Recent and Noteworthy Sequences in the OEIS:® The Illustrations

N. J. A. Sloane,
The OEIS Foundation Inc.,
11 South Adelaide Avenue, Highland Park, NJ 08904, USA,
Email: njasloane@gmail.com

Dec. 15, 2012

Abstract[1].

The article[2] *Recent and Noteworthy Sequences in the OEIS* contains an illustrated account of some new and noteworthy additions to the *On-Line Encyclopedia of Integer Sequences*® (or *OEIS*®), concentrating on sequences that are associated with attractive unsolved problems. The present document contains larger versions of the main illustrations.

Introduction.

The *On-Line Encyclopedia of Integer Sequences* (or *OEIS*) has existed in various forms since it was started by the author in 1964. Since 2009 it has been owned and maintained by *The OEIS Foundation*[3], and since November 2010 it has been on the web as a Wiki.[4] It presently contains about 220000 sequences. New sequences arrive every day. Some come with a complete analysis, giving formulas, asymptotic estimates, computer programs, references, etc. Some, on the other hand, are such that one says "That is a really lovely problem and I wish I had time to work on it". The accompanying article (see Footnote 2) describes a dozen or so sequences of the latter type.

Many more examples could have been included. If problems like this appeal to you, please consider becoming an associate editor[5] of the OEIS: you get to see the new sequences as they arrive, and they often contain lovely problems. There are no formal duties, everything is voluntary, and we badly need more editors to cope with the ever-increasing flow of submissions. Last but not least, please make a donation to the OEIS Foundation to help keep the OEIS running!

---

[1] Prepared for distribution at the AMS/MAA Joint Mathematics Meetings in San Diego, Jan. 2013.

[2] N. A. A. Sloane, *Recent and Noteworthy Sequences in the OEIS*, http://neilsloane.com/doc/sampler2.pdf.

[3] The OEIS Foundation, Inc., http://oeisf.org.

[4] *The On-Line Encyclopedia of Integer Sequences*, http://oeis.org.

[5] If so, send me email: njasloane@gmail.com.

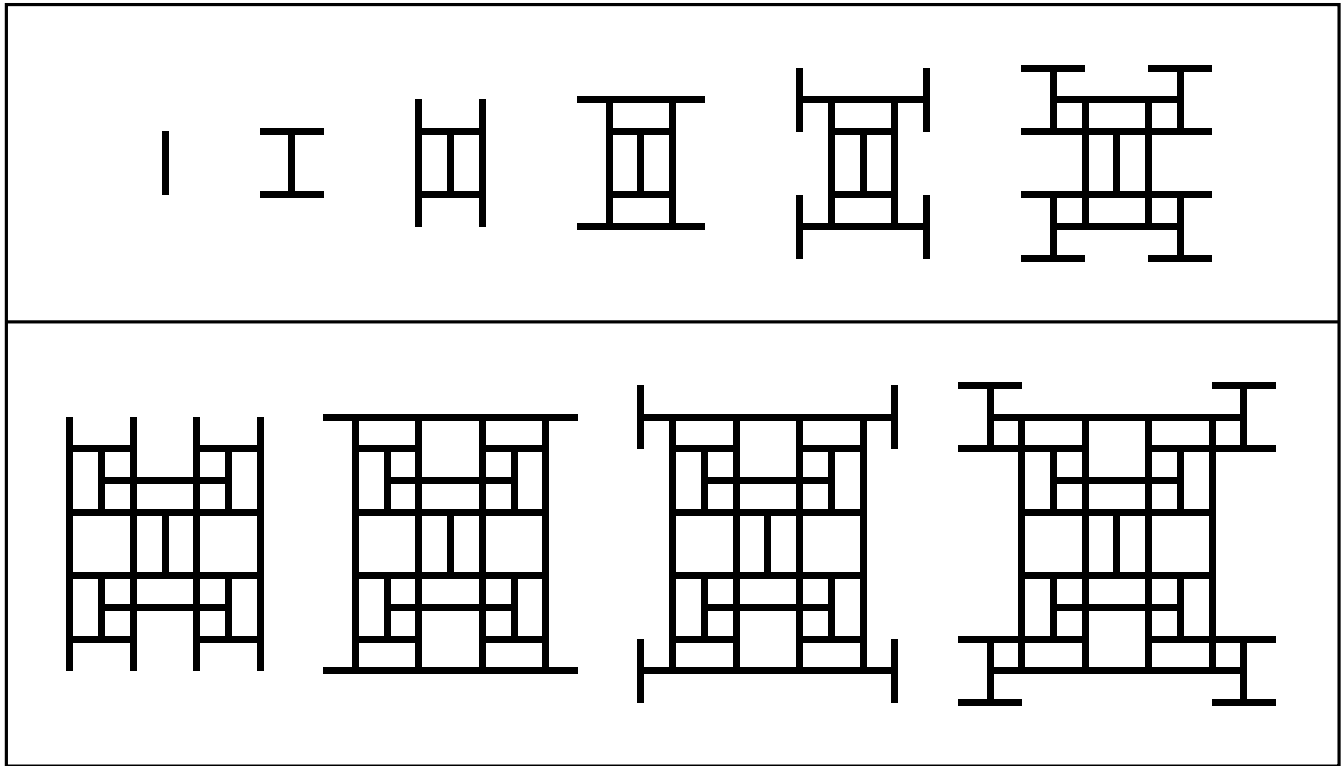# Toothpick structures and the snowflake sequence

Figure 1: Beginning of the evolution of Omar Pol's toothpick structure. The numbers of toothpicks in stages 1 through 10 are $1, 3, 7, 11, 15, 23, 35, 43, 47, 55$ (oeis.org/A139250).
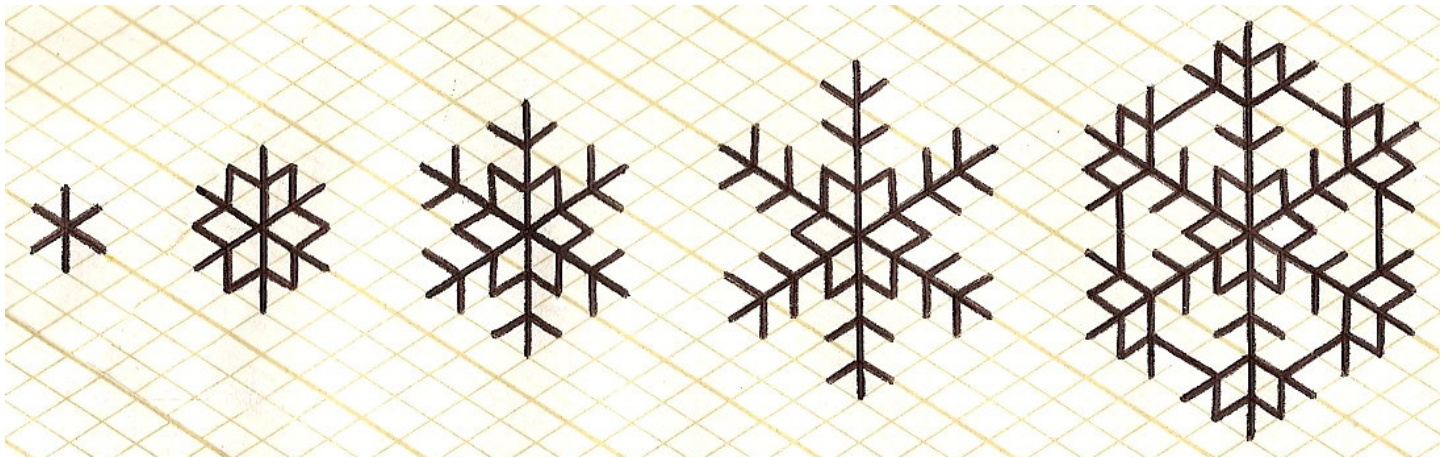

Figure 2: Omar E. Pol's illustration of the first five stages of the E-toothpick (or snowflake) sequence oeis.org/A161330. The first stage consists of two E-toothpicks back-to-back.
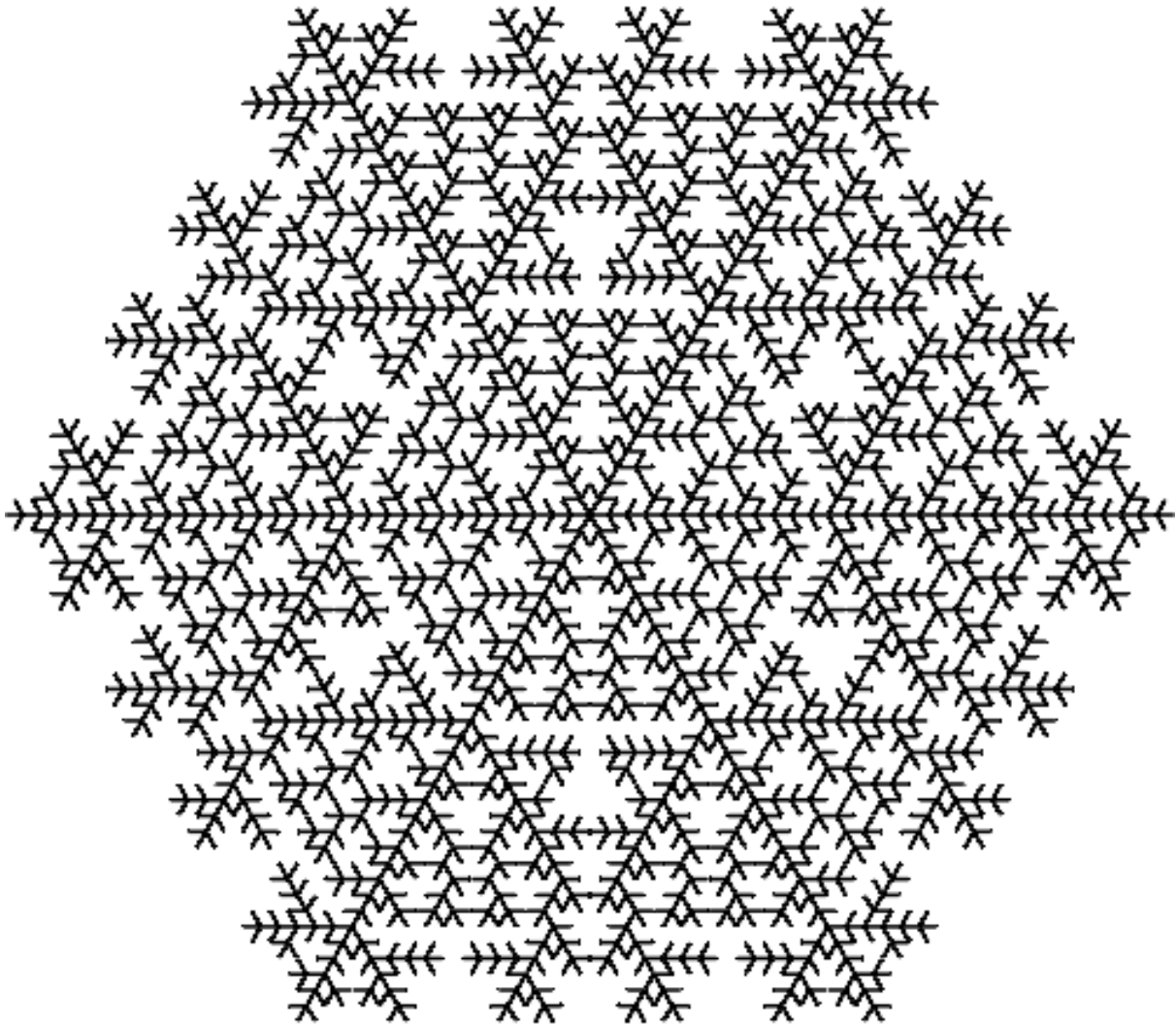
Figure 3: The E-toothpick (or snowflake) sequence oeis.org/A161330 after 32 stages, courtesy of David Applegate. The figure contains 1124 copies of the E-toothpick.

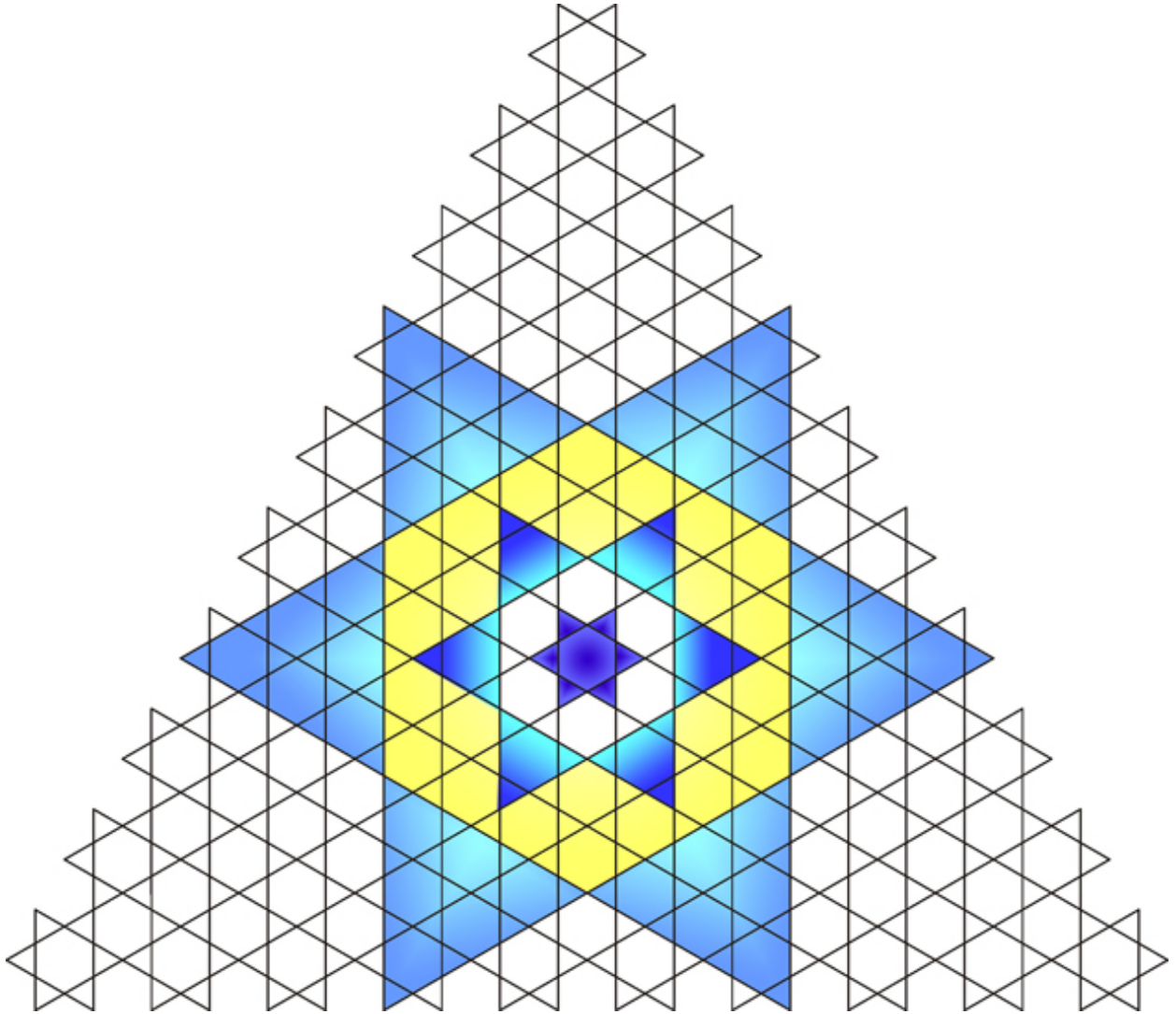# Alice Kleeva's figurate numbers

Figure 4: Alice V. Kleeva's figurate numbers oeis.org/A169720.
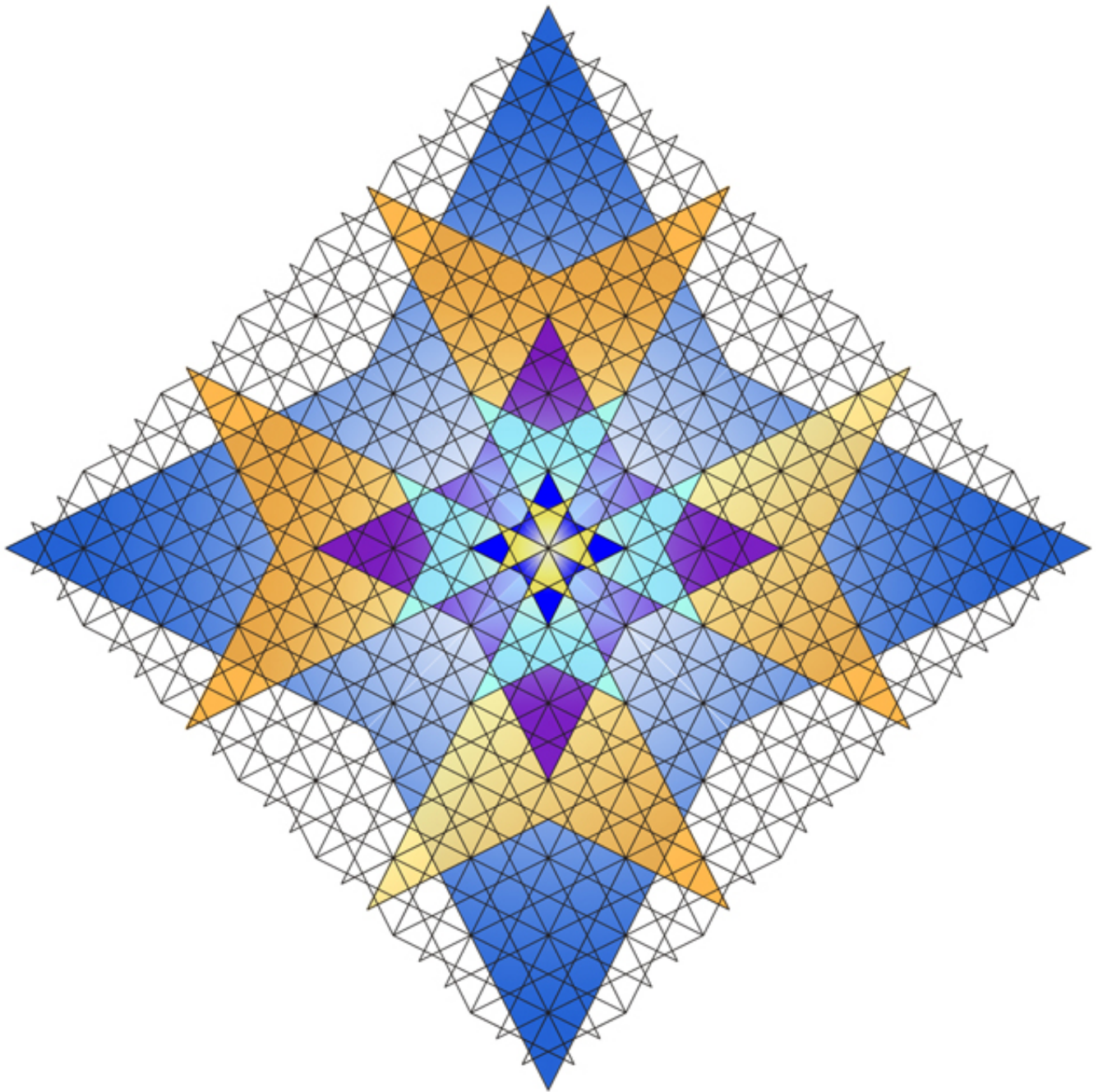
Figure 5: Alice V. Kleeva's figurate numbers oeis.org/A169721.

This is an excerpt from N. J. A. Sloane, *Recent and Noteworthy Sequences in the OEIS: The Illustrations*, http://neilsloane.com/doc/sampler3.pdf.

Dissecting a rectangle into rectangles, etc.

21

1

2

6

}x2 = 8

}x4 = 32

}x8 = 48

———

A189243(5) = 88
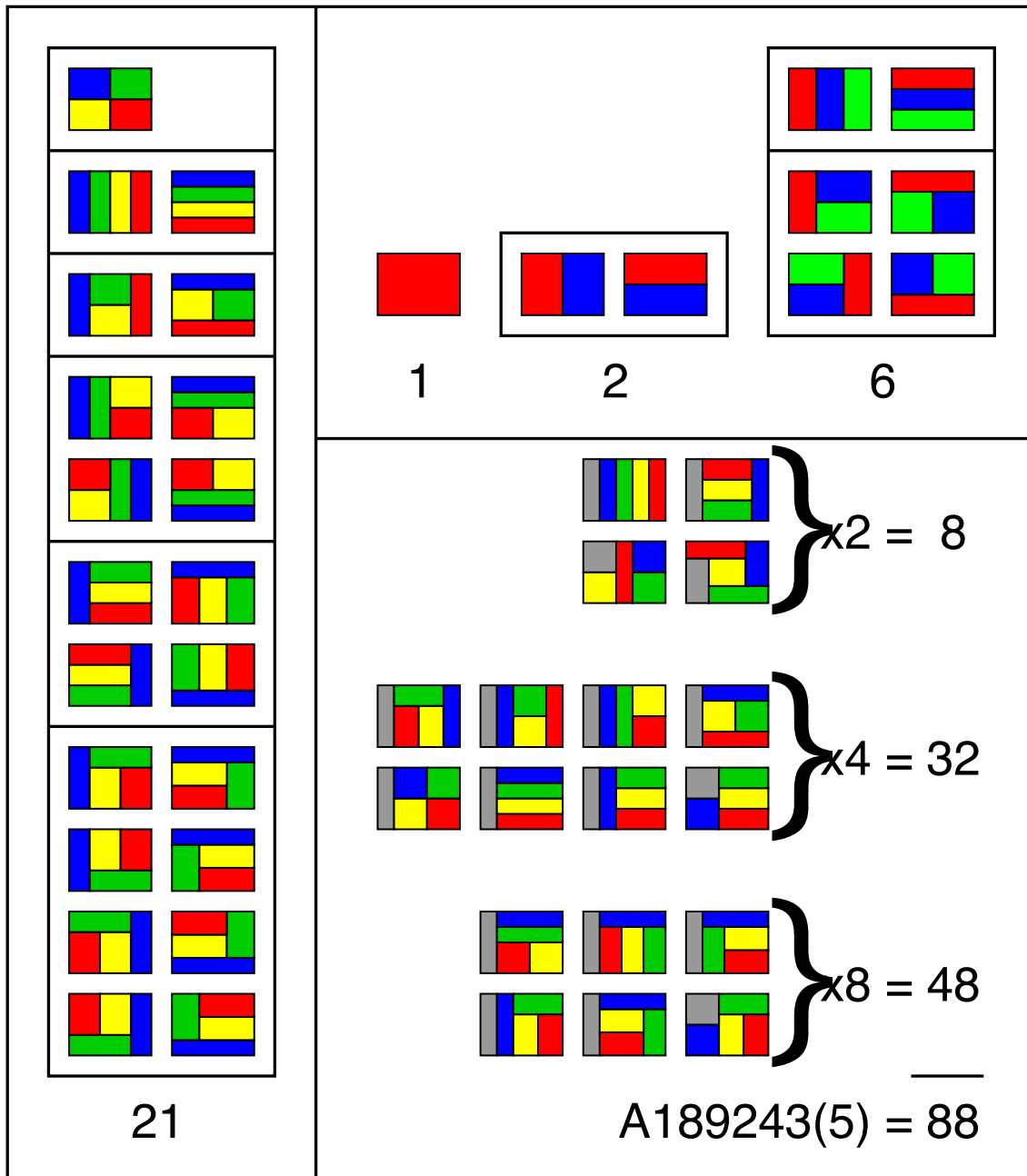
Figure 6: Sequence oeis.org/A189243 gives the number of ways to dissect a non-square rectangle into n rectangles of equal area. Only 18 of the 88 solutions are shown for $n = 5$, the others being obtained by rotations and reflections (and changing the aspect ratio in the case of rotations). Figure courtesy of Geoffrey H. Morley.

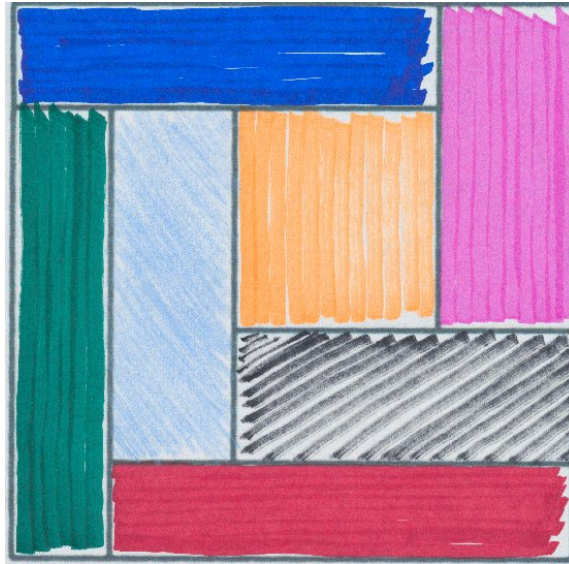This is an excerpt from N. J. A. Sloane, *Recent and Noteworthy Sequences in the OEIS: The Illustrations*, `http://neilsloane.com/doc/sampler3.pdf`.

Figure 7: Blanche Descartes's dissection of a square into seven rectangles of equal area but different proportions (cf. oeis.org/A108066).
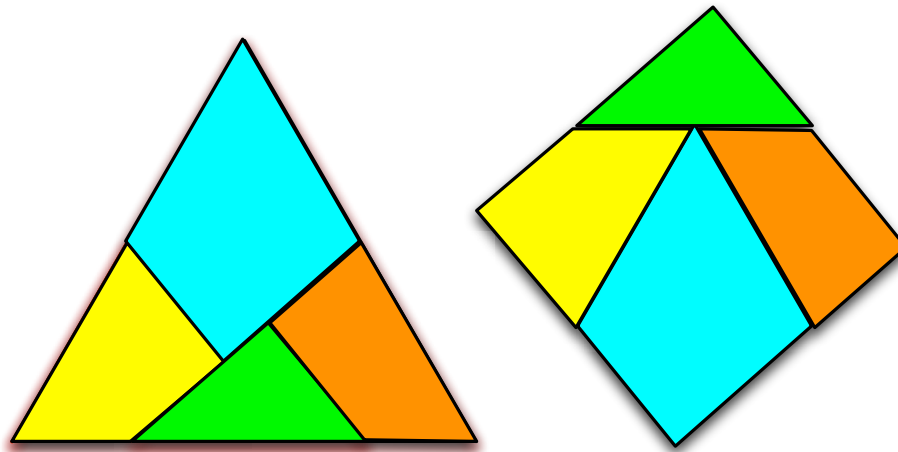

Figure 8: A triangle can be cut into four pieces which can be rearranged to form a square. It is an open question to show this cannot be done using only three pieces (cf. oeis.org/A110312). Figure courtesy of Vinay A. Vaishampayan.

Dominoes
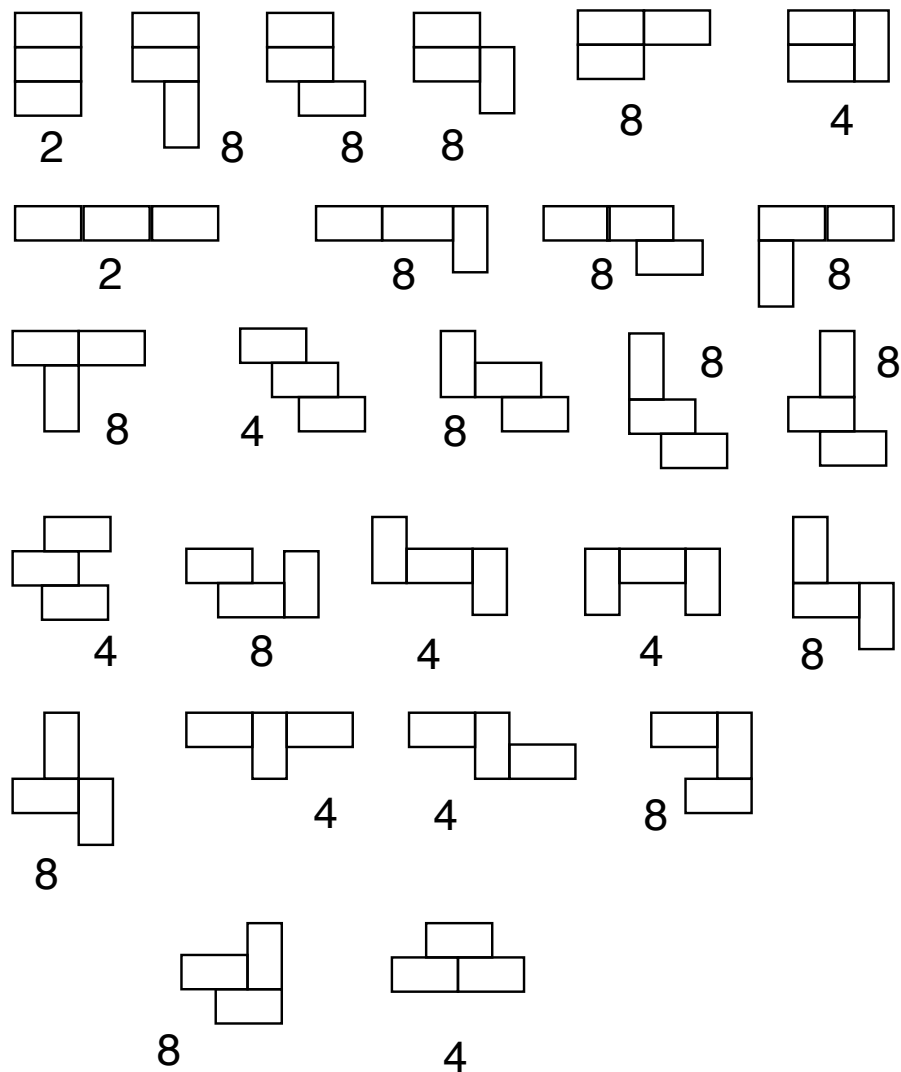

Figure 9: The 26 figures that can be formed with three dominoes (oeis.org/A056786). The figures in the top row all contain two dominoes that share a long edge (cf. A216583), and the two figures in the bottom row have a loop in their adjacency graph (cf. A216492). The figures are labeled with the numbers of their images under rotations and reflections (A216598).
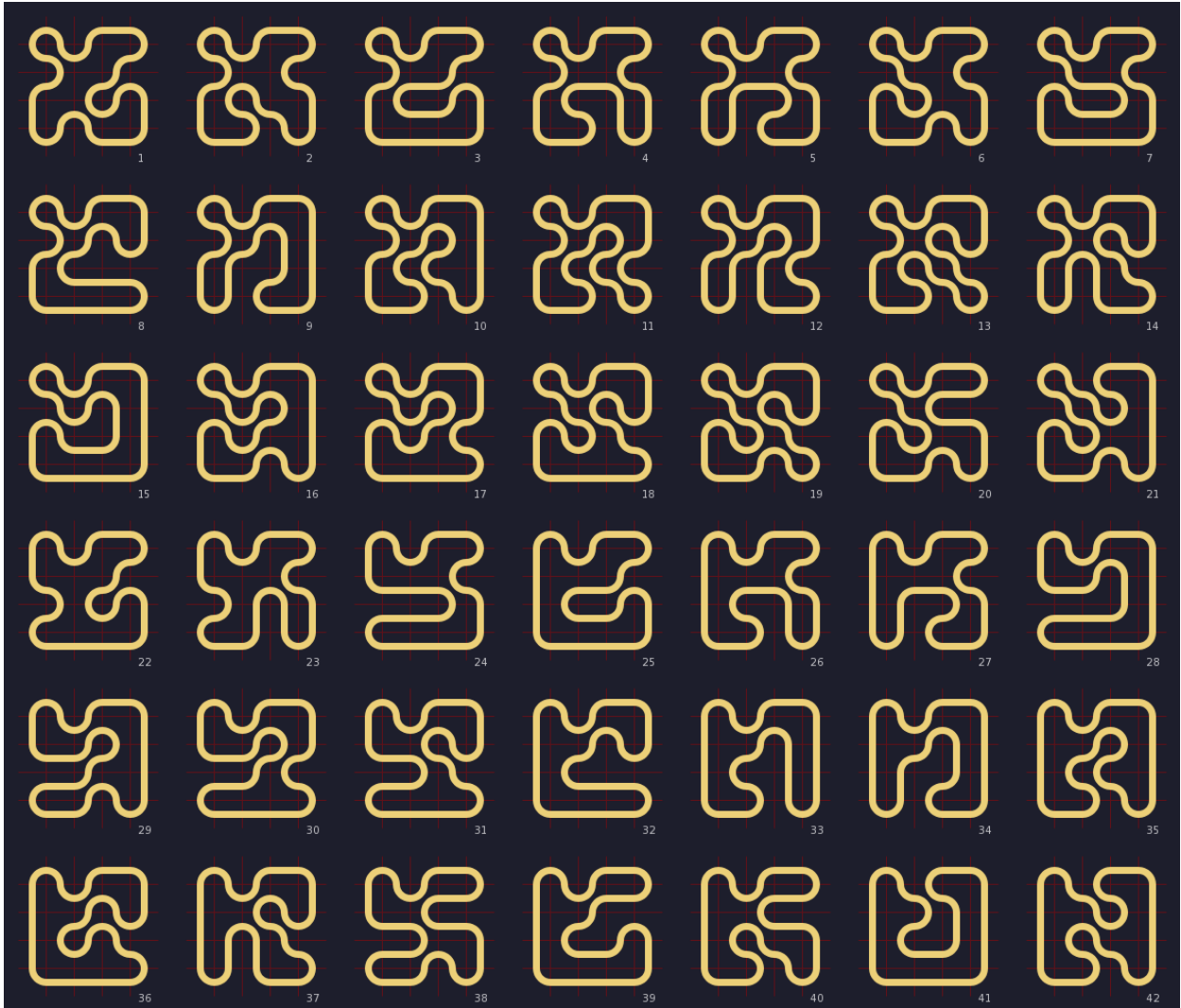
Meanders on a square grid

Figure 10: The 42 non-self-intersecting closed paths that visit every cell of a 4×4 grid at least once and do not cross any edge more than once (cf. oeis.org/A200000). Figure courtesy of Jonathan Wild.

Meanders from circular arcs
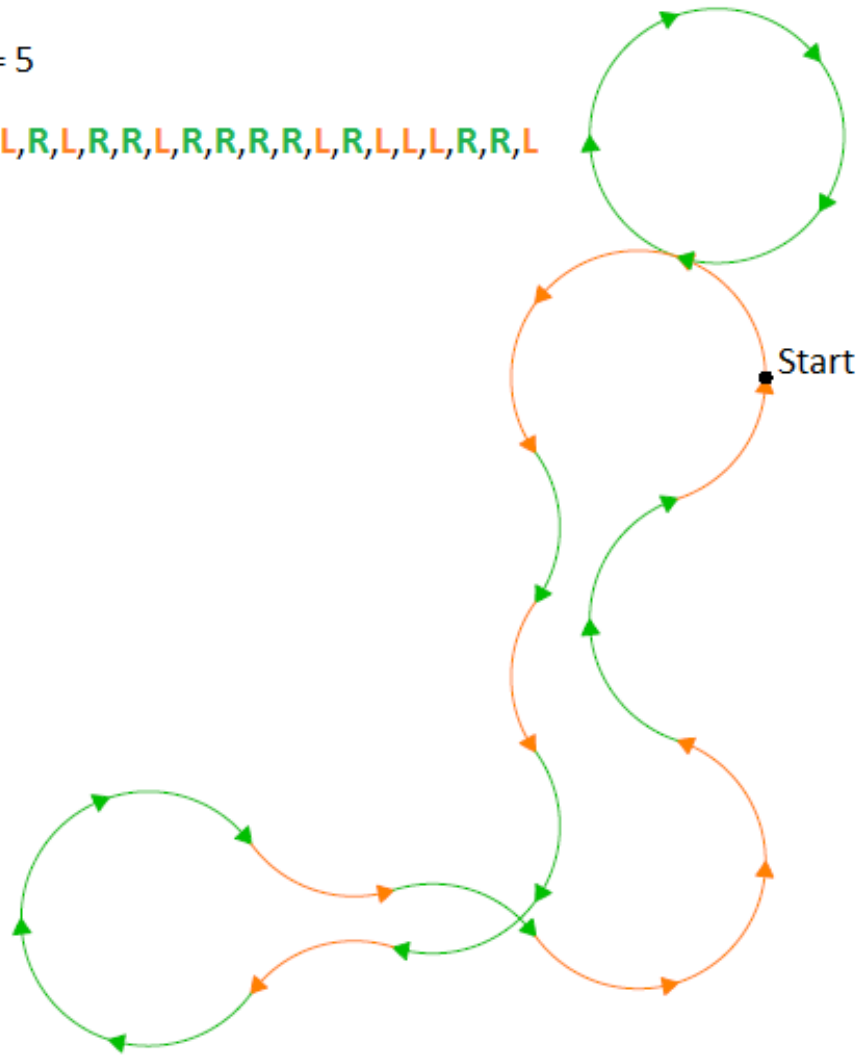
meander, m = 5

L,R,R,R,R,R,L,L,R,L,R,R,L,R,R,R,R,L,R,L,L,L,R,R,L

Figure 11: A meander constructed from 25 circular arcs of angle $2\pi/5$, one of the 13504 meanders counted by oeis.org/A197654(4,1). Figure courtesy of Susanne Wienand.
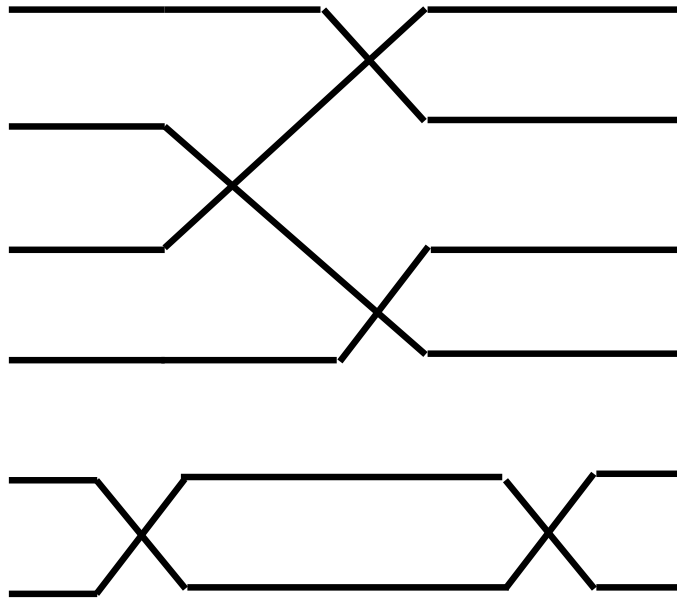
Duraid Madina's braid sequence

Figure 12: Illustrating oeis.org/A200919: five crossings are enough to ensure that every pair out of six wires are adjacent (the top and bottom lines are considered to be adjacent).
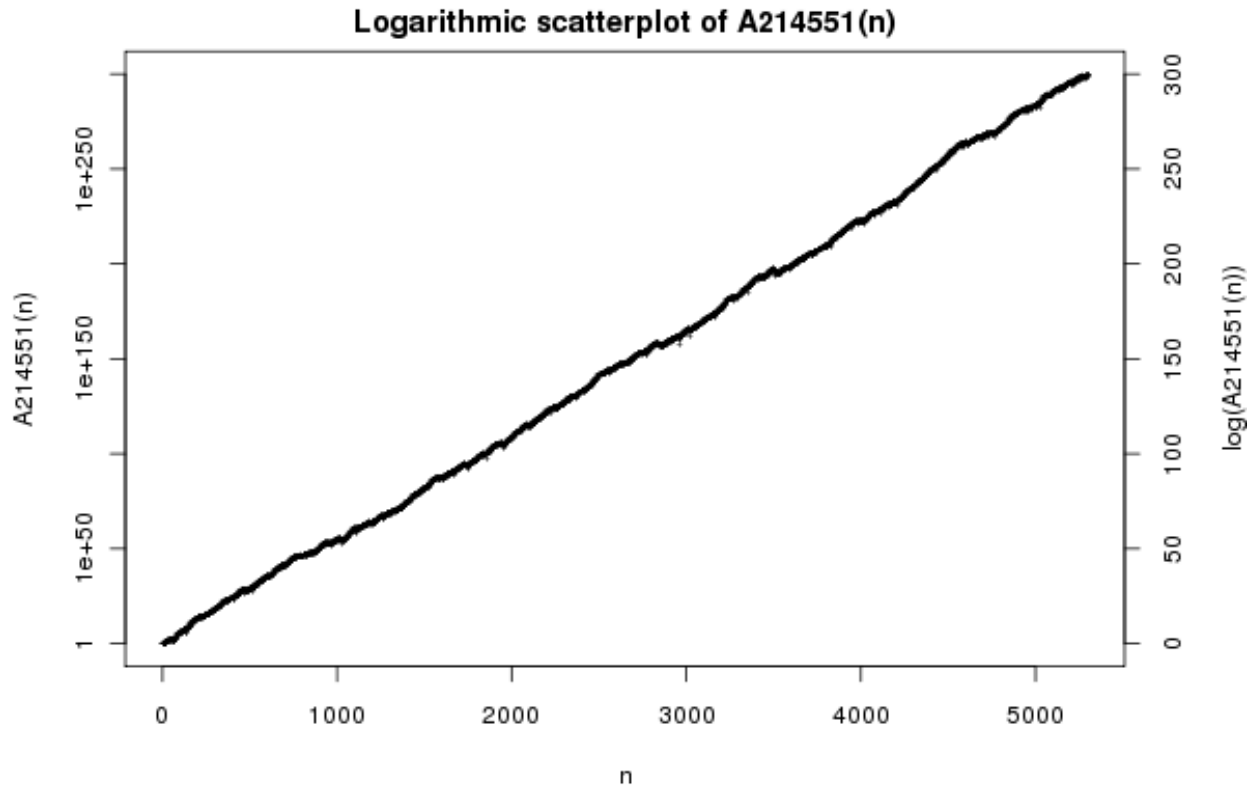
Reed Kelley's sequence

Figure 13: Log-plot of 5000 terms of Reed Kelly's sequence oeis.org/A214551, defined by the recurrence $a(0) = a(1) = a(2) = 1$, $a(n) = (a(n-1) + a(n-3))/\gcd\{a(n-1), a(n-3)\}$. Although the graph is clearly increasing, there are pronounced irregularities. So far nothing has been proved about the rate of growth of this sequence.
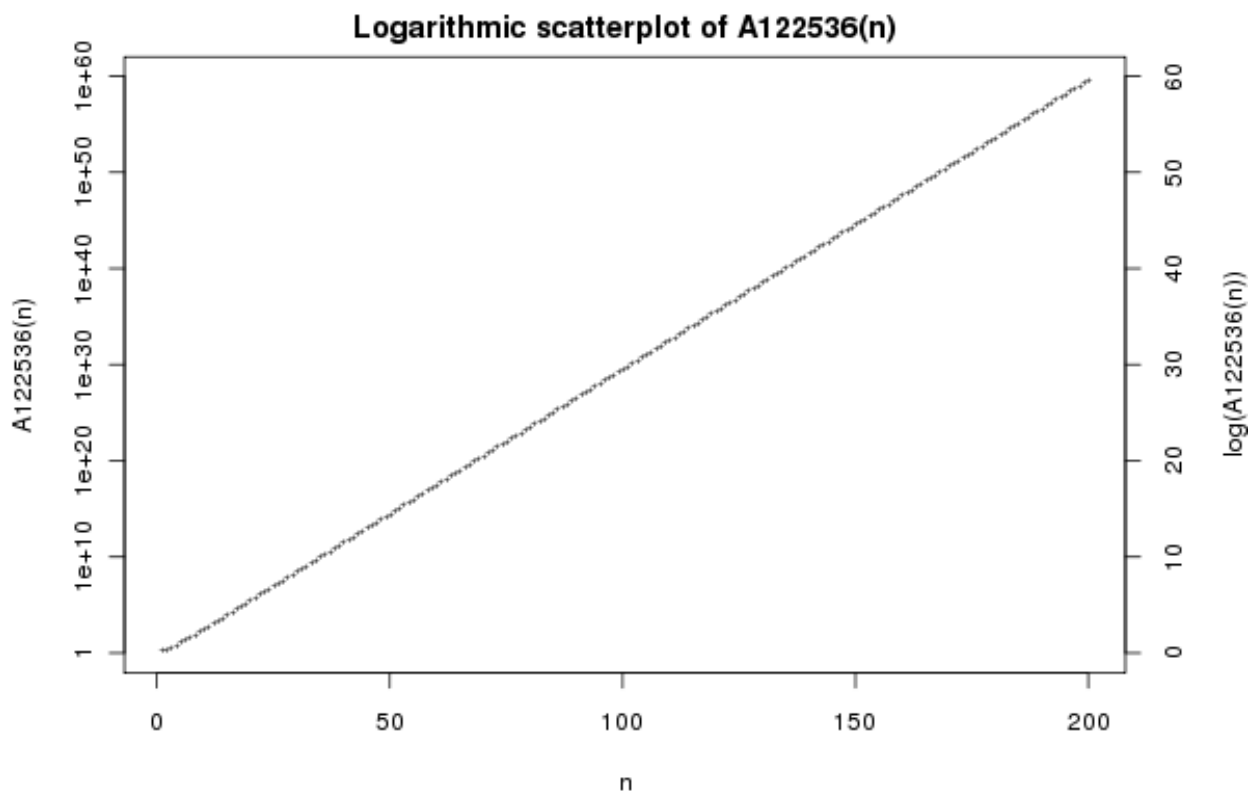
Words with no final repeats

Figure 14: Log-plot of the 200 known values of oeis.org/A122536, the number of binary sequences of length n with no final repeats (or curling number 1). Is there an explicit formula?

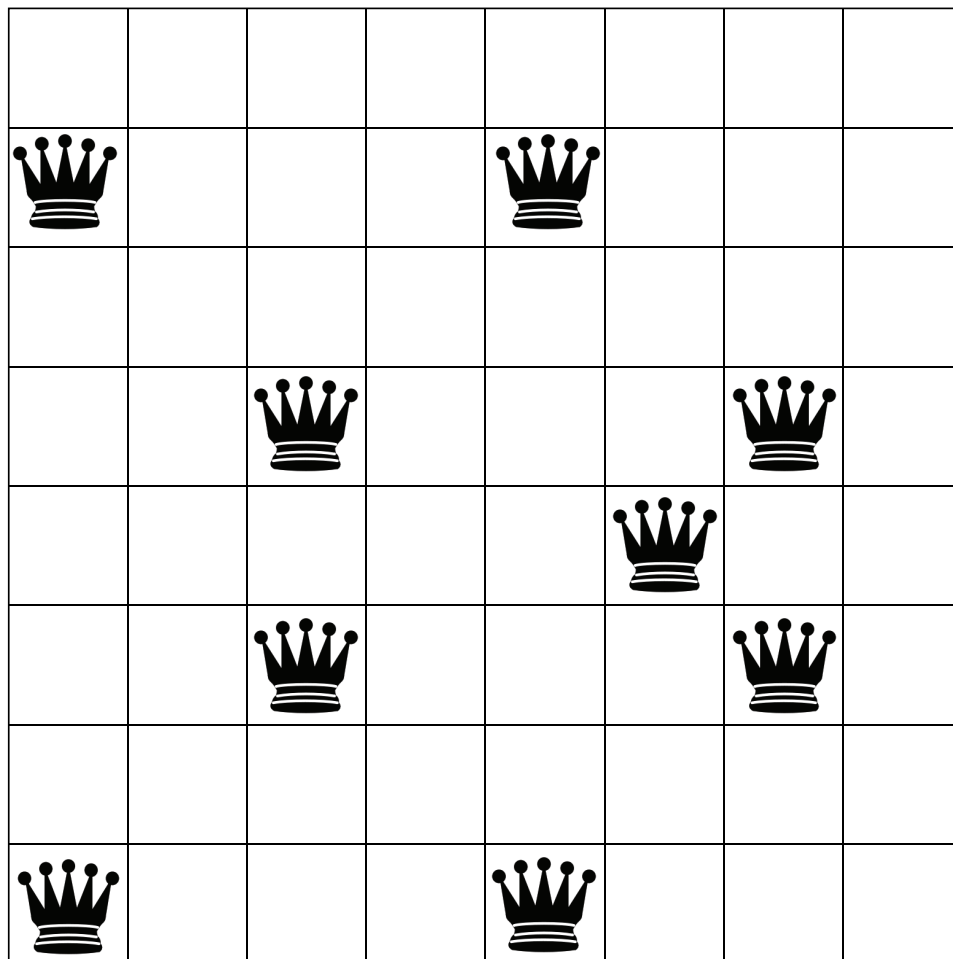# Martin Gardner's minimal no-3-in-line problem

Figure 15: 9 queens on a chessboard, no 3 in a line, such that adding one more queen produces 3 in a line; 9 is minimal (oeis.org/A219760). Figure courtesy of Gregory S. Warrington.

Circulant determinant equals number

$$\begin{vmatrix} 4 & 5 & 6 & 7 & 9 & 0 & 1 & 2 & 3 \\ 3 & 4 & 5 & 6 & 7 & 9 & 0 & 1 & 2 \\ 2 & 3 & 4 & 5 & 6 & 7 & 9 & 0 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 9 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 9 \\ 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 7 & 9 & 0 & 1 & 2 & 3 & 4 & 5 \\ 5 & 6 & 7 & 9 & 0 & 1 & 2 & 3 & 4 \end{vmatrix} = 456790123\,.$$

Figure 16: 456790123 is equal to the circulant determinant formed from its digits (oeis.org/A219324). 247 is the smallest nontrivial number with this property.