OPTIMIZED NET EXCHANGE MONTE CARLO SIMULATION FOR PARTICIPATING MEDIA

P. PEREZ*
M. PAULIN[1] M. EL HAFI[2] R. FOURNIER[3]

* École des Mines d'Albi Carmaux, Campus Jarlard, route de Teillet, Albi, France, UMR/CNRS 2392, e-mail: patperez@enstimac.fr

ABSTRACT

The aim of this paper is to present the use of Monte Carlo method in engineering applications to solve the radiative transfer equation in participating media. Adapted probability density functions that allow to optimize the Monte Carlo algorithm are discussed. 1D and 2D test cases are treated and a validation work has been performed. The 3D implementation is in progress.

Keywords: Monte Carlo, Participating Medium, pdf optimization, spectral properties, radiative heat transfer

1 INTRODUCTION

As a simplification for the resolution of the radiative transfer equation (RTE) in rendering, the interaction between the electomagnetic waves and gaseous media is often neglected. Considering configurations where these effects are not negligible, some researchers tried to include a participating medium in their algorithm [Blinn82a, Rushmeier87a, Arques96a, Pattanaik93a]. Rushmeier[Rushmeier87a] have proposed an extension of the radiosity method to include volume/volume, volume/surface and surface/surface interaction. This method was in fact first introduced by Hottel[Hottel67a] and is well known in heat transfer as the zonal method. In the combustion domain, gas-radiation interactions are crucial for the design of clean combustion systems[Siegel92a, Warnatz98a]. We present in this paper a spectral radiative transfer computation by the Monte Carlo integration method. A first step of formulation has been done, and the method has been applied to simple 1D and 2D geometries. Validation and comparison tests for academic configurations have been done with these geometries. Moreover, the extension of these works in to real complex industrial configurations is in progress.

[1]Institut de Recherche en Informatique de Toulouse

[2]École des Mines d'Albi Carmaux

[3]Laboratoire d'Études des Systèmes et Environnement Thermique de l'Homme, U. Paul Sabatier, Route de Narbonne, Toulouse, France

2 Formulation choices

2.1 Net Exchange Formulation

Green [Green67a] introduced the net exchange formulation (N.E.F.) in 1967 for atmospheric studies. One of its main features is to allow an intrinsic satisfaction of both the energy conservation and reciprocity principle[Cherkaoui96a, Cherkaoui98a, Dufresne98a]. Without entering into any detail, this formulation can be expressed for different pairs of volumes (V-V) or surfaces (S-S) or volume-surface (V-S). For instance, the net exchange between two elements C_i and C_j inside black walls, in non-scattering media, is given by:

$$\varphi_{C_i \leftrightarrow C_j} = \int_0^\infty d\eta \int_{\gamma \in \Gamma(C_i,C_j)} \underbrace{d\gamma}_{\text{geometric}} \underbrace{O(\gamma)}_{\text{optics}} \underbrace{[B_\eta(r_{P_i}(\gamma)) - B_\eta(r_{P_j}(\gamma))]}_{\text{energetics}} \quad (1)$$

Here $\Gamma(C_i, C_j)$ is the space of all optical paths from C_i to C_j and the corresponding differential $d\gamma$ includes all geometrical aspects such as incident angle cosines and solid angles. η is the wave number, k_η the absorption coefficient, B_η the monochromatic black body intensity, and $\tau_\eta(l_{ij}(\gamma))$ is the transmittance function along the straight line between P_i and P_j.

2.2 Spectral properties model

For standard combustion gases, absorption coefficient and transmission functions can be computed from the spectral database proposed by [Soufiani97a]. Average quantities from this database are provided by the Malkmus [Malkmus67a] narrow band statistical model. In this model, the spectral range considered[4] is divided into narrow bands of $25cm^{-1}$ width. The absorption line spectrum within each narrow band (for each given gas specie) is entirely represented by two parameters: κ which is the mean absorption coefficient, and Φ which is the shape parameter, describing the statistical repartition of the lines into the narrow band.

[4]for combustion applications we consider an Infra Red spectral range between $150cm^{-1}$ and $9300cm^{-1}$ that include as high as a million absorption lines

3 Optimization of the Monte Carlo algorithm

3.1 General features and optimization of the MCM

The MCM is used here as a statistical method for numerical computation of multidimensional integrals illustrated in equation (1). It simulates the radiative transfer by generating (for each mesh of the system) a large number of rays (representing photon bundles) that will travel through the medium. In each mesh crossed by a ray, an exchange with the emitting mesh is calculated.
The major convergence difficulties of the MCM are encountered for the simulation of limiting cases, such as optically very thin or very thick media, or when the discretization is non uniform.
Let us Recall the mathematical principles of MCM described in [Hammersley67a].
To estimate $A = \int_D f(v)dv$, a nonzero probability density function (pdf) $p(v)$ is chosen arbitrarily on the domain D. An associated weighting function is then defined as $w(v) = f(v)/p(v)$. N values of v are generated randomly according to $p(v)$ and for each value $v_i$the corresponding weighting factor $w_i = w(v_i)$ is computed. The integral value is estimated as the average of weighting factors: $A \approx \frac{1}{N}\sum_i w_i$. The criterion of accuracy is the variance of $w(v)$. The pdf should be chosen in such a way as to make the variations of $f(v)/p(v)$ minimal. All details may be found in [Lataillade2001a, Lataillade2002].

3.1.1 Spectral integration

The spectral integrations are performed on the basis of the Malkmus model, via a k-distribution reformulation [Domoto74a], randomly generating monochromatic absorption coefficient instead of using narrow-band average transmission functions. Mathematical developments have been performed using inverse Gaussian function properties [Dufresne99a]. These allow the choice of adapted k-distribution functions and lead to efficient spectral average computations.

3.1.2 The optico-geometrics

The k-distribution approach allows one to consider all the optico-geometric integrations as pseudo-monochromatic. Directions and positions can therefore be generated randomly according to simple adapted pdf's depending on optical thickness. Emission positions are then exponentially distributed away from volume boundaries in order to ensure that positions close to the boundaries are favored in the optically thick limit, following the pdf:

$$p(\sigma_i) = \frac{\kappa e^{-\kappa\sigma_i}}{1 - e^{-\kappa\|x_2 - x_1\|}} \qquad (2)$$

where κ is the absorption coefficient. For physical reasons associated to the reciprocity principle, this emission position pdf is the same than that exposed in [Pattanaik93a] for classical light extinction.

To illustrate the behavior of the corresponding Monte Carlo algorithm as a function of optical thickness, a simple monochromatic test case is first studied. The medium is semi-transparent and homogeneous (constant optical properties) inside a slab with black boundaries. The temperature profile is such that the monochromatic blackbody intensity at the considered frequency follows a linear profile from B_0 at the bottom to zero at the top. Three algorithms computing the incident radiative flux at the bottom wall are presented below for comparison in terms of required random generation numbers to reach a one percent relative error (Figure 1). The two first algorithms are standard analog and path-integrated algorithms for which standard deviation may be expressed analytically and further translated into required random generation numbers[Lataillade2001a]. The third algorithm, which uses an adapted pdf to take into account optical thickness variations, shows good numerical behavior. The relative error does not tend to infinity with increasing optical thickness. On the contrary it tends to zero, which is only a satisfactory numerical translation of the fact that radiation is known to tend to its equilibrium value in the optically thick limit.

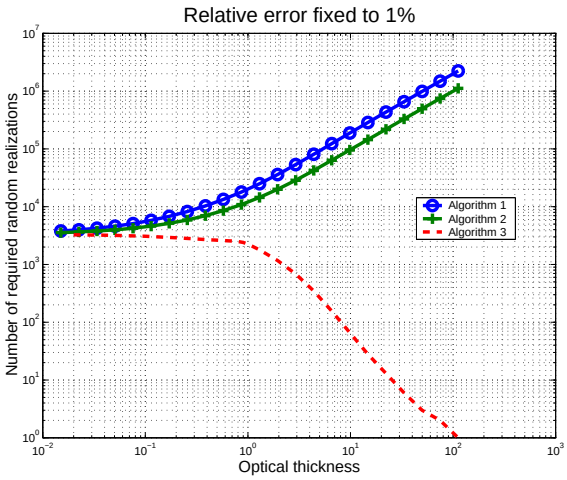

Figure 1: Number of required realizations to produce a 1% estimate of the monochromatic radiative flux at the top of a horizontal 1-D slab with a linear blackbody intensity profile: $B_\eta(T(z)) = B_0(1 - \frac{z}{H})$ where H is the slab thickness. Boundaries are black, there is no scattering and absorption coefficient is uniform.

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