NNTDM 7 (2001) 4, 135-136

ON ONE JACOBSTHAL'S INEQUALITY

Krassimir T. Atanassov

CLBME - Bulg. Academy of Sci., P.O.Box 12, Sofia-1113, Bulgaria
e-mail: krat@bgcict.acad.bg

Let $m \geq 1, r \geq 0, a, b$ be integers. In [1] Ernst Jacobsthal proved that

$$\sum_{i=0}^{r} ([\frac{a+b+i}{m}] + [\frac{i}{m}] - [\frac{a+i}{m}] - [\frac{b+i}{m}]) \geq 0. \tag{1}$$

Bellow we shall generalize this inequality to the following one:

$$\sum_{i=0}^{r} ([\frac{\sum_{j=1}^{n} a_j + (n-1).i}{m}] + [\frac{i}{m}] - \sum_{j=1}^{n} [\frac{a_j + i}{m}]) \geq 0, \tag{2}$$

where $m \geq 1, n \geq 2, r \geq 0, a_1, a_2, ..., a_n$ are integers.

When $n = 2$ we obtain (1). Let us assume that (2) is valid for a certain $n \geq 2$ and for certain $a_1, a_2, ..., a_n$. Let a_{n+1} is an integer. Then,

$$\sum_{i=0}^{r} ([\frac{\sum_{j=1}^{n+1} a_j + n.i}{m}] + [\frac{i}{m}] - \sum_{j=1}^{n+1} [\frac{a_j + i}{m}])$$

$$\geq \sum_{i=0}^{r} ([\frac{\sum_{j=1}^{n} a_j + a_{n+1} + (n-1).i + i}{m}] + [\frac{i}{m}] - [\frac{\sum_{j=1}^{n} a_j + (n-1).i + i}{m}] - [\frac{a_{n+1} + i}{m}])$$

$$= \sum_{i=0}^{r} ([\frac{a+b+i}{m}] + [\frac{i}{m}] - [\frac{a+i}{m}] - [\frac{b+i}{m}])$$

(from (1)).

$$\geq 0,$$

where

$$a = \sum_{j=1}^{n} a_j + (n-1).i$$

and $b = a_{n+1}$.

Is the inequality

$$\sum_{i=0}^{r} \left(\left[\frac{\sum_{j=1}^{n} a_j + i}{m} \right] + \left[\frac{(n-1).i}{m} \right] - \sum_{j=1}^{n} \left[\frac{a_j + i}{m} \right] \right) \geq 0$$

valid, too?

REFERENCES:

[1] Jacobsthal E., Uber eine zahlentheoretische Summe, Kgl. norske vid. selskabs forhandl., 1957, Vol. 30, No. 6, 35-41.