CHAPTER 15
ADIABATIC DEMAGNETIZATION

15.1 *Introduction*

One way to cool a gas is as follows. First compress it isothermally. This means compress it in a vessel that isn't insulated, and wait for the gas to lose any heat that is generated so that it returns to room temperature. Then insulate the vessel and allow the gas to expand adiabatically. We could call this cooling by adiabatic decompression.

You can cool a rubber band as follows. First stretch it isothermally. That means, stretch it slowly, so that it has lots of time to lose any heat that is generated. Then, suddenly de-stretch it, and before it has time to gain any heat from its surrounding, measure its temperature by immediately holding it up to your lips. You will find that it has cooled by adiabatic de-stretching. (If you stretch the band quickly (i.e. adiabatically) and immediately hold it up to your lips, you will find that it is hot. **BUT ...** before you try *that* experiment, **close your eyes tightly**. You don't want the stretched elastic band to break and hit you in the eye. Believe me, you do *not* want that to happen.)

The method of *adiabatic demagnetization* has been used to obtain extremely low temperatures. A sample of a paramagnetic salt (such as cerium magnesium nitrate), already cooled to low temperatures by other means, is magnetized isothermally. The sample is often suspended in an atmosphere of helium, which can conduct away any heat that is produced, and hence keeps the process isothermal. It is then insulated (by pumping out the helium) and suddenly and adiabatically demagnetized. This process of isothermal magnetization followed by adiabatic demagnetization can be repeated over and over again. Temperatures close to 0 K have been reached in this manner. You could actually reach a temperature of absolute zero if you did this an infinite number of times – but not for any fewer.

In the analysis that follows, I shall have to assume that you are familiar with the concepts of **B**, **H**, magnetic moment and magnetization from electricity and magnetism.

In brief, the *magnetic dipole moment* $\mathbf{p}_m$ of a sample is the maximum torque it experiences in unit field **B**. That is, the torque is given by $\boldsymbol{\tau} = \mathbf{p}_m \times \mathbf{B}$. The magnetization **M** of a specimen is defined by $\mathbf{B} = \mu\mathbf{H} = \mu_0(\mathbf{H} + \mathbf{M})$. The magnetization is also equal to the *magnetic moment per unit volume*.

Now consider the following.

If the tension in an elastic string is *F*, the work done **on** the string when its length is increased by *dx* is *F dx*.

If the pressure of a gas is *P*, the work done **on** the gas when its volume is increased by *dV* is −*P dV*.

And the work done *per unit volume* **on** an isotropic sample in increasing its magnetization from M to $M + dM$ in a magnetic field B is BdM. (I am assuming here that the sample is isotropic and that the magnetic moment and the magnetic field are in the same direction, and hence I am no longer using boldface to indicate vector quantities.)

Note that, in all of these examples, the work done is the product of an intensive state variable (P, F, B) and the differential of an extensive state variable (dV, dx, dM).

If we add heat to a magnetizable sample, and do work per unit volume on it by putting it in a magnetic field B and thereby increasing its magnetization by dM, then, provided there is no change in volume, the increase in its internal energy per unit volume is given by

$$dU = TdS + BdM, \qquad\qquad 15.1.1$$

In this magnetic context, we can define state functions H, A and G per unit volume by

$$H = U - BM, \qquad\qquad 15.1.2$$
$$A = U - TS, \qquad\qquad 15.1.3$$
$$G = H - TS = A - BM, \qquad\qquad 15.1.4$$

In differential form, these become

$$dH = TdS - MdB, \qquad\qquad 15.1.5$$
$$dA = -SdT + BdM, \qquad\qquad 15.1.6$$
$$dG = -SdT - MdB. \qquad\qquad 15.1.7$$

Here M is the dipole moment *per unit volume*, in N m T^{-1} m^{-3}, which is the same as the *magnetization*, in A m^{-1}. (Other equivalent units for magnetization would be Pa T^{-1} or T m H^{-1}, but I recommend N m T^{-1} m^{-3} as being the most readily understandable in the present context.)

In Section 15.2 I am going to derive an expression for the lowering of the temperature in an adiabatic decompression, $(\partial T / \partial P)_S$. And then, in Section 15.3, I am going to derive an expression, by exactly the same argument, step-by-step, for the lowering of the temperature in an adiabatic demagnetization, $(\partial T / \partial B)_S$.

15.2 *Adiabatic Decompression*

We are going to calculate an expression for $(\partial T / \partial P)_S$. The expression will be positive, since T and P increase together. We shall consider the entropy as a function of temperature and pressure, and, with the variables
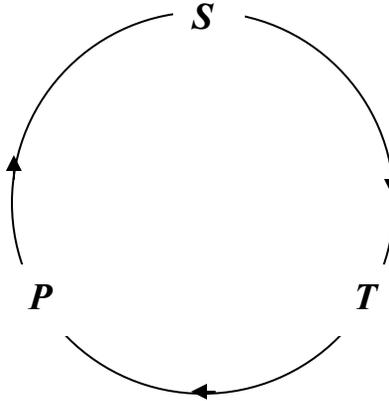
we shall start with the cyclic relation

$$\left(\frac{\partial S}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_S \left(\frac{\partial P}{\partial S}\right)_T = -1. \tag{15.2.1}$$

The middle term is the one we want. Let's find expressions for the first and third partial derivatives in terms of things that we can measure.

In a reversible process $dS = dQ/T$, and, in an isobaric process, $dQ = C_P \, dT$. Therefore $\left(\dfrac{\partial S}{\partial T}\right)_P = \dfrac{C_P}{T}$.

Also, we have a Maxwell relation (equation 12.6.16) $\left(\dfrac{\partial S}{\partial P}\right)_T = -\left(\dfrac{\partial V}{\partial T}\right)_P$. Thus equation 15.2.1 becomes

$$\left(\frac{\partial T}{\partial P}\right)_S = \frac{T}{C_P}\left(\frac{\partial V}{\partial T}\right)_P. \tag{15.2.2}$$

[Check the dimensions of this. Note also that C_P can be total, specific or molar, provided that V is correspondingly total, specific or molar. $(\partial T / \partial P)_S$ is, of course, intensive.]

If the gas is an ideal gas, the equation of state is $PV = RT$, so that $\left(\dfrac{\partial V}{\partial T}\right)_P = \dfrac{R}{P} = \dfrac{V}{T}$.
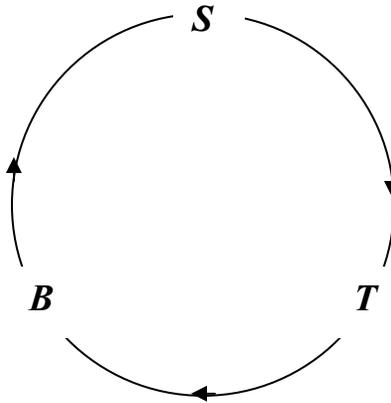
Equation 15.2.2 therefore becomes

$$\left(\frac{\partial T}{\partial P}\right)_S = \frac{V}{C_P}. \qquad\qquad 15.2.3$$

15.3 *Adiabatic Demagnetization*

We are now going to do the same argument for adiabatic demagnetization.

We are going to calculate an expression for $(\partial T / \partial B)_S$. The expression will be positive, since T and B increase together. We shall consider the entropy as a function of temperature and magnetic field, and, with the variables

we shall start with the cyclic relation

$$\left(\frac{\partial S}{\partial T}\right)_B \left(\frac{\partial T}{\partial B}\right)_S \left(\frac{\partial B}{\partial S}\right)_T = -1. \qquad\qquad 15.3.1$$

The middle term is the one we want. Let's find expressions for the first and third partial derivatives in terms of things that we can measure.

In a reversible process $dS = dQ/T$, and, in a constant magnetic field, $dQ = C_B\,dT$.
Here I am taking S to mean the entropy *per unit volume*, and C_B is the heat capacity *per unit volume* (i.e. the heat required to raise the temperature of unit volume by one degree) in a constant magnetic field.

Thus we have $\left(\dfrac{\partial S}{\partial T}\right)_B = \dfrac{C_B}{T}.$

The Maxwell relation corresponding to $\left(\dfrac{\partial S}{\partial P}\right)_T = -\left(\dfrac{\partial V}{\partial T}\right)_P$ is $\left(\dfrac{\partial S}{\partial B}\right)_T = \left(\dfrac{\partial M}{\partial T}\right)_B.$ Thus equation 15.3.1 becomes

$$\left(\frac{\partial T}{\partial B}\right)_S = -\frac{T}{C_B}\left(\frac{\partial M}{\partial T}\right)_B. \qquad\qquad 15.3.2$$

Now for a paramagnetic material, the magnetization, for a given field, is proportional to B and it falls off inversely as the temperature (that's the *equation of state*). That is, $M = aB/T.$ and therefore $\left(\dfrac{\partial M}{\partial T}\right)_B = -\dfrac{aB}{T^2} = -\dfrac{M}{T}.$ Equation 15.3.2 therefore becomes

$$\left(\frac{\partial T}{\partial B}\right)_S = \frac{M}{C_B}. \qquad\qquad 15.3.3$$

You should check the dimensions of this equation.

The cooling effect is particularly effective at low temperatures, when C_B is small.

15.4 *Entropy and Temperature*

Cooling by adiabatic demagnetization involves successive isothermal magnetizations followed by adiabatic demagnetizations, and this suggests that some insight into the process might be obtained by following it on an entropy : temperature ($S : T$) diagram.
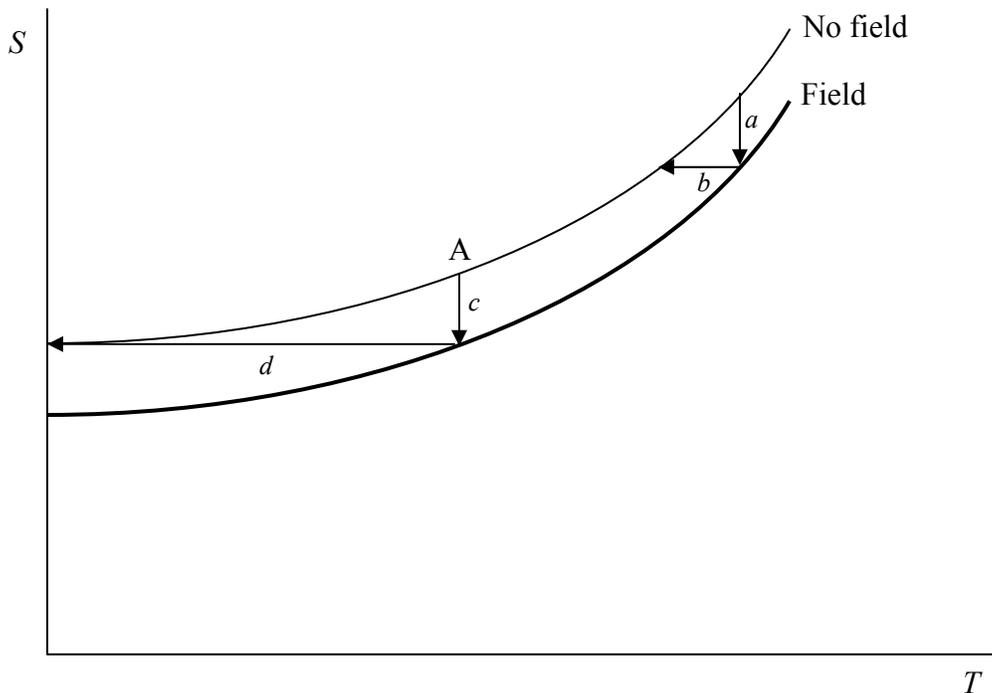

FIGURE XV.1

In figure XV.1 I draw schematically with a thin curve the variation of entropy of the specimen with temperature in the absence of a magnetizing field, and, with a thick curve, the (lesser) entropy of the more ordered state in the presence of a magnetizing field. The process *a* represents an isothermal magnetization, and the process *b* is the following adiabatic (isentropic) demagnetization, and it is readily seen how this results in a lowering of the temperature.

By the time when we reach the point A, an isothermal magnetization is represented by the process *c*, and the following adiabatic demagnetization is the process *d*, which takes us down to the absolute zero of temperature.

We shall find out in the next Chapter, however, that there is a fundamental flaw in this last argument, and that getting down to absolute zero isn't going to be quite so easy.