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REMARK ON JACOBSTHAL NUMBERS

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The n-th Jacobsthal number ($n \geq 0$) is defined by

$$J_n = \frac{2^n - (-1)^n}{3}$$

(see, e.g., [1]).

The first ten values of sequence $\{J_n\}$ are

0	1	2	3	4	5	6	7	8	9
0	1	1	3	5	11	21	43	85	171

Directly it can be checked that for each $n \geq 0$:

$$\begin{aligned} J_{n+1} &= \frac{2^{n+1} - (-1)^{n+1}}{3} = \frac{2.2^n + (-1)^n}{3} \\ &= \frac{2.(2^n - (-1)^n) + 3.(-1)^n}{3} = 2.J_n + (-1)^n = 2.J_n - (-1)^{n+1}. \end{aligned}$$

The arithmetic function ψ is introduced in [2].

Let sequence $a_1, a_2, ...$, with its members being natural numbers, be given, and let

$$c_i = \psi(a_i) \ (i = 1, 2, ...).$$

Hence, we deduce the sequence $c_1, c_2, ...$ from the former sequence. If k and l exist so that $l \geq 0$,

$$c_{i+l} = c_{k+i+l} = c_{2k+i+l} = ...$$

for $1 \leq i \leq k$, then we will say that

$$[c_{l+1}, c_{l+2}, ..., c_{l+k}]$$

is the *base* of the sequence $c_1, c_2, ...$ with length of k and with respect to function ψ.

By induction we can prove directly the following

Theorem: The base of sequence $\{J_n\}$ is $[7, 4, 9, 8, 8, 6, 4]$ (with lenght 7).

References

[1] Ribenboim, P. *The Theory of Classical Variations,* Springer, New York, 1999.

[2] Atanassov K., An arithmetical function and some of its applications. *Bulletin of Number Theory and Related Topics,* Vol. IX (1985), No. 1, 18-27.