Redundant Primes In Goldbach Partitions

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Abstract

Goldbach Strong Conjecture (GSC), still unsolved, states that all even integers $n>2$ can be expressed as the sum of two prime numbers (Goldbach partitions of n). But do we need all primes to satisfy this conjecture? This work is devoted to selection of must-have primes and formulation of stronger version of GSC with reduced set of primes.

1 Problem statement

Goldbach Strong Conjecture (GSC, also called binary) asserts that all positive even integer $n \geq 4$ can be expressed as the sum of two prime numbers. This hypothesis, formulated by Goldbach in 1742 in letter to Euler [1] and then updated by Euler to the form above is one of the oldest and still unsolved problems in number theory. Empirical verification showed that it is true for all $n \leq 4 \text{ x } 10^{18}$ [2] [3].

The expression of a given positive even number n as a sum of two primes p_1 and p_2 is called a Goldbach Partition (GP) of n. Let's denote this relation as $GSC(n, p_1, p_2)$. Then GSC can be written as (1):

$$\forall_{x>1, x \in \mathbb{N}} \exists_{p_1, p_2 \in \mathbb{P}} GSC(2x, p_1, p_2) \tag{1}$$

But maybe we can formulate much stronger version of (1)? The set of prime numbers $\mathbb{P}$ is dense, number of $GP(n)$ is increasing with increasing n, thus question if stronger version GSC is possible, raised in [4], is legitimate (2):

$$\forall_{x>1, x \in \mathbb{N}} \exists_{p_1, p_2 \in \mathbb{R}} GSC(2x, p_1, p_2) \tag{2}$$

where $|\mathbb{R}| < |\mathbb{P}|$. By design $\mathbb{R}$ contains primes only.

2 Algorithm

Elimination of primes must start from the fact that every prime is potentially required (Lemma 1).

Lemma 1. *Every prime is present is some Goldbach partition of even number.*

Proof. Let's assume that exists such prime p that is not present in any Goldbach partition - we will show that this is not possible. $p + p$ is always even number, regardless p is even or odd. This means that we have $GSC(2p, p, p)$ for any prime p (in other words: p is always in Goldbach partition of $2p$) what is in contrary to the initial assumption. □

Further elimination of primes from $\mathbb{P}$ to build $\mathbb{R}$ requires appropriate algorithm A which is able to resign from a given prime, even if it is present in some GPs. Such algorithm A could look as follows:

1. Let $\mathbb{R}$ is empty, a set of $\mathbb{R}_i$ is empty and $n = 2$. Let's assume that we break calculations at $n = n_{max}>2$.
2. **New turn**: $n = n + 2$.
3. Break calculations if $n>n_{max}$.
4. If $\mathbb{R}$ is sufficient to fulfill GSC for all even numbers q ($4 \leq q \leq n$), then go to **New turn**.
5. If we have a set of $\mathbb{R}_i$ and any $\mathbb{R}_j$ belonging to this set is sufficient to fulfill GSC for all even numbers q ($4 \leq q \leq n$), then $\mathbb{R} = \mathbb{R}_j$, we forget all $\mathbb{R}_i$ and go to **New turn**.
6. If not, find all $GP(n)$ and build as many candidates for $\mathbb{R}$ (let's call them $\mathbb{R}_i$) as required. As a base use either (as a first choice) $\mathbb{R}$ (if $\mathbb{R}_i$ does not exist) or all previous $\mathbb{R}_i$ (if they are present).
7. Go to **New turn**.

3 Results

Table 1 presents the very first rounds of algorithm eliminating primes required to satisfy GSC - it is depicting current values of both $\mathbb{R}$ and $\mathbb{R}_i$, plus additional set $\mathbb{E}$ which contains primes that were present in GPs so far but can be eliminated without hurting GSC.

Table 1: Results of the first rounds of A

n	$GP(n)$	$\mathbb{R}$	$\mathbb{R}_i$	$\mathbb{E}$
4	2+2	{2}	∅	∅
6	3+3	{2,3}	∅	∅
8	3+5	{2,3,5}	∅	∅
10	3+7 5+5	{2,3,5}	∅	{7}
12	5+7	{2,3,5,7}	∅	∅
14	3+11 7+7	{2,3,5,7}	∅	{11}
16	5+11 3+13	∅	{2,3,5,7,11} {2,3,5,7,13}	∅
18	5+13 7+11	∅	{2,3,5,7,11} {2,3,5,7,13}	∅
20	3+17 7+13	{2,3,5,7,13}	∅	{11,17}

For instance, if we take into account all positive even numbers $2<n<16$, then we need a set of primes

$\{2, 3, 5, 7\}$ to satisfy GSC for all n checked so far and $\{11\}$ ($GSC(14, 3, 11)$) is our candidate for elimination.

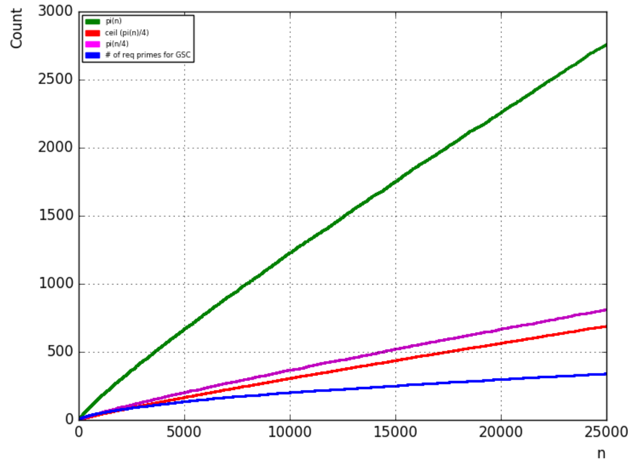

Figure 1: Number of required primes for GSC ($4 \leq n \leq 2.5 \times 10^4$)

Figures 1, 2 and 3 are depicting findings after analyzing even numbers $4 \leq n \leq 2.5 \times 10^4$. First of all, function of required primes increases but slower than $\Pi(n)$.

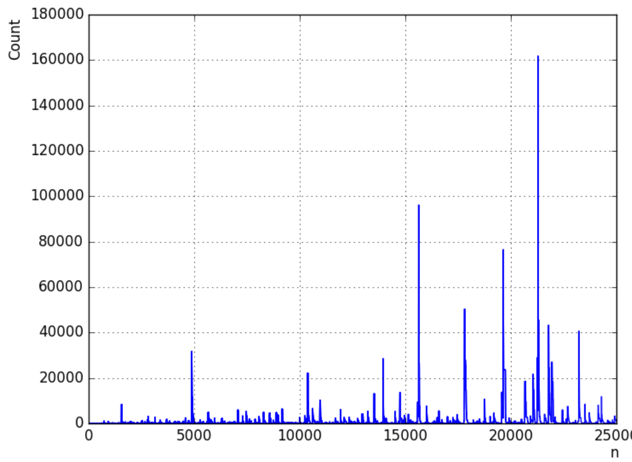

Figure 2: Number of $\mathbb{R}_i$ ($4 \leq n \leq 2.5 \times 10^4$)

It is also interesting that algorithm A has sporadic congestions in terms of number of $\mathbb{R}_i$ - generally its number at a given time is low but there are quite frequent situation when it is high, even exceeding 160000 (this means that we have 160000 subsets containing candidates for $\mathbb{R}$) - it happens when there are still some $\mathbb{R}_i$ and new n requires new prime to be used which is multiplying number of $\mathbb{R}_i$ in the next round of A. Surprisingly, shortly after number of $\mathbb{R}_i$ is decreasing to much smaller values. Congestions could be better handled (from memory utilization standpoint) if in A instead of seperate lists (paths) we have a tree (where branches/leaves represent variable part added on top of a common base).

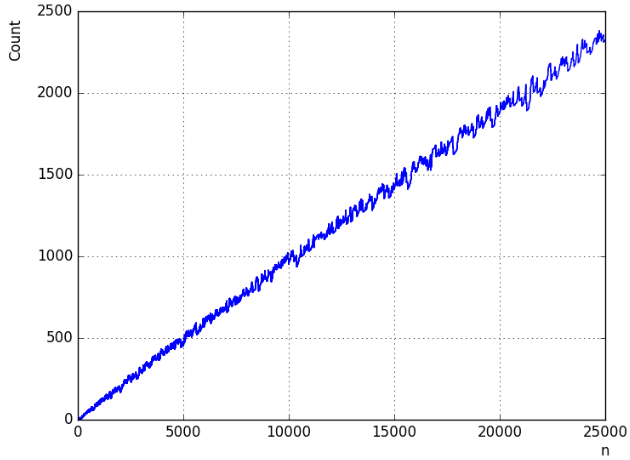

Figure 3: Number of eliminated primes ($4 \leq n \leq 2.5 \times 10^4$)

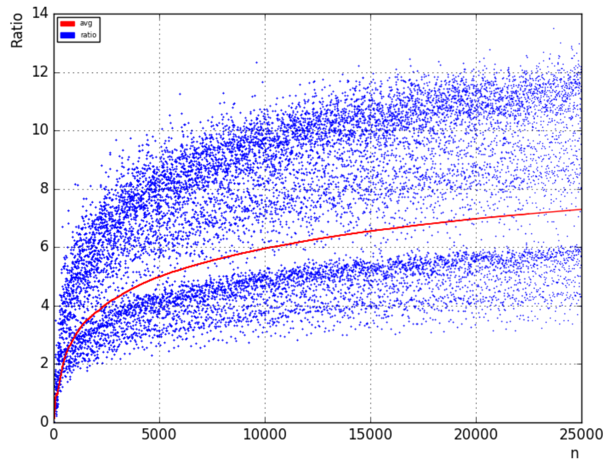

Figure 4: Ratio of number of eliminated primes to number of GPs, including average value ($4 \leq n \leq 2.5 \times 10^4$)

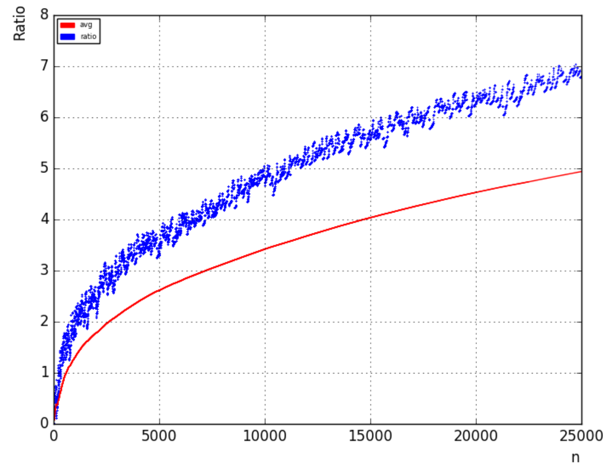

Figure 5: Ratio of number of eliminated primes to number of required primes, including average value ($4 \leq n \leq 2.5 \times 10^4$)

4 Summary and next steps

Executed experiments, run for all even $n \leq 2.5 \times 10^4$, confirmed that we do not need entire set of primes to satisfy

GSC. Appendix A lists eliminated primes after checking all even $n \leq 2.5 \times 10^4$ - the smallest eliminated prime is 17. Of course, exercised cases do not proof that eventually such set exists for all even n but observed trends (Figure 3, Figure 4, Figure 5) give strong foundation that such set exists indeed and conjecture (2) is true.

As a result of this work one integer sequence has been submitted to OEIS database: A328179 [5].

References

[1] Christian Goldbach, *On the margin of a letter to Leonard Euler*, 1742.

[2] Tomás Oliveira e Silva, *Goldbach conjecture verification.* http://sweet.ua.pt/tos/goldbach.html, 2012.

[3] Tomás Oliveira e Silva, Siegfried Herzog, and Silvio Pardi, *Empirical verification of the even Goldbach conjecture and computation of prime gaps up to 4×10^{18}.*, Mathematics of Computation, vol. 83, no. 288, pp. 2033-2060, July 2014 (published electronically on November 18, 2013).

[4] Marcin Barylski *On $6k \pm 1$ Primes in Goldbach Strong Conjecture.*, February 2018.

[5] OEIS Foundation Inc. (2019), The On-Line Encyclopedia of Integer Sequences, https://oeis.org/A328179. Number of distinct primes required to satisfy the Strong Goldbach Conjecture for all even numbers $\leq 2n$.

A List of redundant primes

Based on empirical verification done for all even numbers $4 \leq n \leq 2.5 \times 10^4$:

[17, 29, 43, 67, 71, 73, 83, 89, 103, 107, 131, 137, 157, 163, 167, 173, 179, 181, 193, 199, 229, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 313, 317, 331, 349, 353, 359, 367, 401, 421, 431, 433, 443, 449, 463, 467, 479, 487, 499, 503, 509, 521, 523, 547, 557, 563, 571, 577, 587, 599, 601, 607, 617, 619, 631, 641, 643, 647, 659, 673, 677, 683, 691, 709, 719, 727, 733, 739, 751, 757, 761, 769, 773, 787, 797, 809, 811, 827, 829, 839, 853, 857, 859, 863, 883, 907, 911, 919, 937, 941, 947, 953, 967, 971, 977, 997, 1009, 1013, 1019, 1031, 1039, 1049, 1061, 1087, 1093, 1097, 1103, 1109, 1123, 1129, 1151, 1153, 1163, 1171, 1181, 1187, 1201, 1223, 1229, 1231, 1249, 1259, 1277, 1279, 1283, 1289, 1297, 1301, 1307, 1361, 1367, 1373, 1381, 1399, 1409, 1427, 1429, 1433, 1439, 1447, 1453, 1459, 1471, 1481, 1487, 1489, 1511, 1523, 1549, 1553, 1567, 1571, 1579, 1583, 1597, 1607, 1613, 1621, 1657, 1669, 1697, 1699, 1709, 1721, 1723, 1733, 1747, 1753, 1759, 1777, 1783, 1787, 1789, 1801, 1823, 1831, 1847, 1861, 1871, 1873, 1877, 1889, 1901, 1907, 1933, 1949, 1973, 1979, 1987, 1993, 1997, 2003, 2011, 2017, 2027, 2029, 2039, 2063, 2081, 2083, 2089, 2099, 2111, 2113, 2129, 2131, 2143, 2153, 2161, 2179, 2213, 2221, 2237, 2239, 2243, 2269, 2281, 2287, 2293, 2297, 2309, 2333, 2339, 2341, 2347, 2351, 2357, 2371, 2377, 2389, 2393, 2417, 2423, 2441, 2447, 2459, 2467, 2473, 2477, 2521, 2531, 2539, 2549, 2579, 2591, 2593, 2609, 2617, 2621, 2647, 2663, 2671, 2677, 2683, 2687, 2689, 2699, 2707, 2711, 2713, 2719, 2729, 2731, 2741, 2749, 2753, 2767, 2777, 2791, 2797, 2801, 2803, 2819, 2833, 2837, 2843, 2851, 2857, 2861, 2879, 2887, 2897, 2903, 2917, 2927, 2939, 2953, 2957, 2963, 2969, 2971, 2999, 3001, 3019, 3023, 3041, 3049, 3061, 3083, 3089, 3109, 3119, 3121, 3137, 3163, 3167, 3187, 3209, 3217, 3221, 3229, 3253, 3257, 3259, 3271, 3299, 3301, 3307, 3313, 3319, 3323, 3329, 3331, 3343, 3347, 3361, 3371, 3391, 3433, 3449, 3457, 3461, 3463, 3467, 3511, 3517, 3527, 3529, 3533, 3539, 3541, 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