 1.5D terrain guarding problem can be transformed to the discrete terrain guarding problem with a finite point set X and that if X is two-sided guarded, then T is also two-sided guarded. Through this transformation, we provide an optimal algorithm determining a guard set with minimum cardinality that completely two-sided guards the terrain.

1 Introduction

A 1.5 dimensional(1.5D) terrain T is an x-monotone polygonal chain in $\mathbb{R}^2$ specified by n vertices $V(T) = \{v_1, ..., v_i, ..., v_n\}$, where $v_i = (x_i, y_i)$. The vertices induce $n-1$ edges $E(T) = \{e_1, ..., e_i, ..., e_{n-1}\}$ with $e_i = \overline{v_i v_{i+1}}$.

A point p sees or guards q if the line segment $\overline{pq}$ lies above or on T, or more precisely, does not intersect the open region bounded from above by T and from the left and right by the downward vertical rays emanating from v_1 and v_n.

There are two types of terrain guarding problems: (1) continuous terrain guarding (CTG) problem, with objective of determining a subset of T with minimum cardinality that guards T, and (2) discrete terrain guarding problem, with the objective of determining a subset of U with minimum cardinality guarding X, given that the two point sets U and X are on T.

Many studies have referred to applications of 1.5D terrain guarding in real world [1, 2, 3]. The examples include guarding or covering a road with security cameras or lights and using line-of-sight transmission networks for radio broadcasting.

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1.1 Related Work

Ample research has focused on the 1.5D terrain guarding problem, which can be divided into the general terrain guarding problem and the orthogonal terrain guarding problem.

In a 1.5D terrain, King and Krohn [4] proved that the general terrain guarding problem is NP-hard through planar 3-SAT.

Initial studies on the 1.5D terrain guarding problem discussed the design of a constant-factor approximation algorithm. Ben-Moshe et al. [5] gave the first constant-factor approximation algorithm for the terrain guarding problem and left the complexity of the problem open. King [6] gave a simple 4-approximation, which was later determined to actually be a 5-approximation. Recently, Elbassioni et al. [7] gave a 4-approximation algorithm.

Finally, Gibson et al. [8] considered the discrete terrain guarding problem by finding the minimal cardinality from candidate points that can see a target point [8] and proved the presence of a planar graph that appropriately relating the local and global optima; thus, the discrete terrain guarding problem allows a polynomial time approximation scheme (PTAS) based on local search. Friedrichs et al. [9] revealed that for the continuous 1.5D terrain guarding problem, finite guard and witness sets (G and X, respectively) can be constructed such that an optimal guard cover $G'' \subseteq G$ that covers terrain T is present and when these guards monitor all points in X, the entire terrain is guarded. According to [8], the continuous 1.5D terrain guarding problem can apply PTAS by constructing a finite guard and witness set with the former PTAS.

Some studies have considered orthogonal terrain T. T is called an orthogonal terrain if each edge $e \in E(T)$ is either horizontal or vertical. An orthogonal terrain has four vertex types. If v_i is a vertex of orthogonal terrain and the angle $\angle v_{i-1}v_iv_{i+1} = \pi/2$, then v_i is a convex vertex, otherwise it is a reflex vertex. A convex vertex v_i is left(right) convex if $\overline{v_{i-1}v_i}(\overline{v_iv_{i+1}})$ is vertical. A reflex vertex v_i is left(right) reflex if $\overline{v_{i-1}v_i}(\overline{v_iv_{i+1}})$ is horizontal.

Katz and Roisman [10] gave a 2-approximation algorithm for the problem of guarding the vertices of an orthogonal terrain. The authors constructed a chordal graph demonstrating the relationship of visibility between vertices. On the basis of [11], [10] gave a 2-approximation algorithm and used the minimum clique

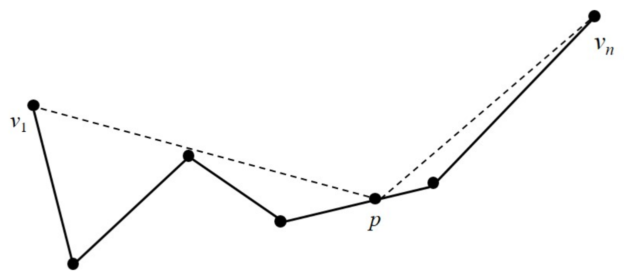

Figure 1: Point p is two-sided guarded by v_1 and v_n.

cover of a chordal graph to solve the right(left) convex vertex guarding problem.

Lyu and Üngör [12] gave a 2-approximation algorithm for the orthogonal terrain guarding problem that runs in $O(n \log m)$, where m is the output size. The authors also gave an optimal algorithm for the right(left) convex vertex guarding problem. On the basis of the vertex type of the orthogonal terrain, the objective of the subproblem is to determine a minimum cardinality subset of $V(T)$ guarding all right(left) convex vertices of $V(T)$; furthermore, the optimal algorithm uses stack operations to reduce time complexity.

The $O(n \log m)$ time 2-approximation algorithm has previously been considered the optimal algorithm for the orthogonal terrain guarding problem. However, some studies have used alternatives to the approximation algorithm.

Durocher et al. [13] gave a linear-time algorithm for guarding the vertices of an orthogonal terrain under a directed visibility model, where a directed visibility mode considers the different visibility for types of vertex. If u is a reflex vertex, then u sees a vertex v of T, if and only if every point in the interior of the line segment uv lies strictly above T. If u is a convex vertex, then u sees a vertex v of T, if and only if $\overline{uv}$ is a nonhorizontal line segment that lies on or above T. Khodakarami et al. [14] considered the guard with guard range. They presented a fixed-parameter algorithm that found the minimum guarding set in time $O(4^k \cdot k^2 \cdot n)$, where k is the terrain guard range.

1.2 Result and Problem Definition

In this paper, we define the CTG problem with two-sided guards and propose an optimal algorithm for the 1.5D CTG problem with two-sided guards. To the best of our knowledge, the 1.5D CTG problem with two-sided guards has never been examined.

Definition 1 (Two-Sided Guarding). A point p on a 1.5D terrain is two-sided guarded if there exist two distinct guards u, which is on or to the left of p, and v, which is on or to the right of p, such that p can be seen by both u and v. Furthermore, the guards u and v are called a left-guard and a right-guard of p.

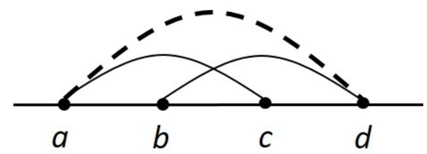

Figure 2: Schematic of Lemma 1.

Fig. 1 illustrates an example where vertex v_1 left-guards p and v_n right-guards p. In this paper, we define the following problem:

Definition 2(CTGTG: Continuous Terrain Guarding with Two-Sided Guards) Given a 1.5D terrain T, find a vertex guard set S of minimum cardinality such that every point of T can be two-sided guarded.

1.3 Paper Organization

Section 2 presents preliminaries, Section 3 demonstrates how to create a finite point set for the CTGTG model, Section 4 gives an algorithm for the CTGTG, along with its proof, and Section 5 presents our conclusions.

2 Preliminaries

Let p and q be two points on a 1.5D terrain, we write $p \prec q$ if p is on the left of q. We denote the visible region of p by $vis(p) = \{v \in V(T) | v \text{ sees } p\}$. For a $vis(p)$, let $L(p)$ be the leftmost vertex in $vis(p)$ and $R(p)$ be the rightmost vertex in $vis(p)$.

Given a CTGTG instance, let $OPT = \{o_1, o_2, ..., o_m\}$ be an optimal guard set, where $o_k \prec o_{k+1}$ for $k = 1, ..., m-1$. For a point p on the terrain, let $O_R(p)$ and $O_L(p)$ be the subsets of OPT such that p is right-guarded by every guard in $O_R(p)$ and left-guarded by every guard in $O_L(p)$. We also define N_i^R as the rightmost point on the terrain that is not right-guarded by $\{o_i, o_{i+1}, ..., o_m\}$ and N_i^L as the leftmost point on the terrain that is not left-guarded by $\{o_1, o_2, ..., o_i\}$.

An important visible property on 1.5D terrains is as follows:

Lemma 1 (Order Claim[5]) *Let a, b, c and d be four points on a terrain T such that $a \prec b \prec c \prec d$. If a sees c and b sees d, then a sees d.*

Fig. 2 is a schematic of Lemma 1. Because T is an x-monotone chain, we use a straight line to demonstrate the relation between x-coordinate of points and an arc to show the visible relation among points on T. In this paper, we use a straight line to simplify the explanations.

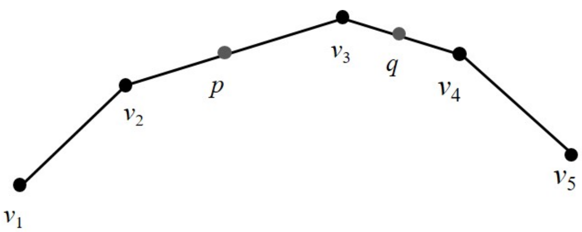

Figure 3: $V(T)$ is right-guarded and left-guarded by $\{v_1, v_2, v_4, v_5\}$, but not T.

Observation 1 *Let e_j be an edge of the terrain and p is on e_j. If p is left-guarded by a guard v, then v also completely guards $\overline{pv_{j+1}}$.*

Observation 2 *Let e_j be an edge of the terrain and p is on e_j. If p is right-guarded by a guard v, then v also completely guards $\overline{v_jp}$.*

3 Discretization

Although $V(T)$ are right-guarded and left-guarded, T is not necessarily right-guarded and left-guarded. In Fig. 3, $V(T)$ is right-guarded and left-guarded by $\{v_1, v_2, v_4, v_5\}$ with minimal cardinality. The vertices v_1 and v_2 are left-guarded by v_1 and right-guarded by v_2. Vertices v_4 and v_5 are left-guarded by v_4 and right-guarded by v_5. Vertex v_3 is left-guarded and right-guarded by v_2 and v_4, respectively. Only v_3 can right-guard p and left-guard q where p is on e_2 and q is on e_3, but $v_3 \notin \{v_1, v_2, v_4, v_5\}$. In our example, we must create a point set X such that if X is right-guarded and left-guarded, then T is also.

Definition 3 (Boundary Point). If line $\overline{v_iv_j}$ and e_k have an intersection point $f \notin \{v_k, v_{k+1}\}$, and v_i and v_j can see f then f is the boundary point.

In Fig. 4, we provide an example with four boundary points: f_1, f_2, f_3 and f_4. Boundary point f_1 is from v_7, f_2 is from v_5; and boundary points f_3 and f_4 are from v_1. We say e_1 has two boundary points, f_1 and f_2; each of e_4 and e_6 has a boundary point.

Lemma 2 *For an edge e_i on terrain T, there exist at most two non-endpoints p and q such that e_i is complete two-sided guarded if p and q are two-sided guarded.*

Proof. According to the number of boundary points on e_i, we may consider the proof under the following cases: edge e_i does not have boundary point or has one, two, or k boundary points (where $k \geq 3$).

In the first case, we assume e_i does not have boundary point. Let point $p \notin \{v_i, v_{i+1}\}$ be on edge e_i. If p is right-guarded and left-guarded, then edge e_i is also right-guarded and left-guarded.

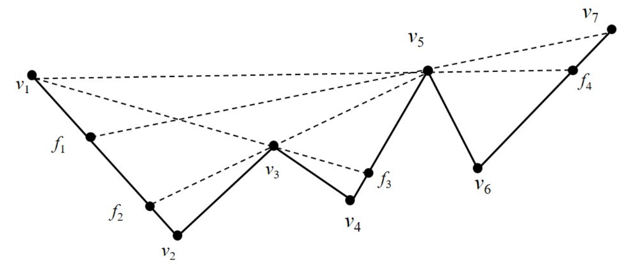

Figure 4: Points f_1, f_2, f_3 and f_4 are boundary points on T.

In the second case, we assume e_i has a boundary point f. We split the edge into two line segments $\overline{v_if}$ and $\overline{fv_{i+1}}$. Then, the first case can be applied to the line segments $\overline{v_if}$ and $\overline{fv_{i+1}}$. Therefore, we create two points $p \notin \{v_i, f\}$ on line segment $\overline{v_if}$ and $q \notin \{f, v_{i+1}\}$ on line segment $\overline{fv_{i+1}}$. If p and q are right-guarded and left-guarded, then e_i is also right-guarded and left-guarded.

In the third case, we assume e_i has two boundary points f_1 and f_2. We split the edge into three line segments $\overline{v_if_1}$, $\overline{f_1f_2}$ and $\overline{f_2v_{i+1}}$. The line segments $\overline{v_if_1}$ and $\overline{f_2v_{i+1}}$ can be reduced to the first case. Therefore, we create two points $p \notin \{v_i, f_1\}$ on line segment $\overline{v_if_1}$ and $q \notin \{f_2, v_{i+1}\}$ on line segment $\overline{f_2v_{i+1}}$. If p and q are left-guarded and right-guarded, then line segment $\overline{f_1f_2}$ is also left-guarded and right-guarded.

In the final case, we assume e_i has k boundary points $f_1, ..., f_k$. We split the edge into $k+1$ line segments $L = \{\overline{v_if_1}, \overline{f_1f_2}, ..., \overline{f_kv_{i+1}}\}$. The line segments $\overline{v_if_1}$ and $\overline{f_kv_{i+1}}$ can be reduced to the first case. Therefore, we create two points: $p \notin \{v_i, f_1\}$ on line segment $\overline{v_if_1}$ and $q \notin \{f_k, v_{i+1}\}$ on line segment $\overline{f_kv_{i+1}}$. If p and q are left-guarded and right-guarded, then each line segment $\overline{f_cf_{c+1}} \in L$ is also left-guarded and right-guarded. □

From the construction of Lemma 2, in order to completely two-sided guard a terrain, it is sufficient to first select a finite subset X of positions from the terrain to be two-sided guarded, such that $|X| \leq 2(n-1)$.

4 An Optimal Algorithm for CTGTG

In this section, we present an optimal algorithm for the CTGTG. The idea of the algorithm follows from Observation 3. In each step of our algorithm, we add a vertex v_i to our result S such that if $v_i \notin OPT$ then v_i can replace a vertex $v_j \in OPT$ and $|S| = |OPT|$.

Observation 3 *The optimal solution of the CTGTG includes v_1 and v_n.*

This is because in the CTGTG for right-guarded and left-guarded T, only v_1 can left-guard v_1 and only v_n can right-guard v_n.

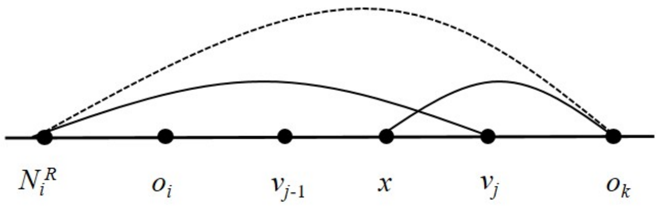

Figure 5: Position of $v_j \in R(N_i^R) \cup O_R(N_i^R)$.

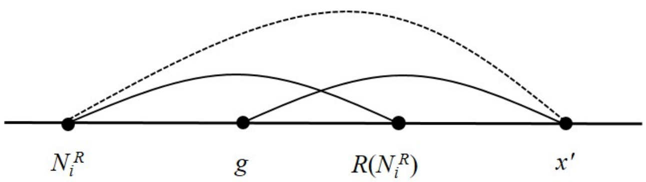

Figure 6: If $g \in O_R(N_i^R)$ left-guard x', then x_k and N_i^R see each other.

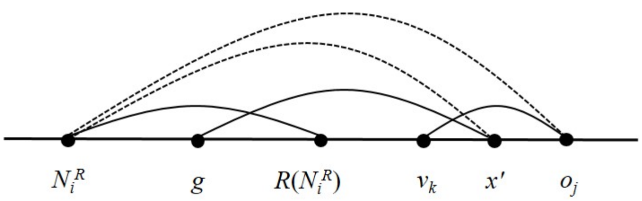

Figure 7: If x' and N_i^R see each other, then o_j right-guards N_i^R.

Lemma 3 *$R(N_i^R)$ and any guard in $O_R(N_i^R)$ do not lie on the right side of o_i.*

Proof. Let x be a point on the edge e_{j-1} such that $N_i^R \prec x$. We assume that $v_j \in R(N_i^R) \cup O_R(N_i^R)$ is on the right side of o_i. We know that x is right-guarded by o_k and o_k is on the right side of v_j. According to Lemma 1, if o_k right-guards x, then N_i^R is right-guarded by o_k. This contradicts the definition of N_i^R and o_k sees N_i^R. The schematic of Lemma 3 is given in Fig 5. $\square$

Lemma 4 *If $R(N_i^R) \notin O_R(N_i^R)$, then any guard in $O_R(N_i^R)$ cannot left-guard $x' \in \{x \in X | R(N_i^R) \prec x\}$.*

Proof. Let g be a guard in $O_R(N_i^R) \setminus R(N_i^R)$ and let x' be a point such that $R(N_i^R) \prec x'$. Therefore, $N_i^R \prec g \prec R(N_i^R) \prec x'$. Consider x' on the edge $e_k = \overline{v_k v_{k+1}}$, there exists a guard o_j that right-guards x'. According to Lemma 1, if x' and g see each other, then x' and N_i^R also see each other. This is illustrated in Fig. 6. Because o_j right-guards x' and sees v_k, if x' sees N_i^R then o_j right-guard N_i^R too, as illustrated in Fig. 7. $\square$

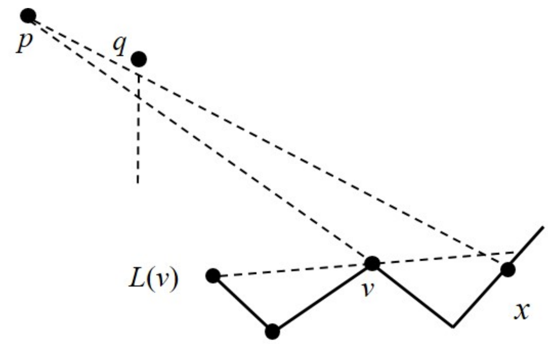

Figure 8: If $L(v)$ cannot see x and v sees x, then $v = L(x)$.

Lemma 5 *If $R(N_i^R) \notin O_R(N_i^R)$, $x \in X$ is right-guarded by o_j and $i \leq j \leq m$, then x cannot lie between $g \in O_R(N_i^R)$ and $R(N_i^R)$.*

Proof. We assume that the point x is on the $e_k = \overline{v_k R(N_i^R)}$ and x is right-guarded by o_j. We know that o_j right-guards v_k by Observation 2. According to Lemma 1, if x is right-guarded by o_j, then $N_R(o_i)$ is right-guarded by o_j. Therefore, we know that if $R(N_i^R) \notin O_R(N_i^R)$, then x cannot lie between $g \in O_R(N_i^R)$ and $R(N_R(o_i))$. $\square$

By Lemma 3, Lemma 4 and Lemma 5, we have the following theorem.

Theorem 6 *If $R(N_i^R) \notin O_R(N_i^R)$, then $R(N_i^R)$ can replace any guard in $O_R(N_i^R)$.*

Proof. Based on Lemma 3, Lemma 4 and Lemma 5, if $R(N_i^R) \notin O_R(N_i^R)$, then $g \in O_R(N_i^R)$ cannot left-guard $x_k \in \{x_j \mid N_i^R \prec x_j\}$. Due to $g \prec R(N_i^R))$, we know $vis(R(N_i^R)) \supseteq vis(g)$ by Lemma 1. $\square$

Similarly, $L(N_i^L)$ can replace any guard in $O_L(N_i^L)$.

Theorem 7 *If $O_L(N_i^L) \notin L(N_i^L)$, then $L(N_i^L)$ can replace any guard in $O_L(N_i^L)$.*

5 Complexity

Because our approach has two phases, we must first discuss the complexity of discretization. We obtain boundary points for a vertex v on $E(T)$ in $O(n)$ time by [15]. Therefore, we compute all boundary points for each vertex of $V(T)$ on each edge $e \in E(T)$ in $O(n^2)$ time. We obtain at most $2|V(T)|$ boundary points in $O(n^2)$ time.

Next, we demonstrate how to compute an optimal solution for the CTGTG. In step 1, we add v_1 and v_n to our solution. In step 2, we compute the $vis(v_1)$ and $vis(v_n)$. In step 3, we add $R(x)$ to our solution, where

$x \in X$ is the nonright-guarded rightmost point. If a point x exists that is not right-guarded, then repeat step 3 until X is right-guarded. In step 4, we add $L(x)$ to our solution, where x is the nonleft-guarded leftmost point. If there exists a point x that is not yet left-guarded, then repeat Step 4 until it is left-guarded. Thus, all points on the terrain are successfully guarded from both sides.

We show our algorithm for the CTGTG runs in $O(n)$ time using two steps. Before the algorithm begins, we can compute $R(x)$ and $L(x)$ for each point of X in $O(n)$ time. After this computation, we proceed to the algorithm in $O(n)$ time. Therefore, our proposed algorithm for the CTGTG runs in $O(n)$ time.

Algorithm 1: Compute all $L(x)$

Input: T: terrain, X: point set
Output: $\{ L(x) | x \in X \}$
$Q \leftarrow X \cup V(T)$
for $q_i \in Q$ *processed from left to right* **do**
  $q_j = q_{i-1}$
  while $L(q_i) = \emptyset$ **do**
    if q_i *sees* $L(q_j)$ **then**
      if $L(q_j)$ *is not* v_1 **then**
        $q_j = L(q_j)$
      else
        $L(q_i) = v_1$
    else
      $L(q_i) = q_j$
for $x \in X$ *processed from left to right* **do**
  Return $L(x)$

Lemma 8 *Let v and x be two points on a terrain T such that $v \prec x$. If $L(v)$ cannot see x and v sees x then $v = L(x)$.*

Proof. Let p, v and x be three points on T such that $p \prec L(v) \prec v \prec x$. We assume that $L(v)$ cannot see x and v can see x. If p sees x and cannot see v, then a vertex q exists and lie above line $\overline{vL(v)}$ and $p \prec q \prec L(v)$, as illustrated in Fig. 8. However, the assumption that $L(v) \neq q$ is contradictory. □

We propose Algorithm 1 to compute $L(x)$ for all points x in X according to Lemma 8 and Lemma 1. We prove that the running time of Algorithm 1 is $O(n)$.

Theorem 9 *Algorithm 1 runs in $O(n)$ time.*

Proof. We count the number of times q_i sees $L(q_j)$ in the algorithm. If q_i sees $L(q_j)$, then the algorithm does not visit the vetrices between q_i and $L(q_j)$. Therefore, the number of times q_i sees $L(q_j)$ is at most once for each point of Q. If q_i does not see $L(q_j)$, then q_i has found $L(q_i)$. Therefore, the number of times q_i does not see $L(q_j)$ is at most once for each point Q. □

After computing $L(x_i)$ and $R(x_i)$ for X, we reach the algorithm for the CTGTG in $O(n)$ time. We divided our algorithm into left-guarding and right-guarding, and therefore we provide the algorithm for left-guarding that can be implemented in $O(n)$ time.

Algorithm 2: Left-guarding

Input: T: terrain, X: point set
Output: S_L : left-guarding set
S_L is null;
Add v_1 to S_L;
$V(T')$=$V(T)$;
for $x_i \in X$ *processed from left to right* **do**
  while $g(x_i)$ *is null* **do**
    s is rightmost vertex in $S_L \cap V(T')$;
    if x_i *is guarded by* s **then**
      $g(x_i)$ is s;
      Remove the vertices between x_i and s from $V(T')$;
    else if $s \prec L(x_i)$ **then**
      $g(x_i)$ be the vertex $L(x_i)$;
      Add $g(v_i)$ to S_L;
      Remove the vertics between x_i and $L(x_i)$ from $V(T')$;
    else
      Remove s from $V(T')$;
return S_L

Theorem 10 *Algorithm 2 runs in $O(n)$ time.*

Proof. For each x_i, we examine whether x_i is guarded by $s \in S_L$ from x_i to $g(x_i)$. If $g(x_i) = v_j$, then Algorithm 2 will not visit the point and vertex between x_i and v_j. We count the number of times x_i is not seen by S_L. We can check s from x_i to $L(x_i)$. If s does not see x_i, then we will not check s for $\{x_k \mid x_i \prec x_k\}$. The number of times X is not seen by S_L is $|V(T)|$, and the number of times X is seen by S_L is $|X|$. Therefore, the algorithm visits the point and vertex at most $2|X| + |V(T)|$ times. After computing all $L(x_i)$, Algorithm 2 runs in $O(n)$ time. □

6 Conclusion

In this paper, we considered the CTGTG problem and devised an algorithm that can determine the minimal cardinality vertex that guards T under two-sided guarding. We showed that the CTGTG problem can be reduced to the discrete terrain guarding problem with at most $2|V(T)|$ points in $O(n^2)$ time and solved the problem using our devised algorithm in $O(n)$ time where n is the number of vertices on T.

References

[1] P. Ashok, F. V. Fomin, K. Sudeshna, S. Saurabh, M. Zehavi, Exact algorithms for terrain guarding, in: 33rd International Symposium on Computational Geometry, 2017.

[2] H. Eliş, A finite dominating set of cardinality $O(k)$ and a witness set of cardinality $O(n)$ for 1.5d terrain guarding problem, Annals of Operations Research (2017) 1–10.

[3] F. Khodakarami, F. Didehvar, A. Mohades, A fixed-parameter algorithm for guarding terrains, Theoretical Computer Science 595 (2015) 134–142.

[4] J. King, E. Krohn, Terrain guarding is np-hard, SIAM Journal on Computing 40 (5) (2011) 1316–1339.

[5] B. Ben-Moshe, M. J. Katz, J. S. Mitchell, A constant-factor approximation algorithm for optimal 1.5 d terrain guarding, SIAM Journal on Computing 36 (6) (2007) 1631–1647.

[6] J. King, A 4-approximation algorithm for guarding 1.5-dimensional terrains, in: Latin American Symposium on Theoretical Informatics, Springer, 2006, pp. 629–640.

[7] K. Elbassioni, E. Krohn, D. Matijević, J. Mestre, D. Ševerdija, Improved approximations for guarding 1.5-dimensional terrains, Algorithmica 60 (2) (2011) 451–463.

[8] M. Gibson, G. Kanade, E. Krohn, K. Varadarajan, An approximation scheme for terrain guarding, in: Approximation, Randomization, and Combinatorial Optimization. Algorithms and Techniques, Springer, 2009, pp. 140–148.

[9] S. Friedrichs, M. Hemmer, C. Schmidt, A ptas for the continuous 1.5d terrain guarding problem, in: Canadian Conference on Computational Geometry, 2014.

[10] M. J. Katz, G. S. Roisman, On guarding the vertices of rectilinear domains, Computational Geometry 39 (3) (2008) 219–228.

[11] F. Gavril, Algorithms for minimum coloring, maximum clique, minimum covering by cliques, and maximum independent set of a chordal graph, SIAM Journal on Computing 1 (2) (1972) 180–187.

[12] Y. Lyu, A. Üngör, A fast 2-approximation algorithm for guarding orthogonal terrains, in: Canadian Conference on Computational Geometry, 2016.

[13] S. Durocher, P. C. Li, S. Mehrabi, Guarding orthogonal terrains., in: Canadian Conference on Computational Geometry, 2015.

[14] F. Khodakarami, F. Didehvar, A. Mohades, 1.5d terrain guarding problem parameterized by guard range, Theoretical Computer Science 661 (2017) 65–69.

[15] M. Löffler, M. Saumell, R. I. Silveira, A faster algorithm to compute the visibility map of a 1.5d terrain, in: Proc. 30th European Workshop on Computational Geometry, 2014.