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Explanation: Sum of s minimum elements of subsets with k many elements of a set with 1 to n elements.

Theorem 1: For $\{1,2,3,...,n\}$, sum of 2 minimum elements of subsets with 3 elements is $3.\binom{n+1}{4}$

Proof: Let the notation for subsets be $\{a,b,c\}$ for $a<b<c$

For "1": for $1<b<c\leq n$ sum is $\binom{n-1}{2}$.

Sum of "1"s: $1.\binom{n-1}{2}$

For "2", two possibilities:

1: for $a<2<c\leq n$, a can have $\binom{1}{1}$ states and c can have $\binom{n-2}{1}$ states. Total number of states: $\binom{1}{1}.\binom{n-2}{1}$

2: for $2<b<c\leq n$ there are $\binom{n-2}{2}$ possible states. So for "2", total number of possible states is $\binom{1}{1}.\binom{n-2}{1}+\binom{n-2}{2}$

Sum of "2"s: $2.\left[\binom{1}{1}.\binom{n-2}{1}+\binom{n-2}{2}\right]$

For "3", two possibilities

1: for $a<3<c\leq n$ a can have $\binom{2}{1}$ possible states and c can have $\binom{n-3}{1}$ possible states.

Total number of states : $\binom{2}{1}.\binom{n-3}{1}$

2: for $3<b<c\leq n$ there are $\binom{n-3}{2}$ possible states. Total number of possible states:

$\binom{2}{1}.\binom{n-3}{1}+\binom{n-3}{2}$

Sum of "3"s: $3.\left[\binom{2}{1}.\binom{n-3}{1}+\binom{n-3}{2}\right]$

similarly,

for r, $2 \le r \le n-2$, two possibilities:

1: for $a < r < c \le n$, a can have $\binom{r-1}{1}$ states and c can have $\binom{n-r}{1}$ possible states.

Total: $\binom{r-1}{1}.\binom{n-r}{1}$

2: for $r < b < c \le n$, there are $\binom{n-r}{2}$ possible states. Total number of possible states:

$\binom{r-1}{1}.\binom{n-r}{1}+\binom{n-r}{2}$

Sum of "r"s: $r.\left[\binom{r-1}{1}.\binom{n-r}{1}+\binom{n-r}{2}\right]$

For $r = n-1$ **and** $a < n-1 < c = n$ **there are** $\binom{n-2}{1}.\binom{1}{1}$ states.

Sum of "$n-1$**"s:** $(n-1).\binom{n-2}{1}.\binom{1}{1}$

If we sum up we get:

$\sum_{r=2}^{n-1} r.\binom{r-1}{1}.\binom{n-r}{1}+\sum_{r=1}^{n-2} r.\binom{n-r}{2}$. Evaluation of these 2 sums:

$$\sum_{r=2}^{n-1} r.\binom{r-1}{1}.\binom{n-r}{1}=\sum_{r=2}^{n-1} r.(r-1).(n-r)=\sum_{r=2}^{n-1}\left((n+1).r^2-n.r-r^3\right)$$

$$=\sum_{r=1}^{n-1}\left((n+1).r^2-n.r-r^3\right)-\left((n+1).1^2-n.1-1^3\right)=(n+1).\frac{n.(n-1)(2n-1)}{6}-n.\frac{(n-1).n}{2}-\left[\frac{(n-1).n}{2}\right]^2$$

$$=\frac{n.(n-1)}{2}\left[\frac{(n+1).(2n-1)}{3}-n-\frac{(n-1).n}{2}\right]=\frac{n.(n-1)}{2}.\left[\frac{4n^2+2n-2-6n-3n^2+3n}{6}\right]$$

$$=\frac{n.(n-1).(n^2-n-2)}{12}=\frac{(n+1).n.(n-2).(n-1)}{12}=2.\binom{n+1}{4} \qquad ....(1.1)$$

$$\sum_{r=1}^{n-2} r.\binom{n-r}{2}=$$

$1.\binom{n-1}{2}+2.\binom{n-2}{2}+3.\binom{n-3}{2}+...+(n-3).\binom{3}{2}+(n-2).\binom{2}{2}$. For this sum,

$$\binom{n-1}{2}+\binom{n-2}{2}+\binom{n-3}{2}+\ldots+\binom{3}{2}+\binom{2}{2}=\binom{n}{3}$$

$$\binom{n-2}{2}+\binom{n-3}{2}+\ldots+\binom{3}{2}+\binom{2}{2}=\binom{n-1}{3}$$

$$\binom{n-3}{2}+\ldots+\binom{3}{2}+\binom{2}{2}=\binom{n-2}{3}$$

…

$$\binom{3}{2}+\binom{2}{2}=\binom{4}{3}$$

$$\binom{2}{2}=\binom{3}{3}$$

summing these equations;

$$1.\binom{n-1}{2}+2.\binom{n-2}{2}+3.\binom{n-3}{2}+\ldots+(n-3).\binom{3}{2}+(n-2).\binom{2}{2}=$$

$$\binom{n}{3}+\binom{n-1}{3}+\binom{n-2}{3}+\ldots+\binom{4}{3}+\binom{3}{3}=\binom{n+1}{4}$$

$$\sum_{r=1}^{n-2} r.\binom{n-r}{2}=\binom{n+1}{4}$$ found. ...(1.2)

Substitution of (1.1) ve (1.2) :

$$\sum_{r=2}^{n-1} r.\binom{r-1}{1}.\binom{n-r}{1}+\sum_{r=1}^{n-2} r.\binom{n-r}{2}=2.\binom{n+1}{4}+\binom{n+1}{4}=3.\binom{n+1}{4}$$

In conclusion, sum of 2 minimum elements of subsets with 3 elements of $\{1,2,3,\ldots,n\}$ is

$$3.\binom{n+1}{4}$$

Example:

For A={1,2,3,4} subsets with 3 elements are {1,2,3},{1,2,4},{1,3,4},{2,3,4}.
Sum of 2 minimum elements of these subsets:
(1+2)+(1+2)+(1+3)+(2+3)=15

For A={1,2,3} subsets with 3 elements is {1,2,3}
Sum of 2 minimum elements of this subset:
1+2=3

Theorem 2: For $\{1,2,3,...,n\}$, sum of 2 minimum elements of subsets with 4 elements is $3.\binom{n+1}{5}$

Proof: Let the notation for 4 elemented subsets be $\{a,b,c,d\}$ for $a<b<c<d$

For "1", as $1<b<c<d\leq n$ sum is $\binom{n-1}{3}$

Sum of "1"s $1.\binom{n-1}{3}$

For "2", two possible states:

1: for $a<2<c<d\leq n$ a can have $\binom{1}{1}$ combinatorial states and c and d can have $\binom{n-2}{2}$ states. Total number of states: $\binom{1}{1}.\binom{n-2}{2}$

2: for $2<b<c<d\leq n$ there are $\binom{n-2}{3}$ possible states. So for "2", total number of possible states is $\binom{1}{1}.\binom{n-2}{2}+\binom{n-2}{3}$

Sum of "2"s: $2.\left[\binom{2}{1}.\binom{n-3}{2}+\binom{n-3}{3}\right]$

For "3", two possibilities:

1: for $a<3<c<d\leq n$ a can have $\binom{2}{1}$ possible states and c can have $\binom{n-3}{2}$ possible states. Total number of states : $\binom{2}{1}.\binom{n-3}{2}$

2: for $3<b<c<d\leq n$ there are $\binom{n-3}{3}$ possible states. Total number of possible states: $\binom{2}{1}.\binom{n-3}{2}+\binom{n-3}{3}$

Sum of "3"s: $3.\left[\binom{2}{1}.\binom{n-3}{2}+\binom{n-3}{3}\right]$

similarly,

for r, $2 \le r \le n-3$**,** two possibilities:

1: for $a < r < c < d \le n$, a can have $\binom{r-1}{1}$ states and c can have $\binom{n-r}{2}$ possible states.

Total: $\binom{r-1}{1}.\binom{n-r}{2}$

2: for $r < b < c < d \le n$, there are $\binom{n-r}{3}$ possible states. Total number of possible states:

$\binom{r-1}{1}.\binom{n-r}{2}+\binom{n-r}{3}$

Sum of "r"s: $r.\left[\binom{r-1}{1}.\binom{n-r}{2}+\binom{n-r}{3}\right]$

For $r = n-2$ **and** $a < n-2 < c < d = n$ **there are** $\binom{n-3}{1}.\binom{2}{2}$ states.

Sum of " $n-2$ **"s:** $(n-2).\binom{n-3}{1}.\binom{2}{2}$

If we sum up we get:

$\sum_{r=2}^{n-2} r.\binom{r-1}{1}.\binom{n-r}{2}+\sum_{r=1}^{n-3} r.\binom{n-r}{3}$ Evaluation of these 2 sums:

$\sum_{r=2}^{n-2} r.\binom{r-1}{1}.\binom{n-r}{2}=\sum_{r=2}^{n-2} 2.\binom{r}{2}.\binom{n-r}{2}$ If we use (1.3) Equation

$$2.\sum_{r=2}^{n-2}\binom{r}{2}.\binom{n-r}{2}=2.\binom{n+1}{5} \qquad ...(2.1)$$

Let sum up $\sum_{r=1}^{n-3} r.\binom{n-r}{3}$

Sum of minimum elements is

$1.\binom{n-1}{3}+2.\binom{n-2}{3}+3.\binom{n-3}{3}+...+(n-4).\binom{4}{3}+(n-3).\binom{3}{3}$.

$$\binom{n-1}{3}+\binom{n-2}{3}+\binom{n-3}{3}+\ldots+\binom{4}{3}+\binom{3}{3}=\binom{n}{4}$$

$$\binom{n-2}{3}+\binom{n-3}{3}+\ldots+\binom{3}{3}+\binom{2}{3}=\binom{n-1}{4}$$

$$\binom{n-3}{3}+\ldots+\binom{4}{3}+\binom{3}{3}=\binom{n-2}{4}$$

…

$$\binom{4}{3}+\binom{3}{3}=\binom{5}{4}$$

$$\binom{3}{3}=\binom{4}{4}$$ summing these equations,

$$1.\binom{n-1}{3}+2.\binom{n-2}{3}+3.\binom{n-3}{4}+\ldots+(n-4).\binom{4}{3}+(n-3).\binom{3}{3}=$$

$$\binom{n}{4}+\binom{n-1}{4}+\binom{n-2}{4}+\ldots+\binom{5}{4}+\binom{4}{4}=\binom{n+1}{5}$$

$$\sum_{r=1}^{n-3} r.\binom{n-r}{3}=\binom{n+1}{5} \qquad \text{... (2.2)}$$

Substitution of (2.1) and (2.2) is

$$\sum_{r=2}^{n-2} r.\binom{r-1}{1}.\binom{n-r}{2}+\sum_{r=1}^{n-3} r.\binom{n-r}{3}=2.\binom{n+1}{5}+\binom{n+1}{5}=3.\binom{n+1}{5}.$$

In conclusion, sum of 2 minimum elements of subsets with 4 elements of $\{1,2,3,\ldots,n\}$ is

$$a_n=3.\binom{n+1}{5}$$

3, 18, 63,168,378, 756, 1386, 2376, 3861, 6006, 9009, 13104, 18564, 25704, 34884, 46512, 61047, 79002, 100947, 127512, 159390, 197340, 242190, 294840, 356265

Example:

For A={1,2,3,4,5} subsets with 4 elements are
{1,2,3,4},{1,2,3,5},{1,2,4,5},{1,3,4,5},{2,3,4,5}
Sum of 2 minimum elements of these subsets:
(1+2)+(1+2)+(1+2)+(1+3)+(2+3)=18

For A={1,2,3,4} subsets with 4 elements is {1,2,3,4}
Sum of 2 minimum elements of this subset:
1+2=3

Theorem 3: For $\{1,2,3,...,n\}$, sum of 2 minimum elements of subsets with **k elements** $(k \geq 2)$ is $3.\binom{n+1}{k+1}$

Proof: Let the notation for **subsets with k elements be** $\{a_1, a_2, a_3, ..., a_k\}$ for $a_1 < a_2 < a_3 < ... < a_k$

For "1", as $1 < a_2 < a_3 < ... \leq a_k$ sum is $\binom{n-1}{k-1}$

Sum of "1"s : $1.\binom{n-1}{k-1}$

For "2", two possiblities (2 can be on first position or second position, as we are summing 2 minimum elements):

1: for $a_1 < 2 < a_3 < ... < a_k \leq n$ a_1 can have $\binom{1}{1}$ combinatorial states and $a_3, a_4 \dots a_k$ can have $\binom{n-2}{k-2}$ states. Total number of states: $\binom{1}{1}.\binom{n-2}{k-2}$

2: for $2 < a_2 < a_3 < ... < a_k \leq n$ there are $\binom{n-2}{k-1}$ possible states. So for "2", total number of possible states is $\binom{1}{1}.\binom{n-2}{k-2} + \binom{n-2}{k-1}$

Sum of "2"s: $2.\left[\binom{2}{1}.\binom{n-2}{k-2} + \binom{n-2}{k-1}\right]$

For "3", two possibilities:

1: for $a_1 < 3 < a_3 < ... < a_k \leq n$, a_1 can have $\binom{2}{1}$ combinatorial states and $a_3, a_4 \dots a_k$ can have $\binom{n-3}{k-2}$ states. Total number of states: $\binom{2}{1}.\binom{n-3}{k-2}$.

2: for $3 < a_2 < a_3 < ... < a_k \leq n$ there are $\binom{n-3}{k-3}$ possible states. So for "3", total number of possible states is $\binom{2}{1}.\binom{n-3}{k-2} + \binom{n-3}{k-3}$

Sum of "3"s: $3.\left[\binom{2}{1}.\binom{n-3}{k-2} + \binom{n-3}{k-3}\right]$

Similarly…

for r, $2 \le r \le n-k+1$**,** two possibilities:

1. : for $a_1 < r < a_3 < \ldots < a_k \le n$ a_1 can have $\binom{r-1}{1}$ states and $a_3, a_4 \ldots a_k$ can have $\binom{n-r}{k-2}$ possible states.

Total : $\binom{r-1}{1}.\binom{n-r}{k-2}$

2. : for $r < a_2 < a_3 < \ldots < a_k \le n$ there are $\binom{n-r}{k-1}$ possible states. So for "r", total number of possible states is $\binom{r-1}{1}.\binom{n-r}{k-2}+\binom{n-r}{k-1}$

Sum of "r"s : $r.\left[\binom{r-1}{1}.\binom{n-r}{k-2}+\binom{n-r}{k-1}\right]$

For $r = n-k+2$ **and** $a_1 < r < a_3 < \ldots < a_k \le n$ there are

$\binom{n-k+1}{1}.\binom{k-2}{k-2}$ states.

Sum of '$n-k+2$'s is $(n-k+2).\binom{n-k+1}{1}.\binom{k-2}{k-2}$

If we sum up we get:

$\sum_{r=2}^{n-(k-2)} r.\binom{r-1}{1}.\binom{n-r}{k-2} + \sum_{r=1}^{n-(k-1)} r.\binom{n-r}{k-1}$ Evaluation of these 2 sums:

$\sum_{r=2}^{n-(k-2)} r.\binom{r-1}{1}.\binom{n-r}{k-2} = 2.\sum_{r=2}^{n-k+2} \binom{r}{2}.\binom{n-r}{k-2}$ If we use (1.4) Equation

$$2.\sum_{r=2}^{n-k+2} \binom{r}{2}.\binom{n-r}{k-2} = 2.\binom{n+1}{k+1} \qquad \ldots(3.1)$$

Evaluation of $\sum_{r=1}^{n-k+1} r.\binom{n-r}{k-1}$:

$$1.\binom{n-1}{k-1}+2.\binom{n-2}{k-1}+3.\binom{n-3}{k-1}+...+(n-k).\binom{k}{k-1}+(n-k+1).\binom{k-1}{k-1}$$

$$\binom{n-1}{k-1}+\binom{n-2}{k-1}+\binom{n-3}{k-1}+...+\binom{k}{k-1}+\binom{k-1}{k-1}=\binom{n}{k}$$

$$\binom{n-2}{k-1}+\binom{n-3}{k-1}+...+\binom{k}{k-1}+\binom{k-1}{k-1}=\binom{n-1}{k}$$

$$\binom{n-3}{k-1}+...+\binom{k}{k-1}+\binom{k-1}{k-1}=\binom{n-2}{k}$$

…

$$\binom{k}{k-1}+\binom{k-1}{k-1}=\binom{k+1}{k}$$

$$\binom{k-1}{k-1}=\binom{k}{k}$$ summing these equations

$$1.\binom{n-1}{k-1}+2.\binom{n-2}{k-1}+3.\binom{n-3}{k-1}+...+(n-k).\binom{k}{k-1}+(n-k+1).\binom{k-1}{k-1}=$$

$$\binom{n}{k}+\binom{n-1}{k}+\binom{n-2}{k}+...+\binom{k+1}{k}+\binom{k}{k}=\binom{n+1}{k+1}$$

$$1.\binom{n-1}{k-1}+2.\binom{n-2}{k-1}+3.\binom{n-3}{k-1}+...+(n-k).\binom{k}{k-1}+(n-k+1).\binom{k-1}{k-1}=\binom{n+1}{k+1}$$

$$\sum_{r=1}^{n-k+1} r.\binom{n-r}{k-1}=\binom{n+1}{k+1} \qquad .... (3.2)$$

Substitution of (3.1) and (3.2) is

$$\sum_{r=2}^{n-(k-2)} r.\binom{r-1}{1}.\binom{n-r}{k-2}+\sum_{r=1}^{n-(k-1)} r.\binom{n-r}{k-1}=3.\binom{n+1}{k+1}$$

In conclusion, sum of 2 minimum elements of subsets with k elements $(k \geq 2)$ of $\{1,2,3,...,n\}$ is $(1+2).\binom{n+1}{k+1}=3.\binom{n+1}{k+1}$ … (3.3)

$a_n = 3.\binom{n+1}{6}$ sum of 2 minimum elements of subsets with 5 elements:

3, 21, 84, 252, 630, 1386, 2772, 5148, 9009, 15015, 24024, 37128, 55692, 81396, 116280, 162792, 223839, 302841, 403788, 531300, 690690, 888030, 1130220, 1425060

n	n+1	C(n+1,6)	3*C(n+1,6)
5	6	1	3
6	7	7	21
7	8	28	84
8	9	84	252
9	10	210	630
10	11	462	1386
11	12	924	2772
12	13	1716	5148
13	14	3003	9009
14	15	5005	15015
15	16	8008	24024
16	17	12376	37128
17	18	18564	55692
18	19	27132	81396
19	20	38760	116280
20	21	54264	162792
21	22	74613	223839
22	23	100947	302841
23	24	134596	403788
24	25	177100	531300
25	26	230230	690690
26	27	296010	888030
27	28	376740	1130220
28	29	475020	1425060

Example:

For $A=\{1,2,3,4,5,6\}$ subsets with 5 elements are
$\{1,2,3,4,5\},\{1,2,3,4,6\},\{1,2,3,5,6\},\{1,2,4,5,6\}.\{1,3,4,5,6\},\{2,3,4,5,6\}$
Sum of 2 minimum elements of these subsets:
(1+2)+(1+2)+(1+2)+(1+2)+(1+3)+(2+3)=21

$a_n = 3.\binom{n+1}{7}$ sum of 2 minimum elements of subsets with 6 elements:

3, 24, 108, 360, 990, 2376, 5148, 10296, 19305, 34320, 58344, 95472, 151164, 232560, 348840, 511632, 735471, 1038312, 1442100, 1973400, 2664090, 3552120, 4682340

n	n+1	C(n+1,7)	3*C(n+1,7)

6	7	1	3
7	8	8	24
8	9	36	108
9	10	120	360
10	11	330	990
11	12	792	2376
12	13	1716	5148
13	14	3432	10296
14	15	6435	19305
15	16	11440	34320
16	17	19448	58344
17	18	31824	95472
18	19	50388	151164
19	20	77520	232560
20	21	116280	348840
21	22	170544	511632
22	23	245157	735471
23	24	346104	1038312
24	25	480700	1442100
25	26	657800	1973400
26	27	888030	2664090
27	28	1184040	3552120
28	29	1560780	4682340

Example:

For A={1,2,3,4,5,6,7} subsets with 6 elements are
{1,2,3,4,5,6},{1,2,3,4,5,7},{1,2,3,4,6,7}{1,2,3,5,6,7},{1,2,4,5,6,7}.{1,3,4,5,6,7},{2,3,4,5,6,7}
Sum of 2 minimum elements of these subsets
(1+2)+(1+2)+(1+2)+(1+2)+(1+2)+(1+3)+(2+3)=24

Theorem 4: For $\{1,2,3,...,n\}$, sum of 3 minimum elements of subsets with **k elements** $(k \geq 3)$ is $6.\binom{n+1}{k+1}$

Proof: Let the notation for subsets with k elements for $a_1 < a_2 < a_3 < ... < a_k$ be $\{a_1, a_2, a_3,..., a_k\}$.

From theorem 3, sum of 2 minimum elements of subsets with k elements is $3.\binom{n+1}{k+1}$.

For $a_1 < a_2 < a_3 < ... < a_k$, evaluation of sum of third minimum elements of subsets of $\{a_1, a_2, a_3,..., a_k\}$:

If “3” is the third minimum element:

For $a_1 < a_2 < 3 < a_4 < ... < a_k$ total number of possible states is $\binom{2}{2}\binom{n-3}{k-3}$.

Sum of "3"s will be: $3.\binom{2}{2}\binom{n-3}{k-3}$.

If "4" is the third minimum element:

For $a_1 < a_2 < 4 < a_4 ... < a_k \leq n$, a_1 and a_2 can have $\binom{3}{2}$ possible states and $a_4, a_5, ..., a_n$ can have $\binom{n-4}{k-3}$ possible states. Total number of possible states is $\binom{3}{2}.\binom{n-4}{k-3}$ hence sum of "4"s is $4.\binom{3}{2}.\binom{n-4}{k-3}$.

If "5" is the third minimum element:

For $a_1 < a_2 < 5 < a_4 ... < a_k \leq n, a_1$ and a_2 can have $\binom{4}{2}$ possible states and $a_4, a_5, ..., a_n$ can have $\binom{n-5}{k-3}$ possible states. Total number of states will be $\binom{4}{2}.\binom{n-5}{k-3}$ and sum of "5"s is $5.\binom{4}{2}.\binom{n-5}{k-3}$

Similarly

If r, $3 \leq r \leq n-k$, is the third minimum element:

For $a_1 < a_2 < r < a_4 ... < a_k \leq n, a_1$ and a_2 can have $\binom{r-1}{2}$ possible states and $a_4, a_5, ..., a_n$ can have $\binom{n-r}{k-3}$ possible states. Total number of states will be $\binom{r-1}{2}.\binom{n-r}{k-3}$ and sum of "r"s will be $r.\binom{r-1}{2}.\binom{n-r}{k-3}$.

To find the total, summing $r.\binom{r-1}{2}.\binom{n-r}{k-3}$ for all possible "r"s:

$$\sum_{r=3}^{n-(k-3)} r.\binom{r-1}{2}.\binom{n-r}{k-3}$$

$$\sum_{r=3}^{n-(k-3)} r.\binom{r-1}{2}.\binom{n-r}{k-3} = 3.\sum_{r=3}^{n-k+3} \binom{r}{3}.\binom{n-r}{k-3}$$ and using equation (1.5);

$$3.\sum_{r=3}^{n-k+2}\binom{r}{3}.\binom{n-r}{k-2}=3.\binom{n+1}{k+1} \qquad ...(3.4)$$

Summing equations (3.1), (3.2) and (3.4);

$$\binom{n+1}{k+1}+2.\binom{n+1}{k+1}+3.\binom{n+1}{k+1}=6.\binom{n+1}{k+1} \qquad ...(3.5)$$

In conclusion, sum of 3 minimum elements of subsets with k elements $(k \geq 3)$ of $\{1,2,3,...,n\}$ is $(1+2+3).\binom{n+1}{k+1}=6.\binom{n+1}{k+1}$

$a_n = 6.\binom{n+1}{4}$ sum of 3 minimum elements of subsets with 3 elements: A033487

6, 30, 90, 210, 420, 756, 1260, 1980, 2970, 4290, 6006, 8190, 10920, 14280, 18360, 23256, 29070, 35910, 43890, 53130, 63756, 75900, 89700, 105300, 122850, 142506

n	n+1	com(n+1,4)	6*com(n+1,4)
3	4	1	6
4	5	5	30
5	6	15	90
6	7	35	210
7	8	70	420
8	9	126	756
9	10	210	1260
10	11	330	1980
11	12	495	2970
12	13	715	4290
13	14	1001	6006
14	15	1365	8190
15	16	1820	10920
16	17	2380	14280
17	18	3060	18360
18	19	3876	23256
19	20	4845	29070
20	21	5985	35910
21	22	7315	43890
22	23	8855	53130
23	24	10626	63756
24	25	12650	75900
25	26	14950	89700
26	27	17550	105300
27	28	20475	122850
28	29	23751	142506

Example:

For A={1,2,3,4} subsets with 3 elements are {1,2,3},{1,2,4},{1,3,4},{2,3,4}

Sum of 3 minimum elements of these subsets:
(1+2+3)+(1+2+4)+(1+3+4)+(2+3+4)=30

$a_n = 6.\binom{n+1}{5}$ sum of 3 minimum elements of subsets with 4 elements:

6, 36, 126, 336, 756, 1512, 2772, 4752, 7722, 12012, 18018, 26208, 37128, 51408, 69768, 93024, 122094, 158004, 201894, 255024, 318780, 394680, 484380, 589680, 712530

n	n+1	com(n+1,5)	6*com(n+1,5)
4	5	1	6
5	6	6	36
6	7	21	126
7	8	56	336
8	9	126	756
9	10	252	1512
10	11	462	2772
11	12	792	4752
12	13	1287	7722
13	14	2002	12012
14	15	3003	18018
15	16	4368	26208
16	17	6188	37128
17	18	8568	51408
18	19	11628	69768
19	20	15504	93024
20	21	20349	122094
21	22	26334	158004
22	23	33649	201894
23	24	42504	255024
24	25	53130	318780
25	26	65780	394680
26	27	80730	484380
27	28	98280	589680
28	29	118755	712530

Example:

For A={1,2,3,4,5} subsets with 4 elements are
{1,2,3,4},{1,2,3,5},{1,2,4,5}{1,3,4,5,},{2,3,4,5}
Sum of 3 minimum elements of these subsets:
(1+2+3)+(1+2+3)+(1+2+4)+(1+3+4)+(2+3+4)=36

$a_n = 6.\binom{n+1}{6}$ sum of 3 minimum elements of subsets with 5 elements:

6, 42, 168, 504, 1260, 2772, 5544, 10296, 18018, 30030, 48048, 74256, 111384, 162792, 232560, 325584, 447678, 605682, 807576, 1062600, 1381380, 1776060, 2260440, 2850120, 3562650, 4417686

n	n+1	com(n+1,6)	6*Com(n+1,6)
5	6	1	6
6	7	7	42
7	8	28	168
8	9	84	504
9	10	210	1260
10	11	462	2772
11	12	924	5544
12	13	1716	10296
13	14	3003	18018
14	15	5005	30030
15	16	8008	48048
16	17	12376	74256
17	18	18564	111384
18	19	27132	162792
19	20	38760	232560
20	21	54264	325584
21	22	74613	447678
22	23	100947	605682
23	24	134596	807576
24	25	177100	1062600
25	26	230230	1381380
26	27	296010	1776060
27	28	376740	2260440
28	29	475020	2850120
29	30	593775	3562650
30	31	736281	4417686

Example:

For A={1,2,3,4,5,6} subsets with 5 elements is
{1,2,3,4,5},{1,2,3,4,6},{1,2,3,5,6},{1,2,4,5,6},{1,3,4,5,6},{2,3,4,5,6}
Sum of 3 minimum elements of these subsets:
(1+2+3)+(1+2+3)+(1+2+3)+(1+2+4)+(1+3+4)+(2+3+4)=42

$a_n = 6.\binom{n+1}{7}$ sum of 3 minimum elements of subsets with 6 elements:

6, 48, 216, 720, 1980, 4752, 10296, 20592, 38610, 68640, 116688, 190944, 302328, 465120, 697680, 1023264, 1470942, 2076624, 2884200, 3946800, 5328180, 7104240, 9364680, 12214800, 15777450

n	n+1	com(n+1,7)	6*Com(n+1,7)
6	7	1	6
7	8	8	48
8	9	36	216
9	10	120	720
10	11	330	1980
11	12	792	4752
12	13	1716	10296
13	14	3432	20592
14	15	6435	38610
15	16	11440	68640
16	17	19448	116688
17	18	31824	190944
18	19	50388	302328
19	20	77520	465120
20	21	116280	697680
21	22	170544	1023264
22	23	245157	1470942
23	24	346104	2076624
24	25	480700	2884200
25	26	657800	3946800
26	27	888030	5328180
27	28	1184040	7104240
28	29	1560780	9364680
29	30	2035800	12214800
30	31	2629575	15777450

Example:

For A={1,2,3,4,5,6,7} subsets with 6 elements are
{1,2,3,4,5,6},{1,2,3,4,5,7},{1,2,3,4,6,7}{1,2,3,5,6,7},{1,2,4,5,6,7}.{1,3,4,5,6,7},{2,3,4,5,6,7}
Sum of 3 minimum elements of these subsets:
(1+2+3)+(1+2+3)+(1+2+3)+(1+2+3)+(1+2+4)+(1+3+4)+(2+3+4)=48