Guarded Negation

Balder ten Cate

UC Santa Cruz

In recent work with Vince Barany and Luc Segoufin [4], we introduce fragments of first-order logic and of fixed-point logic in which all occurrences of negation are required to be guarded by an atomic predicate. In terms of expressive power, the logics in question, called GNFO and GNFP, properly extend the guarded fragment of first-order logic [1] and guarded fixed-point logic [6], respectively. GNFO can be viewed as a natural common generalization of the guarded fragment and the language of unions of conjunctive queries, while GNFP can be viewed as a natural common generalization of guarded fixpoint logic and monadic Datalog. In this sense, the concept of guarded negation provides a unifying perspective on decidable formalisms arising in the areas of modal logic and database theory. The results in [4] (which build on recent results for the guarded fragment and guarded fixed point logic [2,3]) show that GNFO and GNFP are well behaved logics, both computationally and model theoretically.

The aim of the talk is to present the guarded negation logics GNFO and GNFP, and also to describe some of the technical constructions used in the proofs of our main results. In particular, I will discuss two important (known) concepts that are frequently used in this context, and that deserve to belong to the toolkit of any modern modal logician, namely the concepts of *bounded treewidth*, and of *locally acyclic covers*.

Bounded treewidth provides, in some sense, a precise characterization of the dividing line between the decidable world of trees and the undecidable world of grids and tilings. If a class of structures has bounded treewidth, then the first-order theory, and in fact even the guarded second order theory, of that class is decidable, as can be shown by a reduction to the monadic second order theory of trees. If, on the other hand, a class of structures has unbounded treewidth, then it follows from results in graph theory due to Robinson and Seymour, that we can find, inside this class, arbitrarily large grids as graph minors, from which it then follows that the guarded second order theory of the class is undecidable. The decidability of the guarded fragment of first-order logic, as well the decidability of guarded fixed point logic, stems from the fact that these logics are invariant under guarded bisimulations, and every structure is guarded bisimilar to a structure of bounded treewidth namely its "guarded unraveling" [1,5]. A similar analysis applies more generally to the guarded-negation logics GNFO and GNFP.

Locally acyclic structures form finite "approximations" of infinite unravelings. In general, unraveling a structure into a tree-like structure is only possible if one allows for infinite structures (as can be illustrated by the fact that, if a finite Kripke structure contains a cycle, then every finite Kripke structure bisimilar to it must contain a cycle as well, and therefore cannot be a tree). Nevertheless, it is often possible to construct a finite structure that behaves "similarly enough" to an infinite unraveling. In particular, in the case of the basic modal language, every finite Kripke structure is bimilar to a Kripke structure that contains only cycles of length at least k, where k is a natural number that may be chosen arbitrarily large. It follows from locality theorems for first-order logic that such structures cannot be distinguished by means of a first-order formula from structures that are fully acyclic (picking k large enough, depending on the first-order formula in question). Such locally acyclic structures can then be used as a substitute for infinite unravelings in proving finite model-theoretic results [7]. Similar, but more complicated, constructions exist for the guarded fragment [3,8,2], and they are used also to the study of the guarded negation logics GNFO and GNFP.

References

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