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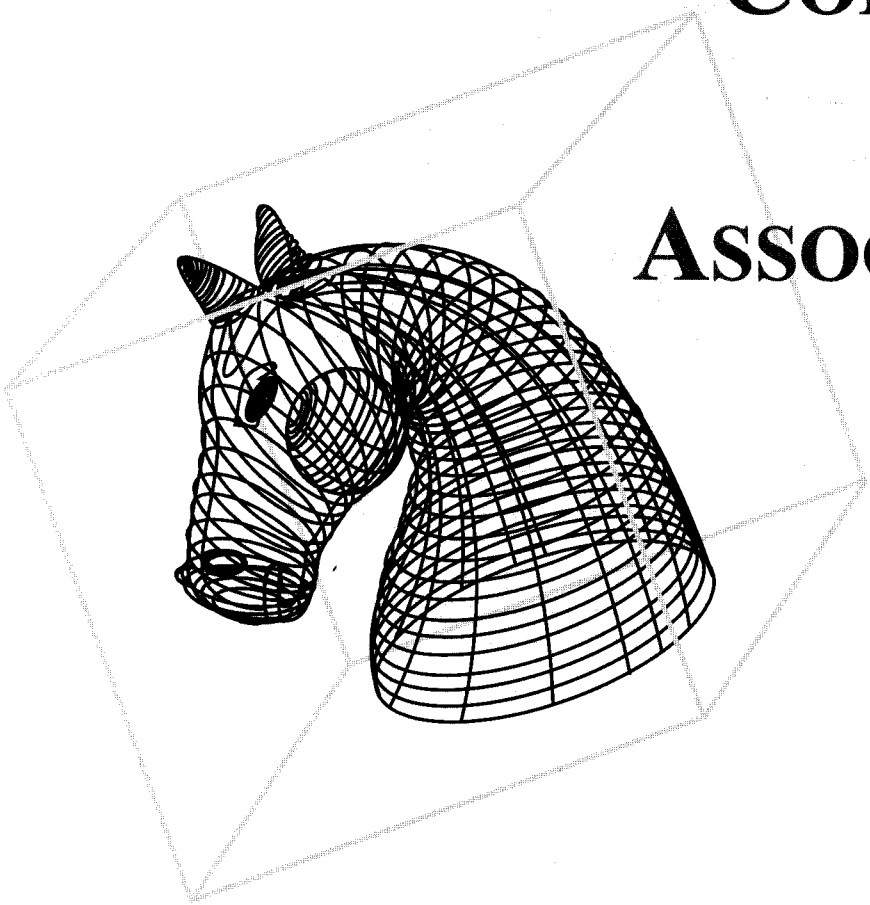

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Ken Thompson Special Issue

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V. Kotěšovec:
King and Two Generalised Knights against King

KING AND TWO GENERALISED KNIGHTS AGAINST KING

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Czech Republic

ABSTRACT

A Knight jumps two squares in one direction and one square in the other. It can be generalised as a *Leaper* which jumps x squares in one direction and y squares in the other. At various times in its history, chess has featured other pieces of this kind, in particular, the mediaeval *Firzan* which moved one square in each direction. Calculations are described which examine the general outcome of the ending "King and two Leapers against King" on a square chessboard of any size. In particular, it is shown that all endings of this kind appear to be drawn on boards larger than 13×13, and that two identical Leapers cannot mate from a general position.

Every practical chess player knows that two Knights are insufficient to force mate against a lone King. But a Knight is merely a particular example of a piece known in generalised forms of chess as an (x,y) *Leaper* which jumps x squares in one direction and y in the other, and ever since my youth the question has been running through my head as to whether there might exist two (x,y) Leapers, not necessarily the same, which could combine with their King to force mate against a bare enemy King from a general position.

The advent of personal computers, and their continually increasing memory capacity, has enabled me to write a program which appears to give a complete answer to this question. The first part of the analysis (Kotěšovec, 1994, 1996) considered boards up to 8×8, and showed that on an 8×8 board there were seven and only seven combinations of two Leapers which could combine with their King to force a win from a general position: (0,1) and (1,2), or one of (0,1)/(1,2) and any one of (1,3)/(1,4)/(1,6). The results were reported in the English-language chess press (Whyld, 1994; Beasley, 1996) and independently confirmed by Smith (1995) and Stewart (Gent, 1996). The second part of the analysis (Kotěšovec, 2000; Beasley, 2001) identifies the winning combinations on boards up to 13×13, and shows with virtual certainty that all endings of this kind are drawn on larger boards. The analysis also shows that no two identical Leapers can mate from a general position.

Table 1 summarises the results. It lists the longest wins in all endings which are won from a general position. Excluded are positions which cannot be reached in play (for example, a (1,6) Leaper on an 8×8 board cannot play to d4) and positions where the lone King can force the capture of one of the Leapers (for example, white Knight on a1, black King on b2, no white man guarding c2 or b3). There are also some interesting drawn positions which might provide ideas for endgame studies, for example white King K on e5, Knight N on b1, (1,3) Leaper X^2 on a1, black King on b3, play **1. Nd2+ Kb2! 2. Xb4 Kc3! 3. Xa1 Kb2! 4. Xb4** and a draw by repetition. But with these exceptions a win is always possible within at most the given number of moves (for example, given a King on a2 and Leapers on a1/b1 against a King on any legal square away from the edge, the enemy King can be pressed back into a corner of the board and there mated).

Some additional endings can be won provided that the enemy King is already penned into a corner of the board. For example, with white K on c3, (1,1) Leaper F on b2, (3,4) Leaper A on g1, black K on a2, there is a mate in 12 by **1. Ac4 Kb1 2. Af8 Ka2 3. Ab5 4. Ae1 5. Ah5 6. Ad2 7. Aa6 8. Ae3 9. Ah7 10. Ad4 11. Aa8/Ah1 Kb1 12. Ae5** (Kotěšovec, 1984). However, there is no win from a general position with this material.

The 1994 analysis generated the first five columns of Table 1, and showed in particular that there were seven generally winning combinations on an 8×8 board. The more recent analysis shows that each of these endings becomes drawn once the board exceeds a certain size. The problem lies in penning the enemy King into a corner. On a larger board the lone King has more room to manoeuvre and can keep the superior side at bay.

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[2] Generalised Knights featuring here: *Wazir* W (0,1), *Firzan* F (1,1), *Knight* N (1,2), *Camel* X (1,3), *Giraffe* G (1,4), *Flamingo* Y (1,6), *Antelope* A (3,4).

Leapers \ Board	4×4	5×5	6×6	7×7	8×8	9×9	10×10	11×11	12×12	13×13	14×14	15×15
# of Leaper-pairs	45	105	210	378	630	990	1485	2145	3003	4095	5460	7140
(0,1) + (0,2)	31	19y	-	45y	-	-	-	-	-	-	-	-
(0,1) + (1,1)	15	18b	45	38b	-	94b	-	-	-	-	-	-
(0,1) + (1,2)	17	22	29	38	45	57	71	101	194	-	-	-
(0,1) + (1,3)	20	30b	40	40b	77	68b	-	-	-	-	-	-
(0,1) + (1,4)	x	38	37	45	55	82	-	-	-	-	-	-
(0,1) + (1,5)	x	x	58	50b	-	-	-	-	-	-	-	-
(0,1) + (1,6)	x	x	x	87	93	-	-	-	-	-	-	-
(0,1) + (2,4)	x	23b	65	43y	-	-	-	-	-	-	-	-
(0,2) + (1,2)	-	19y	55	45y	-	69y	-	-	-	-	-	-
(0,2) + (1,3)	-	21wy	-	44wy	-	67wy	-	-	-	-	-	-
(1,1) + (1,2)	17	19b	36	37b	-	56b	-	-	-	-	-	-
(1,1) + (2,3)	-	-	-	47b	-	82b	-	-	-	-	-	-
(1,1) + (2,4)	x	32z	-	-	-	-	-	-	-	-	-	-
(1,2) + (1,3)	21	21b	33	31b	49	45b	-	67b	-	119b	-	-
(1,2) + (1,4)	x	37	34	36	41	48	61	85	-	-	-	-
(1,2) + (1,5)	x	x	46	36b	-	54b	-	106b	-	-	-	-
(1,2) + (1,6)	x	x	x	48	53	62	78	143	-	-	-	-
(1,2) + (1,7)	x	x	x	x	-	77b	-	-	-	-	-	-
(1,2) + (1,8)	x	x	x	x	x	-	123	-	-	-	-	-
(1,2) + (2,4)	x	26b	60	33y	-	50y	-	-	-	-	-	-
(1,3) + (2,4)	x	35z	-	35wy	-	51wy	-	-	-	-	-	-

An ending is regarded as *won* if King on a2 and Leapers on a1/b1 can force a win against a King on any legal square away from the edge. The number gives the distance to mate in the least favourable winning case, including cases where King and Leapers start on other squares. Endings which are not won in this sense are denoted by "-", and which do not exist by *x*.

Endings marked *b* are won only if the Leaper for which $x+y$ is even runs on black squares, assuming that the board is always coloured so that square a1 is black.

Endings marked *y* are won only if the (0,2) or (2,4) Leaper runs on black squares and additionally is able to reach squares a1, c1 etc. In other cases the ending is drawn: for example, a (0,2) Leaper on b2 can never play to reach a1.

Endings marked *wy* are won only if condition *y* is satisfied and the (1,3) Leaper runs on white squares.

Endings marked *z* are won only if the (1,1) or (1,3) Leaper runs on white squares and the (2,4) Leaper runs on black.

Table 1. Winning endings with King and two Leapers against a bare King.

It is convenient to consider even and odd boards separately. Of the seven endings which are won on an 8×8 board, only three are still won on a 10×10, together with the combination (1,2)+(1,8) which does not exist on the smaller board. The only winning combination on a 12×12 board is (0,1)+(1,2), and even this is not winning on a 14×14. On a 12×12 board, the win with white King on a1, (0,1) Leaper on a12, and (1,2) Leaper on l1 against black King on j4 takes 194 moves! Coming to odd boards, of 15 winning endings on a 9×9 board only five remain winning on an 11×11, and only one, (1,2)+(1,3), on a 13×13. The win with white King on a1, (1,2) Leaper on l13, and (1,3) Leaper on m13 against black King on a3 takes 119 moves. The same combination on a 15×15 board is only drawn.

If an ending is only drawn on a board of side *n*, we can assume that it is also drawn for $n+2$, $n+4$, and so on. (There are two exceptions on small boards: (1,1)+(2,3) is won on a 7×7 board but only drawn on a 5×5 because the (2,3) Leaper has severely impaired mobility, and (0,2)+(1,2) is won on a 6×6 but only drawn on a 4×4 because the White men get in each other's way.) There remains the possibility that there may be wins for combinations which do not exist on smaller boards, as in the example (1,2)+(1,8) on the 10×10. However, in view of the limited powers of movement of such Leapers it is very unlikely that further endings of this kind are won; for example, (1,2)+(1,10) on the 12×12 is drawn. For boards up to 12×12 I have examined all possible combinations of Leapers, for larger boards only those which are won on smaller boards.

Thus it is possible to announce, almost with certainty, that all endings of this kind are drawn on boards larger than 13×13. The examples below show three of the longest maximal wins on the 8×8 board: ° indicates a unique legal move and ' a unique optimal move. In conclusion, one might ask what is the largest board on which a trio of Leapers can combine with their King to force mate against a bare King. Perhaps the resolution of this far from trivial problem will be a task for the next generation.

Example 1. Board 8×8, white K on d2, (0,1) Leaper W on h5, (1,6) Leaper Y on c8, black K on h3.

1. Wg5' Kh4' 2. Wg6 Kh5' 3. Wg7 Kh6' 4. Wf7' Kg6' 5. We7' Kf6' 6. Wd7' Ke6' 7. Wc7' Kd6' 8. Wb7' Kd7 9. Yb2' Kc6' 10. Wb8 Kc7' 11. Wa8' Kc6 12. Kc3 Kc5' 13. Yh1' Kd5' 14. Kb4 Kd6 15. Wa7 Kc6 16. Wa6 Kd5' 17. Wa5 Ke6 18. Kc5 Kf7 19. Kd5 Ke7' 20. Wb5 Kf6' 21. Ke4 Kf7 22. Ke5 Kg6' 23. Wb6 Kf7' 24. Wc6' Kg6' 25. Wd6' Kf7' 26. We6' Ke8' 27. Yg7 Kd7' 28. Kd5 Kc7' 29. Kc5' Kd7' 30. We5' Ke8' 31. Kc6 Ke7 32. Ya6' Kd8' 33. Yg5' Ke8 34. Kd5 Kf8 35. We6 Ke8 36. Ya4' Kf8 37. Yg3' Ke8 38. Ya2' Kf7' 39. Ke5' Ke8 40. Yg1' Kd7' 41. Kd5 Kc7' 42. Wd6' Kd8' 43. Ke6' Kc7' 44. Yh7' Kb7' 45. Kd7' Ka6' 46. Kc6' Ka5' 47. Kc5' Ka4' 48. Kc4' Ka3' 49. Wd5 Kb2' 50. Yb8' Kb1' 51. Kd4 Kc1' 52. Kd3' Kd1' 53. Ke3 Ke1' 54. Wd4' Kf1' 55. Wd3' Kg2' 56. Kf4' Kh3' 57. Yh7' Kg2' 58. Wd2' Kf1' 59. Yb8' Kf2' 60. Ya2' Kf1' 61. Kf3' Ke1' 62. Ke3' Kf1° 63. We2' Kg2' 64. Wf2+' Kh3' 65. Kf4' Kh4' 66. Kf5 Kh5' 67. Wf3 Kh6' 68. Kf6' Kh7 69. Wf4 Kh6' 70. Wg4' Kh5' 71. Kf5' Kh6° 72. Wg5' Kg7' 73. Ke6' Kf8' 74. Yb8' Ke8 75. Yc2' Kf8° 76. Wg6' Ke8' 77. Wg7 Kf8° 78. Wf7+' Ke8' 79. We7+' Kf8° 80. Kf6' Kg8° 81. Wf7 Kh7' 82. Kg5' Kg8' 83. Kg6' Kh8° 84. Yd8 Kg8° 85. Kf6 Kh8' 86. Kg5 Kh7' 87. Kh5' Kh8' 88. Kh6' Kg8° 89. Kg6' Kh8° 90. Ye2' Kg8° 91. Yf8' Kh8° 92. Yg2+' Kg8° 93. Wf8#'.

Example 2. Board 8×8, white K on a1, (0,1) Leaper W on g5, (1,3) Leaper X on h6, black K on f4.

1. Wg6' Kf5' 2. Wg7' Kf6' 3. Wg8' Kf7' 4. Wh8' Kg7 5. Xe7' Kf6' 6. Xf4' Kf5 7. Xc5' Kg6' 8. Xd8' Kf7 9. Ka2 Ke8 10. Xg7 Ke7 11. Kb3 Kf6' 12. Xd8 Ke7' 13. Xc5' Kf7' 14. Xf4' Ke6 15. Kc4 Kf5' 16. Xg7 Kf6 17. Xh4' Kg6 18. Kd5' Kh6 19. Ke5 Kg5 20. Xg7 Kg6 21. Xd6' Kf7' 22. Wh7' Ke8 23. Ke6 Kd8' 24. Wg7' Kc7' 25. Kd5' Kd8 26. Wf7' Ke8' 27. Ke6' Kd8° 28. Xc3' Kc7' 29. Kd5' Kd7' 30. Xf4 Kc7' 31. We7' Kb6' 32. Wd7' Kb5' 33. Wd6 Kb6 34. Wc6+' Ka7 35. Kd6 Kb7' 36. Wc7+ Kb6' 37. Xe7+' Ka5 38. Kd5 Ka6' 39. Kc5' Ka7' 40. Kc6' Ka8' 41. Wc8' Ka7° 42. Wb8' Ka6° 43. Wb7' Ka5° 44. Xf4' Kb4' 45. Kd5' Kb5' 46. Xg1' Ka5' 47. Kc5' Ka6' 48 .Kc6' Ka5° 49. Wb6' Kb4' 50. Kd5' Kc3' 51. Ke4' Kb4' 52. Kd4' Ka5' 53. Kc5' Ka4° 54. Xf4' Kb3' 55. Kd4' Kc2' 56. Ke3' Kb3' 57. Kd3' Kb4' 58. Kd4' Kb3' 59. Wb5' Kc2' 60. Ke3' Kd1' 61. Kd3 Kc1° 62. Wb4' Kd1' 63. Wb3' Kc1° 64. Ke2' Kc2' 65. Wb4' Kc1' 66. Xe7 Kc2' 67. Xd4 Kc3' 68. Xc1' Kb2 69. Kd2' Ka2' 70. Kc3 Kb1' 71. Xd4 Ka1' 72. Kb3' Kb1° 73. Wa4' Ka1° 74. Kc2' Ka2° 75. Xg3 Ka1° 76. Xd2+' Ka2° 77. Wa3#'.

Example 3. Board 8×8, white K on c2, (0,1) Leaper W on h3, (1,2) Leaper N on a8, black K on e1.

{First, the wK takes control of both Leapers} 1.Nc7' Kf2' 2.Ne8' Kg2' 3.Wh4' Kg3' 4.Wh5' Kg4' 5.Ng7' Kf4' 6.Kd3' Ke5' 7.Ne8' Kf5' 8.Wh6' Kg5' 9.Wh7' Kf5' 10.Ke3' Kg6' 11.Wg7+' Kf5' 12.Kd4' Ke6' 13.Kc5' Ke7' 14.Nd6' Kf6' 15.Wf7+' Ke6' 16.Kc6' Ke5' 17.We7' Kf6' 18.Nc8' Ke5 {Now White starts to press the bK into a corner} 19.Kc5' Ke4 20.We6' Kf5 21.Kd5' Kf4' 22.Kd4' Kf5' 23.We5+' Kg6 {23 unique optimal moves by White} 24.Ne7+ Kf6' 25.Nc6' Kg5' 26.Ke4' Kg4' 27.Wf5' Kg3' 28.Ke3' Kg4' 29.Ne7' Kg3 30.Wg5' Kh4' 31.Kf4' Kh3° 32.Kf3' Kh4' 33.Wg6' Kh5' 34.Kf4' Kh4° 35.Nf5+' Kh3' 36.Kf3' Kh2° 37.Kf2' Kh3 38.Wg5' Kh2° 39.Wg4 Kh3 40.Wg3+ Kh2° 41.Kf1 Kh1° 42.Nd6' Kh2° {forcing mate is now easy} 43.Ne4' Kh1° 44.Nf2+' Kh2° 45.Wg2# {or Wh3#}.

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ken

From the beginning, the world of game-playing by machine has been fortunate in attracting contributions from the leading names of computer science. Charles Babbage, Konrad Zuse, Claude Shannon, Alan Turing, John von Neumann, John McCarthy, Alan Newell, Herb Simon and Ken Thompson all come to mind, and each reader will wish to add to this list. Recently, the Journal has saluted both Claude Shannon and Herb Simon.

Ken's retirement from Lucent Technologies' Bell Labs to the start-up Entrisphere is also a good moment for reflection. He is principally known as the father of UNIX and has been the recipient of some six prestigious awards including two IEEE awards, the ACM Turing Award and the National Medal of Technology of the USA. He was also awarded the first Fredkin prize in 1983 when BELLE, ACM and World CC Champion, won the title of U.S. Chess Master. The endgame CDs earned an ICCA Award, and here, the ICCA thanks Ken for his significant and enduring contributions to our community by revisiting some of the themes he developed.

UNIX and C developed in symbiosis and Dennis Ritchie, father of C, leads off by giving us his view from the next desk at Bell. He recreates the special culture of the research community there, simultaneously both liberal and productive, illustrating the sometimes surprising connections between Ken's games-related and other work. Jonathan Schaeffer reviews Ken's three principal contributions to computer game-playing, and Jaap Van den Herik mentions other activities and achievements: ICCA administration, event participation and success, opening-book preparation, intelligent computer vision and player-rating systems.

Ernst Heinz surveys the research inspired by and/or closely related to Ken's pioneering self-play experiments. He announces the results of his own most comprehensive investigation. It appears that statements about the decreasing returns of increasing search may soon be made with high levels of statistical confidence.