PREFACE

The marriage of algebra and topology has produced many beautiful and intricate subjects in mathematics, of which perhaps the broadest is functional analysis. My aim has been to write a textbook with which graduate students can master at least some of the powerful tools of this subject. Because I think that one learns best by doing, I believe that it is critical that the students using this book in a course work the exercises. As an integral part of the book, they have been designed to provide practice in mimicking the techniques that are presented here in the proofs, as well as to lead the novice through fairly elaborate arguments that establish important additional results. The instructor is encouraged and expected to add theorems and examples from his or her own experiences and preferences, for I have quite deliberately restricted this presentation according to my own. My style is to state relatively few theorems, each having a fairly substantial proof, rather than to present a long series of lemmas. The student should read these substantial proofs with pencil in hand, making sure how each step follows from the previous ones and filling in any details that have been left to the reader.

I propose this text for a one-year course. The first six chapters constitute a general study of topological vector spaces, Banach spaces, duality, convexity, etc., concluding with a chapter that contains a number of applications to classical analysis, e.g., convolution, Green's functions, the Fourier transform, and the Hilbert transform.

I assume that the students studying from this book have completed a course in general measure theory, so that terms such as outer measure, σ-algebra, measurability, L^p spaces (including the Riesz representation theorem for $(L^p)^*$), product measures, etc. should be familiar. In addition, I freely use concepts such as separability and completeness from metric space theory (making particular use of the Baire category theorem at several points in Chapter IV), and I employ the general Stone-Weierstrass theorem on several occasions. I also think that many aspects of general topology were in fact invented to support the concepts in functional analysis, and I draw on these results in some rather deep ways. Thinking that those aspects of general topology that are most critical to this subject, e.g., product topologies, weak topologies, convergence of nets, etc., may not be covered in sufficient detail in many elementary topology courses, I go to some effort to explain these notions carefully throughout the text.

I do not intend to include here the most general cases of theorems and definitions, believing that my versions are both hard enough and deep enough for a student's first go at this subject. For example, I consider only locally compact topological spaces that are second countable, measures that are σ-finite, and Hilbert spaces that are separable. Chapter 0 is a kind of catalog for the basic results from linear algebra and topology that will be assumed.

The second half of the book centers on the Spectral Theorem in Hilbert space, the most important theorem of functional analysis in my view. Students with some elementary knowledge of Banach space theory and the Riesz Representation Theorem for $(C(X))^*$ can in fact begin with Chapter VIII, referring to the earlier chapters on those few occasions when more delicate results from locally convex analysis and dual topologies are required. I introduce early on the notion of projection-valued measures and spend some time studying operators that can be expressed as integrals against such a measure. I present the Gelfand approach to the Spectral Theorem for a bounded normal operator, for my sense is that the beauty of that approach is so spectacular that it should be experienced by every analyst, hard or soft. In Chapter XI I spend some time studying the standard classes of operators ordinarily encountered in analysis: compact, Hilbert-Schmidt, trace class, and unbounded selfadjoint. I include only a few of the large number of examples from differential and integral equations that spawned these classes of operators, leaving this addition to the instructor's choice. Indeed, my choice has been to present an introduction to the connection between operator theory and the foundations of quantum mechanics. Thus I devote Chapter VII to a brief presentation of a set of axioms for a mathematical model of experimental science. These axioms are a minor perturbation of those first introduced by G. W. Mackey for Quantum Mechanics, and my aim is to motivate the notions of projection-valued measures and unbounded selfadjoint operators by using them as models of the question-valued measures and observables of this set of axioms. However, this chapter and all references to it can be omitted without any effect on the rest of the material.

Finally, Chapter XII is devoted primarily to a development of the Implicit Function Theorem in infinite dimensions. My experience is that most beginning graduate students can well use another trek through the ideas surrounding this theorem, and what is presented here also provides a basic introduction to nonlinear functional analysis.

The bibliography includes two texts on measure theory and real analysis for reference purposes, several of the standard volumes on functional analysis for different points of view, and a number of books the interested student should consider reading after finishing this one.

It has been said that many functional analysis books are too big. They are encyclopedic; they have everything in them. Having tried to study from them, a student often leaves the course more baffled by the tool box than by the tasks to be solved. I expect students of this small textbook to be masters of the most powerful and commonly used tools.

Many classes of functional analysis students have helped me revise the original version of this book. I valued and welcomed their compliments, their complaints, and their true partnership in developing this material. I thank them all. Many of my faculty colleagues have provided me with alternate proofs, interesting examples, and novel exercises, all of which have enriched the book, and I thank them all, too. Special thanks go to my daughter Molly for her diligent help with the indexing. Finally, I thank my wife Christy for her love, her support, and her true partnership in developing this material. More particularly, I am extremely grateful for her considerable editorial expertise, which has been of continuous help. I count myself extremely lucky to have had such encouragement and support.

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