READING GROUP LEARNING OVERVIEW

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1. General Learning Model

1.1. Rate Based Learning Rule. General form of learning

$$\frac{dw_{ij}}{dt} = F(w_{ij}; v_i, v_j)$$

When F is nice enough (analytic for example), it can be approximated by Taylor expansion

$$\frac{dw_{ij}}{dt} = c_0 + c_1 v_i + c_2 v_j + c_{11} v_i^2 + c_{12} v_i v_j + c_{22} v_j^2 + O(v^3)$$

If $c_{12} > 0$, we call it Hebbian learning. The most naive form of Hebbian learning looks like

$$\frac{dw_{ij}}{dt} = c_{12} v_i v_j$$

This has two problem

(1) w_{ij} is not bounded
(2) w_{ij} cannot decrease

To solve the first problem, we can introduce a soft bound

$$\frac{dw_{ij}}{dt} = r(w_{max} - w_{ij})^\beta v_i v_j, \beta > 0$$

When $\beta \to 0$, this becomes a hard bound. To solve the second problem, we can consider exponential decrease

$$\frac{dw_{ij}}{dt} = r(w_{max} - w_{ij})^\beta v_i v_j - r' w_{ij}, \beta > 0$$

There are two famous learning rules in neuroscience: first, we have Oja's rule which extracts the first principal component and approximate L_2 normalization

$$\frac{dw_{ij}}{dt} = r v_j (v_i - v_j w_{ij})$$

The reason that this approximates L_2 normalization is the following: consider the naive Hebbian rule with hard L_2 normalization

$$\frac{w_{ij} + \lambda v_i v_j}{\|w_j + \lambda v v_j\|_2} = \frac{w_i}{\|w_j\|_2} + \lambda(\frac{v_j v_i}{\|w_j\|_2} - \frac{w_{ij} \sum_k v_j v_k w_{kj}}{\|w_j\|_2^3}) + O(\lambda^2)$$

By plugging in $\|w_j + \lambda v v_j\|_2 = 1, y = \sum_k v_k w_{kj}$, we recover Oja's rule.

Second we have BCM's rule where v_θ is a constant.

$$\frac{dw_{ij}}{dt} = r v_i (v_i - v_\theta) v_j - r' w_{ij}$$

Notice that this first go through depression and then potentiation as v_i increases. This has been shown to induce input selectivity, anything non-important will be surpressed while important signals will be strengthened.

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1.2. **Spike Based Learning Rule.** STPD (Spike Timing Dependent Plasticity) captures the casuality of the spikes. The learning happens when presynaptic spikes happen before post synaptic spikes while the unlearning happens when presynaptic spikes happen after post synaptic spikes. Let $S_i = \sum_f \delta(t - t_i^f), S_j = \sum_f \delta(t - t_j^f)$, We have the following learning rule

$$\frac{dw_{ij}}{dt} = S_j(t)(a_1^{pre} + \int_0^\infty A_-(w_{ij})W_-(s)S_i(t-s)ds) + S_i(t)(a_1^{post} + \int_0^\infty A_+(w_{ij})W_+(s)S_j(t-s)ds)$$

1.3. **Reward Based Learning Rule.** You can similarly consider reward learning rule

$$\frac{de_{ij}}{dt} = -e_{ij} + v_i v_j$$

$$\frac{dw_{ij}}{dt} = M v_i v_j e_{ij}, M = R - \langle R \rangle$$

Where M is the reward modulating signal, immediate reward minus the expected reward.

2. Specific Learning Problem

2.1. **Principal Component.** Using Oja's rule, we can extract the first principal component in a linear nextwork. Consider $x, w \in \mathbb{R}^n, y \in \mathbb{R}$. Now the theorem is

Theorem 2.1. *All eigenvectors of $C = \langle xx^T \rangle$ are fixed points of the learning rule. The first pricipal component is a stable fixed point, the last principal component is an unstable fixed point while all other eigenvectors are saddle points of the system.*

2.2. **Bidirectional Associative Memory.**

2.3. **Associative Memory.**